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Mass, Momentum and Energy of Gravitational Field

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ABSTRACT: The energy of the gravitational field and the mass related to it are calculated. The momentum of the gravitational field of a moving body and the appropriate mass of the field are determined. Comparison of the given masses shows their difference. The reasons of violation of relativity and equivalence principles are discussed.

KEY WORDS: Gravitation; Field Energy; Mass of the Field.

According to the General Theory of Relativity (GTR), energy of all kinds contributes to gravitational mass of a body. The density of gravitational energy in the Lorentz-invariant Theory of Gravity (LITG) according to [1], [2] is equal:

$$u=-\frac{1}{8\pi G}\left(\Gamma^2+c_g^2\,\Omega^2\right), \tag{1}$$

where G – gravitational constant,
$\mathbf{\Gamma}$ – gravitational field strength,
c_g – the speed of gravitation propagation,
$\mathbf{\Omega}$ – gravitational torsion.

Let's find gravitational energy U for a rest round body, when $\Omega=0$. With homogeneous matter density ρ for the gravitational field strength inside and outside of the body it is possible to write down:

$$\Gamma_i=-\frac{4\pi G\rho r}{3}, \qquad \Gamma_o=-\frac{GM}{r^2},$$

where r – current radius,
M – mass of the body.

We substitute the strengths in (1) and then integrate by volume:

$$U=\int u\,dV=-\int_0^R\frac{2\pi G\rho^2 r^2\,dV}{9}-\int_R^\infty\frac{GM^2\,dV}{8\pi r^4}=-\frac{0.6\,GM^2}{R}, \tag{2}$$

here R – radius of the body.

In (2) the basic contribution to gravitational energy U is given by the energy of the field outside the body.

Owing to interrelation between mass and energy we should expect, that negative mass corresponds there to energy (2):

$$m_G = \frac{U}{c^2}, \tag{3}$$

where c – speed of light.

Due to mass m_G there should be reduction of the gravitational mass of the body in relation to initial mass M.

Let's consider now a case when the body is moving with constant speed v along axis *Z*. As far as the body is moving, that is why $\Omega \neq 0$ and there is the vector of momentum density of the gravitational field which is not equal to zero:

$$\boldsymbol{g} = -\frac{1}{4\pi G}[\mathbf{\Gamma} \times \mathbf{\Omega}]. \tag{4}$$

It is convenient to find $\mathbf{\Gamma}$ and $\mathbf{\Omega}$ through scalar ψ and vector $\mathbf{D}$ potentials of the gravitational field. In LITG it is accepted, that:

$$\mathbf{\Gamma} = -\nabla\psi - \frac{\partial \mathbf{D}}{\partial t}, \qquad \mathbf{\Omega} = \nabla \times \mathbf{D}. \tag{5}$$

In their turn, potentials of the field outside the body are set taking into account the delay of gravitational influence and consequently have the Lorentz-invariant form:

$$\psi_o = -\frac{GM}{\sqrt{1-\mathrm{v}^2/c_g^2}\sqrt{x^2+y^2+\frac{(z-\mathrm{v}t)^2}{1-\mathrm{v}^2/c_g^2}}}, \qquad \mathbf{D}_o = \frac{\psi_o \mathbf{v}}{c_g^2}. \tag{6}$$

Substituting (6) in (5), we find:

$$\Gamma_x = -\frac{GMx}{\sqrt{1-\mathrm{v}^2/c_g^2}\left(x^2+y^2+\frac{(z-\mathrm{v}t)^2}{1-\mathrm{v}^2/c_g^2}\right)^{1,5}}, \quad \Gamma_y = -\frac{GMy}{\sqrt{1-\mathrm{v}^2/c_g^2}\left(x^2+y^2+\frac{(z-\mathrm{v}t)^2}{1-\mathrm{v}^2/c_g^2}\right)^{1,5}},$$

$$\Gamma_z = -\frac{GM(z-\mathrm{v}t)}{\sqrt{1-\mathrm{v}^2/c_g^2}\left(x^2+y^2+\frac{(z-\mathrm{v}t)^2}{1-\mathrm{v}^2/c_g^2}\right)^{1,5}}, \quad \Omega_x = -\frac{\mathrm{v}\Gamma_y}{c_g^2}, \quad \Omega_y = \frac{\mathrm{v}\Gamma_x}{c_g^2}, \quad \Omega_z = 0.$$

