

## FORTRAN PROGRAMS FOR SAD MODELS

**Programs are written in fortran 90. They have been successfully compiled using f95 compiler on unix system. No options are needed for compilation.**

**The programs are free for research but should be cited in publications (David et al. JAS 2015, GSE 2017 or Zenodo DOI).**

**The sadmultiuni.f90 combines the saduni.f90 and SADmulti.f90 programs (zenodo DOI: 10.5281/zenodo.192036) which is useful for single trait analysis, multiple trait analysis or single trait analysis with correlated random effects (direct and indirect genetic effects for instance).**

- **The sadmultiuni.f90 is for versions 1 to 3 of ASReml**
- **The sadmultiuni4.f90 is for ASReml4**

For analyzing longitudinal traits using a SAD model you need:

- 1 datafile
- 1 pedigree file if there are genetic effects in your model
- 1 “.as” file (description of the model)
- As many “paraX” files as the number of independent random effects in the model (having a SAD modeling).

## 1. EXPLANATION FOR A SINGLE TRAIT MODEL

Details in : David, I., Ruesche, J., Drouilhet, L., Garreau, H., & Gilbert, H. (2015). Genetic modeling of feed intake. *Journal of animal science*, 93(3), 965-977

For instance, the model is  $\mathbf{y}(t_j) = \boldsymbol{\mu}(t_j) + \mathbf{u}(t_j) + \mathbf{p}(t_j)$

With,  $\boldsymbol{\mu}(t_j)$  the fixed effects at time  $t_j$ ,  $\mathbf{u}(t_j)$  the vector of genetic effects at time  $t_j$ ,  $\mathbf{p}(t_j)$  the vector of pseudo-permanent environmental effects at time  $t_j$ . For a given random effect ( $p$  for instance) with antedependence of order  $\alpha$ , degree  $\beta_i$  for the  $i^{\text{th}}$  antedependence parameter ( $\theta_i$ ) and innovation variance of degree  $\gamma$ :

$$\mathbf{p}(t_j) = \sum_{s=1}^{\alpha} \theta_{sj} \mathbf{p}(t_{j-s}) + \mathbf{e}(t_j),$$

$$\theta_{sj} = \sum_{k=0}^{\beta_s} a_{sk} t_j^k,$$

$$\mathbf{e}(t_j) \sim N(0, I\sigma_{e,j}^2), \sigma_{e,j}^2 = \exp\left(\sum_{k=0}^{\gamma} b_k t_j^k\right)$$

We define a single trait SAD model by “SAD  $\alpha\beta_1\beta_2\dots\beta_\alpha\gamma$ ” for each random term

A SAD111 in then:

$$\mathbf{p}(t_j) = \theta_{1j} \mathbf{p}(t_{j-1}) + \mathbf{e}(t_j),$$

$$\theta_{1j} = a_{10} + a_{11} t_j,$$

$$\mathbf{e}(t_j) \sim N(0, I\sigma_{e,j}^2), \sigma_{e,j}^2 = \exp(b_0 + b_1 t_j)$$

### 1.1. Datafile

In the datafile, in addition to phenotype, fixed effects, animal ID columns..., there must be a column for the variable that indicates the time of measurement (note: for simplicity, we consider the analysis of longitudinal data in this tutorial, so we use the term “time” to define the variable that identifies the different repeated measurements, but one can use SAD model to analyze repeated measurements across different ordered environments for instance). If some records are missing, there is no need to perform data augmentation (i.e. to have the same number of observations per animal).

It is preferable (not mandatory) to sort the data by time-animal. If you do not, check carefully that the time vector used for the analysis is the good one (it appears on the screen when ASreml is running= figures after “time vector”)

Datafile example: successive measurements on 4 days: 1, 3, 8, and 11

time	animal	factor	phenotype
1	1	1	43
1	2	2	21
1	3	1	25
1	4	1	14
3	2	2	52
3	3	2	62
3	4	3	42
8	1	3	51
8	2	1	42
8	3	2	12
8	4	1	45
11	1	3	52
11	2	2	14

### 1.2. Pedigree file (if needed)

As recommended by ASReML.


### 1.3. .as file

Because a pseudo-permanent environmental effect ( $p$ ) is included in the model, there is no classical residual in this model. **However, even if theoretically the residual variance should be fixed to 0, our experience is that a too low value of the residual variance/other variances in the model induces convergence to abnormal parameter values. Thus, we recommend fixing the residual variance to a value around 100 times lower than the other variances of the model.**



If you do not include a pseudo-permanent environmental effect that follows a SAD in your model, then you can consider a classical residual and you do not have to fix its variance. However, you have to keep the !S2==1 option when specifying the model.

Example of “.as” file for a SAD111 for pseudo-permanent environmental and genetic effects

```
unisad
time !A #needed
animal !P
factor !A
phenotype
Pedigree
Datafile !OWN sadmultiuni_exe #link to the fortran executable
Phenotype ~ mu time factor !r time.animal time.ide(animal)
1 1 2
0 0 IDV 0.001 !GF !S2==1 #to fix residual variance to a very low value : 0.001  NOT TOO LOW
time.animal 2
4 time OWN4 0.1 0.1 0.9 0.9 !TCCCC #SAD111 for u: 4 parameters  $a_{10}, a_{11}, b_0, b_1$ , parameter file: para
animal
time.ide(animal) 2
4 time OWN4 0.1 0.1 0.9 0.9 !TCCCC !F2 #SAD111 for p: 4 parameters; + link to para2
ide(animal)
```

For each random effect following a SAD model, the layout of the line indicating the SAD is as follows:

Number of time points -name of the variable indicating the time points-OWNx a b .. !Tyy !Fz with:

- x = the number of parameters to estimate (4 for a SAD111)
- a, b, ...= initial values for the parameters in this order:  
initial values for antedependence parameters ( $a_{sk}$  of  $\theta_s$  starting with parameters of  $\theta_1$  then  $\theta_2$  and so on) , then initial values for the innovation variance parameters ( $b_k$  of  $\sigma_e^2$ )
- y: parameter types: **In the SAD model, we advise to write !TCCCC....**(as many C as parameters)
- z is a number between 1 and 9 that forms the link to the parameter file , if omitted, the parameter file is “para”, if z = 2, the parameter file is “para2”, if z = 3, the parameter file is “para3” and so on. Each random effect must have its own parameter file i.e. even if model is the same for 2 random terms, one has parameter file “para” and the second parameter file “para2”.

#### 1.4. Para file:

The parameter files provide information about the specification of the antedependence and innovation variance (i.e. order, degree...). Their names must be “para”, “para2” or ...”para9”

Example of para file for a SAD111 model ([comments in blue, in bold: unmodifiable words](#))

```

MODEL
SAD
ANTEDEP_ORDER #order of the antedependence  $\alpha$ 
1
MODEL_ANTEDEP #degree of the polynomial functions for the antedependence parameters  $\beta_1 \beta_2 \dots \beta_\alpha$ 
1 # as many figures as the antedep order  $\alpha$ 
MODEL_INNOVATION_VARIANCE #degree of the polynomial function for the innovation variance  $\gamma$ 
1
DATA_FILE #name of the datafile
Datafile
COLUMN_FACTOR # column number in datafile of the variable that indicates the time
1

```

#### 1.5. Results (most important in .asr and .gdg files)

- The .asr file contains the output of the model for the parameter estimates.

Example of .asr file for SAD111 for genetic and permanent environmental effects

```

.....
--- Results from analysis of ADG ---

Source      Model terms  Gamma  Component  Comp/SE  % C
Residual    identity 12384  0.100000E-03  0.100000E-03  0.00  0 F #fixed residual
time.animal  OWNxx      4 -0.126282  -0.126282  -0.45  0 U # $\hat{a}_{10}$  for genetic effect u
time.animal  OWNxx      4 0.193263  0.193263  1.73  0 U # $\hat{a}_{11}$  for genetic effect u
time.animal  OWNxx      4 2.88308  2.88308  11.08  0 U # $\hat{b}_0$  for genetic effect u
time.animal  OWNxx      4 -0.632185  -0.632185  -3.59  0 U # $\hat{b}_1$  for genetic effect u
time.ide(ani  OWNxx      4 0.238707  0.238707  0.54  0 U # $\hat{a}_{10}$  for effect p
time.ide(ani  OWNxx      4 -0.239293  -0.239293  -1.71  0 U # $\hat{a}_{11}$  for effect p
time.ide(ani  OWNxx      4 1.14631  1.14631  3.36  0 U # $\hat{b}_0$  for effect p
time.ide(ani  OWNxx      4 0.240279  0.240279  1.88  0 U # $\hat{b}_1$  for effect p
.....

