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Discussion of "Water Distribution System Analysis: Newton-Raphson Method Revisited" by M. Spiliotis and G. Tsakiris August 2011, Vol. 137, No. 8, pp. 852-855.

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Dear editorial staff (your additional request about tables),

I do not have any tables in my text; (Etable 1 and Etable 2 are submitted as Electronic Annexes to the discussion; do not embed them into the text, instead provide a link to this material). This is possible for technical papers, technical notes and case studies since there is separate item description in editorial system for such purposes. This item description does not exist for discussions. So I have uploaded them as figures. Please, advise me if you have additional requests with these materials.

Dear Editor/Reviewers,

I have accepted all your suggestions. I am not native English speaker and therefore I appreciate your grammar corrections of my text. Thank you!

My text is:

Discussion of “Water Distribution System Analysis: Newton-Raphson Method Revisited” by M. Spiliotis and G. Tsakiris

August 2011, Vol. 137, No. 8, pp. 852-855.

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My discussion contains Supplemental Data; Electronic tables, E-table 1 and E-table 2 (the Excel files with calculations converted in PDF). They can be attached to the electronic version of the discussion in the ASCE Library.

Sincerely yours,

Dejan Brkić, Ph.D.

1 **Discussion of “Water Distribution System Analysis: Newton-Raphson Method**
2 **Revisited” by M. Spiliotis and G. Tsakiris**

3 August 2011, Vol. 137, No. 8, pp. 852-855.

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7

8 Spiliotis and Tsakiris (2011) show the nodal formulation for water distribution system
9 analysis using the head-loss dependent Jacobian. Some issues about the node formulation of
10 the discussed problem will be commented on here. Also, as an addition to the discussed paper,
11 the procedure with pressure drop formulation of the Jacobian will be presented where the
12 method is similar to, but different from Spiliotis and Tsakiris (2011). The procedure uses
13 elements of the methods by Cross (1936), Shamir and Howard (1968) and Epp and Fowler
14 (1970).

15

16 As shown in Brkić (2011a), all methods for the analysis of looped water distribution systems
17 can be divided in two general groups: loop oriented and node oriented methods. To date, all
18 loop oriented methods contained a flow independent Jacobian while all node oriented
19 methods contained a pressure drop (or a head loss) independent Jacobian. In their recent
20 paper, Simpson and Elhay (2011) show an improved loop method with the flow-dependent
21 Jacobian where the problem is how to find the derivative of the implicitly given equation by
22 Colebrook (1939) which is in a common use as a standard for the calculation of flow friction,
23 λ [the Colebrook equation is Eq. 1b in the discussed paper (the Darcy friction is noted as f in
24 the discussed paper and in this discussion as λ)]. The problem can be addressed by using some
25 of the available explicit approximations of the Colebrook equation [Swamee and Jain (1976)

26 approximation is used in Simpson and Elhay (2011)], but as shown in Brkić (201x) it is not
 27 always possible to find a balance between accuracy and complexity among many available
 28 approximate formulas (Brkić 2011b). On the other hand, Spiliotis and Tsakiris (2011) show
 29 the node oriented method with the head-loss dependent Jacobian. In general, the convergence
 30 properties of the nodal formulation are not as good as those of the looped version, but, on the
 31 other hand, the derivative of the implicitly given Colebrook equation used in the proposed
 32 pressure drop or head-loss dependent Jacobian can be easily calculated when the nodal
 33 method is employed. To accomplish the solution of the problem in the pressure drop
 34 dependent form, Eq. 13, is used for non-iterative calculation of flow Q.

$$35 \quad F = Q = \pi \cdot \sqrt{\frac{\Delta p \cdot D^5}{2 \cdot L \cdot \rho}} \cdot \log \left(\frac{2.51 \cdot \nu \cdot \sqrt{L \cdot \rho}}{\sqrt{2 \cdot \Delta p \cdot D^3}} + \frac{\varepsilon}{3.7 \cdot D} \right) \quad (13)$$

36 Eq. 13 is derived combining the Darcy-Weisbach equation, $\lambda = \Delta p \cdot \pi^2 \cdot D^5 / (8 \cdot L \cdot \rho \cdot Q^2)$, and the
 37 equation for the Reynolds number, $Re = 4 \cdot Q / (D \cdot \pi \cdot \nu)$ as in Eq. 14, and then combining the
 38 Colebrook equation and again the Darcy-Weisbach equation to produce Eq. 13.

$$39 \quad Re \cdot \sqrt{\lambda} = \frac{4 \cdot Q}{D \cdot \pi \cdot \nu} \cdot \sqrt{\frac{\Delta p \cdot \pi^2 \cdot D^5}{8 \cdot L \cdot \rho \cdot Q^2}} = \frac{1}{\nu} \cdot \sqrt{\frac{2 \cdot \Delta p \cdot D^3}{L \cdot \rho}} \quad (14)$$

40 In that way, friction factor, λ , is eliminated from Eq. 13 which implies that flow, Q, in a single
 41 pipe can be calculated using the implicit Colebrook equation but also at the same time,
 42 avoiding iteration (Swamee and Jain 1976). On the contrary, standard procedure requires the
 43 iterative calculation of flow friction λ , using the implicitly given Colebrook equation [Eq. 1b
 44 of the discussed paper] and then the Darcy-Weisbach equation rearranged for calculation of
 45 flow $Q = [\Delta p \cdot \pi^2 \cdot D^5 / (8 \cdot L \cdot \rho \cdot \lambda)]^{0.5}$. However, a calculation of flow (or head, i.e. pressure
 46 distribution) in a looped system of pipes still requires an iterative procedure, but by using Eq.
 47 13, a double iterative procedure can be avoided (an iterative procedure for the calculation of a
 48 looped system of pipes still exists while the sub-iterative calculation of a single pipe in a

49 looped system required by the Colebrook equation is not needed). Of course, Eq. 13 has
50 additional value because it can be used for the calculation of the partial derivative
51 $F' = \partial Q(\Delta p) / \partial (\Delta p)$, where pressure drop Δp is treated as the variable.
52 Appropriate partial derivatives, F' , are the terms in the pressure drop dependent Jacobian
53 matrix [matrix [J] in Eq. 15 of this discussion (the relevant head-loss dependent Jacobian
54 matrix is presented by Eq. 9 in the discussed paper)].
55
56 As noted in the discussed paper, this calculation is simple and hence can be performed in
57 Excel. An Excel file converted to PDF are accompanied as an electronic annex to this
58 discussion (Electronic tables; E-tables 1 and 2) [the example network (shown in Fig. 1 of the
59 discussed paper is used as the example]. In this discussion, two versions of the standard
60 procedure with a standard pressure drop independent Jacobian are shown (in E-tables 1 and 2)
61 where convergence properties of both are investigated. The presented procedures start from a
62 pressure-drop distribution which satisfies the second Kirchhoff law for all closed paths in the
63 network of pipes and which is iterated until continuity at each node is approximately satisfied.
64 Initial choices for pressure drops are; for the first loop: $\Delta p_{12} = 1000 \text{ kPa}$, $\Delta p_{25} = 500 \text{ kPa}$, $\Delta p_{56} = -$
65 750 kPa , $\Delta p_{61} = -750 \text{ kPa}$; and for the second loop: $\Delta p_{52} = -500 \text{ kPa}$, $\Delta p_{23} = 600 \text{ kPa}$, $\Delta p_{34} = -600 \text{ kPa}$
66 and $\Delta p_{45} = 500 \text{ kPa}$ [direction of both loops is counterclockwise, i.e. positive]. Also, as shown
67 in Brkić (2011a), the standard procedure with pressure drop independent Jacobian demands
68 assumption of the velocity of water (here 10m/s) required for the Reynolds number, Re , at the
69 start of the procedure where the velocity can be chosen at random in a reasonable range
70 without causing any trouble [the Reynolds number, Re , can be treated as independent of flow
71 (as in E-table 1) or calculated in each iteration using flow Q from the previous iteration (as in
72 E-table 2)]. On the contrary, assumption of the starting velocity is sufficient for the procedure
73 shown in this discussion [using Eq. 13 and related derivatives F'] and in the discussed paper

74 (but note that the Reynolds number, velocity and viscosity still remain connected). The choice
75 of initial pressure drop is not critical but should satisfy the second Kirchhoff law (Gay and
76 Middleton 1971). Kinematic viscosity of water, ν , is $1.004 \cdot 10^{-6} \text{ m}^2/\text{s}$, water density, ρ is
77 $1000\text{kg}/\text{m}^3$ and pipe roughness coefficient, ϵ is $3 \cdot 10^{-6} \text{ m}$ [inner roughness of the third
78 generation polyethylene (PE) pipes]. Pipe lengths ($L_{12}=L_{45}=L_{56}=400\text{m}$, $L_{16}=L_{25}=900\text{m}$,
79 $L_{23}=500\text{m}$, and $L_{34}=800\text{m}$), pipe diameters ($D_{12}=0.3526\text{m}$, $D_{16}=D_{34}=0.2204\text{m}$,
80 $D_{23}=D_{25}=0.2468\text{m}$, $D_{45}=0.1102\text{m}$, and $D_{56}=0.0968\text{m}$) and water demands ($q_1=-130\text{L}/\text{s}$ [input
81 in network], $q_2=15\text{L}/\text{s}$, $q_3=10\text{L}/\text{s}$, $q_4=35\text{L}/\text{s}$, $q_5=40\text{L}/\text{s}$, and $q_6=30\text{L}/\text{s}$) are from Fig. 1 of the
82 discussed paper and in the electronic annex to this discussion [numbers in the index are
83 related to nodes (first number in the index is the starting node and the second is the ending
84 node)]. The network is supplied by water through node 1 where the pressure is 1962kPa [node
85 5 is chosen to be excluded as redundant from the Jacobian matrix to preserve linear
86 independency of rows and columns in the matrix (node 5 is chosen at random, i.e. any other
87 node can be chosen instead of node 5; node 1 is chosen in the discussed paper)]. The
88 calculated corrections of pressure drops in iteration i can be calculated as $\Delta x = \Delta(\Delta p)$, Eq. 15
89 ($[J]x[\Delta x] = [F]$, where: $[J] = [F']$ -the Jacobian matrix (here with symmetrical properties,
90 diagonal terms are positive and non-diagonal terms are negative); $[\Delta x]$ -single column matrix
91 of corrections and $[F]$ -single column matrix of flows where its terms are calculated using Eq.
92 13). The corrections have to be algebraically added to the values of pressure drop in the
93 appropriate pipes in iteration $i-1$ [in the zeroth iteration the initial pressure drop pattern (initial
94 choices) are chosen as explained above]. Algebraic rules for the corrections can be seen in
95 Brkić (2009, 2011a) and in Corfield et al. (1974).

120 with the required number of iterations). Therefore, the new method with pressure drop and
121 head-loss dependent Jacobian should be further evaluated (to avoid unfounded conclusions).

