Analysis of Bohm Diffusions Based on the Ion-Neutral Collisions

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***Abstract*— Three experimental cases of cross-field diffusions in the magnetized plasmas reported to be related with Bohm diffusion are investigated based on the ion current induced by the ion-neutral collisions. High diffusion coefficient for the original Bohm/Simon experiments and the recent experiments of strongly pulsed plasmas can be explained by the gyro-center shift current combined with the short circuit effect, which is different from the turbulence-induced transport of nuclear fusion devices. It can be deduced that Bohm's interpretation of diffusion with $1/B$ dependence came from the fact that the short circuit effect of his experiment was limited by the parallel ion velocity. The ratio of azimuthal current density to the discharge current density measured in the pulsed magnetron experiments is analyzed to be constant and independent from the magnitude of magnetic field due to the maximum condition for the Pedersen conductivity.**

***Index Terms*— Bohm diffusion, ion-neutral collision, plasma confinement, short circuit effect.**

I. Introduction

BOHM'S [1] experimental result on the transport of arc discharges was published, in which surprisingly large magnitude of diffusion is discovered. Simon [2] repeated the experiment and found a similar large amount of diffusion, but with a different analysis from Bohm's explanation. There are two important points in Simon's results: 1) the diffusion coefficient has $1/B^2$ dependence (Bohm analyzed it has $1/B$ dependence) and 2) large amount of diffusion came from the short circuit effect. So far, despite Simon's argument Bohm's diffusion coefficient has been known as an important semiempirical formula since many experimental results were reported close to it [3]. Although many experiments close to Bohm diffusion were reported, there were experiments showed better confinement than Bohm diffusion and the analysis of neo-classical diffusion appeared for the explanation of the large difference of cross-field diffusion from classical diffusion in tokamak research. However, the experimental results easily show $\sim$10 times larger than neo-classical scale so it is generally accepted that this anomalous transport is related with the turbulence activities that are always present in tokamak plasmas. One of recent diffusion analyses showing $B^{-3/2}$ dependence is not applicable to the original Bohm's and Simon's experiments since the basic assumptions of the analysis are not valid in such a low temperature and low magnetic field plasmas [4]. For example, the gyro-radius is larger than Debye length in Bohm's and Simon's experiments. Furthermore, $B^{-3/2}$ dependence is not exactly matches either of them (Bohm's $\sim B^{-1}$, Simon's $\sim B^{-2}$). While the high temperature plasma community focused on the turbulence, simulation studies in the field of low temperature plasmas, which is close to the original Bohm's and Simon's experiments are carried out using Boltzmann equations [5]. The simulation study based on the high frequency electric field instabilities showed matched Bohm diffusion parameter of 0.05 [6] albeit the key parameter in this paper is the ratio of electron gyro-frequency to the electron collisional frequency, which is different from the discussions in this paper since here the key parameter is the ratio of ion gyro-frequency to the ion-neutral collision frequency. In this paper, an alternative explanation for the cases of large amount of cross-field diffusion is introduced, which is based on the gyro-center shift current. The gyro-center shift current is a cross-field current generated by the ion momentum loss at the ion-neutral collisions [7]. After the introduction of gyro-center shift current in Section II, Bohm's and Simon's experiments are explained in Section III, the diffusion limited by parallel ion velocity is discussed in Section IV, and the pulsed magnetron experiments are analyzed in Section V followed by the conclusion.

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II. Ion Current Induced by Ion-Neutral Collisions

In general, most of the plasmas include a certain region where the temperature is lower than the fully ionized state. In this region, there exist non-negligible amount of neutral particles which make collisions with plasma ions. When there are magnetic field and electric field in this region, where the E-field and the pressure gradient are in the same direction (x in Fig. 1), the plasma-neutral collisions make momentum exchange in the direction that is perpendicular to both directions of magnetic field and plasma pressure gradient (y in Fig. 1). The momentum loss of plasma ions at the collisions with neutrals generates $\vec{J} \times \vec{B}$ ($\vec{J}$ is current density and $\vec{B}$ is magnetic field), which leads to the gyro-center shift current. When an ion moves by Lorentz's force and pressure gradient among collisions with neutrals, as shown in Fig. 1, velocity in x-direction can be described by $v_x = \mu(E - v_D B - \nabla P_i / q n_i)$, where the mobility $\mu = q/m_i \nu_{\text{in}}$, q is ion charge, m_i is ion mass, ν_{in} is ion-neutral collision frequency, E is electric field in x-direction, v_D is drift velocity, B is magnetic field