```

- The .gdg file contains the inverse of the variance-covariance matrix (named B) for the random effect sorted by time (i.e the vector  $[p(t_0) \ p(t_1) \ \dots \ p(t_n)]$ ) and its derivatives with respect to each parameter .

If “para” is used, the matrices are in asfilenameB.gdg

If “para2” is used, the matrices are in asfilenameC.gdg

If “para3” is used, the matrices are in asfilenameD.gdg...

Example of .gdg file for a SAD111

```

4 4 1 -1 #number of parameters, number of time points, option, option
0.2750838D+00-.1186720D+000.2958084D+010.2086888D+00 #parameter estimates
1 1 0.4218808934E-01 #value in position [1 :1] of B-1
2 1 -0.1290789340E-02 #value in [2 :1] of B-1
2 2 0.3438416123E-01
3 2 0.2246697433E-02
3 3 0.2865794115E-01
4 3 0.4497389309E-02
4 4 0.2253153920E-01 #value in [4 :4] of B-1
1 1 0.2581578679E-02 #value in [1 :1] of the derivative of B-1relative to the first parameter a10
2 1 -0.3420232981E-01
2 2 -0.4493394867E-02
3 2 -0.2776024304E-01
3 3 -0.8994778618E-02
4 3 -0.2253153920E-01
4 4 0.000000000
1 1 0.5163157359E-02 #value in [1 :1] of the derivative of B-1relative to the second parameter a11
2 1 -0.6840465963E-01
2 2 -0.1348018367E-01
3 2 -0.8328072727E-01
3 3 -0.3597911447E-01
4 3 -0.9012615681E-01
4 4 0.000000000
1 1 -0.4218808562E-01 #value in [1 :1] of the derivative of B-1relative to the third parameter b0
2 1 0.1290789223E-02
2 2 -0.3438415751E-01
3 2 -0.2246697433E-02
3 3 -0.2865794115E-01
4 3 -0.4497389309E-02
4 4 -0.2253153920E-01
1 1 -0.4223679751E-01 #value in cell 1 :1 of the derivative of B-1relative to the fourth parameter b1
2 1 0.2581578447E-02
2 2 -0.6895013899E-01
3 2 -0.6740091834E-02
3 3 -0.8687151968E-01
4 3 -0.1798955724E-01
4 4 -0.9012615681E-01

```

To obtain the co(variance) matrix B for each random effects (time\*time covariance matrix), you have 3 options:

- you use parameter estimates in the .asr file to compute the L and D<sup>-1</sup> matrices (see articles for details) and  $B = (L'D^{-1}L)^{-1}$
- you use the .gdg file to obtain values of the B<sup>-1</sup> matrix and you calculate its inverse.
- you use “matrixSAD.f90” program that reads the .gdg file and computes the inverse of B<sup>-1</sup> for you.

## 2. EXPLANATION FOR A MULTIPLE TRAIT MODEL

Details in :David, I., Garreau, H., Balmisse, E., Billon, Y., & Canario, L. (2017). Multiple-trait structured antedependence model to study the relationship between litter size and birth weight in pigs and rabbits. *Genetics Selection Evolution*, 49(1), 11.

Let's consider several longitudinal traits  $i$ . The model for each trait  $i$  is of the form:

$$\mathbf{y}_i(t_j) = \boldsymbol{\mu}_i(t_j) + \mathbf{u}_i(t_j) + \mathbf{p}_i(t_j)$$

For two traits  $\mathbf{y}_1, \mathbf{y}_2$ , the general form of the multiple-trait SAD model of order  $\alpha, \alpha'$  (for the antedependence) and  $\eta, \eta'$  (for the cross-antedependence) for a given random effect  $\mathbf{p}$  can be written as (for  $j > \max(\alpha, \alpha', \eta, \eta')$ ):

$$\begin{aligned} \mathbf{p}_1(t_j) &= \sum_{s=1}^{\alpha} \boldsymbol{\theta}_{sj} \mathbf{p}_1(t_{j-s}) + \sum_{s=c}^{\eta} \boldsymbol{\delta}_{sj} \mathbf{p}_2(t_{j-s}) + \mathbf{e}(t_j) \\ \mathbf{p}_2(t_j) &= \sum_{s=1}^{\alpha'} \boldsymbol{\theta}'_{sj} \mathbf{p}_2(t_{j-s}) + \sum_{s=c'}^{\eta'} \boldsymbol{\delta}'_{sj} \mathbf{p}_1(t_{j-s}) + \boldsymbol{\varepsilon}(t_j) \end{aligned}$$

With

$$\begin{aligned} \boldsymbol{\theta}_{sj} &= \sum_{k=0}^{\beta_s} a_{sk} t_j^k, \quad \boldsymbol{\theta}'_{sj} = \sum_{k=0}^{\beta'_s} a'_{sk} t_j^k \\ \boldsymbol{\delta}_{sj} &= \sum_{k=0}^{\omega_s} \kappa_{sk} t_j^k, \quad \boldsymbol{\delta}'_{sj} = \sum_{k=0}^{\omega'_s} \kappa'_{sk} t_j^k, \\ \sigma_{e,j}^2 &= \exp\left(\sum_{k=0}^{\gamma} b_k t_j^k\right), \quad \sigma_{\varepsilon,j}^2 = \exp\left(\sum_{k=0}^{\gamma'} b'_k t_j^k\right) \end{aligned}$$

Where, as for the single trait model,  $\boldsymbol{\theta}_{sj}, \boldsymbol{\theta}'_{sj}$  are the  $s^{\text{th}}$  antedependence parameters for time  $j$  for traits 1 and 2, respectively.  $\boldsymbol{\delta}_{sj}, \boldsymbol{\delta}'_{sj}$  are the  $(s-c+1)^{\text{th}}$  (or  $(s-c'+1)^{\text{th}}$ ) cross-antedependence parameters for time  $j$  for traits 1 and 2, respectively. It should be noted that, conversely to the antedependence relationship that starts at time  $t_{j-1}$ , the cross-antedependence relationships show greater flexibility and start at time  $t_{j-c}$  ( $t_{j-c'}$ ) with  $c$  ( $c'$ ) greater or equal to 0. The  $\mathbf{e}(t_j), \boldsymbol{\varepsilon}(t_j)$  parameters are normally distributed random effects with mean 0 and innovation variance  $\sigma_{e,j}^2, \sigma_{\varepsilon,j}^2$ , respectively. The  $\mathbf{e}, \boldsymbol{\varepsilon}$  parameters are assumed to be independent, except if  $c > 0$  and  $c' > 0$  when a correlation between the two can be considered for the first time point.

The multiple-trait SAD model is then defined for two traits by the order of the antedependence for each trait ( $\alpha, \alpha'$ ), the starting points ( $c, c'$ ) and the order of the cross-antedependence ( $\eta - c + 1, \eta' - c' + 1$ ), the degree of the polynomial for each (cross-)antedependence parameter ( $\beta_1$  to  $\beta_\alpha, \beta'_1$  to  $\beta'_\alpha$  for the antedependence  $\varpi_1$  to  $\varpi_{\eta-c+1}, \varpi'_1$  to  $\varpi'_{\eta'-c'+1}$  for the cross antedependence) and the degree of the polynomial for the innovation variance of each trait ( $\gamma, \gamma'$ ), as well as an indicator of the presence of an initial correlation between  $e(t_0), \varepsilon(t_0)$  or not.

For instance, for 2 traits, a SAD111 for the antedependence for  $p_1$ , a SAD111 for the antedependence for  $p_2$ , one way cross antedependence (in this example, we assume that value of  $p_1$  has an effect on  $p_2$  but not the reverse) of order 1, degree 1, and a starting point  $c'=0$  (total: 10 parameters) the model is:

$$\begin{aligned}
 p_1(t_j) &= \theta_{1j} p_1(t_{j-1}) + e(t_j) \\
 p_2(t_j) &= \theta'_{1j} p_2(t_{j-1}) + \delta'_{0j} p_1(t_j) + \varepsilon(t_j)
 \end{aligned}$$

eq1

$$\begin{aligned}
 \theta_{1j} &= a_{10} + a_{11} t_j, & \theta'_{1j} &= a'_{10} + a'_{11} t_j \\
 \delta'_{0j} &= \kappa'_{00} + \kappa'_{01} t_j \\
 \sigma_{e,j}^2 &= \exp(b_0 + b_1 t_j), & \sigma_{\varepsilon,j}^2 &= \exp(b'_0 + b'_1 t_j)
 \end{aligned}$$

## 2.1. Datafile

The same time vector must be used for the different traits. It means that the different traits must be measured at the same time. However, if some records are missing, there is no need to perform data augmentation (i.e. to have the same number of observations per trait and per animal) as long as there are records for each time point within each trait.

