Jack O'Lanterns and integrating spheres: Halloween physics

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Jack O'Lanterns and integrating spheres: Halloween physics

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Although photometry, the measurement of the intensity and distribution of visible light, is important in many areas, most undergraduate physics courses do not include this topic. We present a simple introduction to key concepts in photometry, and as a fun example, we investigate the optics of a Jack O'Lantern.
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I. INTRODUCTION

Our everyday world is filled with light sources and surfaces that redirect light. Our visual perception system has evolved to characterize the nature of such surfaces, which is important to our survival. The measurement of the intensity and distribution of light that is visible to the human eye is broadly termed photometry.

Given its importance, it is surprising that most undergraduate physics students learn nothing about photometry. For example, few students are aware that the terms luminance and illuminance have distinct precise meanings that are as different as momentum and velocity. We present here a conceptually simple introduction to the subject of photometry and, as a fun example, we look at the optics of the inside of a pumpkin, and explain why a pumpkin is ideal for making a Jack O'Lantern.

II. REVIEW OF BASIC PHOTOMETRIC CONCEPTS

We begin with a brief review in which photometry is presented as an important subset of radiometry. Both radiometry and photometry involve measurements of electromagnetic radiation.[1,2] Radiometry and photometry apply to a very common situation which might be called a dynamic steady state, in which the rates of energy flow are constant over time intervals that are longer than the typical field oscillation period and are long enough to be readily measured. In such a situation, the key concept is the rate of energy flow, or power. Sources of electromagnetic energy produce radiation at a fixed rate, which spreads out spatially, such that at any point the time average of the flux is a constant. Radiometric and photometric measurements allow us to quantitatively describe the way in which the radiation propagates.

Students are often confused by the terminology for describing how the radiation can spread out. It is helpful to begin by conceptually considering a plane through which a unique well-defined flow of energy propagates, and to picture the radiant power passing through a very small region of interest on this plane. It is physically impossible for radiant power to be concentrated entirely at one precise point, one precise direction, or at one precise wavelength. Instead, radiant power must always be distributed to some extent in three possible ways: spatially (over area), directionally (over solid angle), and spectrally (over wavelength). Radiometric and photometric measurements describe how the radiation is distributed in these three ways.

A. Radiometric quantities

To describe the key radiometric quantities, we will temporarily ignore the spectral distribution of the radiation, so that we consider radiation that is distributed only over area and solid angle.

There are four key radiometric quantities: Radiant flux, radiant intensity, irradiance, and radiance. To explain these quantities, we start with the conceptual plane through which radiation from a source passes. The radiant flux describes the total power of the radiation from the source regardless of where it strikes the plane, or at what angle it strikes it. This quantity has unit of Watts (W) and is typically denoted by the symbol Φ.

The solid angle is most easily described as the range of angles which, for an observer positioned at the center of a sphere, will project on a given area on the surface of that sphere. The value of the solid angle is equal to the size of that area of interest divided by the square of the radius of the sphere, and its units are given as steradians (sr).

The radiant intensity describes how the radiation is distributed over a solid angle in a particular direction. Radiant intensity has units of Watts/steradian (W/sr). In contrast, the term irradiance describes how the radiation spreads out over an area, where the area is measured in units of Watts/square meter on the notional plane.

To help illustrate these two quantities, consider the case where perfectly collimated radiation passes through the plane, as shown in Fig. 1. In this case, the radiation has an infinite radiant intensity, because it occupies essentially zero solid angle, but it has a finite and well-defined irradiance on the plane. (This situation is depicted for illustrative purposes only; even highly collimated light has a finite divergence, so infinite radiant intensity cannot be achieved in practice.)

In contrast, consider the case where the radiation passes through a single infinitesimally small point on the plane, but within a well-defined solid angle, as shown in Fig. 2. In this case, the radiation has an infinite irradiance, because the area is essentially zero, but it has a finite and well-defined radiant intensity. (Again, this situation is intended for illustrative purposes only, because in a real system the area cannot be equal to zero.)

