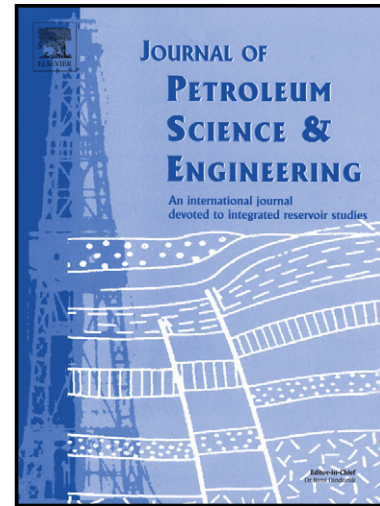


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Discussion of “Gene expression programming analysis of implicit Colebrook–white equation in turbulent flow friction factor calculation” by Saeed Samadianfard [J. pet. sci. Eng. 92-93 (2012), 48–55]

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Discussion of “Gene expression programming analysis of implicit Colebrook–White equation in turbulent flow friction factor calculation” by Saeed Samadianfard [J. Pet. Sci. Eng. 92-93 (2012), 48-55]

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Abstract:

Maximal relative error of the explicit approximation to the Colebrook equation for flow friction presented in the discussed paper by Saeed Samadianfard [J. Pet. Sci. Eng. 92-93 (2012), 48-55; doi. 10.1016/j.petrol.2012.06.005] is investigated. Samadianfard claims that his approximation is very accurate with the maximal relative error of no more than 0.08152%. Here is shown that this error is about 7%. Related comments about the paper are also enclosed.

Keywords: Colebrook equation; Colebrook-White friction factor; Hydraulic resistance; Error analysis; Turbulent flow; Pipes

1. Introduction

Samadianfard (2012) presents an approximate explicit formula as a replacement for the implicitly given Colebrook equation for fluid flow pipe friction. The empirical Colebrook equation (1) [Eq. 1a of the discussed paper] relates the unknown flow friction factor (λ_0) with the known Reynolds number (Re) and the known relative roughness of inner pipe surface (ϵ/D). Widely used empirical and nonlinear Colebrook's equation for calculation of Darcy's friction factor is iterative i.e. implicit in fluid

flow friction factor since the unknown friction factor appears on the both sides of the equation

$[\lambda_0=f(\lambda_0, \text{Re}, \varepsilon/D)]$ (Colebrook 1939).

$$\frac{1}{\sqrt{\lambda_0}} = -2 \cdot \log_{10} \left(\frac{2.51}{\text{Re} \cdot \sqrt{\lambda_0}} + \frac{\varepsilon}{3.71 \cdot D} \right) \quad (1)$$

Equation (1) [Eq. 1a of the discussed paper] is from Colebrook (1939) but where coefficient 3.71 is slightly different, i.e. 3.7.

This unknown friction factor, λ_0 , cannot be extracted to be on the left side of the equal sign analytically, i.e. with no use of some simplifications. Better to say, it can be expressed explicitly only if some kind of approximate calculus takes place, where $\lambda_0 \rightarrow \lambda$. Samadianfard (2012) presents one explicit approximation of the Colebrook equation $[\lambda=f(\text{Re}, \varepsilon/D)]$. His explicit formula is given as (2) [Eq. (29) of the discussed paper].

$$\left. \begin{aligned} \lambda &= \frac{A}{B} + C \cdot D + E \cdot F \\ A &= \text{Re}^{\frac{\varepsilon}{D}} - 0.6315093 \\ B &= \text{Re}^{\frac{1}{3}} + \text{Re} \cdot \frac{\varepsilon}{D} \\ C &= 0.0275308 \\ D &= \left(\frac{6.929841}{\text{Re}} + \frac{\varepsilon}{D} \right)^{\frac{1}{9}} \\ E &= \frac{10^{\frac{\varepsilon}{D}}}{\frac{\varepsilon}{D} + 4.781616} \\ F &= \frac{9.99701}{\text{Re}} + \sqrt{\frac{\varepsilon}{D}} \end{aligned} \right\} \quad (2)$$

Samadianfard (2012) used genetic programming to develop his approximation. Čojbašić and Brkić (2013) also used genetic algorithms to improved accuracy of some of the available approximations (Romeo et al. 2002; Serghides 1984).

