

# **The Dark Side of Cosmology: Dark Matter and Dark Energy**

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## **Abstract**

A simple model with only six parameters (the age of the universe, the density of atoms, the density of matter, the amplitude of the initial fluctuations the scale dependence of this amplitude and the epoch of first star formation) fits all of our cosmological data. While simple, this standard model is strange. The model implies that most of the matter in our Galaxy is in the form of “dark matter”, a new type of particle not yet detected in the laboratory, and most of the energy in the universe is in the form of “dark energy”, energy associated with empty space. Both dark matter and dark energy require extensions to our current understanding of particle physics or point towards a breakdown of General Relativity on cosmological scales.

## **Introduction**

John Archibald Wheeler, my academic great grandfather, succinctly summarized *geometrodynamics*, his preferred name for the theory of General Relativity (1): “Spacetime tells matter how to move; matter tells spacetime how to curve.”

Cosmologists observe the motion of atoms (either in the form of gas or stars) or follow the paths taken by light propagating across the universe and use these observations to infer the curvature of spacetime. They then use these measurements of the curvature of spacetime to infer the distribution of matter and energy in the universe. Throughout this article, I will discuss a variety of observational techniques, but ultimately they all use general relativity to interpret the observations and they all lead to the conclusion that atoms, stuff that we understand, make up only 5% of the matter and energy density of the universe.

## **Standard Cosmological Model Fits But At a Price!**

Observations of the large-scale distribution of galaxies and quasars show that the universe is nearly uniform on its largest scales (2) and that the velocity of a distant galaxy depends on its distance (3). General relativity then implies that we live in an expanding universe that started in a big bang. Because the universe expands, light is “redshifted”, so that light from a distant galaxy appears redder when it reaches us. Hubble’s observations that found a linear relationship between galaxy redshift and distance established the basic model back in the 1920’s.

Our current cosmological standard model assumes that general relativity and the standard model of particle physics has been a good description of the basic physics of the universe throughout its history. It assumes that the large-scale geometry of the universe is flat: the total energy of the universe is zero. This implies that Euclidean geometry, the mathematics taught to most of us in middle school, is valid on the scale of the universe. While the geometry of the universe is simple, its composition is strange: the universe is composed not just of atoms (mostly hydrogen and helium), but also dark matter and dark energy.

The currently most popular cosmological model posits that soon after the big bang, the universe underwent a period of very rapid expansion. During this inflationary epoch, our visible universe expanded in volume by at least 180 e-foldings. The cosmic background radiation is the leftover heat from this rapid expansion. This inflationary expansion also amplifies tiny quantum fluctuations into variations in density. The inflationary model predicts that these fluctuations are “nearly scale-invariant”: the fluctuations have nearly the same amplitude on all scales.

These density variations set off sound waves that propagate through the universe and leave an imprint in the microwave sky and the large-scale distribution of galaxies. Our observations of the microwave background are a window into the universe 380,000 years after the big bang. During this epoch, electron and protons combined to form hydrogen. Once the universe became neutral, microwave background photons could propagate freely, so the sound waves imprint a characteristic scale, the distance that they can propagate in 380,000 years. This characteristic scale, the “baryon acoustic scale”, serves as a cosmic ruler for measuring the geometry of space, thus, determining the density of the universe.

Observations of the temperature and polarization fluctuations in the cosmic microwave background, both from space (4-6) and from ground-based telescopes (7-8), both test this standard cosmological model and determine its basic parameters. Remarkably, a model with only six independent parameters, the age of the universe, the density of atoms, the density of matter, the amplitude of the density fluctuations, their scale dependence, and the epoch of first star formation, provides a detailed fit to all of the statistical properties of the current microwave background measurements. The same model also fits observations of the large-scale distribution of galaxies (9), measurements of the Hubble constant, the expansion rate of the universe (10-11), as well as distance determinations from supernovae (12). The success comes at a price: atoms make up less than 5% of our universe, the standard model posits that dark matter dominates the mass of galaxies and that dark energy, energy associated with empty space, makes up most of the energy density of the universe (see Figure 1).