Radiant intensity is a useful idea because it describes the irradiance that will occur after the light has propagated a specific distance, as depicted in Fig. 3. If the radiant intensity I at a distance d is measured in the perpendicular plane, the irradiance E is given by the inverse square law:

$$E = \frac{I}{d^2}. \tag{1}$$

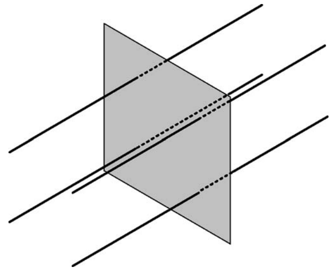

Fig. 1. Collimated radiation passing through a plane.

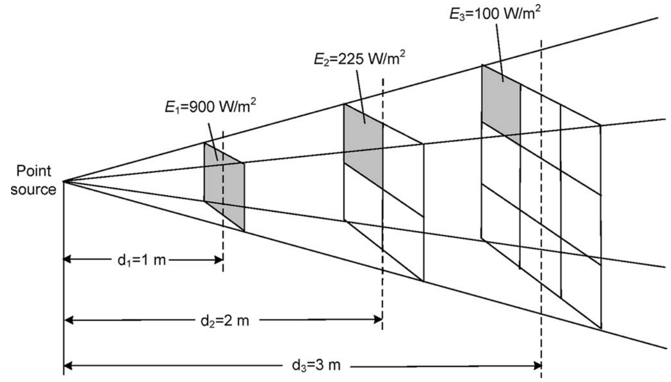

Fig. 3. Irradiance determined by inverse square law.

Finally, consider the case where the radiation passes through the plane over a well-defined area, and also with a well-defined solid angle, as shown in Fig. 4. The radiance L describes this distribution over *both* solid angle and area. This quantity has units of Watts/square meter/steradian and is probably the least intuitive of the four, but in many situations it is the most useful. As will be explained in more detail, the radiance is useful because its value, defined in the plane perpendicular to a ray, remains constant along the ray path, even if it passes through lenses or other nonlossy optical elements that change either the spatial or angular characteristics of the radiation.

It is a well-known (and often frustrating) fact that it is not possible to decrease both the spatial and angular spread of the radiation; decreasing one will automatically increase the other. For instance, it is often desirable to focus the radiation to a small point in space, which would require increasing the solid angle over which the radiation is distributed. Conversely, to use a collimating lens to decrease the solid angle of the radiation, it is first necessary to allow the radiation to spread out spatially so the lens can work effectively. Regardless of how the manner of propagation is manipulated, the radiance of the radiation remains the same, unless it is reduced by lossy optical elements that introduce absorption and/or scattering. A common way of describing this relationship between spatial and angular spread is to note that for a beam of light, the product of the solid angle and the cross-sectional area of the beam is a constant. This product is known as the étendue and is an important characteristic of an optical system.

This situation is analogous to the uncertainty principle in quantum mechanics, which requires that at any given time, either the position or momentum of a particle can be precisely measured, but it not possible to precisely measure both. Similarly, according to the Fourier analysis of a pulse of radiation, it is impossible to have a pulse that is both narrowband in frequency and short in duration.

B. Photometric quantities

To understand the relationship between photometry and radiometry, it is necessary to consider the spectral, or wavelength, characteristics of the radiation. In particular, it is often useful to describe the amount of radiation that lies within a certain band of wavelengths, for example between approximately 400 and 700 nm, for which radiation is visible to the human eye. The perceived brightness varies across this visible band, and is described by the photopic luminous efficiency function $V(\lambda)$ shown in Fig. 5. This curve is a psychophysical quantity—it cannot be determined by objective measurements alone and instead requires comparative subjective assessments reported by human observers.

For any spectral distribution of a radiometric quantity, the equivalent photometric quantity can be obtained by multiplying the radiometric quantity at each wavelength by the value of the photopic curve $V(\lambda)$ at that wavelength and integrating the resulting product function. (A normalization factor is also required, as we will describe.)