122

123 Figure 1. Convergence properties of the proposed method for the solution of the problem of
124 water distribution analysis expressed in the nodal form (both with the independent Jacobian)

125

126 As can be seen from Eqs. 15 [and Eqs. 5 and 10 of the discussed paper], the procedure with
127 the pressure drop dependent Jacobian [or head-loss dependent one from the discussed paper],
128 does not require calculation of the Reynolds number at all (problem from E-tables 1 and 2 is
129 exceeded). On the other hand, Eqs. 15 [and Eq. 4 and 5 of the discussed paper] and its
130 derivative F' [Eq. 10 of the discussed paper] are complex for computing by hand but they can
131 be easily implemented in a computer code.

132

133 In addition to the discrepancy in results from the use of a dependent versus independent
134 Jacobian matrix, a portion of the discrepancy in results of this discussion (in addition to the
135 fact that the discussed paper head, h is in m, and in this discussion pressure drops Δp are in
136 Pa, where $\Delta p = \rho \cdot g \cdot h$) compared with the discussed paper is probably caused by use of different
137 values for the roughness of pipes, ϵ , and kinematic viscosity of water, ν [Spiliotis and Tsakiris
138 (2011) did not report these values].

139

140 **Supplemental Data**

141 The Excel files converted in PDF (Electronic tables; E-tables 1 and 2) used as examples in
142 this discussion is available online in the ASCE Library (www.ascelibrary.org)

143

144 **Notation**

145 The following symbols are used in this discussion:

146 D inner pipe diameter (m)

147 L pipe length (m)

148 Q flow rate (m^3/s)

149 q water demand (m^3/s)

150 Re Reynolds number (-)

151 Δp pressure drop (Pa)

152 Δx correction of pressure drop (Pa)

153 ε pipe roughness coefficient (m)

154 λ Darcy (Darcy-Weisbach) or Moody friction factor (-)

155 ν kinematic viscosity (m^2/s)

156 ρ water density (kg/m^3)

157

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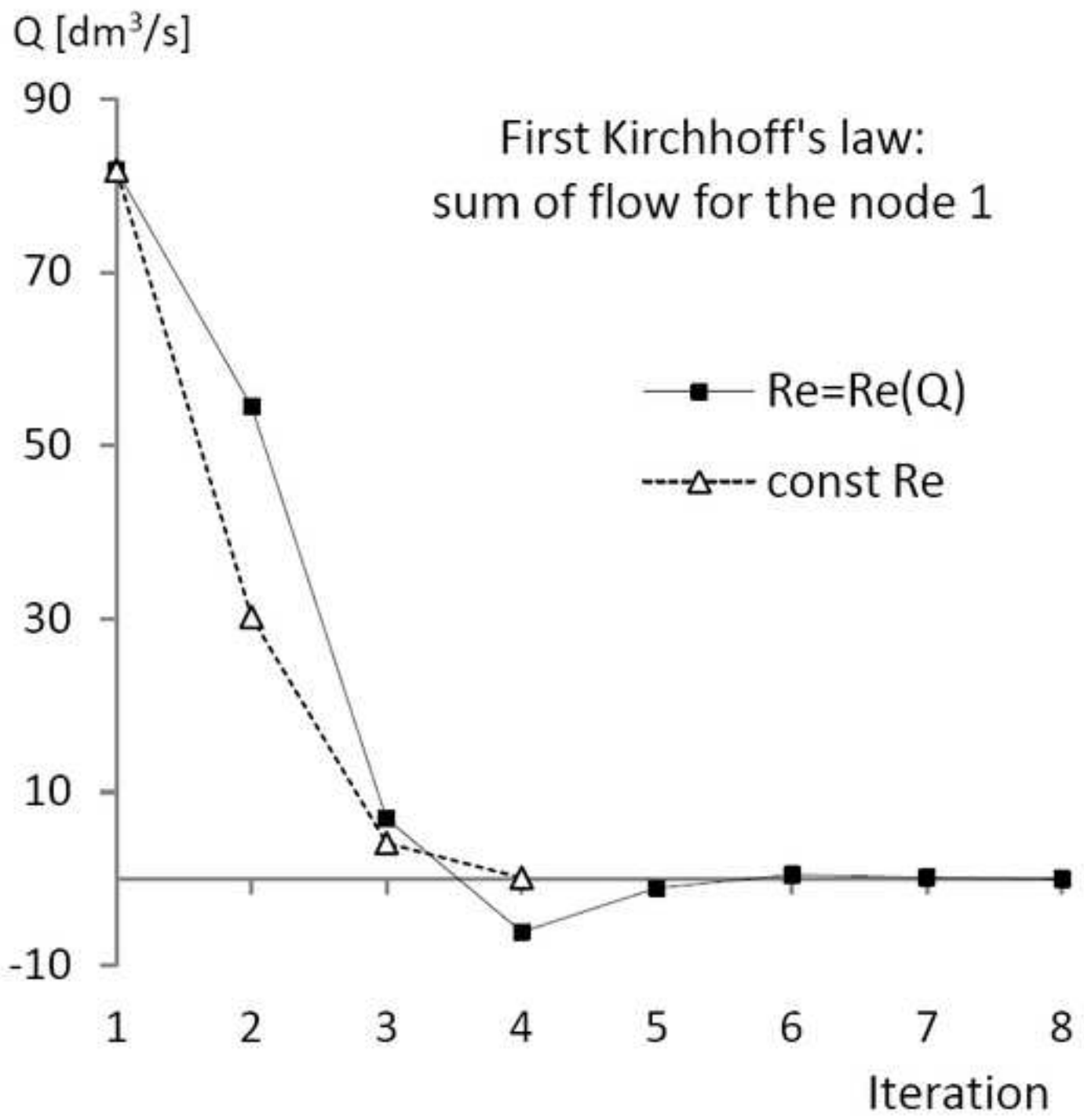
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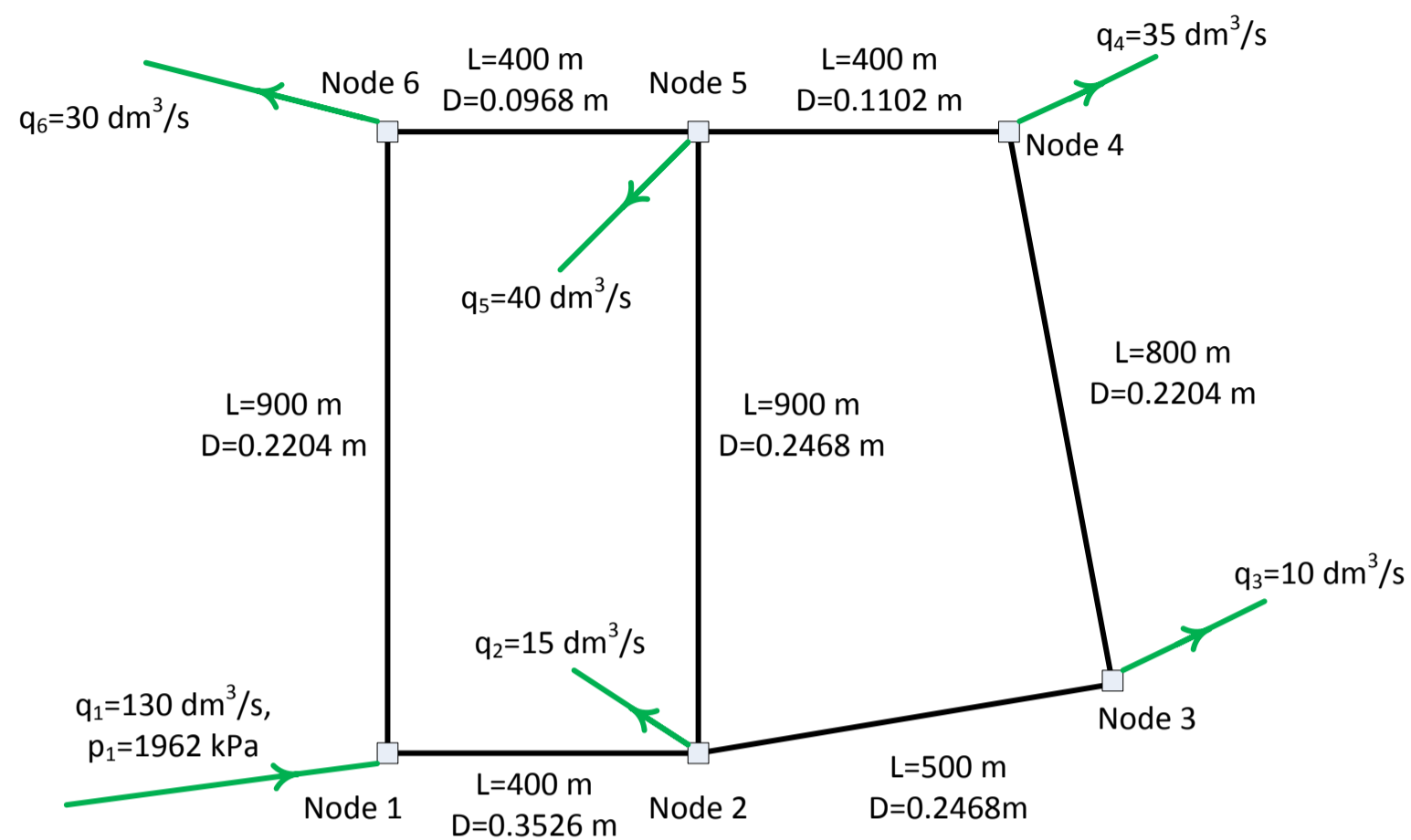
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Figure 1 FB
[Click here to download high resolution image](#)