It is evident, that $\mathbf{\Omega} = \frac{[\mathbf{v} \times \mathbf{\Gamma}]}{c_g^2}$. From all components of the vector $\boldsymbol{g}$ from (4) only the component directed along axis Z is important:

$$g_{zo} = -\frac{1}{4\pi G}\left(\Gamma_x \Omega_y - \Gamma_y \Omega_x\right) = -\frac{\mathrm{v}}{4\pi G c_g^2}\left(\Gamma_x^2 + \Gamma_y^2\right). \tag{7}$$

Let's integrate g_{zo} by all points of the field in space outside the body at the moment of time $t=0$. We should consider the speed v small in order to neglect the Lorentz's factor $1/\sqrt{1-\mathrm{v}^2/c_g^2}$. As the round body is going, it seems flattened in the direction of movement and turns into ellipsoid. In case of small speeds this change of the form can be neglected. It is convenient to use spherical coordinates:

$$x = r\sin Q\cos\varphi, \qquad y = r\sin Q\sin\varphi, \qquad z = r\cos Q.$$

Then for the field momentum outside the body we have:

$$p_{zo} = \int g_{zo}\,dV = -\int_R^\infty \frac{\mathrm{v}GM^2\sin^2 Q\,dV}{4\pi c_g^2 r^4} = -\frac{2\mathrm{v}GM^2}{3c_g^2 R}. \tag{8}$$

Inside the body within the limits of small speed the result is the following:

$$\mathbf{\Gamma}_{\boldsymbol{i}} = -\frac{4\pi G\rho\mathbf{r}}{3}, \qquad \psi_i = -\frac{2\pi G\rho\left(r^2-3R^2\right)}{3}, \qquad \mathbf{D}_{\boldsymbol{i}} = \frac{\psi_i\,\mathbf{v}}{c_g^2}.$$

The component of vector of momentum density of gravitational field inside the body is calculated similarly (7). For the total momentum of the field inside the body we get:

$$p_{zi} = \int g_{zi}\,dV = -\int_0^R \frac{\mathrm{v}4\pi G\rho^2 r^2\sin^2 Q\,dV}{9c_g^2} = -\frac{2\mathrm{v}GM^2}{15c_g^2 R}. \tag{9}$$

The sum of momentums of the field inside and outside the body with the account (2) and (3) equals to:

$$p_z = p_{zo} + p_{zi} = -\frac{4\mathrm{v}GM^2}{5c_g^2 R} = \frac{4\mathrm{v}U}{3c_g^2} = \frac{4m_G\mathrm{v}}{3}. \tag{10}$$

if $c_g = c$.

The factor before the speed v in (10) it is natural to treat as the mass of the moving gravitational field related to the body:

$$m_f = \frac{4m_G}{3}. \tag{11}$$

The mass of the field m_f by absolute value appears to be more, than mass m_G of the gravitational field of a motionless body.

DISCUSSION

As the mass of the field m_f is included into momentum of the gravitational field it can be considered inertial mass. The mass of the field m_G is connected with the energy of motionless potential gravitational field and is related to gravitational mass of the body. The inequality of masses m_f and m_G means inapplicability of the principle of equivalence of inertial and gravitational masses with respect to mass-energy of the gravitational field.

On the other hand, we find out also breaking of the traditional relativity principle. Really, while the observer is motionless towards the body, he fixes potential energy of the gravitational field and correspondingly the mass of the field m_G. As soon as the observer starts to move against the body, he observes the changed mass of the body owing to the presence of the momentum of the body in his reference system. Thereby the effective mass of the body depends smoothly on the speed of movement of the observer against the body, taking into account the Lorentz's factor. But this does not apply to the mass m_f – it at once increases in 4/3 times in relation to m_G.

All aforesaid can be repeated also concerning mass-energy of the electromagnetic field for the body having an electric charge. Maybe equivalence and relativity principles should be fulfilled only for the total mass-energy of the body including the energy of rest of its constituent particles and binding energy of fields? But then we should refuse from the superposition principle of field potentials and its intensities, from the possibility of independent addition of energies of various types and the masses corresponding to them.