2. Error analysis

Samadianfard (2012) claims that his approximation (2) [Eq. (29) of the discussed paper] produces maximal relative error, δ_{\max} of no more than 0.08152% compared with the very accurate iterative solution of the implicit Colebrook equation, λ_0 . Using the procedure from Brkić (2011a), it can be shown that this error, δ_{\max} in the practical range of applicability of the Reynolds number (Re) and the relative roughness of inner pipe surface (ϵ/D) is about 7.4289%¹ [mean (average) relative error is about 1.6198%]. Distribution of the relative error of approximation by Samadianfard (2012) [(2); Eq. (29) of the discussed paper] can be seen in Figure 1. Relative error is defined as $\delta_{\max} = (\lambda_0 - \lambda)/\lambda_0 \cdot 100\%$ as shown by Eq. (26) of the discussed paper where λ_0 is very accurate iterative solution of the implicit Colebrook equation while λ is the solution of the approximation by Samadianfard (2012) [(2); Eq. (29) of the discussed paper]. The error is evaluated twice in MS Excel according to methodology from Brkić (2011a, 2012). To produce Figure 1, entire practical domain of the Reynolds number (Re) and the relative roughness of inner pipe surface (ϵ/D) is covered with 740 point-mesh.

Figure 1. Distribution of the relative error of approximation by Samadianfard (2012) [(2); Eq. (29) of the discussed paper]

Using double check in MS Excel, second mesh of 860 points is formed using 20 values of the relative roughness (ϵ/D) shown in Table 1 [Table 2 of discussed paper] and using 43 values of the Reynolds number (Re); from $4 \cdot 10^3$ to 10^4 with pace 10^3 , from 10^4 to 10^5 with pace 10^4 , from 10^5 to 10^6 with pace 10^5 , from 10^6 to 10^7 with pace 10^6 , and from 10^7 to 10^8 with pace 10^7 . Similarly as in

¹ This discussion was initially submitted on 8 December 2013, and on 12 December 2013 the discussion of Vatankhah (2014) appeared on-line with similar findings regarding this error analysis

Samadianfard (2012), results are shown in Table 1 and compared with the results from Table 2 of the original paper by Samadianfard (2012). According to this second check, the maximal relative error δ_{\max} is about 6.9107% and mean (average) relative error in the range of applicability of equation is about 1.2595%.

Table 1. Distribution of maximal relative error of the approximation by Samadianfard (2012) [(2); Eq. (29) of the discussed paper]

Mean square error as defined by Eq. 27 of the discussed paper, for the mesh of 740 points is about $1.1460 \cdot 10^{-7}$ and for the mesh of 860 points is about $2.2711 \cdot 10^{-7}$. According to Winning and Coole (2013), the approximation by Samadianfard (2012) is in the group with medium value of mean square error (very small is lower than 10^{-11} , small is between 10^{-11} and 10^{-8} , medium is between 10^{-8} and $5 \cdot 10^{-6}$, and large is above $5 \cdot 10^{-6}$). Samadianfard (2012) reported value of mean square error of $1.95 \cdot 10^{-10}$.

3. Source of the error

Regarding the error from Table 1 and from Figure 1 of this discussion, it is useful to mention that actually three level of the accuracy can be introduced; 1. Colebrook's equation is empirical (other maybe more accurate equations can be used to describe related physical processes), 2. Colebrook's equation can be solved very accurately using iterative procedure (term "accurate by default" can be used or better to say, this error can be neglected in many cases; this is λ_0 in this discussion), 3.

Relevant explicit approximations of the Colebrook equation (this is λ in this discussion) can be used to avoid iterative procedure (their error can be estimated very accurately compared with the error of

iterative solution). The estimated error from Table 1 and from Figure 1 of this discussion belongs to the category explained in point 3 (where friction factor from point 2 assumed as accurate). Using “philosophy” from point 2, value λ_0 of friction factor is calculated, while friction factor λ is calculated using point 3. Finally, the maximal relative error was calculate as $\delta_{\max} = (\lambda_0 - \lambda) / \lambda_0 \cdot 100\%$. It is also useful to note that the Moody diagram (Moody 1944) is only graphical interpretation of the Colebrook equation (Colebrook 1939). Moody’s diagram is not less or more accurate compared with Colebrook’s equation. So, inaccurate reading from Moody’s diagram can introduce one more source of error (Yıldırım 2009, Brkić 2011b, Fang et al 2011), which is maybe possible cause of the value of the error estimated in Samadianfard (2012). In this case, source of the error is not then calculative, but rather systematic, where the main source of the error is in inaccuracy of λ_0 (which should be accurate by default).