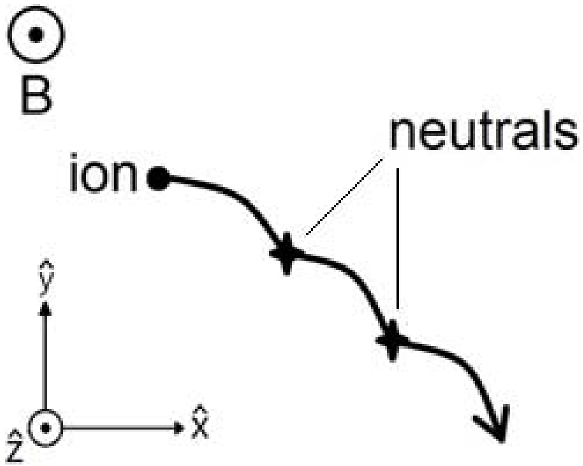

Fig. 1. Ion movement through collisions with neutrals under magnetic field, electric field, pressure gradient, and Lorentz force.

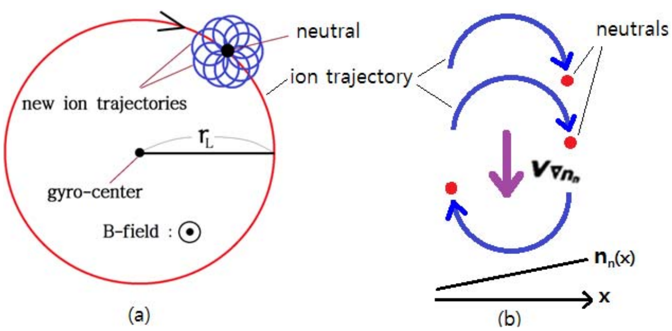

Fig. 2. Third components of ion effective velocity as the source of ion momentum exchange that generates gyro-center shift current. (a) Gyro-center of ion shifts from the center of figure to the location of charge exchange reaction. (b) Third term $v_{\nabla n_n}$ is the effective ion velocity that comes from the neutral density gradient, which was first introduced by Lee [7] as the averaged ion collision velocity over a circle of gyro-motion (as indicated in the figure, the case of collision with up direction is outnumbered by the case of collision with down direction).

in z-direction, ∇P_i is ion pressure gradient in x-direction, and n_i is ion density. The movement of ion in y-direction (v_D) is summation of the velocity induced by Lorenz's force (qv_xB), which is described as: $v_D = \mu^2 B(E - v_D B - \nabla P_i/qn_i)$. Now the current density in x-direction can be induced by momentum change of ion-neutral collision in y-direction by the equation of $\vec{J} \times \vec{B} = n_i m_i \nu_{\rm in} v_D \hat{y}$, therefore

$$J_x = qn_i \frac{\varepsilon_i}{1+\varepsilon_i^2}\left(\frac{E}{B} - \frac{\nabla P_i}{qBn_i}\right), \quad \text{where } \varepsilon_i = \mu B. \tag{1}$$

As shown in Fig. 2, there is a third velocity term to add in the above equation as momentum change of ion at the ion-neutral collisions, which is the component of ion effective velocity induced by the neutral density gradient: $v_{\nabla n_n} = kT_i \nabla n_n / qBn_n$, where kT_i is ion temperature, n_n is neutral density, and ∇n_n is neutral density gradient in x-direction [7]. Since this effect of neutral density gradient is a net value comes from the average over the circle of ion gyro-motion, only ions with mean free path longer than ion gyro-radius contribute to J_x. The ratio of such ions to the total ions is $\exp(-r_L/\lambda_{\rm cx})$, here r_L is ion gyro-radius, $\lambda_{\rm cx}$ is ion mean free path for the collision with neutral. In addition, $\varepsilon_i = \Omega_c/\nu_{\rm in} = \lambda_{\rm cx}/r_L$ (Ω_c is ion gyro-frequency). Therefore, the total gyro-center shift current density by ion is described as follows [8]:

$$J^{\rm GCS} = qn_i \frac{r_L}{\lambda_{\rm cx}}\left[\frac{1}{1+r_L^2/\lambda_{\rm cx}^2}\left(\frac{E}{B} - \frac{\nabla P_i}{qBn_i}\right) + \exp(-r_L/\lambda_{\rm cx})\frac{kT_i \nabla n_n}{qBn_n}\right]. \tag{2}$$