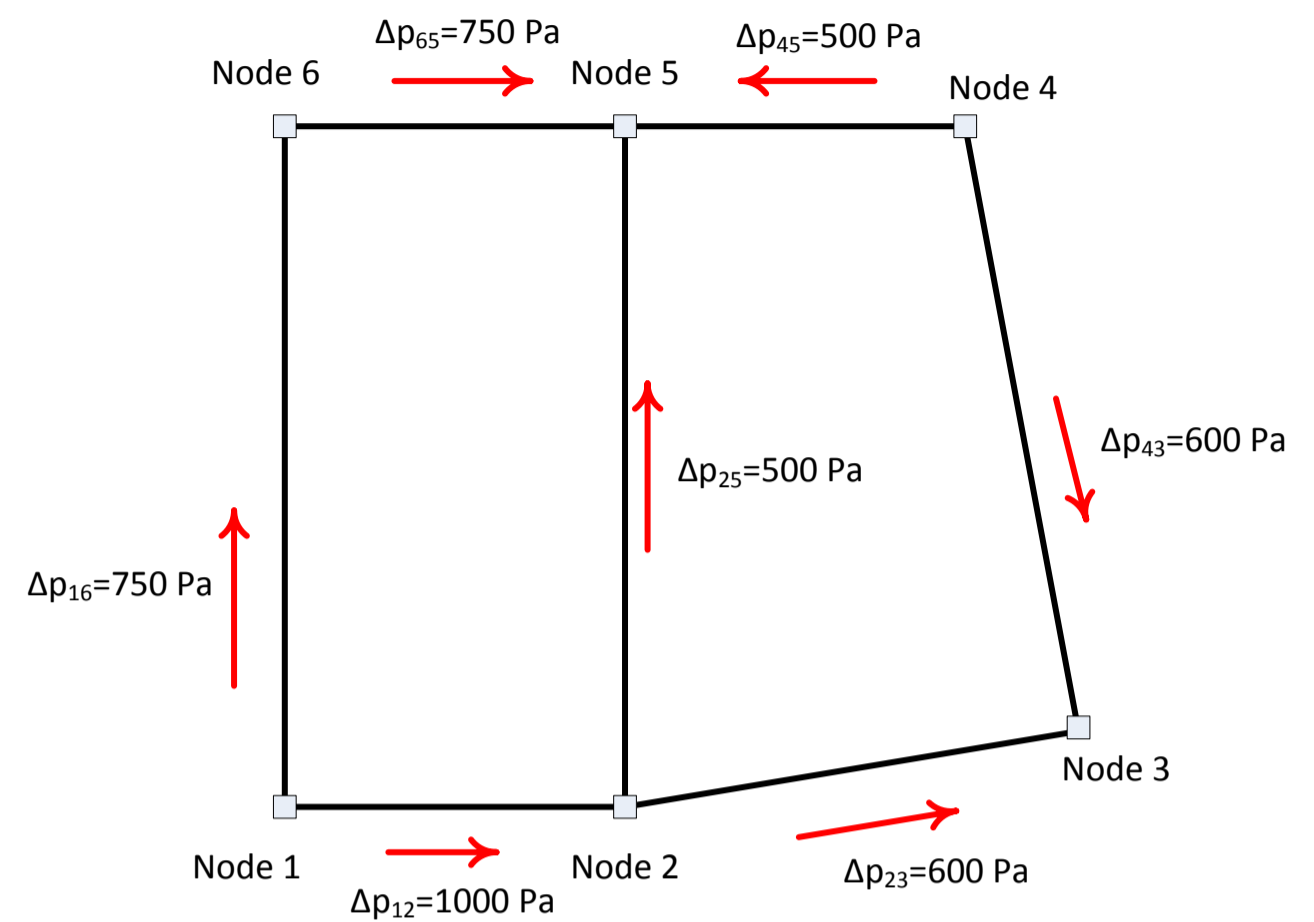
Standard procedure (pressure drop independent Jacobian); Re=const

Parameters of network

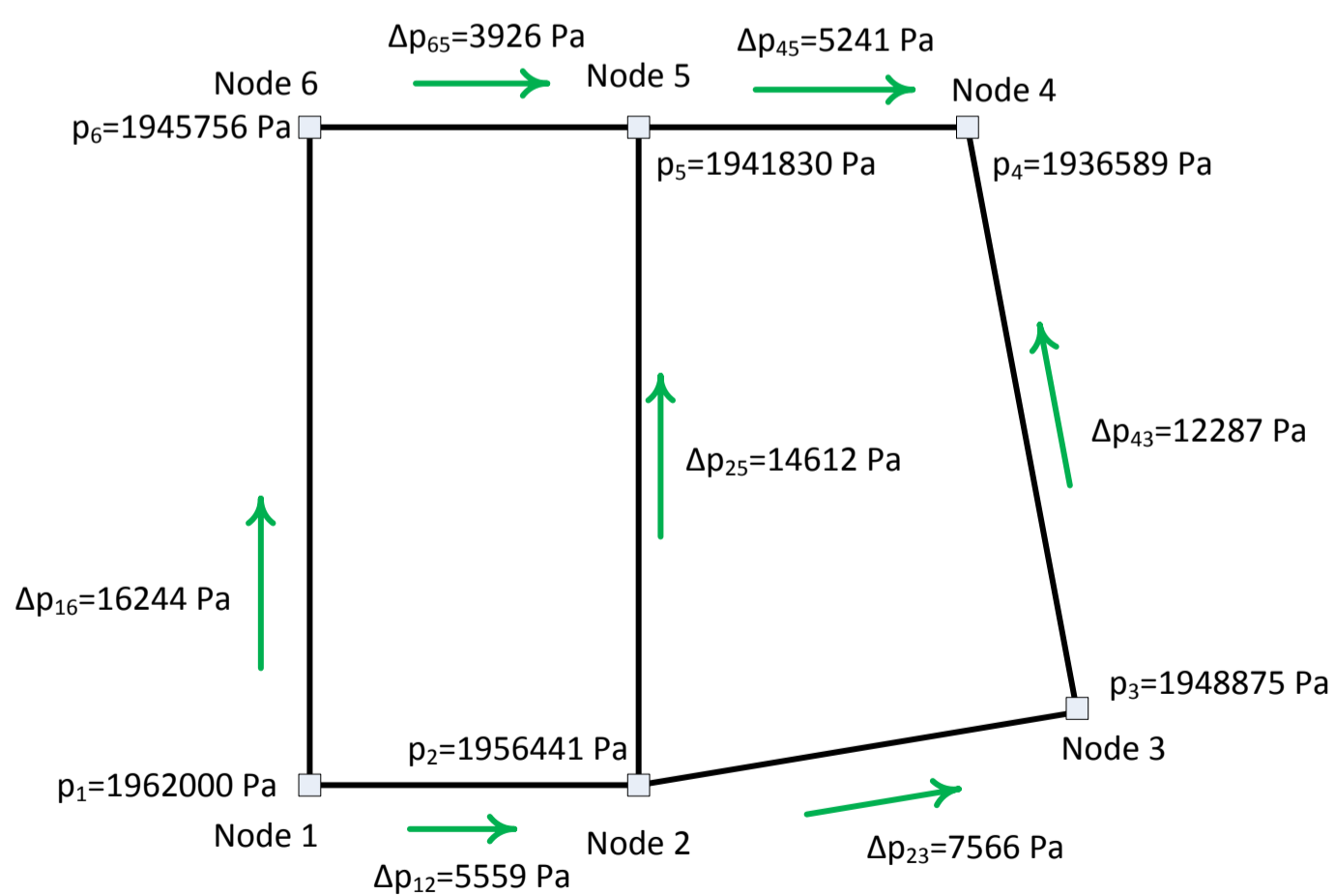


L-length
 D-diameter
 q-water demands and input, Q-flows
 p-pressure, Δp -pressure drop
 ϵ -roughness of pipes; $\epsilon = 3 \cdot 10^{-6} \text{ m}$
 ν -kinematic viscosity of water; $\nu = 1.004 \cdot 10^{-6} \text{ m}^2/\text{s}$
 ρ -water density; $\rho = 1000 \text{ kg/m}^3$

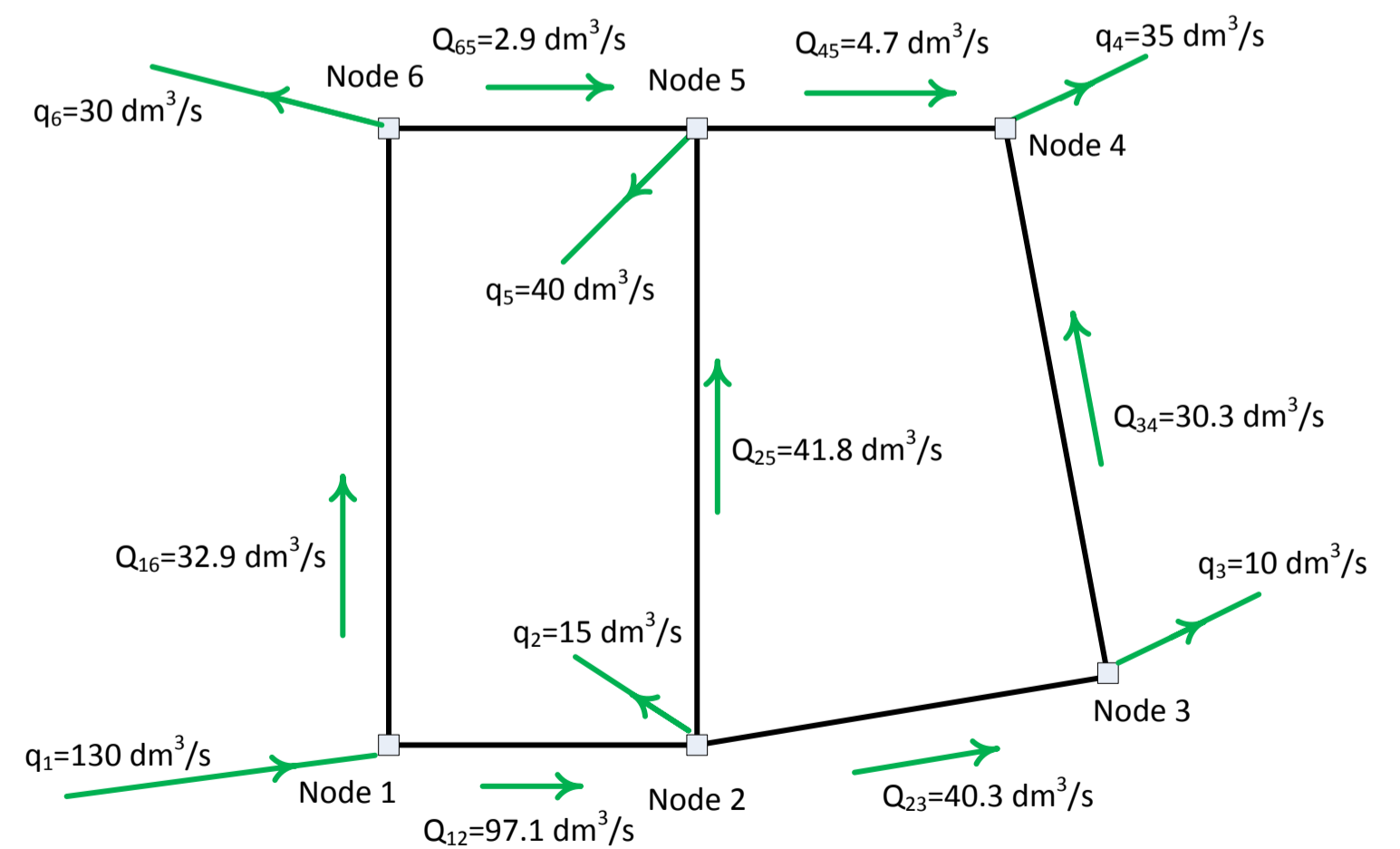
Initial pattern of pressure drops



Final pattern of pressure (after 7 iterations)



Final flows (after 7 iterations)