The datafile must contain two columns to describe the time (time, time2). Time corresponds to the "real" time. Time2 is a vector used to organize the time for each trait. For n time points and k traits it should be numbered from 1 to n\*k (this vector is needed to define the size of the B matrix of each random effects and to sort the random effects by trait and time). Data must be sorted by time2-animal.

Datafile example: successive measurements on 4 days: 1, 3, 8, and 11, and 2 traits

time	Time2	animal	factor	character	phenotype
1	1	1	1	1	43
1	1	2	2	1	21
1	1	3	1	1	25
1	1	4	1	1	14



3	2	2	2	1	52
3	2	3	2	1	62
3	2	4	3	1	42
8	3	1	3	1	51
8	3	2	1	1	42
8	3	3	2	1	12
8	3	4	1	1	45
11	4	1	3	1	52
11	4	2	2	1	14
1	5	1	1	2	1
1	5	2	3	2	5
1	5	3	1	2	6
1	5	4	2	2	2
3	6	1	3	2	3
3	6	3	2	2	4
3	6	4	2	2	5
8	7	1	2	2	2
8	7	3	1	2	1
11	8	1	1	2	2
11	8	2	3	2	5

## 2.2. Pedigree

As recommended by ASReml.

## 2.3. File.as

As for the file.as for single trait analysis, because a pseudo-permanent environmental effect is included ( $p$ ) in the model, there is no classical residual in this model. **However, even if theoretically the residual variance should be fixed to 0, our experience is that a too low value of the residual variance/other variances induces convergence to abnormal parameter values. Thus we recommend fixing the residual variance to a value that is around 100 times lower than the other variances of the model.**



If you do not include a pseudo-permanent environmental effect that follows a SAD in your model, then you can consider a classical residual and you do not have to fix its variance. However, you have to keep the !S2==1 option when specifying the model.

Example of .as for a 2 traits analysis, SAD111 for  $u_1$  and  $u_2$ ,  $u_1$  and  $u_2$  are independent (total: 8 parameters). For  $p$ , model described in eq1

```

multisad
time !A #needed
time2 !A #needed
animal !P
factor !A
character !A
phenotype
Pedigree
Datafile !OWN sadmultiuni_exe #link to the fortran executable
Phenotype ~ character.time character. factor !r time2.animal time2.ide(animal)
1 1 2
0 0 IDV 0.001 !GF !S2==1 #to fix residual variance to a very low value : 0.001 ⚠ NOT TOO LOW
time2.animal 2
8 time2 OWN8 0.1 0.1 0.1 0.1 0.9 0.9 0.9 0.9 !TCCCCCCCC #SAD111 for  $u_1$  and  $u_2$ : 8 parameters
 $a_{10}, a_{11}, a'_{10}, a'_{11}, b_0, b_1, b'_0, b'_1$ , parameter file: para
Animal
time2.ide(animal) 2
8 time2 OWN10 0.6 0.1 0.8 0.4 0.5 0.1 1 2 -0.1 0.1 !TCCCCCCCC !F2 #SAD111 for antedependence
 $p_1, p_2$ , one way cross antedependence order 1, degree 1 : 10 parameters
 $a_{10}, a_{11}, a'_{10}, a'_{11}, \kappa'_{00}, \kappa'_{01}, b_0, b_1, b'_0, b'_1$ , parameter file: para2
ide(anim)

```

For each random effect following a multiple-trait SAD model, the layout of the line indicating the SAD is as follows: Number of levels name of the variable OWNx a b .. !Tyy !Fz with:

- x = the number of parameters to estimate
- a, b, ... initial values for the parameters in this order (for 3 traits):
  - **initial values for antedependence** parameters trait 1, antedependence parameters trait2...
  - **initial values for cross-antedependence** parameters in lower triangular order (in one direction and in the other direction) i.e.
    - Trait1 has an influence on trait2 (cross antedependence  $\delta'$ ) (one direction),
    - Trait2 has an influence on trait1 (cross antedependence  $\delta$ ) (other direction),
    - trait1 → trait3 cross- antedependence parameters,
    - trait3 → trait1 cross- antedependence parameters,
    - trait2 → trait3 cross- antedependence parameters,
    - trait3 → trait2 cross- antedependence parameters

order of the parameters for cross antedependence

	Trait1	Trait2	Trait3
Trait 1			
Trait 2	1 (Tr1 → Tr2) 2 (Tr2 → Tr1)		
Trait3	3 (Tr1 → Tr3) 4 (Tr3 → Tr1)	5 (Tr2 → Tr3) 6 (Tr3 → Tr2)	

- **initial values for the innovation variance** trait 1, for the innovation variance trait 2, for the innovation variance trait3
- **initial correlation value** (if needed)
- y: parameter types: we advise to write !TCCCCC....
- z is a number between 1 and 9 that forms the link with the parameter file , if omitted, the parameter file is “para”, if z = 2, the parameter file is “para2”, if z = 3, the parameter file is “para3” and so on.

#### 2.4. Parameter file

The parameter files provide information about the specification of the (cross-)antedependence and innovation variance (i.e. order, degree...).

Example of parameter file for a random term that follows model in eq1 (antedependence SAD111 for both  $p_1$  and  $p_2$ , cross-antedependence in one direction ( $p_1$  has an influence on  $p_2$ ) of order 1, degree 1 with  $c^2=0$ , no initial correlation between  $e$  and  $\varepsilon$  (comments in blue, in bold: unmodifiable words)

```

NBCHARACTER
2 # number of traits (called "trait" in the following formula)
MODEL
SAD # model type (do not use anything else)
ANTEDEP_ORDER
1 1 # order antedependence:  $\alpha \alpha'$ 
CROSS_ ANTEDEP_ORDER #it is not really the order it is=( order cross antedependence + begin_time) in
triangular inferior in one direction and in the other direction 1->2,2->1,(2traits) + 1->3,3->1,2->3,3->2 (3traits) ...
1 0 #  $\eta'+1$  ,  $\eta + 1$  ...as many figures as possible cross-antedependence (trait*(trait-1)), 0 if no cross
antedependence line 1
BEGIN_ CROSS_ ANTEDEP_ORDER #i.e. "begin time"
0 0 #  $c'$  c as many figures as possible cross-antedependence (trait*(trait-1)) , 0 if no cross antedependence
line 2
MODEL_ ANTEDEP #degree of the polynomial function for each antedependence parameters trait1, trait2...
1 1 #  $\beta_1, \beta'_1$  as many figures as the sum of the antedependence orders
MODEL_ CROSS_ ANTEDEP #degree of the polynomial function for each cross-antedependence
parameters, write nothing if there is no cross-antedependence
1 #  $\varpi'_0$  (as many figures as the sum of cross-antedep order ( $\sum$ (line 1 – line 2))
MODEL_ INNOVATION_ VARIANCE_ COVARIANCE #degree of the polynomial function for each
innovation variance trait1, trait2...
0 0 #  $\gamma, \gamma'$  as many figures as the number of traits
CORRINIT #0 = no initial correlation between error terms, 1 = initial correlation at time t0 (in
triangular inferior order)
0 #as many figures as trait*(trait-1)/2
DATA_ FILE #same as in .as
datafile
COLUMN_ FACTOR #the number of the column containing "time" (real time points)
1

```

## 2.5. Results

- The **.asr** file contains the output of the model for the parameter estimates.