As an example, consider the situation where the radiant flux as a function of wavelength is known. This value can be converted to luminous flux and the corresponding unit could be termed an "effective visible watt." Instead, we use a different unit, with a different size, known as the "lumen." So what is a lumen and why do we use it? The answer is purely historical. When the intensity of light was first quantified, it was not recognized as electromagnetic radiation, so there was little incentive to link it to measurements of electromagnetic energy. Instead, a "standard candle" was used as the standard light source, which led to the first definition of the lumen as $1/4\pi$ times the amount of light radiated from a source which has, in all directions, the luminous intensity of a 1 cm^2 area of molten platinum, measured in the perpen-

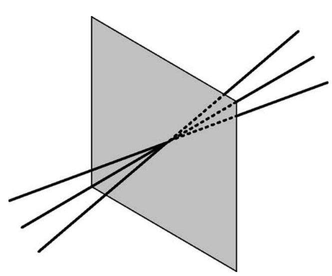

Fig. 2. Radiation passing through a small point on a plane.

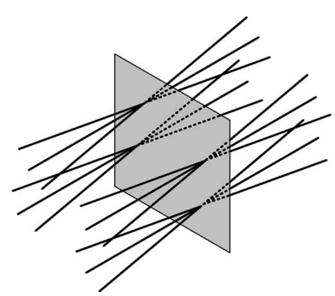

Fig. 4. Radiation passing through a plane with certain area and solid angle.

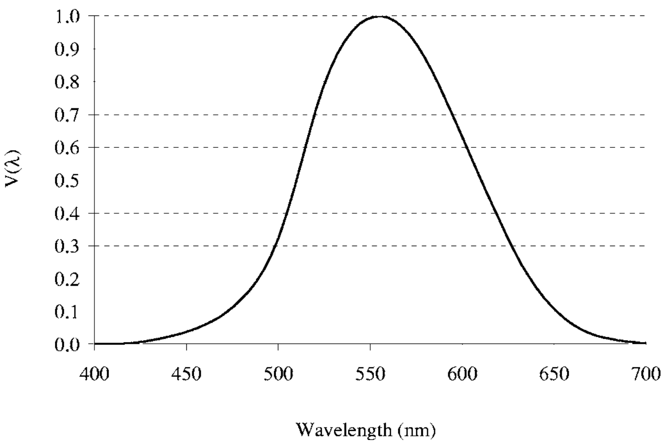

Fig. 5. Photopic luminous efficiency function, $V(\lambda)$.

dicular direction, at its melting point temperature, under 1 atmosphere of air pressure. This well defined, but arbitrary, choice of unit led to the definition of units for the other photometric quantities. A complicated and long story leads to definition that the conventional unit of luminous flux, the lumen, is equivalent of exactly 1/683 effective visible Watts. Another way to say the same thing is that for light at the wavelength of peak sensitivity of the human eye (defined to be light at a frequency of 5.4×10^{14} Hz), 1 W of radiant flux is equivalent to 683 lumens.

The system of photometric units involves the same kind of descriptions as the radiant power based units, that is, radiant flux, irradiance, radiant intensity, and radiance, using the lumen instead the watt. Illuminance is the analog of irradiance, luminous intensity is the analog of radiant intensity, and luminance is the analog of radiance. The unit of illuminance is lumens/square meter (also known as lux), the unit of luminous intensity is lumens/steradian (also known as a candela, cd) and the unit of luminance is lumen/steradian/square meter or candela/square meter (cd/m^2).

As with radiance, luminance is an abstract concept, but it has an extremely important connection to perception, in that the human eye evaluates the luminance of a surface when it looks at it. Luminance is constant along a ray path, and for this reason the apparent brightness of a surface is unrelated to its distance from the eye. We can say that the perceived phenomenon of brightness is closely linked to the precisely defined photometric property of luminance. Although brightness, being a perceived experience, cannot be accurately quantified, it is generally observed that equal luminance corresponds to approximately equal perceived brightness and increased luminance corresponds to increased perceived brightness. Table I provides a summary of the radiometric and photometric terms described above.

