SMARANDACHE SEMIRINGS AND SEMIFIELDS

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Abstract

In this paper we study the notions of Smarandache semirings and semifields and obtain some interesting results about them. We show that not every semiring is a Smarandache semiring. We similarly prove that not every semifield is a Smarandache semifield. We give several examples to make the concept lucid. Further, we propose an open problem about the existence of Smarandache semiring S of finite order.

Keywords: semiring, semifield, semi-algebra, distributive lattice, Smarandache semirings.

Definition [1] :

A non-empty set S is said to be a *semiring* if on S is defined two binary closed operations + and × such that (S, +) is an abelian semigroup with 0 and (S, ×) is a semigroup and multiplication distributes over addition from the left and from the right.

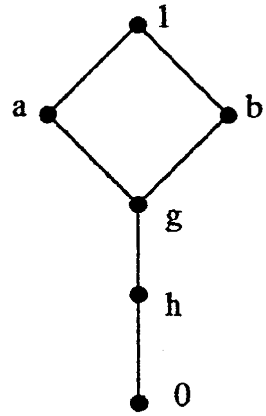
A semiring is a *strict semiring* if $x + y = 0$ implies $x = y = 0$. Semiring is *commutative* if (S, ×) is a commutative semigroup. A commutative semiring is a semifield if (S, ×) has a unit element and $x \times y = 0$ in S if and only if $x = y = 0$. For more properties of semirings please refer [1] , [3] , [4] and [5].

Definition 1:

The *Smarandache semiring* is defined [4] to be a semiring S such that a proper subset A of S is a semifield (with respect to the same induced operation). That is $\phi \neq A \subset$ S.

Example 1: Let $M_{n\times n} = \{(a_{ij})/a_{ij} \in Z^+ \cup \{0\}\}$. Here, Z^+ denotes the set of positive integers. Clearly $M_{n\times n}$ is a semiring with the matrix addition and matrix multiplication. For consider $A = \{(a_{ij}) \mid a_{ij} = 0, i \neq j \text{ and } a_{ii} \in Z^+ \cup \{0\}\}$, that is all diagonal matrices with entries from $Z^+ \cup \{0\}$. Clearly, A is a semifield. Hence $M_{n\times n}$ is a Smarandache semiring.

Example 2: Let S be the lattice given by the following figure. Clearly S is a semiring under min-max operation. S is a Smarandache semiring for A = {1, b, g, h, 0} is a semifield.



Theorem 2:

Every distributive lattice with 0 and 1 is a Smarandache Semiring.

Proof: Any chain connecting 0 and 1 is a lattice which is a semifield for every chain lattice is a semiring which satisfies all the postulates of a semifield. Hence the claim.

Definition 3:

The *Smarandache sub-semiring* [4] is defined to be a Smarandache semiring B which is a proper subset of the Smarandache semiring S.

Example 3: Let $M_{n\times n}$ be the semiring as in Example 1. Clearly $M_{n\times n}$ is a Smarandache semiring. Now,

$$B = \left\{ \left. \begin{pmatrix} a_{11} & 0 & . & . & . & 0 & 0 \\ 0 & 0 & . & . & . & 0 & 0 \\ . & . & & & & . & . \\ . & . & & & & . & . \\ . & . & & & & . & . \\ 0 & . & . & . & . & 0 & 0 \\ 0 & 0 & . & . & . & 0 & a_{nn} \end{pmatrix} \middle/ a_{11} \text{ and } a_{nn} \in Z^{+} \cup \{0\} \right. \right\}$$

is a Smarandache sub-semiring.

Example 4: Let $M_{2\times 2} = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle/ a, b, c, d \in Z^+ \cup \{0\}\right\}$. Clearly $M_{2\times 2}$ under the matrix addition and multiplication is a semiring which is not a semifield. But $M_{2\times 2}$ is a Smarandache semiring for $N = \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \middle/ a, b \in Z^+\right\} \cup \left\{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\right\}$ is a semifield.

Theorem 4:

Not all semirings are Smarandache semirings.

Proof: Let $S = Z^+ \cup \{0\}$. $(S, +, \times)$ is a semiring which has no proper semifield contained in it. Hence the claim.

Definition 5:

The *Smarandache semifield* [4] is defined to be a semifield $(S, +, \times)$ such that a proper subset of S is a K - semi algebra (with respect with the same induced operations and an external operation).

Example 5: Let $S = Z^+ \cup \{0\}$. Now, $(S, +, \times)$ is a semifield. Consider $p \in S$, p any prime. $A = \{0, p, 2p, \ldots\}$ is a k-semi algebra. So $(S, +, \times)$ is a Smarandache semifield.

Consequence 1:

There also exist semifields which are not Smarandache semifields. The following example illustrates the case.

Example 6: Let $S = Q^+ \cup \{0\}$. $(S, +, \times)$ is a semifield but it is not a Smarandache semifield.

Example 7: Let $S = Z^+ \cup \{0\}$. Now $(S, +, \times)$ is a semifield. Let S[x] be polynomial semiring in the variable x. Clearly S[x] is a Smarandache semiring for S is a proper subset of S[x] is a semifield.

Theorem 5:

Let S be any semifield. Every polynomial semiring is a Smarandache semiring.

Proof: Obvious from the fact S is a semifield contained in S[x].

We now pose an open problem about the very existence of finite semirings and Smarandache semirings that are not distributive lattices.

Problem 1: Does there exist a Smarandache semiring S of finite order? (S is not a finite distributive lattice)?

Note:

We do not have finite semirings other than finite distributive lattices. Thus the existence of finite semirings other than finite distributive lattices is an open problem even in semirings.

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