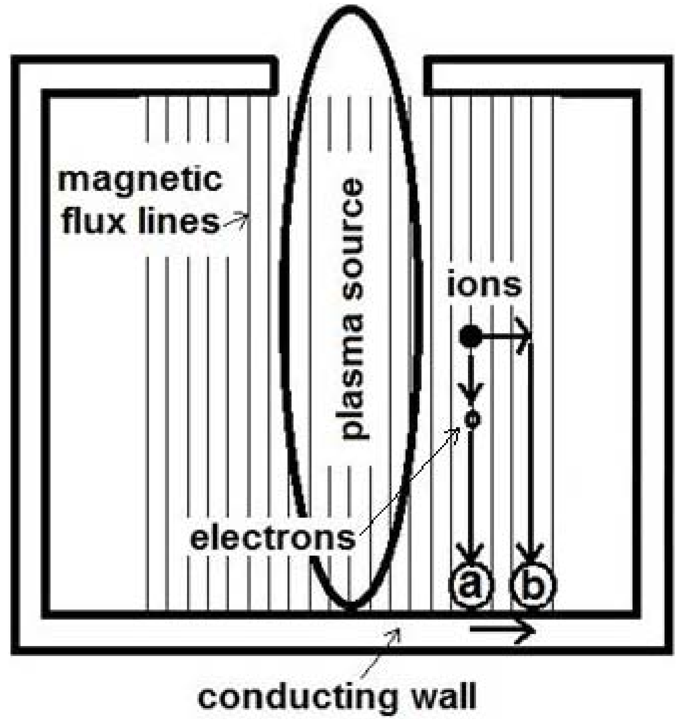

Fig. 3. Ions move cross-field by gyro-center shift current and electrons move along with flux lines through conducting wall to compensate the ion movement. On the wall, at least two kinds of reactions take place simultaneously, which are ionization and hitting of ion/electron to the wall. Larger population of electrons than ions (a in the figure) makes more electrons hit the wall and larger population of ions than electrons (b in the figure) makes more ions hit the wall, which results in a net plasma flow from high density region to the wall.

III. Arc Discharge Experiments by Bohm and Simon

Equation (2) can be used to explain the experimental cases of Bohm and Simon. Similar formula for the gyro-center shift current by electron-neutral collision can be described but it is much smaller than (2) since electron mass is smaller than ion. This means cross-field movement of electron is negligible compared with that of ion when these are generated by the collision with neutral. Usually, this gyro-center shift current is responsible for the spontaneous electric field as described in [7] and [8], however, when there is short circuit effect no spontaneous electric field forms. Fig. 3 is a schematic for the short circuit effect. Thus, in short circuit case, ions move from high density region to low density region by crossing the magnetic field as (2) describes, and electrons also move from high density region to low density region by moving along with the magnetic flux line via conducting wall, which makes a plasma diffusion. For this case, when these ion and electron movements are in the same rate, $E = 0$ in (2) and one can get the plasma flux by $\Gamma = J^{\mathrm{GCS}}/q$ so

$$\Gamma = n_i \frac{r_L}{\lambda_{\mathrm{cx}}}\left[\frac{-1}{1+r_L^2/\lambda_{\mathrm{cx}}^2}\frac{kT_i\nabla n_i}{eBn_i} + \frac{kT_i\nabla n_n}{eBn_n}e^{-r_L/\lambda_{\mathrm{cx}}}\right] \quad (3)$$

here $q = e$ and constant temperature are used.

If we neglect second term in (3), the diffusion coefficient can be described as

$$D = \frac{r_L}{\lambda_{\mathrm{cx}}}\left(\frac{1}{1+r_L^2/\lambda_{\mathrm{cx}}^2}\right)\frac{kT_i}{eB}. \quad (4)$$

This formula is exactly same of the diffusion coefficient driven in [2, eqs. (7)–(9)]. However, there is an additional plasma flux from second term of (3). Measurement of neutral density profile is required to determine the contribution from second term of (3). Since no information for the neutral density profiles of Bohm's or Simon's experiment is available, a simple approximation can be used. In detail, $\nabla n_i/n_i$ is a function of x, so the diffusion coefficient D is not a constant but varying with x too, but when $\nabla n_i/n_i$ is represented by $1/L_i(L_i$ is ion density scale length) diffusion coefficient can be represented by a constant value D^*. In addition, when there are enough collisions between ions and neutrals, so the neutral velocity is saturated to ion velocity, then neutral's penetration into plasma is blocked by ion density. In this case, steep gradient of ion density induces steep gradient of neutral density so $L_i \approx L_n(L_n$ is neutral density scale length representing inverse of $\nabla n_n/n_n)$. This approximation is explained in [9] by example of hyperbolic functions with same scale lengths for ion and neutral densities. Then, the representing diffusion coefficient includes contributions from both first term and second term in (3)

$$D^* = \frac{r_L}{\lambda_{\mathrm{cx}}}\left(\frac{1}{1+r_L^2/\lambda_{\mathrm{cx}}^2} + e^{-r_L/\lambda_{\mathrm{cx}}}\right)\frac{kT_i}{eB}. \quad (5)$$

