

Effects of Electron Inertia and Ion-Streaming on Ion-Acoustic Waves in Quantum Plasma

B. Ghosh^{1*}, H. Sahoo¹, K. K. Mondal²

1. Department of Physics, Jadavpur University, Kolkata-700032, India.

2. Sovarani Memorial College, Jagatballavpur, Howrah-711408, India.

*Corresponding author: Associate Professor. B. Ghosh, E-mail: bsdvgghosh@gmail.com

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ABSTRACT

Using the quantum hydrodynamic (QHD) model both the linear and nonlinear behavior of ion-acoustic waves in quantum plasma have been studied including electron inertia, ion-streaming and quantum diffraction effects. Some of the wave modes are shown to be linearly unstable and growth rate of such instability depends significantly on ion-streaming motion, electron inertia and quantum diffraction effects. Nonlinear analysis shows that the profile of the ion-acoustic KdV solitons also depends significantly on these plasma parameters. The importance of the result has also been pointed out.

Keywords: Ion-acoustic wave; Quantum plasma; Ion-streaming; Electron inertia

INTRODUCTION

Quantum plasma is a relatively new and rapidly growing field of plasma research on account of its potential applications in metal nanostructures [1], semiconductor devices [2-4], dense astrophysical environments [5,6], laser-solid interaction [7,8], cool vibes [9], small electronic devices [10]. The quantum effects may become important in a variety of environments when the plasma temperature is low and particle number density is high. The dispersion caused by strong density correlation due to quantum fluctuations can play important role on wave propagation in quantum plasma [11]. Recently there has been a great deal of interest in the investigation of various collective processes in quantum plasmas by using quantum hydrodynamic (QHD) equations [6] that include the quantum statistical pressure and quantum forces involving tunneling of degenerate plasma particles through the so-called Bohm potential [3,11]. Because of simplicity and numerical efficiency the QHD model has been widely used for studying wave propagation in quantum plasma [12-16]. Using QHD equations, Hass, Garcia et al [17] have investigated the role of quantum diffraction on the propagation of ion-acoustic waves (IAWs) in non-drifting plasma neglecting electron inertia. The effects of the electron inertia and drift motion of ions on small amplitude ion-acoustic soliton have not been widely discussed except by a few authors in classical plasmas. It has been shown that ion-acoustic soliton solutions exist only if ion-drift velocity is less than electron thermal velocity [18, 19]. Roychowdhury, Roychowdhury et al [20] have studied double layers in a relativistic hot plasma considering the contribution of electron inertia from which they got much reduced profile of double layers. Paul, Chattopadhyaya et al [21] have shown that electron inertia and ion drift velocity have significant contribution on the formation of ion acoustic solitary wave and double layers in a multi-component classical plasma. Chattopadhyaya, Bhattacharya et al [22] have shown that drift motion of the ions

have significant contribution on the excitation of ion-acoustic solitary waves and double layers in a negative ion plasma. Thus we find that drift motion of ions and electron-inertia can both play an important role on the propagation of IAWs in the classical plasmas. In quantum plasma we also expect similar results. Electrons because of their lighter mass reach quantum behavior more easily than the heavier ions. But one cannot ignore the role of ions in micro scale and nano-scale systems. So far as we know no one has investigated the effects of both the electron inertia and the ion drift on the propagation of IAWs in quantum plasma. The motivation of the present work is to address this problem.

Basic Formulation

We consider two-species quantum plasma composed of electrons and ions. In the QHD model, we take the continuity and momentum balance equations for both electrons and ions. These equations are supplemented by the Poisson's equation. In this work, we take into account the electron inertia, the ion-streaming and the quantum diffraction effects. For closure we disregard pressure effect for ions and assume an equation of state pertaining to a zero-temperature Fermi gas for the electrons. We also assume that the plasma is unmagnetized and the species are non-relativistic. Under these conditions the dynamics of low phase speed (in comparison with the electron Fermi speed) IAWs in two-component electron-ion quantum plasma is governed by the following equations:

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e u_e)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0 \quad (2)$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{e}{m_e} \frac{\partial \phi}{\partial x} - \frac{1}{m_e n_e} \frac{\partial p_e}{\partial x} + \frac{\hbar^2}{2m_e^2} \frac{\partial}{\partial x} \left[\left(\frac{\partial^2 \sqrt{n_e}}{\partial x^2} \right) / \sqrt{n_e} \right] \quad (3)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{e}{m_i} \frac{\partial \phi}{\partial x} + \frac{\hbar^2}{2m_i^2} \frac{\partial}{\partial x} \left[\left(\frac{\partial^2 \sqrt{n_i}}{\partial x^2} \right) / \sqrt{n_i} \right] \quad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\epsilon_0} (n_e - n_i) \quad (5)$$

Here, n_e , m_e , u_e (n_i , m_i , u_i) are the electron (ion) number density, mass and velocity respectively. ϵ_0 and \hbar denote the free space permittivity and the Planck constant divided by 2π respectively. It is to be noted that the quantum diffraction is taken into account by the terms proportional to \hbar^2 in Eqs. (3) and (4).