Standard procedure (pressure drop independent Jacobian); Re-const

Iteration 1

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity** (m/s)	Re***	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-1000	-1	10	3511952.19	0.0000085082	10.04640976	0.009907823	-0.04119	0.00000205950	-7261.262805	4540.579691	-3721
1	16	900	0.2204	-750	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.00707	0.0000047108	-7261.262805	1803.307753	-6208
stopping criterion $\Sigma=0; \Sigma$											0.081743812410				
											Σ	0.0000253058			
2	21	400	0.3526	1000	1	10	3511952.19	0.0000085082	10.04640976	0.009907823	0.04119	0.00000205950	-4540.579691	7261.262805	3721
2	23	500	0.2468	-600	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.01136	0.0000094654	-4540.579691	1577.81829	-3563
2	25	900	0.2468	-500	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.00773	0.0000077285	-4540.579691		-5041
stopping criterion $\Sigma=0; \Sigma$											0.007103023684				
											Σ	0.0000377889			
3	32	500	0.2468	600	1	10	2458167.33	0.0000121556	9.755779122	0.010506936	0.01136	0.0000094654	-1577.81829	4540.579691	3563
3	34	800	0.2204	600	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.00670	0.0000055863	-1577.81829	-4885.424419	-5863
stopping criterion $\Sigma=0; \Sigma$											0.008062080158				
											Σ	0.0000150517			
4	43	800	0.2204	-600	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.00670	0.0000055863	4885.424419	1577.81829	5863
4	45	400	0.1102	-500	-1	10	1097609.56	0.0000272232	9.100473186	0.01207458	-0.00144	0.0000014407	4885.424419		4385
stopping criterion $\Sigma=0; \Sigma$											-0.043144310939				
											Σ	0.0000070270			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	750	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.00707	0.0000047108	-1803.307753	7261.262805	6208
6	65	400	0.0968	-750	-1	10	964143.43	0.0000309917	8.995308178	0.012358561	-0.00126	0.0000008409	-1803.307753		-2553
stopping criterion $\Sigma=0; \Sigma$											-0.024195064118				
											Σ	0.0000055517			

Jacobian

0.0000253058	-0.0000205950	0	0	-0.0000047108
-0.0000205950	0.0000377889	-0.0000094654	0	0
0	-0.0000094654	0.0000150517	-0.0000055863	0
0	0	-0.0000055863	0.0000070270	0
-0.0000047108	0	0	0	0.0000055517

inverse Jacobian

145888.8662	102387.5583	91335.74753	72609.7091	123792.6379
102387.5583	105934.6875	94499.9962	75125.21022	86880.00844
91335.74753	94499.9962	178543.7473	141937.9586	77502.09742
72609.7091	75125.21022	141937.9586	255144.5743	61612.29202
123792.6379	86880.00844	77502.09742	61612.29202	285169.4619

[F]

0.0817438124
0.0071030237
0.0080620802
-0.0431443109
-0.0241950641

[Δx]

7261.262805
4540.579691
1577.81829
-4885.424419
1803.307753

*chosen initial pattern

**assumed velocity 10m/s

***constant value of the Reynolds number

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Standard procedure (pressure drop independent Jacobian); Re-const

Iteration 2

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re**	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)	
1	12	400	0.3526	-3721	-1	10	3511952.19	0.0000085082	10.04640976	0.009907823	-0.07945	0.0000106770	-8824.264907	7167.672128	-5377	
1	16	900	0.2204	-6208	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.02033	0.0000016374	-8824.264907	1171.096558	-13861	
stopping criterion $\Sigma=0; \Sigma$ 0.030218689494												Σ 0.0000123144				
2	21	400	0.3526	3721	1	10	3511952.19	0.0000085082	10.04640976	0.009907823	0.07945	0.0000106770	-7167.672128	8824.264907	5377	
2	23	500	0.2468	-3563	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.02768	0.0000038844	-7167.672128	3955.948689	-6774	
2	25	900	0.2468	-5041	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.02454	0.0000024341	-7167.672128		-12208	
stopping criterion $\Sigma=0; \Sigma$ 0.012234887370												Σ 0.0000169955				
3	32	500	0.2468	3563	1	10	2458167.33	0.0000121556	9.755779122	0.010506936	0.02768	0.0000038844	-3955.948689	7167.672128	6774	
3	34	800	0.2204	-5863	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.02096	0.0000017870	-3955.948689	-1191.181637	-11010	
stopping criterion $\Sigma=0; \Sigma$ -0.003277436616												Σ 0.0000056714				
4	43	800	0.2204	5863	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.02096	0.0000017870	1191.181637	3955.948689	11010	
4	45	400	0.1102	4385	1	10	1097609.56	0.0000272232	9.100473186	0.01207458	0.00427	0.0000004865	1191.181637		5577	
stopping criterion $\Sigma=0; \Sigma$ -0.009777583049												Σ 0.0000022735				
5	omitted to preserve linear independency of Jacobian															
6	61	900	0.2204	6208	1	10	2195219.12	1.36116E-05	9.663698962	0.01070812	0.02033	0.0000016374	-1171.096558	8824.264907	13861	
6	65	400	0.0968	-2553	-1	10	964143.43	3.09917E-05	8.995308178	0.012358561	-0.00233	0.0000004557	-1171.096558		-3724	
stopping criterion $\Sigma=0; \Sigma$ -0.011997517539												Σ 0.0000020931				

Jacobian

0.0000123144	-0.0000106770	0	0	-0.0000016374
-0.0000106770	0.0000169955	-0.0000038844	0	0
0	-0.0000038844	0.0000056714	-0.0000017870	0
0	0	-0.0000017870	0.0000022735	0
-0.0000016374	0	0	0	0.0000020931

inverse Jacobian

390078.41	309443.8036	281712.0059	221433.0028	305149.0192
309443.8036	319775.9225	291118.1789	228826.5008	242070.4933
281712.0059	291118.1789	499398.791	392540.508	220376.5706
221433.0028	228826.5008	392540.508	748396.167	173221.7469
305149.0192	242070.4933	220376.5706	173221.7469	716469.2059

[F]

0.03022
0.01223
-0.00328
-0.00978
-0.01200

[Δx]

8824.264907
7167.672128
3955.948689
-1191.181637
1171.096558

*from the previous iteration

**constant value of the Reynolds number

Dejan Brkic, Strumicka 88, 11050 Belgrade, Serbia; dejanrgf@tesla.rcub.bg.ac.rs, dejanbrkic0611@gmail.com

Standard procedure (pressure drop independent Jacobian); Re-const

Iteration 3

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re**	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-5377	-1	10	3511952.19	0.0000085082	10.04640976	0.009907823	-0.09552	0.0000088814	-2484.809459	2304.674894	-5557
1	16	900	0.2204	-13861	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03038	0.0000010958	-2484.809459	196.7879626	-16149
stopping criterion $\Sigma=0$; Σ 0.004107031176												Σ 0.0000099772			
2	21	400	0.3526	5377	1	10	3511952.19	0.0000085082	10.04640976	0.009907823	0.09552	0.0000088814	-2304.674894	2484.809459	5557
2	23	500	0.2468	-6774	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.03817	0.0000028169	-2304.674894	1539.595414	-7540
2	25	900	0.2468	-12208	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.03819	0.0000015641	-2304.674894		-14513
stopping criterion $\Sigma=0$; Σ 0.004159970650												Σ 0.0000132624			
3	32	500	0.2468	6774	1	10	2458167.33	0.0000121556	9.755779122	0.010506936	0.03817	0.0000028169	-1539.595414	2304.674894	7540
3	34	800	0.2204	-11010	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.02872	0.0000013041	-1539.595414	308.7516734	-12241
stopping criterion $\Sigma=0$; Σ -0.000550070355												Σ 0.0000041210			
4	43	800	0.2204	11010	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.02872	0.0000013041	-308.7516734	1539.595414	12241
4	45	400	0.1102	5577	1	10	1097609.56	0.0000272232	9.100473186	0.01207458	0.00481	0.0000004314	-308.7516734		5268
stopping criterion $\Sigma=0$; Σ -0.001471912869												Σ 0.0000017355			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	13861	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03038	0.0000010958	-196.7879626	2484.809459	16149
6	65	400	0.0968	-3724	-1	10	964143.43	0.0000309917	8.995308178	0.012358561	-0.00281	0.0000003773	-196.7879626		-3921
stopping criterion $\Sigma=0$; Σ -0.002432933964												Σ 0.0000014731			

Jacobian

0.0000099772	-0.0000088814	0	0	-0.0000010958
-0.0000088814	0.0000132624	-0.0000028169	0	0
0	-0.0000028169	0.0000041210	-0.0000013041	0
0	0	-0.0000013041	0.0000017355	0
-0.0000010958	0	0	0	0.0000014731

inverse Jacobian

550959.5984	455776.4985	408740.2066	307136.7631	409834.8885
455776.4985	470180.4494	421657.6648	316843.2374	339032.3193
408740.2066	421657.6648	696502.7013	523368.0997	304044.0669
307136.7631	316843.2374	523368.0997	969483.9882	228465.6832
409834.8885	339032.3193	304044.0669	228465.6832	983690.4851

[F]

0.0041070312
0.0041599706
-0.0005500704
-0.0014719129
-0.0024329340

[Δx]

2484.809459
2304.674894
1539.595414
308.7516734
196.7879626

*from the previous iteration

**constant value of the Reynolds number

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Standard procedure (pressure drop independent Jacobian); Re-const

Iteration 4

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re**	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-5557	-1	10	3511952.19	0.0000085082	10.04640976	0.009907823	-0.09710	0.0000087363	-99.91325401	98.4984645	-5559
1	16	900	0.2204	-16149	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03279	0.0000010152	-99.91325401	4.825012366	-16244
stopping criterion $\Sigma=0$; Σ 0.000108893461												Σ 0.0000097515			
2	21	400	0.3526	5557	1	10	3511952.19	0.0000085082	10.04640976	0.009907823	0.09710	0.0000087363	-98.4984645	99.91325401	5559
2	23	500	0.2468	-7540	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.04026	0.0000026702	-98.4984645	71.81068141	-7566
2	25	900	0.2468	-14513	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.04164	0.0000014345	-98.4984645		-14611
stopping criterion $\Sigma=0$; Σ 0.000200197582												Σ 0.0000128409			
3	32	500	0.2468	7540	1	10	2458167.33	0.0000121556	9.755779122	0.010506936	0.04026	0.0000026702	-71.81068141	98.4984645	7566
3	34	800	0.2204	-12241	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03028	0.0000012368	-71.81068141	26.4205966	-12287
stopping criterion $\Sigma=0$; Σ -0.000015124132												Σ 0.0000039070			
4	43	800	0.2204	12241	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03028	0.0000012368	-26.4205966	71.81068141	12287
4	45	400	0.1102	5268	1	10	1097609.56	0.0000272232	9.100473186	0.01207458	0.00468	0.0000004439	-26.4205966		5241
stopping criterion $\Sigma=0$; Σ -0.000044410182												Σ 0.0000016806			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	16149	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03279	0.0000010152	-4.825012366	99.91325401	16244
6	65	400	0.0968	-3921	-1	10	964143.43	0.0000309917	8.995308178	0.012358561	-0.00288	0.0000003677	-4.825012366		-3926
stopping criterion $\Sigma=0$; Σ -0.000094759151												Σ 0.0000013829			