For the Bohm's experiments, $r_L/\lambda_{\mathrm{cx}}$ varies roughly in the range of $1/13 \sim 1/40$ and diffusion coefficient is in the range of $kT_i/7eB \sim kT_i/20eB$, here the number coefficient (1/7 or 1/20) is a function of magnetic field and pressure since $r_L \propto 1/B$ and $\lambda_{\mathrm{cx}} \propto 1/n_n$. Simon's interpretation on Bohm's conclusion seems reasonable, which is that Bohm missed $1/B^2$ dependence of coefficient and made the Bohm diffusion formula as

$$D_B = \frac{1}{16}\frac{kT_e}{eB}. \quad (6)$$

Misusing T_e instead of T_i increased uncertainty of Bohm diffusion analysis; although Simon regarded T_e as equal to T_i, for example, if T_e is much higher than T_i then (6) represents a diffusion that is higher than (5). In the Simon's experiment, $r_L/\lambda_{\mathrm{cx}}$ varies in the range of $1/8 \sim 1/45$ and $1/B$ dependence in $r_L/\lambda_{\mathrm{cx}}$ is successfully investigated to discover $D \propto 1/B^2$. The pressure dependence due to $\lambda_{\mathrm{cx}} \propto 1/n_n$ in diffusion coefficient was also confirmed by Simon [10]. Simon's calculated diffusion coefficient (0.175 $\mathrm{m^2/s}$) for 0.4 T case, which excluded the plasma flux generated by second term of (3), was smaller than measured value 0.92 $\mathrm{m^2/s}$ [2], however, if the diffusion coefficient from gyro-center shift, (5) is used for the same case, it gives the diffusion coefficient as 0.87 $\mathrm{m^2/s}$, which is very close to the experimental measurement. Basically, the origin of diffusions for Bohm and Simon experiments can be explained as the plasma diffusion induced by the combination of ion gyro-center shift current and the short circuit effect.

IV. Current Limit by Parallel Ion Velocity

In the analysis of short circuit effect, the compensation current going through conducting wall can be limited by the parallel ion velocity (in the vicinity of wall where the compensation current is carried by ions, their movement is limited by the ion thermal velocity). This limitation takes place because the compensation current at the point ⓑ of Fig. 3 is generated by the excessive number of ions that move in parallel with the magnetic field toward the wall and this movement is limited by the ion thermal velocity. For the inner plasma section facing point ⓐ of Fig. 3, same function is carried out by parallel electrons but the electron thermal velocity is much faster than ion. Thus, it is needed to check whether gyro-center shift current is larger than the current limited by the ion thermal velocity, because if the gyro-center shift current is larger than this current then it cannot be neutralized and the diffusion would be reduced. When the gyro-center shift current is larger than the limited current then the change of charge distribution by the gyro-center shift current start to generate the electric field so the E-field in the first term of (2) increases. Since the polarity of induced E-field is opposite for the direction of current comparing with the other two terms (ion current from plasma accumulates plus charge at the outside of plasma and that charge will block the current), increase of E-field will decrease the current. As time goes by the decrease of current saturates at the current which is same as limited current by the ion thermal velocity and E-field also saturates at the value that keeps this equilibrium. This process is similar to the saturation of E-field at the tokamak boundary at the equilibrium of return current (in the tokamak case the return current is induced by the turbulence) [11]. The diffusion process is originated from the cross-field movement of ion (v^{GCS}) by which the excessive ions are accumulated in the box of Fig. 4(a). The number of ions accumulated in the box per second is $\Gamma^{\mathrm{GCS}} = n_e(v^{\mathrm{GCS}}(x) - v^{\mathrm{GCS}}(x+\delta x))\mathrm{WH}$, here W is the width of plasma column and H is the height of plasma column in Fig. 4. Then, these excessive ions will diffuse along with B-field into the wall and make recombinations with electrons that came through the conducting wall. In the case of unlimited, all the excessive ions can reach the wall. However, in the case of limited, not all the excessive ions can reach the wall because ions cannot move faster than ion thermal velocity in the area close to the wall. Thus, there is limiting ion flow $\Gamma^{\mathrm{Lim}} = n_e v^{\mathrm{pi}} W\delta x$, here v^{pi} is parallel ion velocity which is same of ion thermal velocity $= (kT_i/m_i)^{1/2}$. If Γ^{Lim} is larger than Γ^{GCS}, it is unlimited case. Here, is a question about how to set the value of δx. If we set δx as whole length of the wall Γ^{Lim} becomes large, and the comparison can lead a misunderstanding that Γ^{Lim} is larger than Γ^{GCS} and it is unlimited. Because