The electrons are assumed to obey the equation of state relating to a one-dimensional zero-temperature Fermi gas [6]:

$$p_e = \frac{m_e v_{Fe}^2}{3n_0^2} n_e^3 = \left(\frac{2k_B T_{Fe}}{3n_0^2} \right) n_e^3 \quad (6)$$

where, n_0 is the equilibrium density of the electrons and ions, v_{Fe} is the electron Fermi velocity, T_{Fe} is the electron Fermi temperature and k_B is the Boltzmann constant.

Now we introduce the following normalizations:

$$\bar{x} = \frac{\omega_{pi} x}{c_s}, \quad \bar{t} = \omega_{pi} t, \quad \bar{n}_e = \frac{n_e}{n_0}, \quad \bar{n}_i = \frac{n_i}{n_0}, \quad \bar{u}_e = \frac{u_e}{c_s}, \quad \bar{u}_i = \frac{u_i}{c_s}, \quad \bar{\phi} = \frac{e\phi}{2k_B T_{Fe}} \quad (7)$$

where $\omega_{pi} = (n_0 e^2 / m_i \epsilon_0)^{1/2}$ denotes the ion plasma frequency and $c_s = (2k_B T_{Fe} / m_i)^{1/2}$ is the quantum ion-acoustic speed.

Using the above normalizations (7) and dropping bars for simplifying the notations, we obtain from Eqs. (1)- (5)

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e u_e)}{\partial x} = 0 \quad (8)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0 \quad (9)$$

$$q \left(\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} \right) = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2} \frac{\partial}{\partial x} \left[\left(\frac{\partial^2 \sqrt{n_e}}{\partial x^2} \right) / \sqrt{n_e} \right] \quad (10)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} + q \frac{H^2}{2} \frac{\partial}{\partial x} \left[\left(\frac{\partial^2 \sqrt{n_i}}{\partial x^2} \right) / \sqrt{n_i} \right] \quad (11)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i \quad (12)$$

where $q = m_e/m_i$ is a measure of electron inertia and $H = \hbar \omega_{pe} / 2k_B T_{Fe}$ is a non-dimensional quantum parameter in

which $\omega_{pe} = (n_0 e^2 / m_e \epsilon_0)^{1/2}$ is the electron plasma frequency. Physically, H is the ratio of electron plasmon energy to the electron Fermi energy.

Linear Instability

In order to linearize Eqs. (8)- (12) we expand the plasma variables around their unperturbed values as follows.

$$n_i = 1 + n_{i1}, \quad n_e = 1 + n_{e1}, \quad u_i = u_{i0} + u_{i1}, \quad u_e = u_{e1}, \quad \phi = \phi_1 \quad (13)$$

Linearizing Eqs. (8)- (12) and assuming space-time dependence of the first order perturbations to be proportional to $\exp[i(kx - \omega t)]$ where ω and k are the normalized wave frequency and wave number respectively, we obtain the following linear dispersion law:

$$k^2 = \frac{1}{q \left(\frac{\omega}{k} \right)^2 - \left(1 + \frac{k^2 H^2}{4} \right)} + \frac{1}{\left(\frac{\omega}{k} - u_{i0} \right)^2 - \frac{q k^2 H^2}{4}} \quad (14)$$

If one considers a non-drifting quantum plasma and disregards electron inertia effect this dispersion law reduces to that given by Hass, Garcia et al [17]. Furthermore in the limit $H \rightarrow 0$, the dispersion law (14) reduces to that for classical ion-acoustic waves (with the ion-acoustic speed replaced by quantum ion-acoustic speed). The dispersion law given by (14) can also be expressed as

$$a k^8 + b k^6 + c k^5 + d k^4 + e k^3 + f k^2 + g k + h = 0 \quad (15)$$

$$\text{Where, } a = \frac{q H^4}{4}, \quad b = -(q + u_{i0}^2) H^2, \quad c = 2\omega u_{i0} H^2,$$

$$d = -4u_{i0}^2 - H^2 \omega^2 (q^2 + 1) + H^2 (1 + q^2), \quad e = 8\omega u_{i0},$$

$$f = 4(q\omega^2 u_{i0}^2 - \omega^2 - u_{i0}^2 + 1), \quad g = 8u_{i0} \omega (-q\omega^2 + 1)$$

$$\text{and } h = 4\omega^2 [q(\omega^2 - 1) - 1] \quad (16)$$

It is an eighth order algebraic equation in k indicating different modes of propagation of the IAWs.