We obtained the inequality of masses m_f and m_G on the basis of the theory LITG. The equations of gravitational field in the theory are the following:

$$\nabla \cdot \mathbf{\Gamma} = -4\pi G\rho, \qquad \nabla \cdot \mathbf{\Omega} = 0, \tag{12}$$

$$\nabla \times \mathbf{\Gamma} = -\frac{\partial \mathbf{\Omega}}{\partial t}, \qquad c_g^2\, \nabla \times \mathbf{\Omega} = -4\pi G\,\mathbf{J} + \frac{\partial \mathbf{\Gamma}}{\partial t},$$

where $\mathbf{\Gamma}$ – the vector of gravitational field strength,

$\mathbf{\Omega}$ – the solenoidal vector of gravitational torsion or simply torsion,

G – the gravitational constant,

ρ – the mass density,

$\mathbf{J} = \rho \mathbf{V}$ – the vector of density of the mass current, dependent on the speed of motion $\mathbf{V}$ of the mass element.

The equations (12) to within signs coincide with Maxwell equations for an electromagnetic field. The density of energy of the field in electromagnetism is equal:

$$w = \frac{\varepsilon_0}{2}\left(E^2 + c^2 B^2\right), \tag{13}$$

where $\mathbf{E}$ – the vector of electric field strength,

$\mathbf{B}$ – the vector of magnetic induction,

ε_0 – the electric constant,

c – the speed of propagation of electromagnetic field (speed of light).

From comparison (13) and (1) it is visible, that these expressions also have the identical form.

We should remind that in LITG the gravitational field is a real physical field of fundamental type similar to electromagnetic field.

According to LITG, GTR has the function of describing the phenomena in noninertial reference systems. In order to find the correct metrics of space-time deviated by fields, it is necessary in equations of GTR to add energy-momentum stress tensor of gravitational field. This tensor is defined in LITG in covariant form. The metrics found this way does not specify the gravitational field, but the degree of deviation from the flat Minkowski space-time.

By the way in standard GTR the gravitational field is replaced with the metric field having geometrical meaning. Then, perhaps, in standard GTR there will be no difference between m_f and m_G? However, as it was already shown in a number of works, for example in [3], [4], equations of GTR within the limits of a weak field coincide with equations of LITG (12). Hence, the problem remains in GTR too.

Most likely, the inequality of masses of the gravitational field is connected neither with the special theory of relativity, nor with GTR. Probably, the reason lies in the essence of the gravitational field. Let's assume that gravitation between bodies is created due to effect of gravitons streams. The fact, that the gravitation force and the mass of bodies does not depend on their movement against streams of gravitons, we fix as the principle of relativity. But the principle of relativity concerning mass of the field does not work in the case when bodies do not change the movement against streams of gravitons, but the observer does it. In this case the mass-energy of the field connected with the body can not depend on the state of movement of the observer. At the same time, the inequality of masses m_f and m_G can reflect the fact, that the inertial mass m_f contains additional mass of the field. This additional mass, according to (11) equal to $\frac{m_G}{3}$, is connected with the moving body. Probably, it represents the energy mass of excitation of the gravitational field, which is necessary to transfer a body from the condition of rest in relation to the streams of gravitons, to a certain condition of movement.

Let's notice, that finding the correlation between masses m_f and m_G we did not specify the initial state of the body. Due to the principle of relativity, it was insignificant for calculation m_G, whether there the body was motionless in relation to isotropic reference systems of gravitons streams or was moving together with the observer against this reference system. But from the point of view of the gravitation theory which is based on the concept of gravitons, it is important. As when the body is moving against the streams of gravitons these streams become nonisotropic, what can become the reason appearing of the gravitational field momentum and of the additional mass of the field $\frac{m_G}{3}$. But for an observer who is motionless in relation to the body, the additional field mass $\frac{m_G}{3}$ can not be found from the equations of the field (this is the consequence of the relativity principle). From the stated above it follows, that the distinction of inertial and gravitational masses of the gravitational field can be explained by the existence of the specific isotropic reference system. The feature of such reference system then is isotropy of gravitons streams which are responsible for gravitation.

On falling of a test body in the gravitational field of a massive body the graviton streams are obviously not isotropic. In this case it is necessary to count, that the mass of the gravitational field of the test body for an external observer is equal to m_f. For preservation of relativity and equivalence principles it is also necessary to draw certain conclusions for the observer who is motionless in relation to the test body. Though this observer finds the mass of own gravitational field of the test body m_G, but it is necessary to add to this mass the mass of the field $\frac{m_G}{3}$ as the consequence of movement of the test body under influence of attraction of the massive body.

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