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Dear Editor/Reviewers,

My text is (the second revised version):

**Discussion of "Jacobian Matrix for Solving Water Distribution System Equations with the Darcy-Weisbach Head-Loss Model" by Angus Simpson and Sylvan Elhay**

June 2011, Vol. 137, No. 6, pp. 696-700.

DOI: 10.1061/(ASCE)HY.1943-7900.0000341


It contains 1783 words in total (including the title, notations and references).

Sincerely yours,

Dejan Brkić, Ph.D.

**Discussion of "Jacobian Matrix for Solving Water Distribution System Equations with the Darcy-Weisbach Head-Loss Model" by Angus Simpson and Sylvan Elhay**

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The general recommendation by Simpson and Elhay (2011) that the Hazen-Williams formula be replaced by the Darcy-Weisbach is heartily applauded. Today, this fact cannot be disputed. The main purpose of this discussion is to recommend more appropriate formulas for the calculation of the Darcy friction factor, $\lambda$ which are more suitable for computation of the derivatives in a Jacobian matrix, and also to clarify a few points in the calculation of the flow-dependent Jacobian. The Darcy-Weisbach formula contains the Darcy friction factor, $\lambda$ which in general is dependent on the Reynolds number and, therefore, in general on flow, i.e. $\lambda(Re) \rightarrow \lambda(Q)$. This fact can be incorporated in the Jacobian matrix [Eq. (6) in the discussed paper] as clearly shown in Simpson and Elhay (2011). On the other hand, flow, Q, in the formula for flow friction, $\lambda$, is not treated as a variable in EPANET (Computer software, Washington, DC, U.S. Environmental Protection Agency) during the computation of the Jacobian matrix (probably following the analogy with the Hazen-Williams scheme). Use of the Darcy friction factor dependent on flow $\lambda = \lambda(Q)$ in the Jacobian matrix is more accurate but also more complicated. A certain balance between accuracy and complexity in engineering systems has to be carefully evaluated. The procedure shown in the discussed paper is more accurate mathematically and also can be more easily implemented in a computer code. This increased computing burden of the improved procedure has to be

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carefully investigated where the ratio between complexity and accuracy will decide whether this method will be used in every day engineering practice (Giustolisi et al 2011).

The pressure drop model $\Delta p$ (Pa) for the Darcy-Weisbach formula $\Delta p = 8 \cdot L \cdot \rho \cdot Q^2 \cdot \lambda / (\pi^2 \cdot D^5)$ is used in this discussion instead of the head-loss model $\Delta H$ (m) which was used in the discussed paper where the connection between them is simple, i.e. $\Delta p = \rho \cdot g \cdot \Delta H$. Also, the Darcy friction factor noted as $\lambda$ is used as in Colebrook (1939) where the Darcy friction is noted as f in the discussed paper (one should not confuse the Darcy friction factor used in this discussion and the discussed paper with the Fanning friction factor which is numerically equal to one-fourth the Darcy friction). The Darcy friction factor $\lambda$ for the turbulent zone (turbulent flow is the most common in pipe networks) can be calculated using the implicit Colebrook Eq. (16), using some of the explicit approximations to the Colebrook equation (Brkić 2011a) or using some other suitable formulas [EPANET uses the approximation by Swamee and Jain (1976) which is shown in the next formula on the far right side of the equation].

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log \left( \frac{2.51}{\text{Re} \cdot \sqrt{\lambda}} + \frac{\varepsilon}{3.7 \cdot D} \right) \approx -2 \cdot \log \left( \frac{5.74}{\text{Re}^{0.9}} + \frac{\varepsilon}{3.7 \cdot D} \right) \qquad (16)$$

The first derivative of the Darcy-Weisbach equation (appropriate terms in the Jacobian matrix [Eq. (6) in the discussed paper]) has to be calculated using the fact that the Darcy friction factor, $\lambda$ contains flow, Q, which now has to be treated as a variable, i.e., $\lambda = \lambda(Q)$ and not as constant anymore (e.g. flow is treated as constant in Brkić (2011b) and in EPANET). Explicit approximations of Colebrook's equation shown in Brkić (2011a) are more suitable for the calculation of the derivatives used in the Jacobian matrix since the original Colebrook equation is implicit in the Darcy friction factor, $\lambda$. The implicitly given form of the original Colebrook function complicates the calculation of the first derivative or even makes it impossible. The Reynolds number in the flow term $\text{Re} = 4 \cdot Q / (D \cdot \pi \cdot \nu)$ has to be placed in the

equation for the Darcy friction factor, λ (the Swamee and Jain approximation in Eq. 17) and

the first derivative where flow, Q, being variable, is treated as the variable and calculated

from Eq. (17).