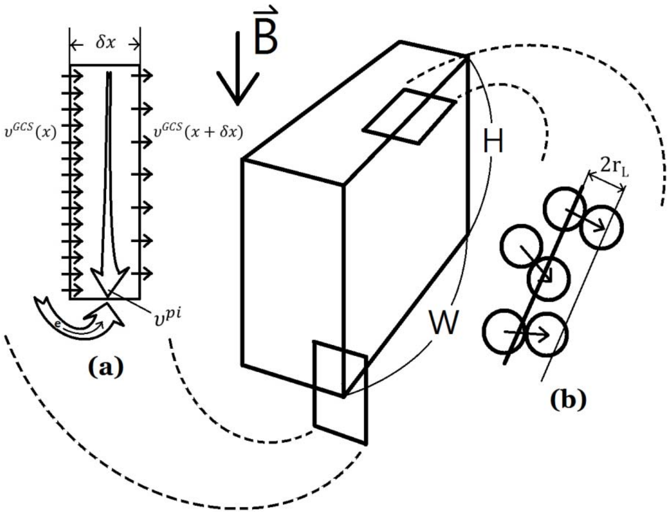

Fig. 4. Comparison of parallel ion velocity (v^{pi}) with gyro-center shift velocity (v^{GCS}). Plasma column is in the middle and its bottom is the conducting wall. (a) Magnified from right bottom corner of plasma column. (b) Magnified from the top view of plasma column.

Γ^{GCS} and Γ^{Lim} are localized functions of x, at some point Γ^{Lim} can be smaller than Γ^{GCS} and it is limited at that point. Thus, the best way of comparison is to be between the maximum of Γ^{GCS} and the minimum of Γ^{Lim}. If minimum of Γ^{Lim} is larger than maximum of Γ^{GCS}, it is safe to consider as unlimited. To make maximum of Γ^{GCS}, $v^{\mathrm{GCS}}(x+\delta x)$ is set as zero. In addition, to make minimum of Γ^{Lim}, δx should be set as small as possible. But we cannot set δx smaller than $2r_L$ because the mechanism of v^{GCS} is ion gyro-center shift, as shown in Fig. 4(b). Thus, if we set δx smaller than $2r_L$ then those ions coming out of left side in Fig. 4(a) will not remain in the box, which means δx must be $2r_L$. By using $v^{\mathrm{GCS}} \approx D^*\nabla n_i/n_i$ from (5), v^{GCS} can be calculated for both of Bohm's and Simon's cases (here $\nabla n_i/n_i = L_i^{-1}$). In addition, the ratio of gyro-center shift current to the maximum current by ion thermal velocity becomes $v^{\mathrm{GCS}}H/2v^{\mathrm{pi}}r_L$. Calculations of this ratio for two cases are summarized in Table I. The gyro-center shift currents for Simon's experiment is smaller than the limit by ion thermal velocity, but for Bohm's experiment, gyro-center shift current is twice larger than the limit. According to this calculation, diffusion measurement for Bohm's experiment should be half of the calculation by (5). Indeed, using parameters for Bohm's experiment in Table I, (5) gives 0.8 m^2/s, and experimental value [by (6): Bohm diffusion] is 0.34 m^2/s. It is understandable that why Bohm regarded the diffusion proportional to $1/B$ instead of $1/B^2$ since for the Bohm's case, the limiting current is proportional to $v^{\mathrm{pi}}r_L = kT_i/eB$ and the diffusion is proportional to kT_i/eB.