C. The integrating sphere

In photometry experiments, it is common to measure the total luminous flux of a light source by measuring the luminous intensity at many different angles and summing over all angles to determine the total flux.[3–5] To achieve accuracy such measurement procedures are extremely time consuming, generate large amounts of data, and require a sophisticated data collection system. An alternate approach that requires only a single measurement is often used. This approach uses a spherical structure known as an integrating sphere[6–8] (also known as an Ulbricht sphere), as depicted in Fig. 6.

The inner surface of the structure is a highly reflective, white material that causes multiple reflections of light within the sphere so that the inner surface is very uniformly illuminated. The resulting luminance of the inner wall of the sphere is measured by viewing with an appropriate luminance detector through a small hole in the side of the sphere, typically called the exit port. To calibrate the device a calibration lamp of known luminous flux is placed in the sphere and the ratio of the detector response to the lamp output is determined in several spectral bands. These data can then be used to calibrate a measurement of the radiant flux within each wavelength band for a test lamp placed in the structure. In this way the luminance measurement at a single position inside the sphere is known to be accurately proportional to the flux emitted from the lamp.

If the inner surface is highly reflective over a wide range of wavelengths, the integrating sphere is a very forgiving device because the light will reflect many times so that the inner surface of the sphere achieves highly uniform luminance regardless of the exact shape of the structure, minor defects in the surface, or the size and distribution characteristics of the source. A typical integrating sphere has a reflectance of greater than 0.8 compared to a perfect reflector. This reflectance value is about the same as white paper. In contrast, black ink printed on a page has a reflectance of about 0.05.

The fraction of the integrating sphere surface occupied by the exit port is denoted by f:

Table I. Summary of radiometric and photometric terms.

Radiation spreading over	Radiometric term Symbol (units)	Photometric term Symbol (units)
Nothing	Radiant flux ϕ (W)	Luminous flux ϕ_v (lm=1/683 effective visible W)
Solid angle	Radiant intensity I (W/sr)	Luminous intensity I_v (lm/sr=cd)
Area	Irradiance E (W/m^2)	Illuminance E_v (lm/m^2=lux)
Solid angle and area	Radiance L ($W/m^2/sr$)	Luminance L_v ($lm/m^2/sr=cd/m^2$)

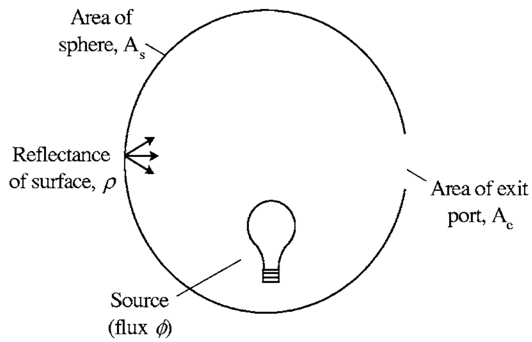

Fig. 6. An integrating sphere.

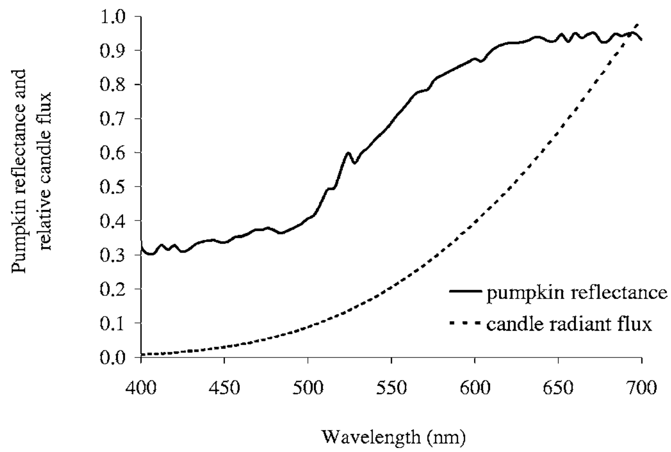

Fig. 7. Reflectance inside pumpkin and relative radiant flux of candle.