Example of .asr file for antedependence SAD111 for independent  $u_1$  and  $u_2$ . Antedependence SAD111 for both  $p_1$  and  $p_2$ , cross-antedependence in one direction ( $p_1$  has an influence on  $p_2$ ) of order 1, degree 1 with  $c'=0$ , no initial correlation between  $e$  and  $\varepsilon$  (see equation 1)

Source	Model terms	Gamma	Component	Comp/SE	% C	
Residual	identity 12384	0.100000E-03	0.100000E-03	0.00	0 F	#fixed residual
time.animal	OWNxx	8 -0.126282	-0.126282	-0.45	0 U	# $\hat{a}_{10}$ for genetic effect u1
time.animal	OWNxx	8 0.193263	0.193263	1.73	0 U	# $\hat{a}_{11}$ for genetic effect u1
time.animal	OWNxx	8 -0.226282	-0.226282	-1.45	0 U	# $\hat{a}'_{10}$ for genetic effect u2
time.animal	OWNxx	8 0.93263	0.93263	1.39	0 U	# $\hat{a}'_{11}$ for genetic effect u2
time.animal	OWNxx	8 2.88308	2.88308	11.08	0 U	# $\hat{b}_0$ for genetic effect u1
time.animal	OWNxx	8 -0.632185	-0.632185	-3.59	0 U	# $\hat{b}_1$ for genetic effect u1
time.animal	OWNxx	8 2.30811	2.30811	1.08	0 U	# $\hat{b}'_0$ for genetic effect u2
time.animal	OWNxx	8 -0.32185	-0.32185	-1.59	0 U	# $\hat{b}'_1$ for genetic effect u2
time.ide(ani	OWNxx	10 0.238707	0.238707	0.54	0 U	# $\hat{a}_{10}$ for effect p1
time.ide(ani	OWNxx	10 -0.239293	-0.239293	-1.71	0 U	# $\hat{a}_{11}$ for effect p1
time.ide(ani	OWNxx	10 0.38707	0.38707	5.4	0 U	# $\hat{a}'_{10}$ for effect p2
time.ide(ani	OWNxx	10 -0.39293	-0.39293	-7.1	0 U	# $\hat{a}'_{11}$ for effect p2
time.ide(ani	OWNxx	10 0.707	0.707	1.54	0 U	# $\hat{\kappa}'_{00}$ for p1 → p2
time.ide(ani	OWNxx	10 -1.239293	-1.239293	-1.1	0 U	# $\hat{\kappa}'_{01}$ for p1 → p2
time.ide(ani	OWNxx	10 1.14631	1.14631	3.36	0 U	# $\hat{b}_0$ for effect p1
time.ide(ani	OWNxx	10 0.240279	0.240279	1.88	0 U	# $\hat{b}_1$ for effect p1
time.ide(ani	OWNxx	10 1.4631	1.4631	3.6	0 U	# $\hat{b}'_0$ for effect p2
time.ide(ani	OWNxx	10 0.40279	0.40279	0.88	0 U	# $\hat{b}'_1$ for effect p2

- The **.gdg** file contains the inverse of the variance-covariance matrix for each random term. The matrix is sorted by trait-time i.e. corresponds to the matrix for the vector

$$[p_1(t_0) \ p_1(t_1) \ \dots \ p_1(t_n) \ p_2(t_0) \ p_2(t_1) \ \dots \ p_2(t_n)]$$

If "para" is used, the matrix is in asfilenameB.gdg

If "para2" is used, the matrix is in asfilenameC.gdg

If "para3" is used, the matrix is in asfilenameD.gdg...

To obtain the co(variance) matrix for each random effects (time x trait\*time x trait covariance matrix, named B), you have 3 options:

- you use parameter estimates in the **.asr** file to compute the L and  $D^{-1}$  matrices (see articles for details) and  $B = (L'D^{-1}L)^{-1}$
- you use the **.gdg** file to obtain values of the  $B^{-1}$  matrix and you calculate its inverse.
- you use "matrixSAD.f90" program that reads the **.gdg** file and computes the inverse of  $B^{-1}$  for you.

### **3. EXPLANATION FOR A SINGLE TRAIT MODEL WITH CORRELATED RANDOM EFFECTS**

*Details in :David I., Sanchez J-P., Piles M., (Submitted 2017). Longitudinal analysis of direct and indirect effects on the average daily gain in growing rabbits using a structured antedependence model .*

This program is useful to analyze trait with direct and indirect genetic effects. For instance, suppose that animals are raised in pen of two, the model for animal  $i$  living with animal  $l$  is

$$y_i(t_j) = \mu_i(t_j) + d_i(t_j) + s_l(t_j) + p_i(t_j)$$

With,  $\mu(t_j)$  the fixed effect at time  $t_j$ ,  $d(t_j)$  the vector of direct genetic effects at time  $t_j$ ,  $s(t_j)$  the vector of indirect genetic effects at time  $t_j$ ,  $p(t_j)$  the vector of pseudo-permanent environmental effect at time  $t_j$ .

Specification of the SAD model for  $p$  is the same as in the single trait SAD model.

Specification of the SAD model for correlated  $s$  and  $d$  is performed using the multiple trait SAD model with constraint ( $d$  and  $s$  are two correlated terms as the random terms of different traits in a multiple trait model that are also correlated). Constraints that need to be done on the multiple trait SAD model to consider correlated random effects within trait are: cross-antedependence in one direction only (the one you want), the order of the cross antedependence is 1 and  $c$  (or  $c'$  if you chose the other direction) is 0 (consequently, no initial correlation). The correlated-effects SAD model of antedependence order  $\alpha$ ,  $\alpha'$  can then be written as (for  $j > \max(\alpha, \alpha')$ ):

$$\begin{aligned} d(t_j) &= \sum_{s=1}^{\alpha} \theta_{sj} d(t_{j-s}) + \delta_j s(t_j) + e_d(t_j) & \theta_{sj} &= \sum_{k=0}^{\beta_s} a_{sk} t_j^k, \theta'_{sj} = \sum_{k=0}^{\beta'_s} a'_{sk} t_j^k \\ s(t_j) &= \sum_{s=1}^{\alpha'} \theta'_{sj} s(t_{j-s}) + e_s(t_j) & \text{with } \delta_j &= \sum_{k=0}^{\omega} \kappa_k t_j^k \\ & & \sigma_{ed,j}^2 &= \exp\left(\sum_{k=0}^{\gamma} b_k t_j^k\right), \sigma_{es,j}^2 = \exp\left(\sum_{k=0}^{\gamma'} b'_k t_j^k\right) \end{aligned}$$

Thus a correlated effects SAD model is defined (2 random effects) by: the order of the antedependence for each random effect ( $\alpha, \alpha'$ ), the degree of the polynomial for the antedependence parameter ( $\beta_1$  to  $\beta_\alpha$ ,  $\beta'_1$  to  $\beta'_{\alpha'}$ ), the degree of the polynomial for the cross-antedependence parameter ( $\varpi_1$  or  $\varpi'_1$ ) and the degree of the polynomial for the innovation variance of each random effect ( $\gamma, \gamma'$ ).

For instance, a correlated effects SAD model, SAD111 for antedependence effect 1, SAD111 for antedependence effect 2 and a cross antedependence of degree 1 is:

$$\begin{aligned} \mathbf{d}(t_j) &= \theta_{1j} \mathbf{d}(t_{j-1}) + \delta_j \mathbf{s}(t_j) + \mathbf{e}_d(t_j) \\ \mathbf{s}(t_j) &= \theta'_{1j} \mathbf{s}(t_{j-1}) + \mathbf{e}_s(t_j) \end{aligned}$$

eq. 2

$$\theta_{1j} = a_{10} + a_{11}t_j, \quad \theta'_{1j} = a'_{10} + a'_{11}t_j$$

$$\delta_j = \kappa_0 + \kappa_1 t_j$$

$$\sigma_{ed,j}^2 = \exp(b_0 + b_1 t_j), \quad \sigma_{es,j}^2 = \exp(b'_0 + b'_1 t_j)$$

### 3.1. Datafile

The datafile is the same as for the single trait analysis with an additional column containing the ID of the co-mate. It is preferable to sort data by time animal (not mandatory).

Datafile example for correlated SAD model: successive measurements on 4 days: 1, 3, 8, and 11

time	animal	Co-mate	factor	phenotype
1	1	2	1	43
1	2	3	2	21
1	3	4	1	25
1	4	1	1	14
3	2	3	2	52
3	3	4	2	62
3	4	1	3	42
8	1	2	3	51
8	2	3	1	42
8	3	4	2	12
8	4	1	1	45
11	1	2	3	52
11	2	3	2	14

### 3.2. Pedigree

As recommended by ASReml.

### 3.3. File.as


As for the file.as for single trait analysis and multiple trait analysis, because a pseudo-permanent environmental effect is included ( $p$ ) in the model, there is no classical residual in this model. **However, even if theoretically the residual variance should be**



**fixed to 0, our experience is that a too low value of the residual variance/other variances induces convergence to abnormal parameter values. Thus we recommend fixing the residual variance to a value that is around 100 times lower than the other variances of the model.**

If you do not include a pseudo-permanent environmental effect that follows a SAD in your model, then you can consider a classical residual and you do not have to fix its variance. However, you have to keep the !S2==1 option when specifying the model.