V. Experiments of Pulsed Magnetron Sputtering Systems

Recently, another study related with Bohm diffusion is reported in which various cases of pulsed plasmas are investigated [12], where the ratio of azimuthal current density to the discharge current density (J_ϕ/J_D) found close to a constant value 2.7 over two orders of magnitude in magnetic field

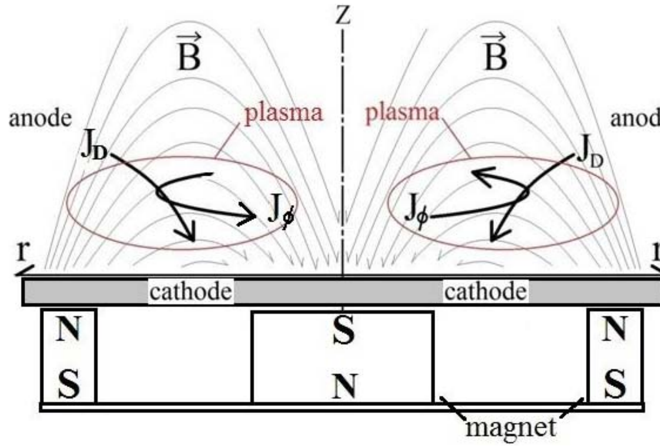

Fig. 5. Schematic of pulsed sputtering magnetron J_D is discharge current and J_φ is the azimuthal current. Reversed electric field is found at the plasma near the cathode.

strength, and the cross-field diffusion is five times larger than Bohm scale. The analysis in [12] follows the assumption that most currents are carried by electrons, however, the discharge current close to the cathode is due mostly to ions as pointed out in the discussion of early paper [13]. Thus, it is reasonable to express Pedersen conductivity (coefficient of discharge current proportional to the electric field) by current due to ions, an example is in [14, eq. (6)]. This Pedersen conductivity (σ_P) can be also induced from the component of J^{GCS} induced by the electric field in (2), so

$$\sigma_P = \frac{en_i}{B}\frac{r_L}{\lambda_{\mathrm{cx}}}\frac{1}{1+r_L^2/\lambda_{\mathrm{cx}}^2}. \tag{7}$$

The discharge current (J_D) in Fig. 5 can be expressed by (2). The azimuthal current (J_φ) in Fig. 5 has two components, one is Hall current and the other is diamagnetic current. The ion velocity that contributes to the azimuthal current is the ion drift velocity for (1), which is $v_i^\varphi = (E/B - \nabla P/eBn_i)/(1+r_L^2/\lambda_{\mathrm{cx}}^2)$ and electron velocity for azimuthal current can be described as $v_e^\varphi = (E/B - \nabla P/eBn_e)/(1+r_{\mathrm{Le}}^2/\lambda_{\mathrm{en}}^2)$, here r_{Le} is electron gyro-radius and λ_{en} is mean free path of electron-neutral collision. The total azimuthal current density (J_φ) can be induced from $\mathrm{e}n_e(v_i^\varphi - v_e^\varphi)$, which is

$$-en_i\left[\frac{r_L^2/\lambda_{\mathrm{cx}}^2}{1+r_L^2/\lambda_{\mathrm{cx}}^2}\frac{E}{B} + \left(1+\frac{1}{1+r_L^2/\lambda_{\mathrm{cx}}^2}\right)\frac{\nabla p}{eBn_e}\right]. \tag{8}$$

Here $n_e \approx n_i$ and $r_{\mathrm{Le}}/\lambda_{\mathrm{en}} \ll 1$ are used. Thus, the ratio of azimuthal current density to discharge current density can be described as (8) divided by (2)

$$\frac{J_\varphi}{J_D} \approx \frac{\frac{\theta^2}{1+\theta^2}v_{\mathrm{ExB}} + \left(1+\frac{1}{1+\theta^2}\right)v_{\nabla P}}{\frac{\theta}{1+\theta^2}\left(v_{\nabla P} - v_{\mathrm{ExB}}\right) + \theta e^{-\theta}v_{\nabla P}}. \tag{9}$$

Here, $\theta = r_L/\lambda_{\mathrm{cx}}$, $v_{\mathrm{ExB}} = E/B$, $v_{\nabla P} = \nabla P/eBn_e$, and $kT_i\nabla n_n/eBn_n \approx -v_{\nabla P}$ is assumed.

The magnetic field distributions in Fig. 5 indicates the possibility of short circuit effect since it has magnetic field lines going into the conducting area of cathode, however, the short circuit effect is not perfect since reversed electric

TABLE I

COMPARISON OF PHYSICAL PARAMETERS FOR BOHM'S AND SIMON'S EXPERIMENTS. σ_{cx} IS CHARGE EXCHANGE CROSS SECTION, L_i IS DENSITY SCALE LENGTH, AND RE IS REYNOLDS NUMBER (RE IS DISCUSSED IN [11])