$$f = \frac{A_e}{A_s}, \tag{2}$$

where A_e is the area of the exit port and A_s is the total surface area of the sphere. The luminous intensity at the exit port can be predicted from the sphere multiplier, M, which is determined by the surface reflectance ρ and the port fraction f^2.

$$M = \frac{\rho}{1 - \rho(1-f)}. \tag{3}$$

This multiplier quantifies the expected increase in luminance at the exit port as a result of the multiple reflections within the sphere. If the exit port is small, for example, $f \leqslant 0.05$, and the surface reflectance is high, the value of this multiplier can be very high, often greater than 20.

Alternatively, the output efficiency η of the sphere as

$$\eta = fM. \tag{4}$$

The total luminous flux diffusely reflected into the sphere, ϕ_s, can be quantified by knowing the spectral radiant flux of the source, $\phi_{\text{radiant}}(\lambda)$, and the spectral reflectance of the surface $\rho(\lambda)$ from:

$$\phi_s = \sum_{\lambda=400}^{\lambda=700} \phi_{\text{radiant}}(\lambda)\rho(\lambda)V(\lambda). \tag{5}$$

The total luminous flux that leaves the exit port is then given by:

$$\phi_e = \eta\phi_s. \tag{6}$$

The luminance at the exit port L_e can be determined by the relation for diffuse illumination,

$$L_e = \frac{\phi_e}{\pi A_e}, \tag{7}$$

which is based on the effective solid angle over which the diffuse light is distributed being π steradians. [The value of π arises from integration of the $\cos(\theta)$ dependence of the irradiance per unit area of a diffuse emitter over the half sphere of emitted directions.]

It is interesting to compare this luminance to what would be expected in the absence of a highly reflective inner surface, in which case the luminance of the exit port would be

$$L = \frac{\phi_s f}{\pi A_s}. \tag{8}$$

Equation (8) results from the fact that in the absence of a highly reflective inner surface, the input flux is spread over the entire sphere A_s, but only the flux that strikes the small exit port f will leave the sphere. Multiple reflections result in a value of L_e that is much greater than would otherwise be the case.

III. PUMPKIN PHOTOMETRICS

Consider the Halloween Jack O'Lantern, which has many of the same features as an integrating sphere—a diffusely reflective interior, a light source, and an exit aperture. To determine a Jack O'Lantern's efficiency, the reflectance of the inner surface of a pumpkin was measured using a spectrophotometer to quantify the spectral radiance of a sample of pumpkin interior under diffuse illumination with an incandescent source, compared to the same spectral radiance measurements for a calibrated diffuse white reflectance standard. The reflectance of the pumpkin was calculated from the ratio of the pumpkin radiance to that of the standard. The result is shown by the solid line in Fig. 7. The reflectance varies smoothly as a function of wavelength and is surprisingly high for wavelengths greater than 600 nm, which explains the bright orange color of the pumpkin's interior.

The spectral distribution of a typical candle also was measured using the spectrophotometer. Because the candle is an incandescent source, the relative flux also varies smoothly with wavelength. The relative radiant flux of the candle (normalized so that the value at 700 nm is equal to 1) is shown as the dotted line in Fig. 7. The relatively strong contributions from wavelengths greater than 600 nm give candlelight and other common incandescent light sources their warm visual appearance; this appearance is seen not only for illuminating a Jack O'Lantern, but for lighting in general, because the slightly reddish light is known to give a more pleasant color rendering of skin.