Example of .as for a correlated effects SAD model. SAD111 for d, s and p, cross antedependence between d and s of degree 1 (total: 14 parameters)

```
sadcorrelated
time !A #needed
animal !P
comate !P
factor !A
character !A
phenotype
Pedigree
Datafile !OWN sadmultiuni_exe #link to the fortran executable
Phenotype ~ character.time character. factor !r ![ time.animal time.comate !] time.ide(animal) #one G matrix (8*8) will be used for time.animal time.comate
1 1 2
0 0 IDV 0.001 !GF !S2==1 #to fix residual variance to a very low value : 0.001  NOT TOO LOW
time.animal 2
8 0 OWN8 OWN10 0.6 0.1 0.8 0.4 0.5 0.1 1 2 -0.1 0.1 !TCCCCCCCCC #SAD111 for antedependence s and d, one way cross antedependence order 1 (constraint), degree 1, 10 parameters
a10, a11, a'10, a'11, κ00, κ01, b0, b1, b'0, b'1 file: para
animal
time.ide(animal) 2
4 time OWN4 0.1 0.1 0.9 0.9 !TCCCC !F2 #SAD111 for p 4 parameters a10, a11, b0, b1, parameter file: para2
ide(anim)
```

For each random effect following a SAD model, the layout of the line indicating the SAD is as for the single and multiple trait cases.

### 3.4. Parameter file

The parameter files provide information about the specification of the (cross-)antedependence and innovation variance (i.e. order, degree...).

Example of parameter file for genetic correlated effects (s,d) described in eq2 (antedependence SAD111 for both s and d, cross-antedependence in one direction (s has an influence on d) of degree 1 with c=0, (comments in blue, in bold: unmodifiable words)

```

NBCHARACTER
2 # number of traits (called "trait" in the following formula)
MODEL
SAD # model type (do not use anything else)
ANTEDEP_ORDER
1 1 # order antedependence:  $\alpha \alpha'$ 
CROSS_ANTEDEP_ORDER #for 2 correlated effects, 1 0 or 0 1 (one direction or the other, not both)...
0 1 #  $\eta'+1, \eta +1$ ...as many figures as possible cross-antedependence (trait*(trait-1)), if no cross
antedependence: 0 line 1
BEGIN_CROSS_ANTEDEP_ORDER #i.e. "begin time"
0 0 #a line with trait*(trait-1) "0" line 2
MODEL_ANTEDEP #degree of the polynomial function for each antedependence parameters trait1, trait2...
1 1 #  $\beta_1, \beta'_1$  as many figures as the sum of the antedependence orders
MODEL_CROSS_ANTEDEP #degree of the polynomial function for the cross-antedependence
parameters, write nothing if there is no cross-antedependence
1 #  $\varpi_0$  (as many figures as the sum of cross-antedep order ( $\sum(\text{line 1} - \text{line 2})$ ))
MODEL_INNOVATION_VARIANCE_COVARIANCE #degree of the polynomial function for each
innovation variance trait1, trait2...
1 1 #  $\gamma, \gamma'$  as many figures as the number of traits
CORRINIT #for correlated effects 0 = no initial correlation between error terms
0 #as many "0" as trait*(trait-1)/2
DATA_FILE #same as in .as
datafile
COLUMN_FACTOR #the number of the column containing "time" (real time points)
1

```

### 3.5. Results

- The **.asr** file contains the output of the model for the parameter estimates (see .1.4 and 2.4 for details)
- The **.gdg** file contains the covariance matrix (inverse and derivatives). For the correlated random effects, the B matrix is given for the vector: [d1,d2,d3...s1,s2,s3] corresponding to the order in which correlated effects appear in the line specifying model (i.e. Phenotype ~ character.time character. factor !r !**[ time.animal (=d) time.comate (=s) !]** time.ide(animal))



#### **4. EXPLANATION TO COMPUTE B MATRIX USING matrixSAD\_exe**

This program uses the results in the .gdg file to calculate the variance covariance matrix B and the variance correlation matrix for each random term. For the executable to work properly, the parameter estimates have to be on the same line in the .gdg file. If it is not the case, put them on the same line before running matrixSAD\_exe

Type matrixSAD\_exe on the command line

The program will ask you the name of the .gdg file

Then  $B^{-1}$ , B and the corresponding correlation matrix (variance on the diagonal, correlation elsewhere) will appear on the screen. B (var-covariance) and B (var-correlation) will be written in a "resumat" file also.

## 5. EXAMPLES

### 5.1. Examples with .as, datafiles, parafiles available

#### 5.1.1. Ex1.as example

Study of average daily gain (simulated data) for 960 animals raised in pen of 8, 5 measurements per animal (week 1 to 5).

The datafile **dataex1** is of the form:

#simulation	#generation	Animal ID	Comate1 ID	...	Comate7 ID	pen	week	ADG	group
1	1	2279	2283	...	2940	1	1	31.49	1
1	1	2279	2283	...	2940	1	2	31.72	1

The pedigree file is **pedex1**

#### 5.1.1.1. Single trait analysis

- The model is:

$$ADG(t_i) = week_i + u_i + p_i + g_i$$

$u$  the direct genetic effects for week  $i$  that follows a SAD111 ( $t_i=i$  because week=[1,2,3,4,5])

$$u_1 = e_{u,1}, \quad u_i = (a_{u0} + a_{u1} * i)u_{i-1} + e_{u,i} \quad \text{and} \quad \sigma_{e_{u,i}}^2 = \exp(b_{u0} + b_{u1} * i)$$

$p$  the pseudo-permanent environmental effects for week  $i$  that follows a SAD111:

$$p_1 = e_{p,1}, \quad p_i = (a_{p0} + a_{p1} * i)p_{i-1} + e_{p,i} \quad \text{and} \quad \sigma_{e_{p,i}}^2 = \exp(b_{p0} + b_{p1} * i)$$

$g$  the group effects for week  $i$  that follows a SAD111 :

$$g_1 = e_{g,1}, \quad g_i = (a_{g0} + a_{g1} * i)g_{i-1} + e_{g,i} \quad \text{and} \quad \sigma_{e_{g,i}}^2 = \exp(b_{g0} + b_{g1} * i)$$

- The .as is:

See ex1.as PATH1

- The parafiles are :

Parafiles are the same (with different name) for all the random effects because they follow the same SAD model

See “para1ex1”, “para2ex1” and “para3ex1” (to use them, rename to “para”, “para2” and “para3”)

o Results:

Source	Model terms	Gamma	Component	Comp/SE	% C
Residual	identity 4800	0.100000E-02	0.100000E-02	0.00	0 F
week.group	OWNxx 5	-0.223349	-0.223349	-0.88	-1 U
week.group	OWNxx 5	0.637139E-01	0.637139E-01	0.95	0 U
week.group	OWNxx 5	2.29012	2.29012	11.38	0 U
week.group	OWNxx 5	0.818158E-01	0.818158E-01	1.36	0 U
week.aniID	OWNxx 5	1.24177	1.24177	10.72	0 U
week.aniID	OWNxx 5	-0.105572	-0.105572	-3.49	0 U
week.aniID	OWNxx 5	3.95421	3.95421	19.45	0 U
week.aniID	OWNxx 5	-0.691299	-0.691299	-5.55	0 U
week.ide(aniID)	OWNxx 5	0.574581E-02	0.574581E-02	0.05	0 U
week.ide(aniID)	OWNxx 5	0.123505E-01	0.123505E-01	0.44	0 U
week.ide(aniID)	OWNxx 5	3.03829	3.03829	31.14	0 U
week.ide(aniID)	OWNxx 5	0.203402	0.203402	7.93	0 U

Estimated correlation (variance on the diagonal) matrix for week.group:

	Week1	Week 2	Week 3	Week 4	Week 5
Week 1	10.72	-0.09	0.00	0.00	0.00
Week 2	-0.09	11.73	-0.03	0.00	0.00
Week 3	0.00	-0.03	12.64	0.03	0.00
Week 4	0.00	0.00	0.03	13.71	0.09
Week 5	0.00	0.00	0.00	0.09	14.99

Estimated correlation (variance on the diagonal) matrix for week.aniID

26.12	0.82	0.76	0.72	0.68
0.82	40.83	0.92	0.87	0.83
0.76	0.92	41.50	0.95	0.90
0.72	0.87	0.95	31.15	0.95
0.68	0.83	0.90	0.95	17.52

Estimated correlation (variance on the diagonal) matrix for week.ide(aniID)

25.58	0.03	0.00	0.00	0.00
0.03	31.37	0.04	0.00	0.00
0.00	0.04	38.47	0.05	0.00
0.00	0.00	0.05	47.20	0.06
0.00	0.00	0.00	0.06	57.92

### 5.1.1.2. Single trait analysis with correlated random effects

This time we include the social effects in the model.