4. Analysis of the equation

It is worth to mention that the approximation by Samadianfard (2012) does not contained a single logarithmic expression. This structure of the equation is valuable contribution since the logarithmic expression uses a lot of computational resources in a computer environment (Giustolisi et al. 2011; Clamond 2009). Only, three other available approximations do not use logarithms; Moody (1947), Wood (1966) and Chen (1984), and all three are less accurate than the approximation by Samadianfard (2012). But also, Samadianfard (2012) uses non-integer power such as $Re^{\epsilon/D}$ which in computer environment usually means $\exp[(\epsilon/D) \cdot \ln(Re)]$ where ‘ln’ is natural (Napierian) logarithm (Clamond 2009).

5. Reference notes

Finally, it has to be mentioned that source of the review article of Brkić (2011a) is not Petroleum Science and Technology [ISSN 1091-6466 (Print), 1532-2459 (Online)] as Samadianfard (2012) cited in the discussed paper but Journal of Petroleum Science and Engineering [ISSN 0920-4105]. In the first mentioned journal Brkić (2011c) published an article with his explicit approximation of Colebrook's equation, which possible makes confusion here.

Also, it can be mentioned that Colebrook equation is published solely by Colebrook (1939) and not in collaboration with White as often cited. Colebrook (1939) acknowledged contribution of White in a footnote of his paper with the reference of their previous joint work (Colebrook and White 1937).

6. Notation remark

In this discussion, the Darcy friction factor is labeled as λ while in the discussed paper as f . This is because f should be rather used for Fanning friction factor. This can make confusion since the physical meaning is equal but where the Darcy friction factor is 4 times greater than Fanning's friction factor.

7. Conclusion

According to the classification proposed by Samadianfard (2012), his approximation with the relative error of about 7% (7.43% after first evaluation and 6.91% after second evaluation; both in MS Excel), have to be characterized as less accurate method (group of methods with relative error of more than 3%) and should be placed in Figure 3 of the discussed paper (or even in Figure 2 of the discussed paper with the non-advisable approximations) and not in Figure 6 of the discussed paper with the extremely accurate approximations.

References:

- Brkić, D., (2011a). Review of explicit approximations to the Colebrook relation for flow friction. *J. Petrol. Sci. Eng.* 77(1), 34-48. doi. 10.1016/j.petrol.2011.02.006
- Brkić, D., (2011b). New explicit correlations for turbulent flow friction factor. *Nucl. Eng. Des.* 241(9), 4055-4059. doi. 10.1016/j.nucengdes.2011.07.042
- Brkić, D., (2011c). An explicit approximation of the Colebrook equation for fluid flow friction factor. *Petrol. Sci. Tech.* 29(15), 1596-1602. doi. 10.1080/10916461003620453
- Brkić, D., (2012). Determining friction factors in turbulent pipe flow. *Chem. Eng. (New York)* 119(3), 34-39.
- Chen, J.J.J., (1984). A simple explicit formula for the estimation of pipe friction factor. *Proc. Inst. Civ. Eng.*, 77, 49–55. doi. 10.1680/iicp.1984.1272
- Clamond, D., (2009). Efficient resolution of the Colebrook equation. *Ind. Eng. Chem. Res.* 48(7), 3665-3671. doi. 10.1021/ie801626g
- Ćojbašić, Ž., Brkić, D., (2013). Very accurate explicit approximations for calculation of the Colebrook friction factor. *Int. J. Mech. Sci.* 67, 10-13. doi. 10.1016/j.ijmecsci.2012.11.017
- Colebrook, C.F., (1939). Turbulent flow in pipes with particular reference to the transition region between the smooth and rough pipe laws. *J. Inst. Civil. Eng. (London)*, 11(4), 133-156. doi. 10.1680/ijoti.1939.13150
- Colebrook, C.F., White, C.M., (1937). Experiments with fluid friction in roughened pipes. *Proc. Roy. Soc. Ser. A Math. Phys. Sci.* 161(906), 367-381. doi. 10.1098/rspa.1937.0150