	Bohm's experiment	Simon's experiment	Pulsed magnetron
Ion/neutral species	argon	nitrogen	argon
B (T)	0.37	0.4	0.01
σ_{cx} (m^2)	6.2×10^{-19}	7×10^{-19}	6.2×10^{-19}
λ_{cx} (m)	4.6×10^{-2}	2.9×10^{-2}	0.09
r_L/λ_{cx}	1/13	1/11	1.2
L_i (m)	3×10^{-3}	7×10^{-3}	0.02
n_n (/m^3)	3.5×10^{19}	5×10^{19}	1.8×10^{19}
υ^{GCS} (m/sec)	2.7×10^{2}	1.1×10^{2}	N.A.
H (m)	1.9×10^{-1}	1.8×10^{-1}	N.A.
υ^{pi} (m/sec)	3.8×10^{3}	4.5×10^{3}	N.A.
r_L (m)	3.5×10^{-3}	2.7×10^{-3}	1.1×10^{-1}
$\upsilon^{GCS}H/2\upsilon^{pi}r_L$	**1.9**	**0.81**	N.A.
$\mathrm{Re}\approx\lambda_{cx}/L_n$	15	4.3	4.5

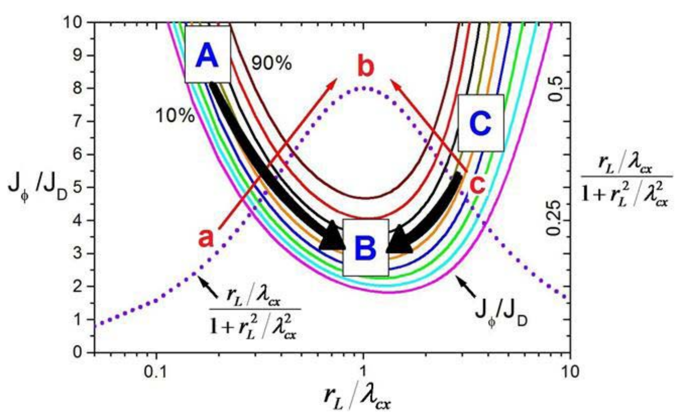

Fig. 6. J_ϕ/J_D and $(r_L/\lambda_{cx})/(1+r_L^2/\lambda_{cx}^2)$ as function of r_L/λ_{cx}.

field is found [15], which means there is some electric field generated by (2) to compensate other driving components in part. Thus, υ_{ExB} is not zero since it is found but υ_{ExB} is not so large as $\upsilon_{\nabla P}$ because there is a short circuit effect. Fig. 6 includes nine cases of (9) for the different electric fields from the case υ_{ExB} is 10% of $\upsilon_{\nabla P}$ to the case υ_{ExB} is 90% of $\upsilon_{\nabla P}$. One of the differences in pulsed plasma from steady-state is that pulsed plasma may not be in the thermal equilibrium with surroundings and the plasma temperature can be higher than the steady-state case. For the example of pulsed magnetron sputtering, applied high voltage generates plasma and continues on heating. The heating power is proportional to the discharge current, and the discharge current is proportional to the Pedersen conductivity of (7). At the beginning of discharge, plasma temperature is low and ion gyro-radius is small (r_L/λ_{cx} is also small), so J_ϕ/J_D is in the [A] region in Fig. 6. As time goes by ion temperature increases and so does r_L/λ_{cx}, which increase conductivity (a $\to$ b in the figure) and accelerates the heating to reach [B] region of J_ϕ/J_D. However, once it passed [B] region further increase of temperature has decreasing effect of conductivity (b$\to$c in the figure) then the heating becomes weaker and J_ϕ/J_D goes back to [B] region. This mechanism indicates that J_ϕ/J_D of pulsed plasmas has tendency of settle in [B] region, where r_L/λ_{cx} is close to 1 regardless of each parameter's value in r_L/λ_{cx}, such as magnetic field strength. Fig. 6 indicates [B] region for $r_L/\lambda_{cx} = 1$ covers $1.8 < J_\phi/J_D < 4.7$, which is in agreement with experimental result in [12] that covers factor of 1.6 from the common average of 2.7 ($1.7 < J_\phi/J_D < 4.3$). These values are different from a previously published analysis that follows Bohm's empirical diffusion coefficient; it predicted the ratio of J_ϕ/J_D to be 16 [12]. This analysis including ion-neutral collisions also explains large amount ($\sim$5 times larger than Bohm diffusion) of diffusion found in the experiments. Although the short circuit effect performs only in part for the pulsed plasma cases, the diffusion coefficients by small part of (5) for these cases are already larger than the scale of Bohm diffusion since r_L/λ_{cx} is close to 1. For example, if the partial short circuit effect reduced gyro-center shift current down to 40% for r_L/λ_{cx} is 1.2 case (Table I), then (5) yields $D = 0.34kT_i/eB$, which is 5.4 times larger than Bohm diffusion.