Astronomical observations and cosmological theory suggest that the composition of the universe is remarkably rich and complex. As Figure 1 shows, the current best estimates of the universe’s composition (5-8) suggest that dark energy, dark matter, atoms, three different types of neutrinos and photons all make an observable

contribution to the energy density of the universe. While black holes are an unlikely candidate for the dark matter (13), their contribution to the mass density of the universe is roughly 0.5% of the stellar density (14).

Figure 1 shows the multiple components that compose our universe

### **Astronomical Evidence for Dark Matter**

The evidence for dark matter long predates our observations of the microwave background, supernova observations, and measurements of large-scale structure. In a prescient article published in 1933, Fritz Zwicky (15) showed that the velocities of galaxies in the Coma cluster was much higher than expected from previous estimates of galaxy masses, thus implying that there was a great deal of additional mass in the cluster. In the 1950s, Kahn and Woltjer (16) argued that the Local Group of galaxies could be dynamically stable only if it contained significant amounts of unseen matter. By the 1970s, astronomers argued that mass in both clusters (17) and galaxies (18) increased with radius and did not trace light. Theoretical arguments that showed that disk stability required dark matter halos (19) buttressed these arguments. Astronomers studying the motion of gas in the outer regions of galaxies found evidence in an ever-increasing number of systems for the existence of massive halos (20-24). By the 1980s, dark matter had become an accepted part of the cosmological paradigm.

What do we know about dark matter from astronomical observations today? Microwave background and large-scale structure observations imply that dark matter is five times more abundant than ordinary atoms (4-8). The observations also imply that the dark matter has very weak (or no) interactions with photons, electrons and protons. If the dark matter was made of atoms today, then in the early universe, it would have been made of ions and electrons and would have left a clear imprint on the microwave sky. Thus, dark matter must be non-baryonic and “dark”.

Observations of large-scale structure and simulations of galaxy formation imply that the dark matter must also be “cold”: the dark matter particles must be able to cluster on small scales. Simulations of structure formation with cold dark matter (and dark energy) are generally successful at reproducing the observations of the large-scale distribution of galaxies (25). When combined with hydro-dynamical simulations that model the effects of cooling and star formation, the simulations can reproduce the basic observed properties of galaxies (26,27).

Supermassive clusters are important laboratories for studying dark matter properties. These clusters are thought to be “fair samples” of the universe with the ratio of dark matter to ordinary matter observed in the clusters is very close to the cosmological value (28). X-ray observations directly trace the distribution of

ordinary (“baryonic matter”) as most of the atoms in the cluster gas have been ionized. As Fritz Zwicky (29) first discussed, observations of gravitational lensing of background galaxies directly trace the total distribution of matter in the clusters. Today, over 75 years after Zwicky’s prescient suggestion, astronomers use large-format cameras on the Hubble Space Telescope to make detailed maps of the cluster dark matter distribution (30). These observations reveal significant amounts of dark matter substructure in the clusters, generally consistent with the predictions of numerical simulations (31).

At much smaller scales, dwarf galaxies are another important astronomical testing ground for theories of dark matter. The gravitational potential wells of these dark matter dominated systems are quite shallow, so the predicted properties of dwarf galaxy halos are quite sensitive to dark matter properties. Several groups (32,33) have argued that the observed properties of dwarf galaxies do not match the predictions of numerical simulations. While some astrophysicists argue that improved models of star formation feedback can reconcile this discrepancy (34), others suggest that dark matter self-interactions are needed to match simulations to observations (35).

All of the astronomical arguments for the existence of dark matter assume General Relativity is valid on galactic scales. Alternative gravity theories, such as Modified Newtonian Dynamics (MOND) (36), obviate the need for dark matter by changing the physics of gravity. While these models have some phenomenological success on the galaxy scale (37), they have great difficulties fitting the microwave background fluctuation observations (4-8,38) and observations of clusters, particularly the bullet cluster (39). Most theorists also consider these alternative models as lacking motivation from fundamental physics.