- The model is:

$$ADG(t_i) = week_i + u_i + p_i + \sum_1^7 s_i + g_i$$

$p, g$  are the same as in the previous exemple.

$u, s$  are the correlated direct and social genetic effects for week i that both follows a SAD111, the cross antedependence is of degree 1.

$$u_1 = e_{u,1}, \quad u_i = (a_{u0} + a_{u1} * i)u_{i-1} + e_{u,i} \quad \text{and} \quad \sigma_{e_{u,i}}^2 = \exp(b_{u0} + b_{u1} * i)$$

$$s_1 = (\kappa_0 + \kappa_1 * i)u_1 + e_{s,1}, \quad s_i = (\kappa_0 + \kappa_1 * i)u_i + (a_{s0} + a_{s1} * i)s_{i-1} + e_{s,i} \quad \sigma_{e_{s,i}}^2 = \exp(b_{s0} + b_{s1} * i)$$

- The .as is:

See ex1.as, PATH 2

- The parafiles :

Parafiles are the same as in the previous example for p and g, for u and s it is “para4ex1” :

(to use them, rename to “para”, “para3”, “para4”)

- Results:

Source	Model terms	Gamma	Component	Comp/SE	% C
Residual	identity 4800	0.100000E-02	0.100000E-02	0.00	0 F
week.group	OWNxx 5	0.613686E-01	0.613686E-01	0.23	0 U
week.group	OWNxx 5	-0.147983E-01	-0.147983E-01	-0.20	0 U
week.group	OWNxx 5	2.13100	2.13100	8.19	0 U
week.group	OWNxx 5	0.833258E-01	0.833258E-01	1.04	0 U
week.aniID	OWNxx 10	1.40529	1.40529	10.96	0 U
week.aniID	OWNxx 10	-0.155471	-0.155471	-4.60	0 U
week.aniID	OWNxx 10	-1.99535	-1.99535	-5.32	0 U
week.aniID	OWNxx 10	0.726111	0.726111	5.43	0 U
week.aniID	OWNxx 10	-0.275731	-0.275731	-5.48	0 U
week.aniID	OWNxx 10	0.641934E-01	0.641934E-01	3.72	0 U
week.aniID	OWNxx 10	3.49594	3.49594	17.07	0 U
week.aniID	OWNxx 10	-0.530704	-0.530704	-4.67	0 U
week.aniID	OWNxx 10	-0.782813	-0.782813	-0.39	0 U
week.aniID	OWNxx 10	-0.945158	-0.945158	-0.84	0 U
week.ide(aniID)	OWNxx 5	0.344759E-01	0.344759E-01	0.30	0 U
week.ide(aniID)	OWNxx 5	0.314436E-02	0.314436E-02	0.11	0 U
week.ide(aniID)	OWNxx 5	2.98907	2.98907	29.25	0 U
week.ide(aniID)	OWNxx 5	0.210869	0.210869	7.90	0 U

Estimated correlation (variance on the diagonal) matrix week.group

9.16	0.03	0.00	0.00	0.00
0.03	9.96	0.02	0.00	0.00
0.00	0.02	10.82	0.00	0.00
0.00	0.00	0.00	11.76	-0.01
0.00	0.00	0.00	-0.01	12.78

Estimated genetic correlation (variance on the diagonal) matrix

Direct genetic effects					Social genetic effects				
Week	Week	Week	Week	Week	Week	Week	Week	Week	Week
1	2	3	4	5	1	2	3	4	5
19.39	0.82	0.74	0.68	0.62	-0.91	-0.32	-0.68	-0.68	-0.66
0.82	34.63	0.91	0.84	0.76	-0.75	-0.71	-0.89	-0.88	-0.87
0.74	0.91	37.24	0.92	0.84	-0.68	-0.64	-0.95	-0.95	-0.93
0.68	0.84	0.92	26.81	0.91	-0.62	-0.59	-0.88	-0.90	-0.86
0.62	0.76	0.84	0.91	12.90	-0.57	-0.54	-0.80	-0.82	-0.75
-0.91	-0.75	-0.68	-0.62	-0.57	1.05	0.14	0.59	0.60	0.58
-0.32	-0.71	-0.64	-0.59	-0.54	0.14	0.41	0.72	0.70	0.71
-0.68	-0.89	-0.95	-0.88	-0.80	0.59	0.72	0.37	0.98	0.98
-0.68	-0.88	-0.95	-0.90	-0.82	0.60	0.70	0.98	0.43	0.99
-0.66	-0.87	-0.93	-0.86	-0.75	0.58	0.71	0.98	0.99	0.88

Estimated pseudo permanent environmental correlation (variance on the diagonal) matrix

24.53	0.04	0.00	0.00	0.00
0.04	30.33	0.04	0.00	0.00
0.00	0.04	37.46	0.04	0.00
0.00	0.00	0.04	46.26	0.05
0.00	0.00	0.00	0.05	57.14

### 5.1.2. Ex2 exemple: multiple trait analysis

We consider 2 traits, 6 measurements per trait (time  $t_0$  to  $t_5$ : 1,2,3,4,5,10).note: simulated data without any subjective interpretation.

The datafile **dataex2** is as follow (factor: 10 levels):

animal ID	permanent ID	factor	week	week2	character	phenotype
35	1	2	1	1	1	2.1621369
35	1	5	2	2	1	-0.24649745
35	1	9	3	3	1	3.97515901
35	1	5	4	4	1	2.48136414
35	1	8	5	5	1	0.8077682
35	1	5	10	6	1	1.82469341
36	2	2	1	1	1	6.35244978
36	2	6	2	2	1	3.40466406
36	2	8	3	3	1	4.05081936
36	2	7	4	4	1	1.94876435
36	2	5	5	5	1	8.20519782
36	2	10	10	6	1	3.6706812
...	...	...	...	...	...	...
35	1	2	1	7	2	1.33369744
35	1	5	2	8	2	-1.05635543
35	1	9	3	9	2	1.32508293
35	1	5	4	10	2	1.58862245
35	1	8	5	11	2	3.75727085
35	1	5	10	12	2	7.28011657
36	2	2	1	7	2	3.68366083
36	2	6	2	8	2	2.79504194
36	2	8	3	9	2	0.7385331
36	2	7	4	10	2	2.52987967
36	2	5	5	11	2	1.07048596
36	2	10	10	12	2	-2.46401598

The pedigree is **ped2ex**

- The model is:

$$y_1(t_i) = factor_1 + u_{1i} + p_{1i}$$

$$y_2(t_i) = factor_2 + u_{2i} + p_{2i}$$

With :

SAD100 for  $u_1$  and  $u_2$ , a one way cross antedependence of order 1 with  $c=1$  and an initial correlation:

$$\begin{aligned}
\mathbf{u}_{1,t_0} = \mathbf{e}_{u1,0}, \quad \mathbf{u}_{1,i} &= (a_{u1})\mathbf{u}_{1,t_{i-1}} + (\kappa_{u1,0} + \kappa_{u1,1}t_i)\mathbf{u}_{2,t_{i-1}} + \mathbf{e}_{u1,i} & \sigma_{e_{u1,i}}^2 &= \exp(b_{u1}) \\
\mathbf{u}_{2,t_0} = \mathbf{e}_{u2,0}, \quad \mathbf{u}_{2,i} &= (a_{u2})\mathbf{u}_{2,t_{i-1}} + \mathbf{e}_{u2,i} & \text{with } \sigma_{e_{u2,i}}^2 &= \exp(b_{u2}) \\
& & \text{cov}(e_{u1,0}, e_{u2,0}) &= \text{constant}
\end{aligned}$$

7 parameters

SAD100 for  $p_1$ , SAD101 for  $p_2$ ,  $p_1$  and  $p_2$  are independent:

$$\begin{aligned}
\mathbf{p}_{1,t_0} = \mathbf{e}_{p1,0}, \quad \mathbf{p}_{1,i} &= (a_{p1})\mathbf{p}_{1,t_{i-1}} + \mathbf{e}_{p1,i} & \sigma_{e_{p1,i}}^2 &= \exp(b_{p1}) \\
\mathbf{p}_{2,t_0} = \mathbf{e}_{p2,0}, \quad \mathbf{p}_{2,i} &= (a_{p2})\mathbf{p}_{2,t_{i-1}} + \mathbf{e}_{p2,i} & \text{with } \sigma_{e_{p2,i}}^2 &= \exp(b_{p2,0} + b_{p2,1}t_i)
\end{aligned}$$

5 parameters

- The .as is:

See ex2.as

- The parafiles are:

See paraex2 para2ex2 (to use them, rename to para para2)

- Results:

Source	Model terms	Gamma	Component	Comp/SE	% C
Residual	identity	2160	0.100000E-02	0.100000E-02	0.00 0 F
week2.anilD	OWNxx	12	0.862148	0.862148	2.47 0 U
week2.anilD	OWNxx	12	0.632569	0.632569	4.32 0 U
week2.anilD	OWNxx	12	-1.19147	-1.19147	-1.75 0 U
week2.anilD	OWNxx	12	0.347955	0.347955	1.42 0 U
week2.anilD	OWNxx	12	-0.403212	-0.403212	-0.40 0 U
week2.anilD	OWNxx	12	-2.02764	-2.02764	-0.85 0 U
week2.anilD	OWNxx	12	3.35422	3.35422	1.55 0 U
week2.perm	OWNxx	12	0.491627	0.491627	13.98 0 U
week2.perm	OWNxx	12	0.312983	0.312983	9.92 0 U
week2.perm	OWNxx	12	3.02883	3.02883	58.12 0 U
week2.perm	OWNxx	12	4.08358	4.08358	50.14 0 U
week2.perm	OWNxx	12	-0.814920E-01	-0.814920E-01	-5.30 0 U

estimated genetic correlation (variance on the diagonal) matrix

Trait 1						Trait 2					
T0	t1	T2	T3	T4	T5	T0	T1	T2	T3	T4	T5
0.66	-0.62	-0.60	-0.36	-0.03	0.46	0.99	0.97	0.92	0.83	0.69	0.51
-0.62	1.13	0.79	0.59	0.32	-0.13	-0.63	-0.62	-0.59	-0.53	-0.44	-0.33
-0.60	0.79	1.87	0.78	0.48	-0.03	-0.61	-0.61	-0.58	-0.52	-0.43	-0.32
-0.36	0.59	0.78	1.79	0.74	0.25	-0.37	-0.37	-0.33	-0.30	-0.25	-0.18
-0.03	0.32	0.48	0.74	1.89	0.61	-0.03	-0.02	0.02	0.08	0.07	0.05
0.46	-0.13	-0.03	0.25	0.61	4.46	0.46	0.48	0.53	0.62	0.73	0.55
0.99	-0.63	-0.61	-0.37	-0.03	0.46	7.66	0.98	0.93	0.84	0.69	0.52
0.97	-0.62	-0.61	-0.37	-0.02	0.48	0.98	3.19	0.95	0.86	0.71	0.53
0.92	-0.59	-0.58	-0.33	0.02	0.53	0.93	0.95	1.41	0.90	0.74	0.55
0.83	-0.53	-0.52	-0.30	0.08	0.62	0.84	0.86	0.90	0.69	0.82	0.61
0.69	-0.44	-0.43	-0.25	0.07	0.73	0.69	0.71	0.74	0.82	0.41	0.74
0.51	-0.33	-0.32	-0.18	0.05	0.55	0.52	0.53	0.55	0.61	0.74	0.29

estimated pseudo-permanent correlation (variance on the diagonal) matrix

20.68	0.44	0.21	0.10	0.05	0.03	0.00	0.00	0.00	0.00	0.00	0.00
0.44	25.67	0.48	0.23	0.12	0.06	0.00	0.00	0.00	0.00	0.00	0.00
0.21	0.48	26.88	0.49	0.24	0.12	0.00	0.00	0.00	0.00	0.00	0.00
0.10	0.23	0.49	27.17	0.49	0.24	0.00	0.00	0.00	0.00	0.00	0.00
0.05	0.12	0.24	0.49	27.24	0.49	0.00	0.00	0.00	0.00	0.00	0.00
0.03	0.06	0.12	0.24	0.49	27.26	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	54.71	0.31	0.10	0.03	0.01	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.31	55.79	0.32	0.11	0.03	0.01
0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.32	51.95	0.33	0.11	0.04
0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.11	0.33	47.94	0.33	0.12
0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.11	0.33	44.19	0.38
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.12	0.38	30.61



## 5.2. Other examples

### 5.2.1. Single trait analysis, 5 time points ( $t_1, t_2, t_3, t_4, t_5$ )

$$\text{Model } y(t_j) = \mu(t_j) + u(t_j) + p(t_j)$$

- structure for genetic effects  $u$  (initial values in brackets)

SAD 1-10 (0.6 -0.1, 2)

total: 3 parameters

$$u(t_j) = (a_{u0} + a_{u1}t_j)u(t_{j-1}) + e_u(t_j), \quad \sigma_{e,j}^2 = \exp(b_{u0})$$

- structure for permanent effects

SAD 2-021 (0.66 0.7-0.11 0.01, 2.1 -0.1)

total: 6 parameters

$$p(t_j) = (a_{p,10})p(t_{j-1}) + (a_{p,20} + a_{p,21}t_j + a_{p,22}t_j^2)p(t_{j-2}) + e_p(t_j), \quad \sigma_{ep,j}^2 = \exp(b_{p0} + b_{p1}t_j)$$

- file.as, the model

(in blue: antedependence parameters, green: innovation variance parameters)

```
....
Phenotype ~ fixed effects !r time.animal time.ide(animal)
1 1 2
0 0 IDV 0.001 !GF !S2==1 #to fix residual variance to a very low value : 0.001
time.animal 2
5 time OWN3 0.6 -0.1 2 !TCCC
Animal
time.ide(animal) 2
5 time OWN6 0.66 0.7 -0.11 0.01 2 -0.1 !TCCCCC !F2
ide(animal)
```

- para (for the genetic effects) :

```
MODEL
SAD
ANTEDEP_ORDER #order of the antedependence  $\alpha$ 
1
MODEL_ANTEDEP #degree of the polynomial functions for the antedependence parameters  $\beta_1 \beta_2 \beta_3 \dots \beta_\alpha$ 
1
MODEL_INNOVATION_VARIANCE #degree of the polynomial function for the innovation variance  $\gamma$ 
0
DATA_FILE
Datafile
COLUMN_FACTOR
1
```

- Para2 (for the permanent effects) :

```
MODEL
SAD
ANTEDEP_ORDER #order of the antedependence  $\alpha$ 
2
```

**MODEL\_ANTEDEP** #degree of the polynomial functions for the antedependence parameters  $\beta_1 \beta_2 \beta_3 \dots \beta_\alpha$   
0 2  
**MODEL\_INNOVATION\_VARIANCE** #degree of the polynomial function for the innovation variance  $\gamma$   
1  
**DATA\_FILE**  
Datafile  
**COLUMN\_FACTOR**  
1

### 5.2.2. Multiple trait analysis: 2 traits, 5 time points

Model

$$y_1(t_j) = \mu_1(t_j) + u_1(t_j) + p_1(t_j)$$

$$y_2(t_j) = \mu_2(t_j) + u_2(t_j) + p_2(t_j)$$

- structure for genetic effects (initial values in brackets)

trait1: antedependence SAD 1-10 (0.6 -0.1 , 2)

trait2: antedependence SAD1-11 (0.66 -0.11, 1 -0.1)

(4 parameters antedependence, 3 parameters innovation variances)

crossantedependence 1->2: SAD1-0, starting point:2, initial correlation between error terms (2 parameters: 0.8, 0.5)

crossantedependence 2->1: nothing

Total: 9 parameters

$$u_1(t_j) = (a_{u1,0} + a_{u1,t_j})u_1(t_{j-1}) + e_{u1}(t_j) \quad \sigma_{eu1,j}^2 = \exp(b_{u1,0})$$

$$u_2(t_j) = (a_{u2,0} + a_{u2,t_j})u_2(t_{j-1}) + (\kappa_{u2,0})u_1(t_{j-2}) + e_{u2}(t_j) \quad \sigma_{eu2,j}^2 = \exp(b_{u2,0} + b_{u2,t_j})$$

And  $corr(e_{u1}(t_0), e_{u2}(t_0)) = \text{constant}$

- structure for permanent effects

trait1: antedependence SAD 1-21 (0.6 -0.1 0.01, 2 -0.1)

trait2: antedependence SAD1-21 (0.66 -0.11 0.11, 1 -0.11)

crossantedependence 1->2: SAD1-0, starting point:0 (0.8)

crossantedependence 2->1: nothing

total: 11 parameters

$$p_1(t_j) = (a_{p1,0} + a_{p1,t_j} + a_{p1,t_j^2})p_1(t_{j-1}) + e_{p1}(t_j) \quad \sigma_{ep1,j}^2 = \exp(b_{p1,0} + b_{p1,t_j})$$

$$p_2(t_j) = (a_{p2,0} + a_{p2,t_j} + a_{p2,t_j^2})p_2(t_{j-1}) + (\kappa_{p2,0})p_1(t_j) + e_{p2}(t_j) \quad \sigma_{ep2,j}^2 = \exp(b_{p2,0} + b_{p2,t_j})$$