Jacobian

0.0000097515	-0.0000087363	0	0	-0.0000010152
-0.0000087363	0.0000128409	-0.0000026702	0	0
0	-0.0000026702	0.0000039070	-0.0000012368	0
0	0	-0.0000012368	0.0000016806	0
-0.0000010152	0	0	0	0.0000013829

inverse Jacobian

584493.0924	488088.6072	434889.9307	320034.2533	429069.5712
488088.6072	503170.712	448328.1782	329923.4211	358300.1615
434889.9307	448328.1782	733149.9848	539522.9721	319247.6327
320034.2533	329923.4211	539522.9721	992047.358	234933.4177
429069.5712	358300.1615	319247.6327	234933.4177	1038072.878

[F]

0.0001088935
0.0002001976
-0.0000151241
-0.0000444102
-0.0000947592

[Δx]

99.91325401
98.4984645
71.81068141
26.4205966
4.825012366

*from the previous iteration

**constant value of the Reynolds number

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Standard procedure (pressure drop independent Jacobian); Re-const

Iteration 5

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re**	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-5559	-1	10	3511952.19	0.0000085082	10.04640976	0.009907823	-0.09711	0.0000087351	-0.143966437	0.143953852	-5559
1	16	900	0.2204	-16244	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03289	0.0000010122	-0.143966437	0.003325347	-16244
stopping criterion $\Sigma=0; \Sigma$												0.000000142470			
Σ												0.0000097474			
2	21	400	0.3526	5559	1	10	3511952.19	0.0000085082	10.04640976	0.009907823	0.09711	0.0000087351	-0.143953852	0.143966437	5559
2	23	500	0.2468	-7566	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.04034	0.0000026655	-0.143953852	0.108162659	-7566
2	25	900	0.2468	-14611	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.04178	0.0000014297	-0.143953852		-14612
stopping criterion $\Sigma=0; \Sigma$												0.000000301095			
Σ												0.0000128303			
3	32	500	0.2468	7566	1	10	2458167.33	0.0000121556	9.755779122	0.010506936	0.04034	0.0000026655	-0.108162659	0.143953852	7566
3	34	800	0.2204	-12287	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03034	0.0000012345	-0.108162659	0.039799441	-12287
stopping criterion $\Sigma=0; \Sigma$												-0.000000011007			
Σ												0.0000039000			
4	43	800	0.2204	12287	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03034	0.0000012345	-0.039799441	0.108162659	12287
4	45	400	0.1102	5241	1	10	1097609.56	0.0000272232	9.100473186	0.01207458	0.00466	0.0000004450	-0.039799441		5241
stopping criterion $\Sigma=0; \Sigma$												-0.000000066684			
Σ												0.0000016795			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	16244	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03289	0.0000010122	-0.003325347	0.143966437	16244
6	65	400	0.0968	-3926	-1	10	964143.43	0.0000309917	8.995308178	0.012358561	-0.00289	0.0000003675	-0.003325347		-3926
stopping criterion $\Sigma=0; \Sigma$												-0.0000001411			
Σ												0.0000013797			

Jacobian

0.0000097474	-0.0000087351	0	0	-0.0000010122
-0.0000087351	0.0000128303	-0.0000026655	0	0
0	-0.0000026655	0.0000039000	-0.0000012345	0
0	0	-0.0000012345	0.0000016795	0
-0.0000010122	0	0	0	0.0000013797

inverse Jacobian

585703.1321	489301.4772	435821.8724	320349.9922	429692.3916
489301.4772	504404.3594	449274.0418	330237.9823	358968.7513
435821.8724	449274.0418	734332.2032	539769.4115	319734.2346
320349.9922	330237.9823	539769.4115	992184.6484	235020.0072
429692.3916	358968.7513	319734.2346	235020.0072	1040013.305

[F]

0.0000001425
0.0000003011
-0.0000000110
-0.0000000667
-0.0000001411

[Δx]

0.143966437
0.143953852
0.108162659
0.039799441
0.003325347

*from the previous iteration

**constant value of the Reynolds number

Dejan Brkic, Strumicka 88, 11050 Belgrade, Serbia; dejanrgf@tesla.rcub.bg.ac.rs, dejanbrkic0611@gmail.com

Standard procedure (pressure drop independent Jacobian); Re-const

Iteration 6

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re**	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-5559	-1	10	3511952.19	0.0000085082	10.04640976	0.009907823	-0.09711	0.0000087351	-0.0000003050	0.0000003050	-5559
1	16	900	0.2204	-16244	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03289	0.0000010122	-0.0000003050	0.0000000006	-16244
0.13000															
stopping criterion $\Sigma=0$; Σ 0.000000000000															
Σ 0.0000097474															
2	21	400	0.3526	5559	1	10	3511952.19	0.0000085082	10.04640976	0.009907823	0.09711	0.0000087351	-0.0000003050	0.0000003050	5559
2	23	500	0.2468	-7566	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.04034	0.0000026655	-0.0000003050	0.0000002361	-7566
2	25	900	0.2468	-14612	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.04178	0.0000014297	-0.0000003050		-14612
-0.01500															
stopping criterion $\Sigma=0$; Σ 0.000000000001															
Σ 0.0000128303															
3	32	500	0.2468	7566	1	10	2458167.33	0.0000121556	9.755779122	0.010506936	0.04034	0.0000026655	-0.0000002361	0.0000003050	7566
3	34	800	0.2204	-12287	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03034	0.0000012345	-0.0000002361	0.0000000836	-12287
-0.01000															
stopping criterion $\Sigma=0$; Σ 0.000000000000															
Σ 0.0000038999															
4	43	800	0.2204	12287	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03034	0.0000012345	-0.0000000836	0.0000002361	12287
4	45	400	0.1102	5241	1	10	1097609.56	0.0000272232	9.100473186	0.01207458	0.00466	0.0000004450	-0.0000000836		5241
-0.03500															
stopping criterion $\Sigma=0$; Σ 0.000000000000															
Σ 0.0000016795															
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	16244	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03289	0.0000010122	-0.0000000006	0.0000003050	16244
6	65	400	0.0968	-3926	-1	10	964143.43	0.0000309917	8.995308178	0.012358561	-0.00289	0.0000003675	-0.0000000006		-3926
-0.03000															
stopping criterion $\Sigma=0$; Σ 0.000000000000															
Σ 0.0000013797															

Jacobian

0.0000097474	-0.0000087351	0	0	-0.0000010122
-0.0000087351	0.0000128303	-0.0000026655	0	0
0	-0.0000026655	0.0000038999	-0.0000012345	0
0	0	-0.0000012345	0.0000016795	0
-0.0000010122	0	0	0	0.0000013797

inverse Jacobian

585704.8442	489303.2155	435823.2102	320350.4172	429693.2006
489303.2155	504406.1292	449275.4013	330238.406	358969.6532
435823.2102	449275.4013	734333.8498	539769.6809	319734.8835
320350.4172	330238.406	539769.6809	992184.7735	235020.0745
429693.2006	358969.6532	319734.8835	235020.0745	1040015.954

[F]

0.0000000000
0.0000000000
0.0000000000
0.0000000000
0.0000000000

[Δx]

0.0000003050
0.0000003050
0.0000002361
0.0000000836
0.0000000006

*from the previous iteration

**constant value of the Reynolds number

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Standard procedure (pressure drop independent Jacobian); Re-const

Iteration 7

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re**	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	Pipe	Final pressure drop (Pa)	Final flow distribution (dm ³ /s)
1	12	400	0.3526	-5559	-1	10	3511952.19	0.0000085082	10.04640976	0.009907823	-0.09711	0.0000087351	0	0	12	-5559	-97.1
1	16	900	0.2204	-16244	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03289	0.0000010122	0	0	16	-16244	-32.9
stopping criterion $\Sigma=0$; Σ 0.00000000000000000000000000000000												Σ 0.0000097474					
2	21	400	0.3526	5559	1	10	3511952.19	0.0000085082	10.04640976	0.009907823	0.09711	0.0000087351	0	0	21	5559	97.1
2	23	500	0.2468	-7566	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.04034	0.0000026655	0	0	23	-7566	-40.3
2	25	900	0.2468	-14612	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.04178	0.0000014297	0		25	-14612	-41.8
stopping criterion $\Sigma=0$; Σ 0.00000000000000000000000000000000												Σ 0.0000128303					
3	32	500	0.2468	7566	1	10	2458167.33	0.0000121556	9.755779122	0.010506936	0.04034	0.0000026655	0	0	32	7566	40.3
3	34	800	0.2204	-12287	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.03034	0.0000012345	0	0	34	-12287	-30.3
stopping criterion $\Sigma=0$; Σ 0.00000000000000000000000000000000												Σ 0.0000038999					
4	43	800	0.2204	12287	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03034	0.0000012345	0	0	43	12287	30.3
4	45	400	0.1102	5241	1	10	1097609.56	0.0000272232	9.100473186	0.01207458	0.00466	0.0000004450	0		45	5241	4.7
stopping criterion $\Sigma=0$; Σ 0.00000000000000000000000000000000												Σ 0.0000016795					
5	omitted to preserve linear independency of Jacobian																
6	61	900	0.2204	16244	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.03289	0.0000010122	0	0	61	16244	32.9
6	65	400	0.0968	-3926	-1	10	964143.43	0.0000309917	8.995308178	0.012358561	-0.00289	0.0000003675	0		65	-3926	-2.9
stopping criterion $\Sigma=0$; Σ 0.00000000000000000000000000000000												Σ 0.0000013797					

Jacobian

0.0000097474	-0.0000087351	0	0	-0.0000010122
-0.0000087351	0.0000128303	-0.0000026655	0	0
0	-0.0000026655	0.0000038999	-0.0000012345	0
0	0	-0.0000012345	0.0000016795	0
-0.0000010122	0	0	0	0.0000013797

inverse Jacobian

585704.8442	489303.2155	435823.2102	320350.4172	429693.2006
489303.2155	504406.1292	449275.4013	330238.406	358969.6532
435823.2102	449275.4013	734333.8498	539769.6809	319734.8835
320350.4172	330238.406	539769.6809	992184.7735	235020.0745
429693.2006	358969.6532	319734.8835	235020.0745	1040015.954

[F]