The dispersion law given by Eq. (15) has been solved numerically.

It is shown that for a particular set of values of q , ω , H and u_{i0} , some roots are real and some are complex. It is also observed that the real roots may be both 'positive' and 'negative'. The positive real roots correspond to the forward moving wave and negative real roots correspond to the backward moving wave. Instability of the wave, when it propagates through the plasma medium is determined by the imaginary part of the complex roots. Again positive values of the imaginary parts signify a decaying mode and negative values of the imaginary part signify growing instability of the wave. The growth rate of instability is determined by $|k_i|$, where k_i is the imaginary part of k .

In Fig.-1, we show the variation of $|k_i|$ (for one typical mode) with ion streaming velocity u_{i0} for different values of the quantum parameter H . We find that $|k_i|$ increases with u_{i0} for a given H ; physically it means that the growth rate of instability increases with increase in ion-streaming velocity. From Fig.1 we also note that the growth rate of linear instability decreases with increase in H . Figure 2 shows the dependence of instability growth rate on H for different values of electron inertia factor q . It shows that the

instability growth rate is high for low values of electron inertia factor.

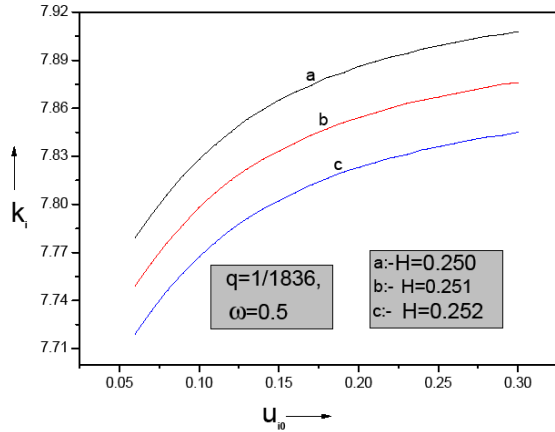


Fig.1 Variation of linear instability growth rate with u_{i0} (ion streaming velocity) for different value of H .

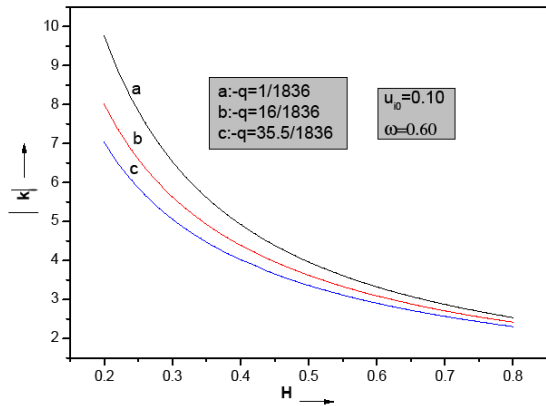


Fig.2 Variation of linear instability growth rate with H for different value of electron inertia factor (q).

Nonlinear Wave Analysis

To investigate the nonlinear properties of the quantum ion-acoustic waves including ion-streaming and electron inertia, we use the following rescaling of space and time variables:

$$\xi = \varepsilon^{\frac{1}{2}} (x - t) \quad \text{and} \quad \tau = \varepsilon^{\frac{3}{2}} t \quad (17)$$

where ξ and τ are two new independent coordinates and ε is a small parameter indicating smallness of perturbation. Equations (8)-(12) are written in terms of the stretched coordinates ξ and τ and then we introduce the following perturbation expansions for the field quantities about their equilibrium values:

$$\begin{aligned} n_i &= 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \dots \\ n_e &= 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \dots \\ u_i &= u_{i0} + \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \dots \\ u_e &= \varepsilon u_{e1} + \varepsilon^2 u_{e2} + \varepsilon^3 u_{e3} + \dots \\ \phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \dots \end{aligned} \quad (18)$$