$$\frac{\partial \Delta p(Q)}{\partial Q} = \frac{16 \cdot L \cdot \rho \cdot Q}{\pi^2 \cdot D^5} \cdot \frac{\partial(\lambda)}{\partial Q} = \frac{\frac{16 \cdot L \cdot \rho \cdot Q}{\pi^2 \cdot D^5} \cdot \partial \left( \left[ \log_{10} \left( \frac{5.74}{\left( \frac{4 \cdot Q}{\pi \cdot v \cdot D} \right)^{0.9}} + \frac{\varepsilon}{3.7 \cdot D} \right) \right]^{-2} \right)}{\partial Q} \tag{17}$$

This procedure is not simple and hence the practical achievement in accuracy can be disputed

knowing the empirical nature of the Colebrook and similarly-related formulas (with an error

of a few percent or more compared with experimental data).


Further, the Hagen-Poiseuille equation used for the laminar regime in the discussed paper

[Table 2 and Eq. (2) in the original paper] is nothing more than the Darcy-Weisbach formula

where the suitable Darcy friction factor λ=64/Re is included (Eq. 18).

$$\Delta p = \frac{8 \cdot L \cdot \rho \cdot Q^2}{\pi^2 \cdot D^5} \cdot \frac{64}{\text{Re}} = \frac{128 \cdot L \cdot v \cdot \rho \cdot Q}{\pi \cdot D^4} = r \cdot Q \tag{18}$$

The fact that hydraulic resistance, r, from Eq. (18) is independent of flow [as shown in Eq. (2)

of the discussed paper] implies that a water network in which laminar flow takes place (in all

pipes) can be solved using a non-iterative procedure. On the other hand, the Darcy friction

factor, λ in the laminar zone depends only on the Reynolds number, Re, which further

depends on flow Q. The fact that laminar resistance, r is independent of flow, Q, is not crucial

for the presented calculation [where λ=λ(Q) and r≠r(Q)]. The use of separate equations for

hydraulic resistance, r or at least separate comments for the laminar and turbulent regimes in

Eqs. (1-3) of the discussed paper should be avoided (better to say the parameter, λ should

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always be used and not r). However, in laminar, turbulent zone and also for the unstable transient zone between them (Table 2 and 4 of the discussed paper) it is certain that the Darcy friction factor, λ depends on the Reynolds number [λ=λ(Re, ε/D)] and subsequently in general on flow [where Re=Re(Q)]. This is not true for the fully turbulent regime (fully developed flow) where the Darcy friction factor, λ depends only on relative roughness ε/D and not on the Reynolds number Re and where, therefore the influence of the flow Q can be neglected).

Finally, the flow dependent Jacobian, as recommended by Simpson and Elhay (2011), and the Darcy-Weisbach scheme should be used. Furthermore, EPANET also should be revised in light of the following discussion. In the laminar zone the Darcy friction factor should be calculated using λ=64/Re. For the turbulent zone, the Swamee and Jain approximation [used in EPANET, in the discussed paper and in Spiliotis and Tsakiris (2011)] should be replaced with one of the more accurate approximations of the Colebrook equation [where λ=λ(Re, ε/D)]. The Swamee and Jain approximation produces maximal error of 2% compared to the iterative solution of the Colebrook equation as shown in the recent study by Brkić (2011a). For example, the approximation proposed by Romeo et al. (2002) with a maximal error up to 0.1345% is more accurate. Also, approximations by Buzzelli (2008) and Serghides (1984) or by Zigrang and Sylvester (1982) are very accurate with an error of no more than 0.1385%. Vatankhah and Kouchakzadeh (2008) also show very accurate approximation with a relative error up to 0.1472%. There are at least ten approximations more accurate than the Swamee-Jain equation. These very accurate approximations are sometimes complex but they can be simply included in a computer code (Matlab, Excel, etc.) without causing significant additional computational time. Also, between the laminar and turbulent zones it is more