VI. DISCUSSION AND CONCLUSION

The detailed investigation of the influence of neutral density profile for the gyro-center shift current remains as an important future task of experimental plasma transport research, since for most cases of plasma experiments, neutral density is an unknown parameter. The theory of gyro-center shift is developed by comparing with the experimental cases and produced agreed radial electric fields to the experiments which were mainly generated by the neutral density gradient [7], [8]. In this analysis, the most important term that generates radial current is the neutral density gradient, since it is newly introduced as another source of momentum exchange adding to well-known terms, such as $E \times B$ drift and diamagnetic drift. Although the approximation of $L_i \approx L_n$ is used to make a formula that expresses two components of plasma transport (one by the ion density gradient and the other by the neutral density gradient) as one diffusion term in this analysis, detail measurement of neutral density profile will bring new physics research by the contribution of neutral density gradient to the transport and current of plasma boundary. There are cases where the approximation of $L_i \approx L_n$ is not valid. For example, in a weakly ionized plasma, L_n can be much larger than L_i, in this case, the second term of (3) vanishes and the diffusion coefficient will be reduced by half. The contribution from the inhomogeneity of neutrals has been studied through the concept of neutral depletion [16]. However, neutral density gradient is more often ignored while mobility part is well investigated through the terms of Pedersen and Hall conductivities in weakly ionized plasmas [14]. As of now, there is no unanimous agreement on the validity of short circuit effect. Thus, it can be said that the electron cross-field mobility is dominant for the diffusion without short circuit effect. This possibility is well discussed as an ambipolar diffusion [17]. From the view point of gyro-center shift theory, the role of neutral density gradient for the case without short circuit effect is to form the electric field, and for the ideal case this electric field can be an ambipolar electric field. However, the return current by E-field [the first term of (2)] doesn't compensate GCS current completely. One of the reasons for this is that the real system must be analyzed in full three dimensions. Equation (2) is only for 1-D approximation and for most cases there are other current paths in addition to (2) to make the divergence free current. Therefore, the electric field is not the case of ambipolar electric field. The detail investigation of 3-D approach remains as a future work. In the history of plasma physics, the Bohm's diffusion coefficient ($kT_e/16eB$) remained as important formula because Princeton C-stellarator showed diffusion very close to it. The large diffusion coefficient proportional to kT_e/eB in Princeton C-stellarator can be interpreted as $2(\partial n_e/n_e)^2 kT_e/\pi eB$, which is the diffusion coefficient induced by the turbulence [11], [18]. This means fully ionized cases of stellarators and tokamaks must be analyzed by a different process than Bohm's or Simon's experiments, in which turbulence activity is important; one indication is the low Reynolds number (Re) in Table I (lower than critical Reynolds number). Another example of large diffusion under the name of Bohm diffusion is the cases of pulsed plasmas [12]. In addition, large diffusion in the pulsed magnetrons can be explained by the combination of gyro-center shift and short circuit effect. According to the analysis of cross-field transport in the magnetized plasmas based on the gyro-center shift, both Bohm's analysis and Simon's analysis missed the contribution from the neutral density gradient so that Simon's calculation was less than half of the measurement. There is a difference between Bohm's experiment and Simon's experiment for the parallel ion velocity limited current comparing with ion gyro-center shift current. The ion limited current in Bohm's experiment is smaller than the ion gyro-center shift current and the particle transport is limited by the ion thermal velocity and the ion gyro-radius so that the B-field dependence of diffusion coefficient in Bohm's experiment is $1/B$, while the diffusion coefficient in Simon's experiment is proportional to $1/B^2$. One of the under lying meanings revealed by the validity of short circuit effect (another recent analysis using short circuit effect can be found in [19]) is that the plasma is not self-neutralized by the quasi-neutrality. If the quasi-neutrality is effective then the ions cannot make the charge separation at the beginning and there is no cause for the short circuit effect. Thus, there is a reason for the quasi-neutrality is not valid [20]. There are many cases of anomalous transports reported as Bohm diffusion, which can be explained in two categories, one is the diffusion by gyro-center shift current with short circuit effect, (4) for the cases with low Reynolds numbers, and the other is the transport by turbulence induced diffusion for the cases with high Reynolds numbers. Therefore, this analysis suggested that there are two quite different cross-field diffusion mechanisms which were reported under the name of Bohm diffusion.

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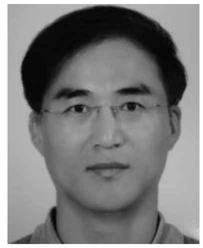

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