Solving the lowest order equations in ε with the boundary condition that, $n_{e1}, u_{e1}, n_{i1}, u_{i1}$ and $\phi_1 \rightarrow 0$ as $|\xi| \rightarrow \infty$ we obtain the following solutions:

$$u_{e1} = n_{e1} = \frac{\phi_1}{1-q} \quad (19)$$

$$u_{i1} = \frac{\phi_1}{1-u_{i0}} \quad (20)$$

$$n_{i1} = \frac{\phi_1}{(1-u_{i0})^2} \quad (21)$$

Going to the next higher order terms in equations:

$$-\frac{\partial n_{e2}}{\partial \xi} + \frac{\partial n_{e1}}{\partial \tau} + u_{e1} \frac{\partial n_{e1}}{\partial \xi} + \frac{\partial u_{e2}}{\partial \xi} + n_{e1} \frac{\partial u_{e1}}{\partial \xi} = 0 \quad (22)$$

$$-\frac{\partial n_{i2}}{\partial \xi} + \frac{\partial n_{i1}}{\partial \tau} + u_{i0} \frac{\partial n_{i2}}{\partial \xi} + u_{i1} \frac{\partial n_{i1}}{\partial \xi} + \frac{\partial u_{i2}}{\partial \xi} + n_{i1} \frac{\partial u_{i1}}{\partial \xi} = 0 \quad (23)$$

$$\begin{aligned} & -q \frac{\partial u_{e2}}{\partial \xi} - 3qn_{e1} \frac{\partial u_{e1}}{\partial \xi} + qu_{e1} \frac{\partial u_{e1}}{\partial \xi} + q \frac{\partial u_{e1}}{\partial \tau} \\ & = \frac{\partial \phi_2}{\partial \xi} + 3n_{e1} \frac{\partial \phi_1}{\partial \xi} - \frac{\partial n_{e2}}{\partial \xi} - 4n_{e1} \frac{\partial n_{e1}}{\partial \xi} + \frac{H^2}{4} \frac{\partial^3 n_{e2}}{\partial \xi^3} \end{aligned} \quad (24)$$

$$\begin{aligned} & -\frac{\partial u_{i2}}{\partial \xi} - 3n_{i1} \frac{\partial u_{i1}}{\partial \xi} + u_{i1} \frac{\partial u_{i1}}{\partial \xi} + \frac{\partial u_{i1}}{\partial \tau} + u_{i0} \frac{\partial u_{i2}}{\partial \xi} + 3u_{i0}n_{i1} \frac{\partial u_{i1}}{\partial \xi} \\ & = -\frac{\partial \phi_2}{\partial \xi} - 3n_{i1} \frac{\partial \phi_1}{\partial \xi} + \frac{qH^2}{4} \frac{\partial^3 n_{i1}}{\partial \xi^3} \end{aligned} \quad (25)$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = n_{e2} - n_{i2} \quad (26)$$

After proper elimination of $n_{e2}, n_{i2}, u_{i2}, u_{e2}$ and ϕ_2 from Eqs. (22)- (26) and using lowest order solutions (19)-(21), we obtain the desired KdV equation

$$\frac{\partial \phi_1}{\partial \tau} + a_1 \phi_1 \frac{\partial \phi_1}{\partial \xi} + b_1 \frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \quad (27)$$

where

$$a_1 = \frac{3 + \frac{4}{(1-q)^2}}{\frac{2q}{1-q} + \frac{2}{1-u_{i0}}} \quad \text{and} \quad b_1 = \frac{[(1-q) - \frac{H^2(1+q)}{4(1-q)}]}{\frac{2q}{1-q} + \frac{2}{1-u_{i0}}} \quad (28)$$

To find the solution of Eq. (27) we transform the independent variables ξ and τ into one variable $\mu = \xi - M\tau$ where M is the normalized constant speed of the wave frame. Applying the

boundary conditions that as $\mu \rightarrow \pm\infty, \phi, \frac{\partial \phi}{\partial \mu}, \frac{\partial^2 \phi}{\partial \mu^2} \rightarrow 0$ the

possible stationary solution of Eq. (27) is obtained as:

$$\phi_1 = A \operatorname{sech}^2 \left[\frac{\mu}{\delta} \right] \quad (29)$$

where the amplitude A and width δ of the soliton are given by

$$A = \frac{3M}{a_1} \quad \text{and} \quad \delta = \frac{2\sqrt{b_1}}{\sqrt{M}}$$

The solitary wave structure is formed due to a delicate balance between dispersive and nonlinear effects. Hence the coefficients a_1 and b_1 corresponding to nonlinear effect and dispersive effect play significant role in determining the solitary structure. Note that the coefficient a_1 of the nonlinear term in the KdV equation (27) depends on electron inertia factor q and ion-streaming velocity u_{i0} . Whereas the coefficient b_1 of the dispersive term depends on q , u_{i0} as well as the quantum diffraction parameter H . Thus the nonlinear and the dispersive terms get modified by electron inertia and ion-streaming effect in quantum plasma.