convenient to model it as a sharp jump (sudden transition to turbulence) instead of using a cubic spline (explained in the discussed paper in Table 2; also instead of a cubic spline, the simple equation $\lambda=0.0025 \cdot Re^{0.33}$ is more convenient). At last but not less important, Churchill (1977) also provides a single formula which covers all three cases in the discussed paper (Table 2 in the discussed paper of Simpson and Elhay (2011), i.e. used for the laminar and turbulent zone as well as for the unstable transient zone between them as described in EPANET). The equation by Churchill (1977) perfectly fits the laminar regime and provides a reasonable transition to turbulence (cubic spline in the discussed paper). Finally it approximates the implicit Colebrook function (suitable only for the turbulent zone) at least equally as accurate as the Swamee-Jain approximation. Simpson and Elhay (2011) use Matlab and hence for the readers' convenience, Matlab code for the well known equation by Churchill (1977) is given:

```
function Lambda = churchilllamapp(Re,RelEpsilon)
Lambda = (2.457.*ln(1./((7./Re).^0.9+0.27.*RelEpsilon))).^16;
Lambda = 8.*((8./Re).^12+(1./(Lambda+(37530./Re).^16)^1.5)).^(1/12);
```

Clamond (2009) provides Matlab code for the iterative solution of the implicit Colebrook equation.

**Acknowledgment**

**Notation**

The following symbols are used in this discussion:

D - inner pipe diameter (m)

Q – flow rate (m$^3$/s)

Δp – pressure drop (Pa)

L – pipe length (m)

λ – Darcy (Darcy-Weisbach) or Moody friction factor (-) ["Lambda" in the presented Matlab code]

r – hydraulic resistance defined in text

ε/D – relative roughness of inner pipe surface (-) ["RelEpsilon" in the presented Matlab code]

Re – Reynolds number (-) ["Re" in the presented Matlab code]

ν – kinematic viscosity (m$^2$/s)

ρ – density (kg/m$^3$)

π ~ 3.1415

**References**:

Brkić, D. (2011a), "Review of explicit approximations to the Colebrook relation for flow friction," *J. Petrol. Sci. Eng.,* 77(1), 34-48.

Brkić, D. (2011b), "Spreadsheet-based pipe networks analysis for teaching and learning purpose," *J. Pipeline Syst. Eng. Pract. ASCE*, 2(4), doi: 10.1061/(ASCE)PS.1949-1204.0000080

Buzzelli, D. (2008). "Calculating friction in one step," *Mach. Des.,* 80(12), 54-55.

Churchill, S. W. (1977). "Friction-factor equation spans all fluid flow regimes," *Chem. Eng.,* 84(24), 91–92.

Clamond, D. (2009). "Efficient resolution of the Colebrook equation," *Ind. Eng. Chem. Res.,* 48(7), 3665–3671.

Colebrook, C. F. (1939). "Turbulent flow in pipes with particular reference to the transition region between the smooth and rough pipe laws," *J. Inst. Civil. Eng. (London)*, 11(4), 133-156.

Giustolisi, O., Berardi, L. and Walski, T.M. (2011), "Some explicit formulations of Colebrook–White friction factor considering accuracy vs. computational speed," *J. Hydroinform.*, 13(3), 401-418.

Romeo, E., Royo, C., and Monzon, A. (2002). "Improved explicit equation for estimation of the friction factor in rough and smooth pipes," *Chem. Eng. J.*, 86(3), 369–374.

Serghides, T. K. (1984). "Estimate friction factor accurately," *Chem. Eng.,* 91(5), 63–64.

Simpson, A., and Elhay, S. (2011). "Jacobian matrix for solving water distribution system equations with the Darcy-Weisbach head-loss model," *J. Hydraul. Eng. ASCE*, 137 (6), 696-700.

Spiliotis, M. and Tsakiris, G. (2011). "Water distribution system analysis: Newton-Raphson method revisited," *J. Hydraul. Eng. ASCE*, 137 (8), 852-855.

Swamee, P. K., and Jain, A. K. (1976). "Explicit equations for pipe flow problems," *J. Hydraul. Div. ASCE*, 102 (HY5), 657–664.

Vatankhah, A. R., and Kouchakzadeh, S. (2008). "Discussion of 'Turbulent flow friction factor calculation using a mathematically exact alternative to the Colebrook-White equation' by Jagadeesh R. Sonnad and Chetan T. Goudar." *J. Hydraul. Eng. ASCE*, 134 (8), 1187.

Zigrang, D. J., and Sylvester, N. D. (1982). "Explicit approximations to the solution of Colebrook friction factor equation," *AIChE J.*, 28 (3), 514–515.

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Dejan Brkić, Ph.D.