- Fang, X., Xu, Y., Zhou, Z., (2011). New correlations of single-phase friction factor for turbulent pipe flow and evaluation of existing single-phase friction factor correlations. *Nucl. Eng. Des.* 241(3), 897-902. doi. 10.1016/j.nucengdes.2010.12.019
- Giustolisi, O., Berardi, L., Walski, T.M., (2011). Some explicit formulations of Colebrook–White friction factor considering accuracy vs. computational speed. *J. Hydroinf.* 13(3), 401-418. doi. 10.2166/hydro.2010.098
- Moody, L.F., (1944). Friction factors for pipe flow. *Trans. ASME* 66(8), 671-684.
- Moody, L.F., (1947). An approximate formula for pipe friction factors. *Trans. ASME* 69(12), 1005–1011.
- Romeo, E., Royo, C., Monzon, A., (2002). Improved explicit equation for estimation of the friction factor in rough and smooth pipes. *Chem. Eng. J.* 86(3), 369-374. doi. 10.1016/S1385-8947(01)00254-6
- Samadianfard, S., (2012). Gene expression programming analysis of implicit Colebrook–White equation in turbulent flow friction factor calculation. *J. Petrol. Sci. Eng.* 92-93, 48–55. doi. 10.1016/j.petrol.2012.06.005
- Serghides, T.K., (1984). Estimate friction factor accurately. *Chem. Eng. (New York)* 91(5), 63-64.
- Vatankhah, A.R., (2014). Comment on “Gene expression programming analysis of implicit Colebrook–White equation in turbulent flow friction factor calculation”. *J. Petrol. Sci. Eng.* (in press, accepted manuscript); doi. 10.1016/j.petrol.2013.12.001
- Winning, H.K., Coole, T., (2013). Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. *Flow Turbulence Combust.* 90(1), 1-27. doi. 10.1007/s10494-012-9419-7
- Wood, D.J., (1966). An explicit friction factor relationship. *Civil. Eng.* 36(12), 60–61.

Yıldırım, G., (2009). Computer-based analysis of explicit approximations to the implicit Colebrook-White equation in turbulent flow friction factor calculation. *Adv. Eng. Softw.* 40(11), 1183-1190. doi. 10.1016/j.advengsoft.2009.04.004

Accepted manuscript

Research highlights

-Discussion of paper by Saeed Samadianfard [J. Pet. Sci. Eng. 92-93 (2012), 48-55].

-Samadianfard (2012) claims that his approximation is very accurate with the maximal relative error of no more than 0.08152%.

-Here is shown that this error is about 7%.

-Related comments about the paper are also enclosed.

Accepted manuscript

Table 1. Distribution of maximal relative error of the approximation by Samadianfard (2012) [(2); Eq. (29) of the discussed paper]

Relative roughness (ϵ/D)	Relative error (%)			
	Samadianfard		This discussion	
	Max δ_{\max}	Mean δ	Max δ_{\max}	Mean δ
$1 \cdot 10^{-6}$	0.049	0.012	6.3210	2.9289
$5 \cdot 10^{-6}$	0.063	0.02	6.9107	3.0483
$1 \cdot 10^{-5}$	0.07	0.024	6.2389	3.1781
$5 \cdot 10^{-5}$	0.076	0.035	4.6612	2.7105
$1 \cdot 10^{-4}$	0.077	0.039	3.9580	2.0012
$2 \cdot 10^{-4}$	0.078	0.043	3.1845	1.3767
$4 \cdot 10^{-4}$	0.078	0.047	2.1375	0.9998
$6 \cdot 10^{-4}$	0.078	0.049	1.3145	0.7962
$8 \cdot 10^{-4}$	0.079	0.051	1.1123	0.6961
$1 \cdot 10^{-3}$	0.079	0.052	1.2409	0.6857
$2 \cdot 10^{-3}$	0.079	0.055	2.4851	1.1169
$4 \cdot 10^{-3}$	0.079	0.06	4.0676	1.1633
$6 \cdot 10^{-3}$	0.079	0.062	4.6207	1.0094
$8 \cdot 10^{-3}$	0.079	0.065	4.6890	0.8434
$1 \cdot 10^{-2}$	0.079	0.066	4.5401	0.6976
$1.5 \cdot 10^{-2}$	0.079	0.07	3.8766	0.5606
$2 \cdot 10^{-2}$	0.079	0.072	3.1906	0.4685
$3 \cdot 10^{-2}$	0.079	0.074	2.0914	0.3450
$4 \cdot 10^{-2}$	0.079	0.075	1.2864	0.2831
$5 \cdot 10^{-2}$	0.08	0.076	0.6478	0.2804