RESULTS AND DISCUSSIONS

Using nonlinear quantum fluid model equations and the standard reductive perturbation technique both the linear and nonlinear properties of IAWs have been investigated including electron inertia and ion-streaming effects. A general type dispersion relation is obtained which shows the possibilities of excitation of a number of wave modes. Some of these modes are shown to be linearly unstable and growth rate of this instability depends in a significant way on electron inertia, ion-streaming and quantum diffraction effects.

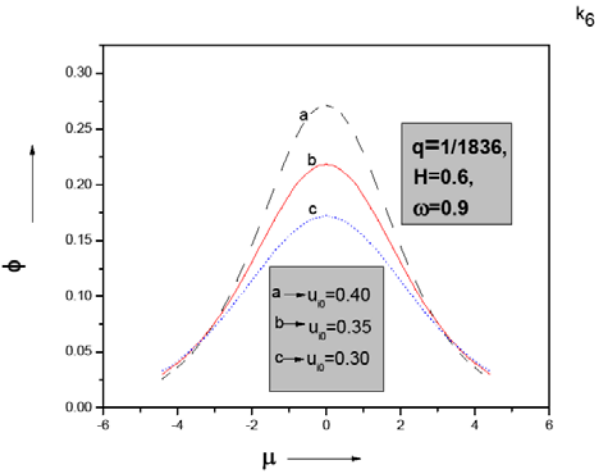


Fig.3 Solitary wave profile for different values of streaming velocity (u_{i0})

To study the nonlinear behavior of the wave a KdV equation has been derived in which the coefficients of the nonlinear and dispersive terms are shown to get modified due to inclusion of electron inertia, ion-streaming and quantum diffraction effects. The model plasma under consideration can support only compressive type of soliton. The soliton amplitude and width are found to depend significantly on the ion-streaming velocity and

quantum diffraction parameter (Figs. 3-5). Figure 3 shows the KdV solitary wave profile for different values of ion-streaming velocity u_{i0} . Obviously the solitons amplitude increases and width decreases with increase in ion-streaming (Fig. 4). Figure 5 shows the KdV solitary wave profile for different values of quantum diffraction parameter H . It shows an increase in soliton amplitude with increase in quantum diffraction effect. The electron inertia though has significant effect on the linear behavior of the wave; it has not so significant effect on the nonlinear behavior of the wave.

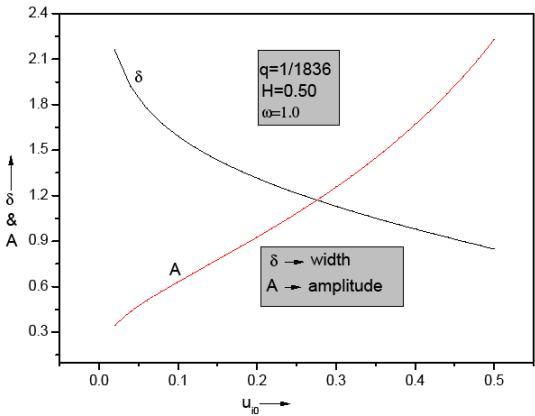


Fig.4 Variation of soliton amplitude and width with streaming velocity (u_{i0})

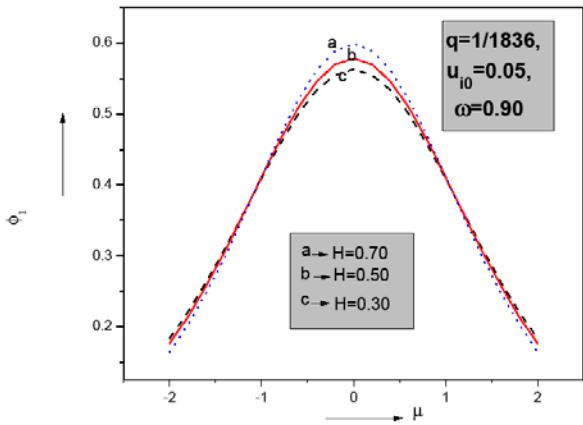


Fig.5 Solitary wave profile for different values of quantum parameter (H)

CONCLUSION

Linear and nonlinear propagation of IAWs in dense electron-ion quantum plasma have been investigated including electron inertia and ion-streaming effects. Our investigation shows that inclusion of electron inertia and ion-streaming can change the linear and nonlinear properties of IAWs in quantum plasma in a significant way. The results presented in the paper may be useful in understanding the nonlinear solitary structures, the origin of electrostatic fluctuations and associated energy

transport in dense plasmas such as that found in white dwarfs, neutron stars and in many other astrophysical plasma systems.

This work may be easily extended to three component quantum plasma like electron-ion-negative ions plasma and electron-ion-positron plasma.

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