### **What is the dark matter?**

The existence of non-baryonic dark matter implies that there must be new physics beyond the standard model of particle physics. Particle physicists have suggested a wealth of possibilities, some motivated by ideas in fundamental physics and others by a desire to explain astronomical phenomena (40).

The early universe was an incredibly powerful particle accelerator. At the high temperatures and densities of the early moments of the big bang, the cosmic background radiation created an enormous number of particles. Cosmic microwave background experiments (5-8) have detected the observational signatures of the copious number of neutrinos produced in the early first moments of the universe. These early moments could have also created the dark matter particles.

Supersymmetry, the most studied extension of our current understanding of particle physics, provides potential candidates for dark matter. Particles can be divided into two types: fermions and bosons. Fermions obey the Pauli exclusion principle: only one particle can be found in each state. Multiple bosons can be found

in the same quantum state. Electrons are fermions, while photons are bosons. Supersymmetry would be a new symmetry of nature that links each boson to a fermionic partner and vice versa. This symmetry implies a plethora of new particles: the photon would have a fermionic partner, the photino, and the electron would have a bosonic partner, the selectron. One of the goals of the Large Hadron Collider (LHC) is to search for these yet undiscovered supersymmetric particles.

The lightest supersymmetric particle (LSP) can be stable. These particles would have been produced copiously in the first moments after the big bang. For certain parameters in the supersymmetric model, the abundance of the LSP is just what is needed to explain the observed abundance of dark matter. This success is an example of the “WIMP miracle” of cosmology: a Weakly Interacting Massive Particle (WIMP), a particle that interacts through exchanging particle with masses comparable to the Higgs mass, has the needed properties to be the dark matter.

Particle physics suggests other well-motivated dark matter candidates in addition to the including the axion (41) and “asymmetric dark matter”(42), particles whose abundances are not set by their cross-section but by an asymmetry between particles and anti-particles.

If WIMPs are the dark matter, then they could be detected through several different routes: dark matter could be created at an accelerator or seen either in deep underground experiments or through astronomical observations (40,43). These possibilities have led to an active program of searching for dark matter. This search has had many exciting moments. There are currently a number of intriguing signals that might turn out to be the first detection of dark matter:

- The Gran Sasso DArk MAtter (DAMA) experiment has seen an annual modulation in the event rate in its detector (44) with just the theoretical predicted form (45). The interpretation of this result is controversial as other experiments have failed to detect dark matter and seem to be in contradiction with this detection claim (46,47).
- There have been multiple claims of excess gamma rays signals coming from the center of our Galaxy at a range of potential dark matter masses (48,49). Because of the high dark matter density in the galactic center, it is potentially the brightest source of high-energy photons produced through dark matter self-annihilation. However, the galactic center also contains a wealth of astrophysical sources that emit high-energy photons. Searches in external galaxies have also suggested the existence of dark matter with yet a different mass (50). This claim is also controversial (51). Cosmologists hope that observations of nearby dwarfs could provide a less ambiguous signal (52).
- Dark matter annihilation in our Galaxy could potentially produce positrons. Cosmic ray experiments have been search for these signals (53). The challenge for these experiments is to separate this signal from astrophysical sources of cosmic rays, such as pulsars and production from secondary collisions.

Hopefully, future experiments will verify one of these results.

The discovery of the dark matter particle would resolve a long-standing mystery in astronomy, provide insights into dark matter's role in galaxy formation and structure, and be the first signature of new physics beyond the Higgs.

## **Dark Energy**

When Einstein introduced his theory of General Relativity, he added a cosmological constant term. This term generated a repulsive force that countered the pull of gravity and kept the universe static and stable. In the 1920s, Hubble's discoveries showed that the universe was expanding and physicists dropped the cosmological constant term.