$$\lambda = \frac{A}{B} + C \cdot D + E \cdot F$$

$$A = \text{Re}^b - 0.6315093$$

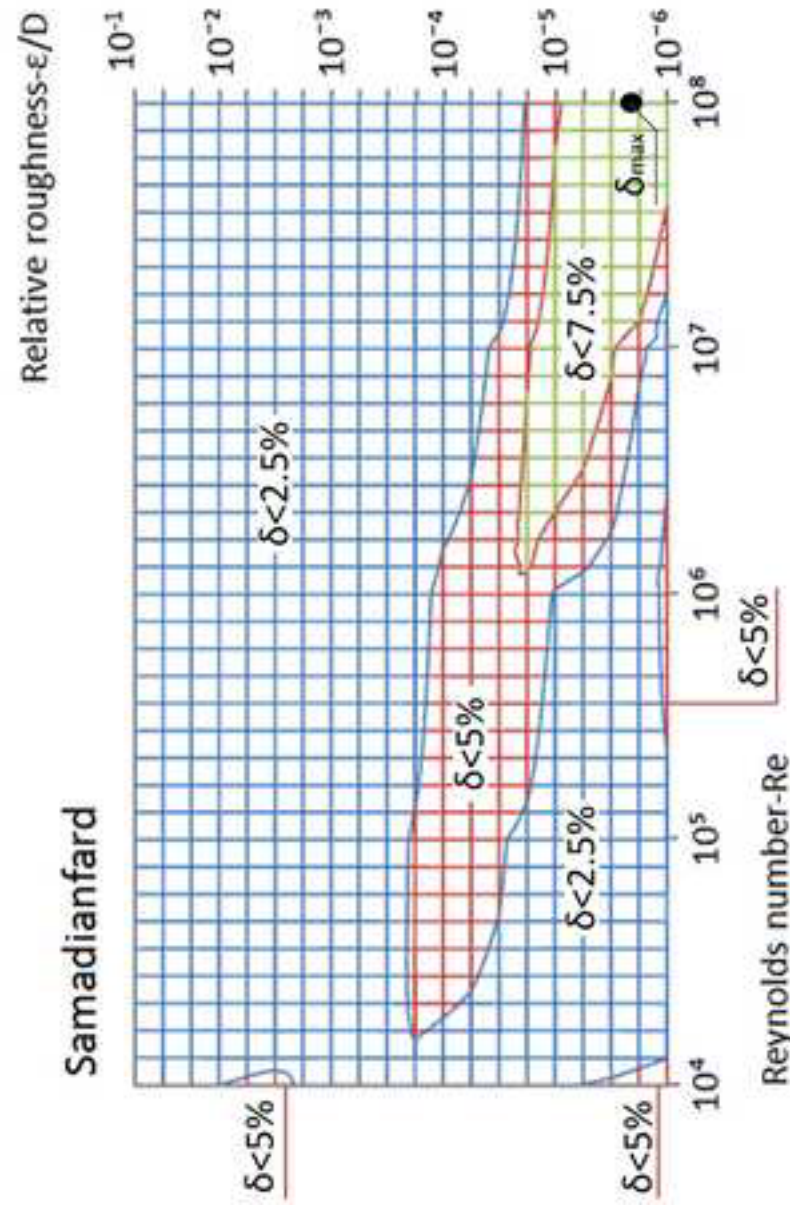
$$B = \text{Re}^{\frac{1}{3}} + \text{Re} \cdot \frac{\varepsilon}{D}$$

$$C = 0.0275308$$

$$D = \left(\frac{6.929841}{\text{Re}} + \frac{\varepsilon}{D} \right)^{\frac{1}{6}}$$

$$E = \frac{10^b}{\frac{\varepsilon}{D} + 4.781616}$$

$$F = \frac{9.99701}{\text{Re}} + \sqrt{\frac{\varepsilon}{D}}$$



λ -Darcy friction factor (-); δ -relative error (%); Re-Reynolds number (-); ε/D -relative roughness (-)