As an example of using the relations in Table I to determine the photometric quantities for this candle, we assume that the irradiance at a distance of 1 m from the candle is 5 mW/m^2 in a 10 nm wide wavelength band centered on 555 nm, the peak of the visible band. If we multiply by 4π

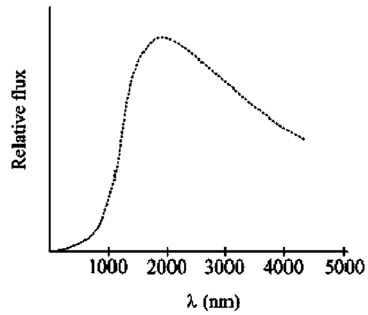

Fig. 8. Emission spectrum of a candle due to blackbody radiation.

Fig. 10. Comparison of a Jack O'Lantern with normal pumpkin reflectance (left) to one with an interior reflectance of 0.1 (right).

steradians, the solid angle of the full sphere over which the candle emits light, we obtain a total radiant flux of about 63 mW in this band.

The radiant flux of the candle shown in Fig. 7 represents only a small part of the emitted radiation. A candle, like all other objects, has a very special property that it emits electromagnetic radiation with a distribution of wavelengths that depends on its temperature. Only a fraction of the emitted radiation is at the visible wavelengths as shown by the solid portion of the curve in Fig. 8; the complete emission spectrum is depicted by the dotted line.

We now employ these measured values to determine the output efficiency of a typical Jack O'Lantern.

A. Understanding and quantifying a Jack O'Lantern

A typical pumpkin has an inner diameter of approximately 0.3 m, resulting in a surface area $A_s = 0.28\ \mathrm{m}^2$. For the following calculations, we assume that the face of the Jack O'Lantern, which represents the exit ports of the integrating sphere, occupies an area fraction $f = 0.05$. The efficiency was calculated at each wavelength using Eqs. (3) and (4) and the measured spectral reflectance function for the pumpkin, as shown in Fig. 9. As expected, the pumpkin demonstrates the highest efficiency for wavelengths greater than 600 nm.

By using Eq. (5) and (6) and assuming a typical candle luminous flux of 10 lumens, the total luminous flux that leaves the Jack O'Lantern face is calculated to be 2.54 lumens. It follows from Eq. (7) that the luminance of the face is approximately 60 $\mathrm{cd/m}^2$. This luminance level is comparable to that of an image produced by a television set; a picture of an orange, for example, displayed on television has the same luminance value as a typical Jack O'Lantern. A harvest moon at moonrise has approximately this luminance value.

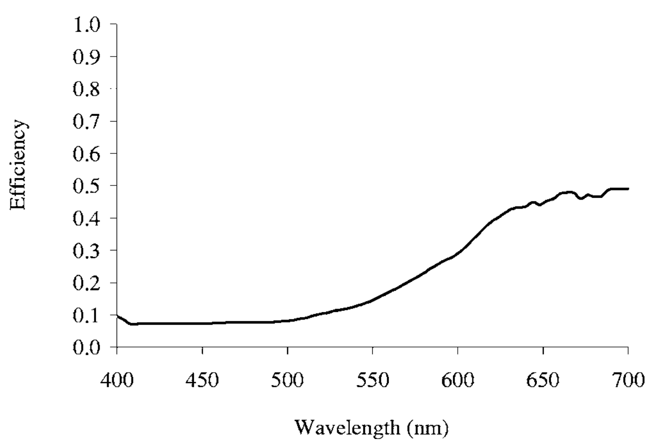

Fig. 9. Efficiency of pumpkin as an integrating sphere.

The effect of different exit port sizes can be demonstrated by removing the lid of the Jack O'Lantern and observing the corresponding reduction in the luminance of the face.

If the inside of the pumpkin were absorptive instead of reflective, for instance with values of $\rho(\lambda)$ of about 0.1, Eq. (8) would yield a luminance value of only 0.6 $\mathrm{cd/m}^2$, two orders of magnitude less than the normal Jack O'Lantern, as illustrated in the sketch in Fig. 10. With such a low reflectance interior, it would be difficult to see the glowing features even in pitch dark, whereas the usual Jack O'Lantern is clearly visible in even partial daylight and exhibits a very dramatic contrast at night.

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