0.0000000000
0.0000000000
0.0000000000
0.0000000000
0.0000000000

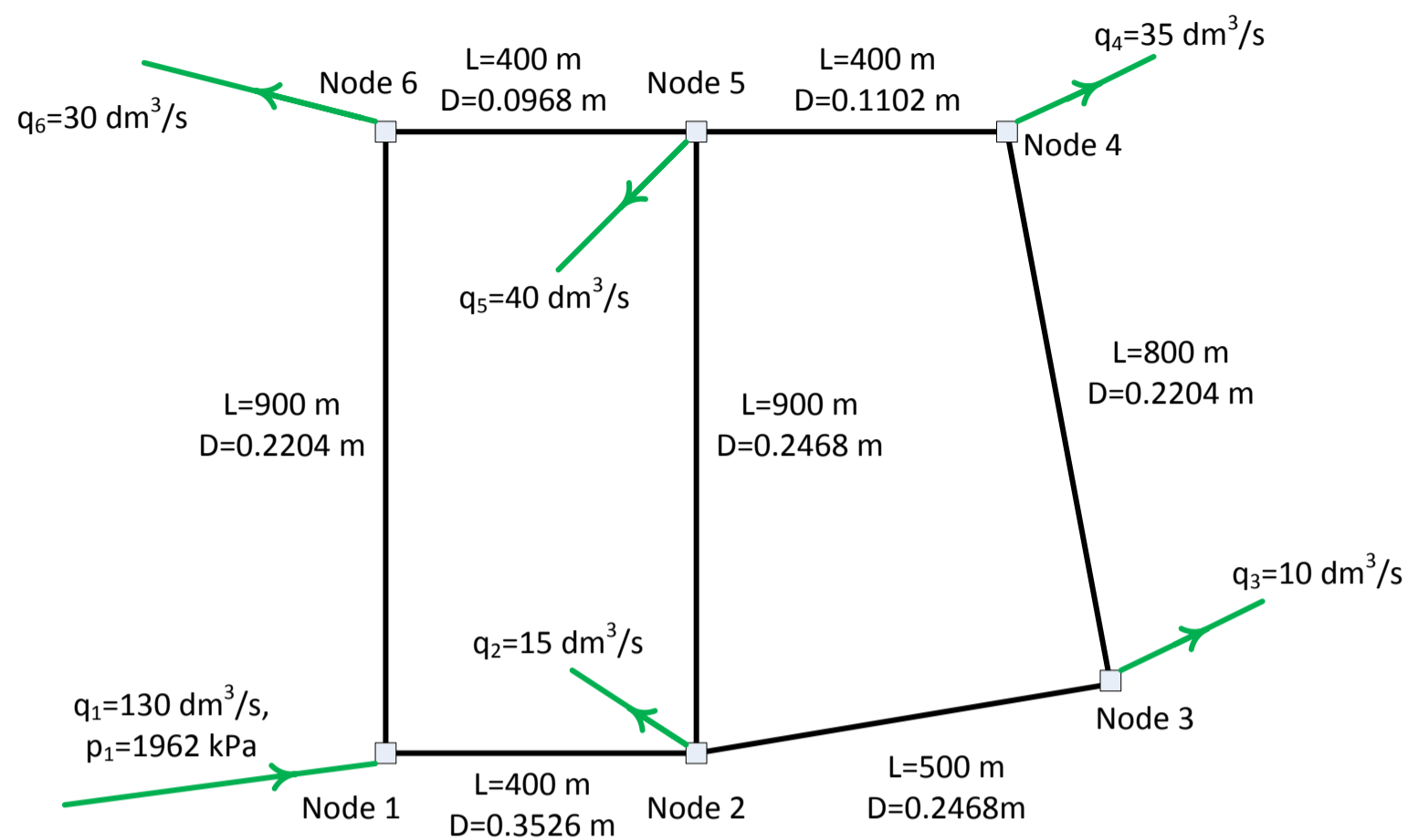
[Δx]

0
0
0
0
0

*from the previous iteration
 **constant value of the Reynolds number
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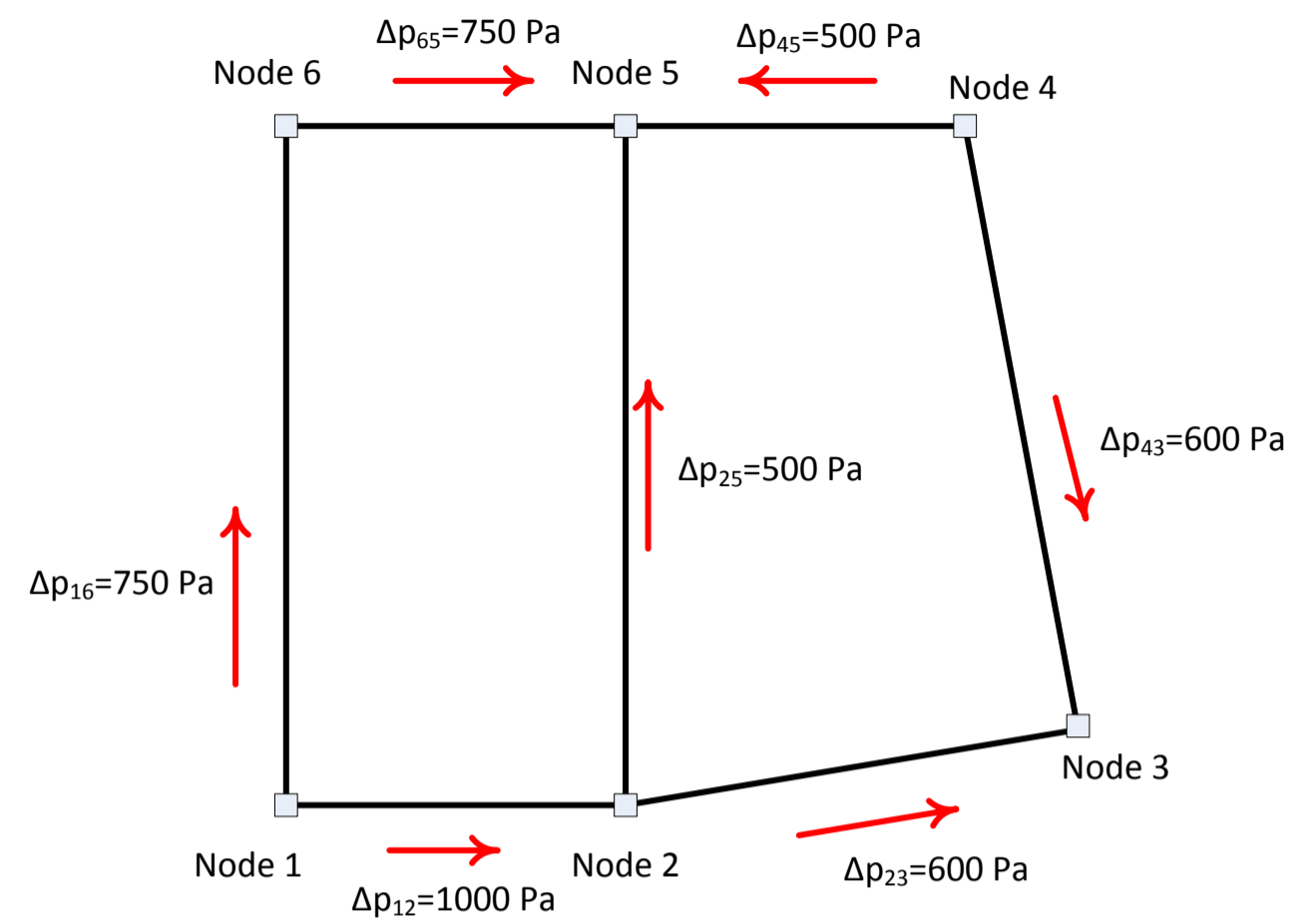
Standard procedure (pressure drop independent Jacobian); $Re=Re(Q)$

Parameters of network

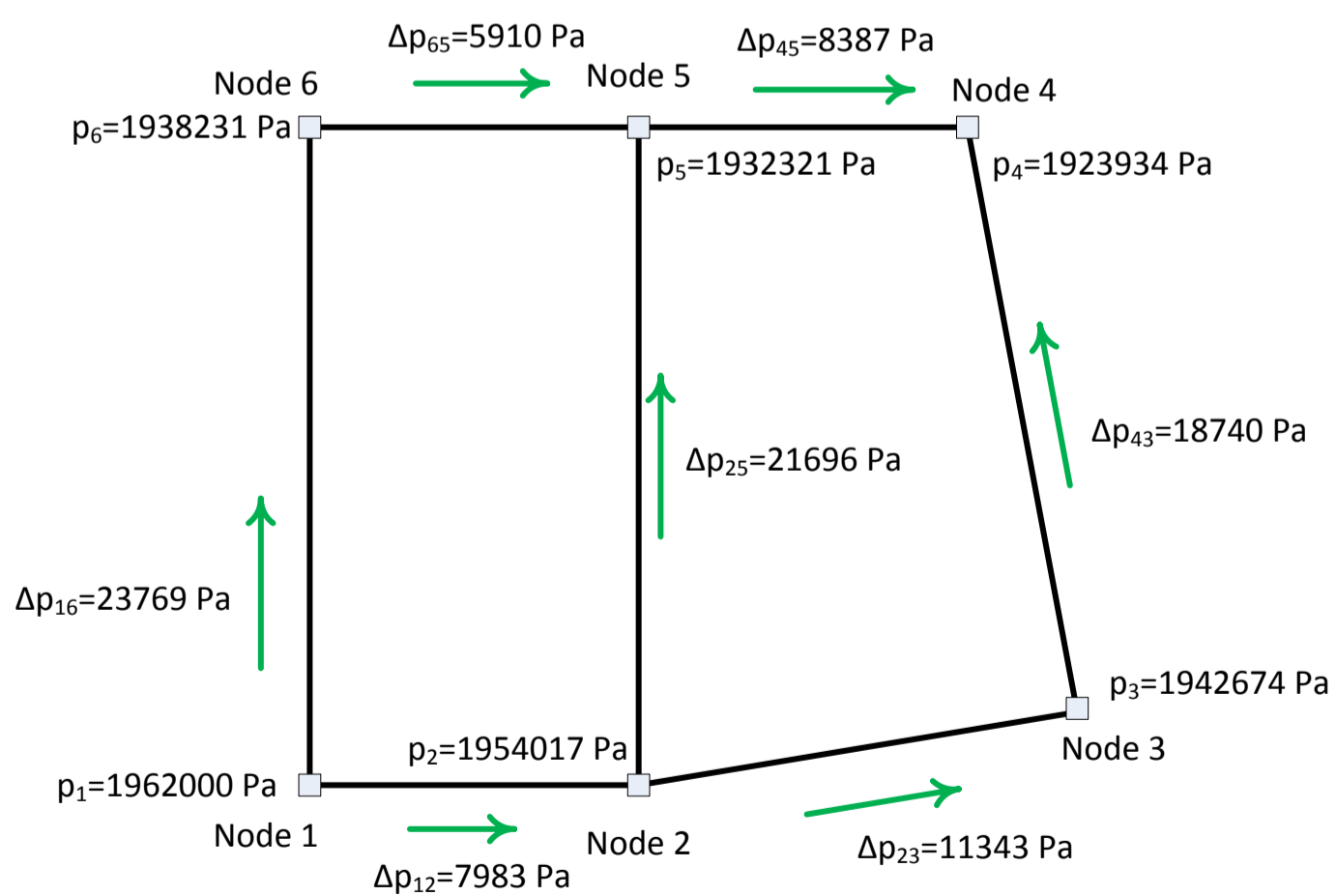


L-length
 D-diameter
 q-water demands and input, Q-flows
 p-pressure, Δp -pressure drop
 ϵ -roughness of pipes; $\epsilon=3 \cdot 10^{-6} \text{ m}$
 ν -kinematic viscosity of water; $\nu=1.004 \cdot 10^{-6} \text{ m}^2/\text{s}$
 ρ -water density; $\rho=1000 \text{ kg/m}^3$