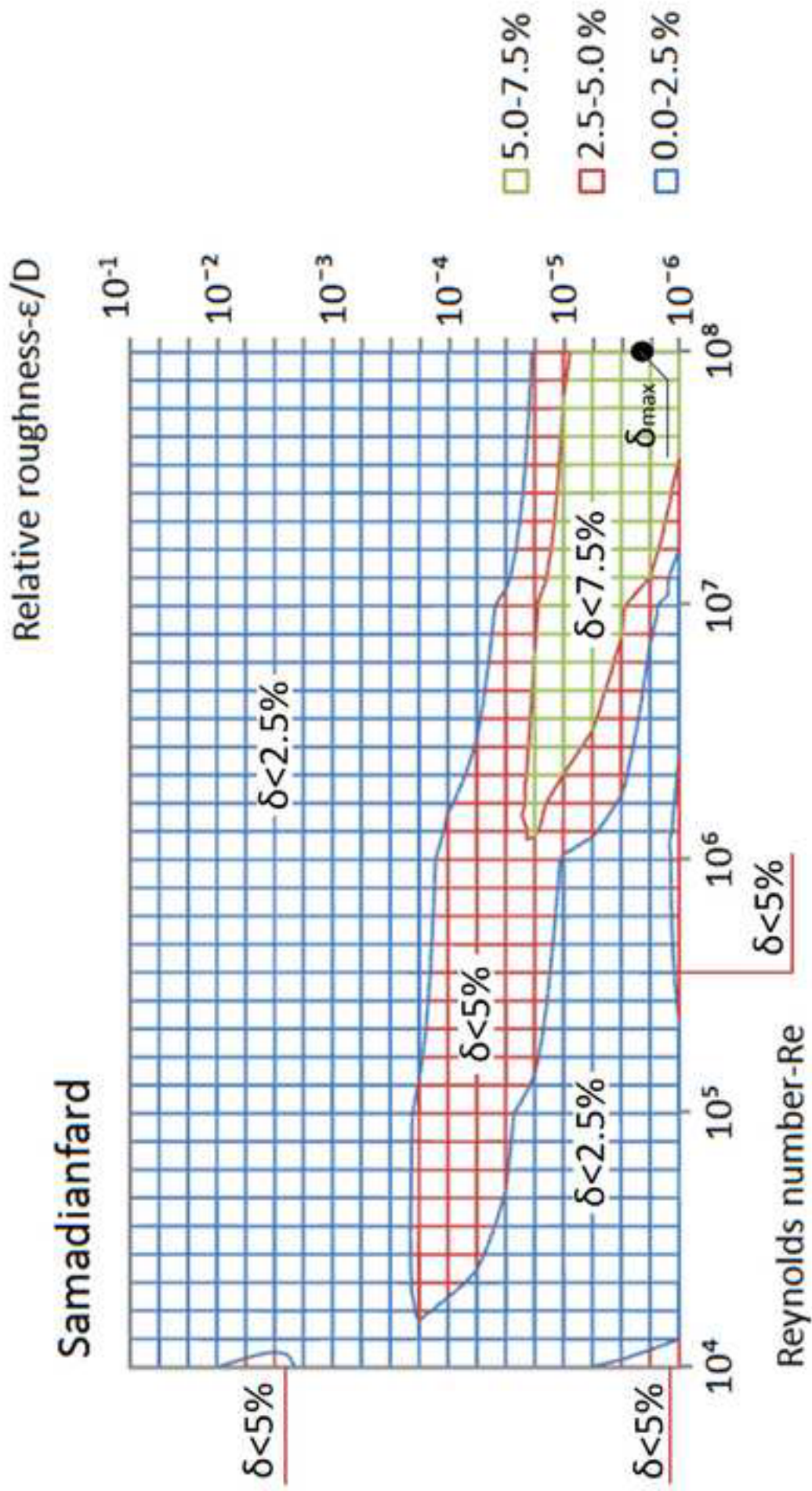


Figure 1 DB