- file.as, the model

(in blue: antedependence parameters, red:cross antedependence parameters, green: innovation variance parameters, black:initial correlation)

```
Phenotype ~ fixed effects !r time2.animal time2.ide(animal)
1 1 2
0 0 IDV 0.001 !GF !S2==1 #to fix residual variance to a very low value : 0.001
time2.animal 2
10 time2 OWN9 0.6 -0.1 0.66 -0.11 0.8 2 1 -0.1 0.5 !TCCCCCCCCC
Animal
```

```
time2.ide(animal) 2
10 time2 OWN11 0.6 -0.1 0.01 0.66 -0.11 0.11 0.8 2 -0.1 1 -0.11 !TCCCCCCCCCCC !F2
ide(animal)
```

- *Para (for the genetic effects)*

```
NBCHARACTER
2
MODEL
SAD
ANTEDEP_ORDER
1 1
CROSS_ ANTEDEP_ORDER
3 0
BEGIN_ CROSS_ ANTEDEP_ORDER
2 0
MODEL_ ANTEDEP
1 1
MODEL_ CROSS_ ANTEDEP
0
MODEL_ INNOVATION_ VARIANCE_ COVARIANCE
0 1
CORRINIT
1
DATA_ FILE #same as in .as
data
COLUMN_ FACTOR
1
```

- *para2 (for the permanent effects)*

```
NBCHARACTER
2
MODEL
SAD
ANTEDEP_ORDER
1 1
CROSS_ ANTEDEP_ORDER
1 0
BEGIN_ CROSS_ ANTEDEP_ORDER
0 0
MODEL_ ANTEDEP
2 2
MODEL_ CROSS_ ANTEDEP
0
MODEL_ INNOVATION_ VARIANCE_ COVARIANCE
1 1
CORRINIT
0
DATA_ FILE #same as in .as
data
COLUMN_ FACTOR
1
```

### 5.2.3. Multiple trait analysis: 3 traits, 10 time points

$$y_1(t_j) = \mu_1(t_j) + u_1(t_j) + p_1(t_j)$$

Model  $y_2(t_j) = \mu_2(t_j) + u_2(t_j) + p_2(t_j)$

$$y_3(t_j) = \mu_3(t_j) + u_3(t_j) + p_3(t_j)$$

**Note:** it is a very complex model to provide an example; it may not converge.

- structure for genetic effects (initial values in brackets)

trait1: antedependence SAD 1-10 (0.6 -0.1 , 2)

trait2: antedependence SAD1-11 (0.66 -0.11, 1 -0.1)

trait3:antedependence: SAD2-100 (0.7 -0.1, 0.5, 2.1)

(7 parameters antedependence, 4 parameters innovation variances)

crossantedependence trait 1->2: no relationship, no initial correlation between error terms

crossantedependence trait 2->1: no relationship

crossantedependence traits 1->3: recursive relationship (trait1 has an effect on trait3 but not the reverse) of order 1, starting point:  $c'=0$ , degree 1. no initial correlation between error terms (0.8 -0.2)

(2 parameters)

crossantedependence traits 3->1: no relationship

cross antedependence traits 2->3 no relationship, no initial correlation between error terms

cross antedependence traits 3->2 no relationship, no initial correlation between error terms

total: 13 parameters

$$u_1(t_j) = (a_{u1,0} + a_{u1,1}t_j)u_1(t_{j-1}) + e_{u1}(t_j),$$

$$u_2(t_j) = (a_{u2,0} + a_{u2,1}t_j)u_2(t_{j-1}) + e_{u2}(t_j),$$

$$u_3(t_j) = (a_{u3,10} + a_{u3,11}t_j)u_3(t_{j-1}) + (a_{u3,20})u_3(t_{j-2}) + (\kappa_{u3,0} + \kappa_{u3,1}t_j)u_1(t_j) + e_{u3}(t_j),$$

$$\sigma_{eu1,j}^2 = \exp(b_{u1,0})$$

$$\sigma_{eu2,j}^2 = \exp(b_{u2,0} + b_{u2,1}t_j)$$

$$\sigma_{eu3,j}^2 = \exp(b_{u3,0})$$

- structure for permanent effects

trait1: antedependence SAD 1-21 (0.6 -0.1 0.01, 2 -0.1)

trait2: antedependence SAD1-21 (0.66 -0.11 0.11, 1 -0.11)

trait3:antedependence: SAD1-11 (0.7 -0.2, 2 -0.2)

crossantedependence 1->2: SAD1-0, starting point:0 (0.8)

crossantedependence 2->1: -

crossantependence 1->3: SAD1-0, starting point:1, initial correlation between error terms (0.3, 0.5)

crossantependence 3->1: SAD1-0, starting point:1 (0.2)

crossantependence 2->3: SAD1-1, starting point:0 (0.5 -0.2)

crossantependence 3->2: SAD1-1, starting point:0 (0.6 -0.1)

total: 22 parameters

$$p_1(t_j) = (a_{p1,0} + a_{p1,1}t_j + a_{p1,1}t_j^2)p_1(t_{j-1}) + (\omega_{p1,0})p_3(t_{j-1}) + e_{p1}(t_j)$$

$$p_2(t_j) = (a_{p2,0} + a_{p2,1}t_j + a_{p2,1}t_j^2)p_2(t_{j-1}) + (\omega_{p2,0})p_1(t_j) + (\omega'_{p2,0} + \omega'_{p2,1}t_j)p_3(t_j) + e_{p2}(t_j)$$

$$p_3(t_j) = (a_{p3,0} + a_{p3,1}t_j)p_3(t_{j-1}) + (\omega_{p3,0})p_1(t_{j-1}) + (\omega'_{p3,0} + \omega'_{p3,1}t_j)p_2(t_j) + e_{p3}(t_j)$$

$$\sigma_{ep1,j}^2 = \exp(b_{p1,0} + b_{p1,1}t_j)$$

$$\sigma_{ep2,j}^2 = \exp(b_{p2,0} + b_{p2,1}t_j)$$

$$\sigma_{ep3,j}^2 = \exp(b_{p3,0} + b_{p3,1}t_j)$$

$$\text{corr}(e_{p1}(t_0), e_{p3}(t_0)) = \text{constant}$$

o file.as, the model

(in blue: antependence parameters, red:cross antependence parameters, green: innovation variance parameters, black:initial correlation)

```
Phenotype ~ fixed effects !r time2.animal time2.ide(animal)
1 1 2
0 0 IDV 0.00001 !GF !S2==1 #to fix residual variance to a very low value : 0.00001
time2.animal 2
30 time2 OWN13 0.6 -0.1 0.66 -0.11 0.7 -0.1 0.5 0.8 -0.2 2 1 -0.1 2.1 !TCCCCCCCCCCCC
Animal
time2.ide(animal) 2
30 time2 OWN22 0.6 -0.1 0.01 0.66 -0.11 0.11 0.7 -0.2 0.8 0.3 0.2 0.5 -0.2 0.6 -0.1 2 -0.1 1 -0.11 2 -0.2
0.5 !TCCCCCCCCCCCC !F2
ide(animal)
```

o Para (for the genetic effects)

```
NBCHARACTER
3
MODEL
SAD
ANTEDEP_ORDER
1 1 2
CROSS_ANTEDEP_ORDER
0 0 1 0 0
BEGIN_CROSS_ANTEDEP_ORDER
0 0 0 0 0
MODEL_ANTEDEP
1 1 1 0
MODEL_CROSS_ANTEDEP
1
```

```
MODEL_INNOVATION_VARIANCE_COVARIANCE
0 1 0
CORRINIT
0 0 0
DATA_FILE #same as in .as
data
COLUMN_FACTOR
1
```

- *para2 (for the permanent effects)*

```
NBCHARACTER
3
MODEL
SAD
ANTEDEP_ORDER
1 1 1
CROSS_ANTEDEP_ORDER
1 0 2 2 1 1
BEGIN_CROSS_ANTEDEP_ORDER
0 0 1 1 0 0
MODEL_ANTEDEP
2 2 1
MODEL_CROSS_ANTEDEP
0 0 0 1 1
MODEL_INNOVATION_VARIANCE_COVARIANCE
1 1 1
CORRINIT
0 1 0
DATA_FILE #same as in .as
data
COLUMN_FACTOR
1
```