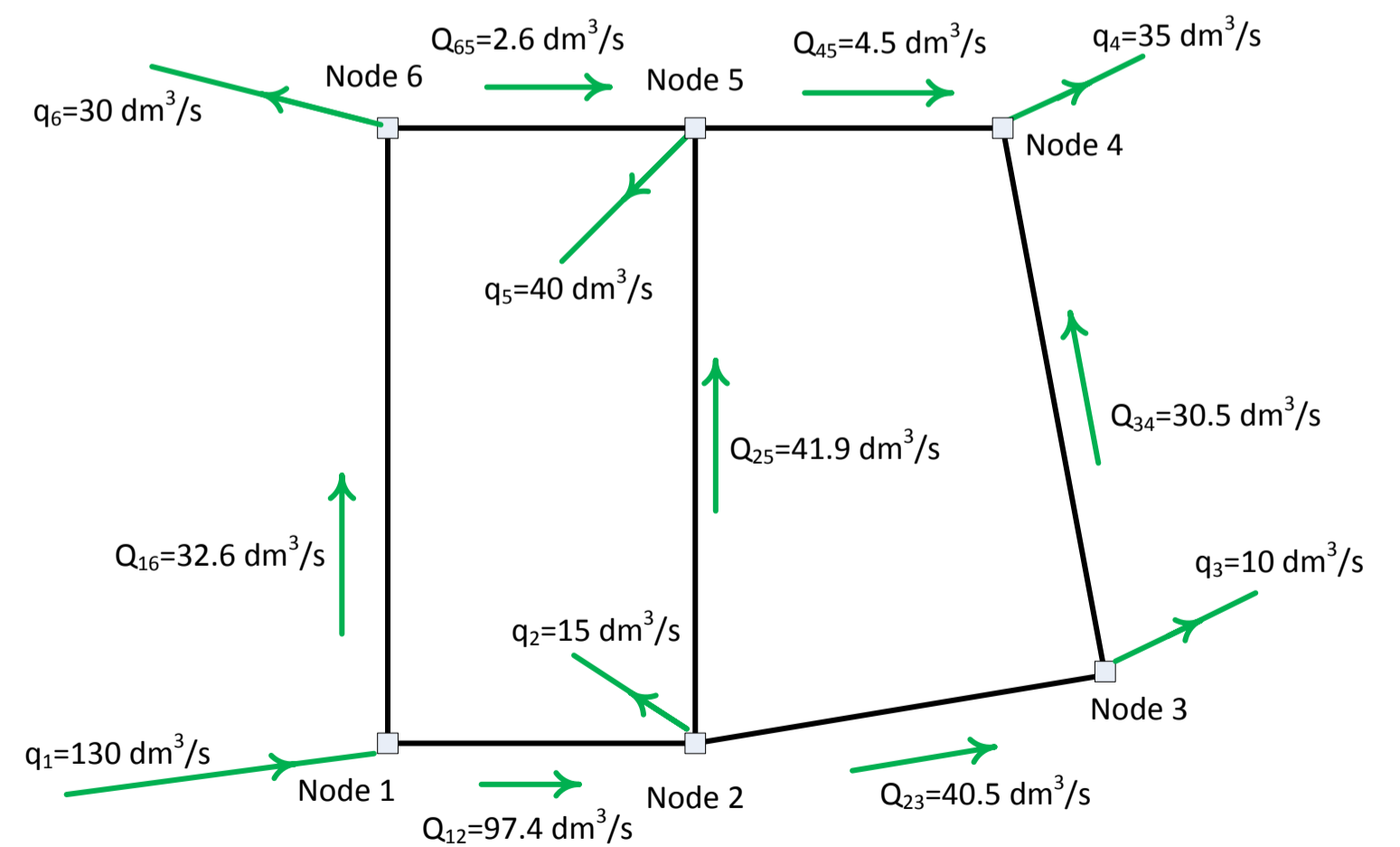
Initial pattern of pressure drops



Final pattern of pressure (after 12 iterations)



Final flows (after 12 iterations)



Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 1

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity** (m/s)	Re***	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-1000	-1	10	3511952.19	0.0000085082	10.04640976	0.009907823	-0.04119	0.0000205950	-7261.262805	4540.579691	-3721
1	16	900	0.2204	-750	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.00707	0.0000047108	-7261.262805	1803.307753	-6208
stopping criterion $\Sigma=0$; Σ 0.081743812410												Σ 0.0000253058			
2	21	400	0.3526	1000	1	10	3511952.19	0.0000085082	10.04640976	0.009907823	0.04119	0.0000205950	-4540.579691	7261.262805	3721
2	23	500	0.2468	-600	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.01136	0.0000094654	-4540.579691	1577.81829	-3563
2	25	900	0.2468	-500	-1	10	2458167.33	0.0000121556	9.755779122	0.010506936	-0.00773	0.0000077285	-4540.579691		-5041
stopping criterion $\Sigma=0$; Σ 0.007103023684												Σ 0.0000377889			
3	32	500	0.2468	600	1	10	2458167.33	0.0000121556	9.755779122	0.010506936	0.01136	0.0000094654	-1577.81829	4540.579691	3563
3	34	800	0.2204	600	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.00670	0.0000055863	-1577.81829	-4885.424419	-5863
stopping criterion $\Sigma=0$; Σ 0.008062080158												Σ 0.0000150517			
4	43	800	0.2204	-600	-1	10	2195219.12	0.0000136116	9.663698962	0.01070812	-0.00670	0.0000055863	4885.424419	1577.81829	5863
4	45	400	0.1102	-500	-1	10	1097609.56	0.0000272232	9.100473186	0.01207458	-0.00144	0.0000014407	4885.424419		4385
stopping criterion $\Sigma=0$; Σ -0.043144310939												Σ 0.0000070270			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	750	1	10	2195219.12	0.0000136116	9.663698962	0.01070812	0.00707	0.0000047108	-1803.307753	7261.262805	6208
6	65	400	0.0968	-750	-1	10	964143.43	0.0000309917	8.995308178	0.012358561	-0.00126	0.0000008409	-1803.307753		-2553
stopping criterion $\Sigma=0$; Σ -0.024195064118												Σ 0.0000055517			

Jacobian

0.0000253058	-0.0000205950	0	0	-0.0000047108
-0.0000205950	0.0000377889	-0.0000094654	0	0
0	-0.0000094654	0.0000150517	-0.0000055863	0
0	0	-0.0000055863	0.0000070270	0
-0.0000047108	0	0	0	0.0000055517

inverse Jacobian

145888.8662	102387.5583	91335.74753	72609.7091	123792.6379
102387.5583	105934.6875	94499.9962	75125.21022	86880.00844
91335.74753	94499.9962	178543.7473	141937.9586	77502.09742
72609.7091	75125.21022	141937.9586	255144.5743	61612.29202
123792.6379	86880.00844	77502.09742	61612.29202	285169.4619

[F]

0.0817438124
0.0071030237
0.0080620802
-0.0431443109
-0.0241950641

[Δx]

7261.262805
4540.579691
1577.81829
-4885.424419
1803.307753

*chosen initial pattern

**assumed velocity 10m/s for the start of calculation

***the Reynolds number calculated for velocity of 10m/s

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Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 2

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity** (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-3721	-1	0.42	148144.54	0.0000085082	7.748505571	0.016655746	-0.06128	0.0000082349	-19290.31926	14899.69072	-8111
1	16	900	0.2204	-6208	-1	0.19	40658.51	0.0000136116	6.752406402	0.021932233	-0.01421	0.0000011441	-19290.31926	3243.047121	-22255
stopping criterion $\Sigma=0$; Σ 0.054516105450											Σ 0.0000093790				
2	21	400	0.3526	3721	1	0.42	148144.54	0.0000085082	7.748505571	0.016655746	0.06128	0.0000082349	-14899.69072	19290.31926	8111
2	23	500	0.2468	-3563	-1	0.24	58364.88	0.0000121556	7.029694664	0.020236112	-0.01994	0.0000027990	-14899.69072	7571.140532	-10891
2	25	900	0.2468	-5041	-1	0.16	39712.27	0.0000121556	6.735132125	0.022044881	-0.01694	0.0000016804	-14899.69072		-19940
stopping criterion $\Sigma=0$; Σ 0.009393981992											Σ 0.0000127143				
3	32	500	0.2468	3563	1	0.24	58364.88	0.0000121556	7.029694664	0.020236112	0.01994	0.0000027990	-7571.140532	14899.69072	10891
3	34	800	0.2204	-5863	-1	0.18	38572.05	0.0000136116	6.71218502	0.022195869	-0.01456	0.0000012412	-7571.140532	-5239.473291	-18674
stopping criterion $\Sigma=0$; Σ -0.004611303413											Σ 0.0000040402				
4	43	800	0.2204	5863	1	0.18	38572.05	0.0000136116	6.71218502	0.022195869	0.01456	0.0000012412	5239.473291	7571.140532	18674
4	45	400	0.1102	4385	1	0.15	16579.55	0.0000272232	6.066929703	0.027168276	0.00284	0.0000003243	5239.473291		9625
stopping criterion $\Sigma=0$; Σ -0.017600196387											Σ 0.0000015655				
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	6208	1	0.19	40658.51	0.0000136116	6.752406402	0.021932233	0.01421	0.0000011441	-3243.047121	19290.31926	22255
6	65	400	0.0968	-2553	-1	0.17	16523.84	0.0000309917	6.063556479	0.027198512	-0.00157	0.0000003072	-3243.047121		-5796
stopping criterion $\Sigma=0$; Σ -0.017363570708											Σ 0.0000014513				