Motivated by observational evidence favoring a low density universe and theoretical prejudice that favored a flat universe, enthusiasm for a cosmological constant revived in the 1970s and 1980s in the astronomy community (54-56). Physicists recognized that the value of the cosmological constant was a profound problem in fundamental physics (57).

A universe dominated by a cosmological constant is a strange place to live. We think of gravity as an attractive force. If you throw a ball upwards, gravity slows its climb out of the Earth's gravitational well. Similarly, gravity (in the absence of a cosmological constant) slows the expansion rate of the universe. Imagine your surprise if you threw a ball upwards and it started to accelerate! This is the effect that a cosmological constant has on the universe's rate of expansion.

Supernova observations provided critical evidence for the universe's acceleration. Supernovae are bright stellar explosions of nearly uniform peak luminosities (58). Thus, they serve as beacons that can be used to determine the light travel distance to their host galaxies. By determining distance as a function of galaxy redshift, the supernova observations measure the expansion rate of the universe as a function of time. In the late 1990s, supernova observers reported the surprising result that the expansion rate of the universe is accelerating (59,60)!

Over the past fifteen years, the observational evidence for cosmic acceleration continued to grow. Measurements of the baryon acoustic scale, both in the microwave background (3-8) and in the galaxy distribution (9) as a function of redshift traced the scale of the universe back to redshift of 1100. Measurements of the growth rate of structure as a function of redshift also reinforced the case for cosmic acceleration.

Why is the universe accelerating? The most studied possibility is that the cosmological constant (or equivalently, the vacuum energy of empty space) is driving cosmic acceleration. Another possibility is that there is an evolving scalar field that fills space (like the Higgs field or the inflaton field that drove the rapid early expansion of the universe (61)). Both of these possibilities are lumped together in "dark energy". Since all of the evidence for dark energy uses the equations of General Relativity to interpret our observations of the universe's expansion and evolution, an alternative conclusion is that a new theory of gravity is needed to explain the observations (38). Possibilities include modified gravity theories with extra-dimensions (62).

Future observations can determine the source of cosmic acceleration and determine the nature of dark energy. Our observations can measure two different effects: the relationship between distance and redshift and the growth rate of structure (63). If General Relativity is valid on cosmological scales, then these two measurements should be consistent. These measurements will also determine the basic properties of the dark energy.

Astrophysicists are currently operating several ambitious experiments that aim to use measurements of galaxy clustering and supernova observations to measure distance and gravitational lensing observations to measure the growth rate of structure<sup>1</sup>. These are complemented by microwave background<sup>2</sup> observations that will provide independent measurements of gravitational lensing and more precise measurements of cosmic structure. In the next decade, even more powerful observations will map the large-scale structure of the universe over the past ten billion years and trace the distribution of matter over much of the observable sky<sup>3</sup>. These observations will provide deeper insights into the source of cosmic acceleration.

### Conclusions

While General Relativity is now a hundred year old theory, it remains a powerful, and controversial idea in cosmology. It is one of the basic assumptions behind our current cosmological model: a model that is both very successful in matching observations, but implies the existence of both dark matter and dark energy. They signify that our understanding of physics is incomplete. We will likely need a new idea as profound as General Relativity to explain these mysteries and likely require more powerful observations and experiments to light the path towards our new insights.

### Acknowledgements

DNS is supported by grants from the National Science Foundation and NASA.

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<sup>1</sup> <https://www.sdss3.org/future/eboss.php>, <http://www.darkenergysurvey.org>, <http://sumire.ipmu.jp/en/3002>

<sup>2</sup> <http://www.princeton.edu/act/>, <http://pole.uchicago.edu>, <http://bolo.berkeley.edu/polarbear/>

<sup>3</sup> <http://www.lsst.org/lsst/>, <http://wfirst.gsfc.nasa.gov>, <http://www.euclid-ec.org>

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