Jacobian

0.0000093790	-0.0000082349	0	0	-0.0000011441
-0.0000082349	0.0000127143	-0.0000027990	0	0
0	-0.0000027990	0.0000040402	-0.0000012412	0
0	0	-0.0000012412	0.0000015655	0
-0.0000011441	0	0	0	0.0000014513

inverse Jacobian

556645.4698	451580.4812	413586.0364	327909.4576	438822.2534
451580.4812	464860.4479	425748.6719	337552.5376	355996.0066
413586.0364	425748.6719	717143.8627	568583.6425	326043.7142
327909.4576	337552.5376	568583.6425	1089553.347	258501.9998
438822.2534	355996.0066	326043.7142	258501.9998	1034976.103

[F]

0.0545161055
0.0093939820
-0.0046113034
-0.0176001964
-0.0173635707

[Δx]

19290.31926
14899.69072
7571.140532
-5239.473291
3243.047121

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

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Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 3

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-8111	-1	0.63	220396.17	0.0000085082	8.053574983	0.015417806	-0.09404	0.0000057969	-6197.128544	5614.524376	-8694
1	16	900	0.2204	-22255	-1	0.37	81735.41	0.0000136116	7.286294946	0.018835906	-0.02902	0.0000006520	-6197.128544	737.810175	-27715
stopping criterion $\Sigma=0$; Σ 0.006936969187												Σ 0.0000064489			
2	21	400	0.3526	8111	1	0.63	220396.17	0.0000085082	8.053574983	0.015417806	0.09404	0.0000057969	-5614.524376	6197.128544	8694
2	23	500	0.2468	-10891	-1	0.42	102481.12	0.0000121556	7.460882598	0.017964684	-0.03701	0.0000016990	-5614.524376	3530.014773	-12976
2	25	900	0.2468	-19940	-1	0.35	87049.07	0.0000121556	7.335888597	0.018582089	-0.03670	0.0000009202	-5614.524376		-25555
stopping criterion $\Sigma=0$; Σ 0.005331117729												Σ 0.0000084162			
3	32	500	0.2468	10891	1	0.42	102481.12	0.0000121556	7.460882598	0.017964684	0.03701	0.0000016990	-3530.014773	5614.524376	12976
3	34	800	0.2204	-18674	-1	0.38	83750.33	0.0000136116	7.304920399	0.018739976	-0.02827	0.0000007569	-3530.014773	515.961554	-21688
stopping criterion $\Sigma=0$; Σ -0.001260223488												Σ 0.0000024560			
4	43	800	0.2204	18674	1	0.38	83750.33	0.0000136116	7.304920399	0.018739976	0.02827	0.0000007569	-515.961554	3530.014773	21688
4	45	400	0.1102	9625	1	0.30	32733.95	0.0000272232	6.581436002	0.023086531	0.00457	0.0000002375	-515.961554		9109
stopping criterion $\Sigma=0$; Σ -0.002158906865												Σ 0.0000009944			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	22255	1	0.37	81735.41	0.0000136116	7.286294946	0.018835906	0.02902	0.0000006520	-737.810175	6197.128544	27715
6	65	400	0.0968	-5796	-1	0.21	20551.49	0.0000309917	6.228113731	0.025780239	-0.00243	0.0000002094	-737.810175		-6534
stopping criterion $\Sigma=0$; Σ -0.003405176411												Σ 0.0000008615			

Jacobian				
0.0000064489	-0.0000057969	0	0	-0.0000006520
-0.0000057969	0.0000084162	-0.0000016990	0	0
0	-0.0000016990	0.0000024560	-0.0000007569	0
0	0	-0.0000007569	0.0000009944	0
-0.0000006520	0	0	0	0.0000008615

inverse Jacobian				
933294.7716	786307.8105	710695.2482	540972.7189	706413.7612
786307.8105	807808.4521	730128.3552	555764.9674	595158.8659
710695.2482	730128.3552	1191889.071	907251.6987	537927.4787
540972.7189	555764.9674	907251.6987	1696210.772	409463.96
706413.7612	595158.8659	537927.4787	409463.96	1695512.422

[F]
0.0069369692
0.0053311177
-0.0012602235
-0.0021589069
-0.0034051764

[Δx]
6197.128544
5614.524376
3530.014773
515.961554
737.810175

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

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Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 4

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)		
1	12	400	0.3526	-8694	-1	0.96	338227.57	0.0000085082	8.380949593	0.014236838	-0.10132	0.0000058269	4093.117544	-3421.522911	-8022		
1	16	900	0.2204	-27715	-1	0.76	166993.74	0.0000136116	7.831619812	0.016304099	-0.03481	0.0000006280	4093.117544	-566.7642687	-24188		
stopping criterion $\Sigma=0$; Σ											-0.006127957496						
												Σ	0.0000064549				
2	21	400	0.3526	8694	1	0.96	338227.57	0.0000085082	8.380949593	0.014236838	0.10132	0.0000058269	3421.522911	-4093.117544	8022		
2	23	500	0.2468	-12976	-1	0.77	190171.01	0.0000121556	7.933209399	0.015889205	-0.04295	0.0000016551	3421.522911	-1948.574579	-11503		
2	25	900	0.2468	-25555	-1	0.77	188580.17	0.0000121556	7.926813297	0.015914857	-0.04489	0.0000008784	3421.522911		-22133		
stopping criterion $\Sigma=0$; Σ											-0.001530003977						
												Σ	0.0000083604				
3	32	500	0.2468	12976	1	0.77	190171.01	0.0000121556	7.933209399	0.015889205	0.04295	0.0000016551	1948.574579	-3421.522911	11503		
3	34	800	0.2204	-21688	-1	0.74	162662.02	0.0000136116	7.81162916	0.016387653	-0.03258	0.0000007511	1948.574579	798.7639351	-18941		
stopping criterion $\Sigma=0$; Σ											0.000374436202						
												Σ	0.0000024062				
4	43	800	0.2204	21688	1	0.74	162662.02	0.0000136116	7.81162916	0.016387653	0.03258	0.0000007511	-798.7639351	-1948.574579	18941		
4	45	400	0.1102	9109	1	0.48	52606.86	0.0000272232	6.940828154	0.020757613	0.00469	0.0000002574	-798.7639351		8310		
stopping criterion $\Sigma=0$; Σ											0.002269130779						
												Σ	0.0000010085				
5	omitted to preserve linear independency of Jacobian																
6	61	900	0.2204	27715	1	0.76	166993.74	0.0000136116	7.831619812	0.016304099	0.03481	0.0000006280	566.7642687	-4093.117544	24188		
6	65	400	0.0968	-6534	-1	0.33	31805.22	0.0000309917	6.558142742	0.02325082	-0.00271	0.0000002077	566.7642687		-5967		
stopping criterion $\Sigma=0$; Σ											0.002096943068						
												Σ	0.0000008357				

Jacobian				
0.0000064549	-0.0000058269	0	0	-0.0000006280
-0.0000058269	0.0000083604	-0.0000016551	0	0
0	-0.0000016551	0.0000024062	-0.0000007511	0
0	0	-0.0000007511	0.0000010085	0
-0.0000006280	0	0	0	0.0000008357

inverse Jacobian				
956117.2803	810109.0778	726010.6185	540687.0449	718505.1279
810109.0778	831808.3481	745457.2598	555169.6802	608782.5611
726010.6185	745457.2598	1209528.934	900781.0746	545584.0649
540687.0449	555169.6802	900781.0746	1662387.64	406316.696
718505.1279	608782.5611	545584.0649	406316.696	1736514.579

[F]
-0.0061279575
-0.0015300040
0.0003744362
0.0022691308
0.0020969431

[Δx]
-4093.117544
-3421.522911
-1948.574579
798.7639351
-566.7642687

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

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Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 5

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-8022	-1	1.04	364397.76	0.0000085082	8.437656496	0.014046118	-0.09798	0.0000061069	822.7871268	-726.6128476	-7926
1	16	900	0.2204	-24188	-1	0.91	200300.90	0.0000136116	7.969712731	0.015743985	-0.03309	0.0000006841	822.7871268	-105.5573975	-23471
stopping criterion $\Sigma=0$; Σ -0.001077990309												Σ 0.0000067910			
2	21	400	0.3526	8022	1	1.04	364397.76	0.0000085082	8.437656496	0.014046118	0.09798	0.0000061069	726.6128476	-822.7871268	7926
2	23	500	0.2468	-11503	-1	0.90	220714.24	0.0000121556	8.046465161	0.015445065	-0.04102	0.0000017830	726.6128476	-439.5999529	-11216
2	25	900	0.2468	-22133	-1	0.94	230681.41	0.0000121556	8.079990194	0.015317163	-0.04259	0.0000009621	726.6128476	0	-21407
stopping criterion $\Sigma=0$; Σ -0.000623468032												Σ 0.0000088520			
3	32	500	0.2468	11503	1	0.90	220714.24	0.0000121556	8.046465161	0.015445065	0.04102	0.0000017830	439.5999529	-726.6128476	11216
3	34	800	0.2204	-18941	-1	0.85	187458.07	0.0000136116	7.919447902	0.015944474	-0.03087	0.0000008148	439.5999529	-0.015372369	-18501
stopping criterion $\Sigma=0$; Σ 0.000153569358												Σ 0.0000025978			
4	43	800	0.2204	18941	1	0.85	187458.07	0.0000136116	7.919447902	0.015944474	0.03087	0.0000008148	0.015372369	-439.5999529	18501
4	45	400	0.1102	8310	1	0.49	53972.04	0.0000272232	6.960221573	0.0206421	0.00449	0.0000002703	0.015372369		8310
stopping criterion $\Sigma=0$; Σ 0.000358174487												Σ 0.0000010851			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	24188	1	0.91	200300.90	0.0000136116	7.969712731	0.015743985	0.03309	0.0000006841	105.56	-823	23471
6	65	400	0.0968	-5967	-1	0.37	35558.24	0.0000309917	6.642491483	0.022664074	-0.00263	0.0000002201	105.56		-5862
stopping criterion $\Sigma=0$; Σ 0.0004674254												Σ 0.0000009042			

Jacobian

0.0000067910	-0.0000061069	0	0	-0.0000006841
-0.0000061069	0.0000088520	-0.0000017830	0	0
0	-0.0000017830	0.0000025978	-0.0000008148	0
0	0	-0.0000008148	0.0000010851	0
-0.0000006841	0	0	0	0.0000009042

inverse Jacobian

884762.7501	745142.0769	668991.5563	502354.9599	669376.2731
745142.0769	765462.4548	687235.2734	516054.4179	563744.8303
668991.5563	687235.2734	1120534.279	841424.6002	506132.3781
502354.9599	516054.4179	841424.6002	1553416.841	380061.7632
669376.2731	563744.8303	506132.3781	380061.7632	1612333.78

[F]

-0.0010779903
-0.0006234680
0.0001535694
0.0003581745
0.0004674254

[Δx]

-822.7871268
-726.6128476
-439.5999529
-0.015372369
-105.5573975

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

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Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 6

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-7926	-1	1.00	352408.69	0.0000085082	8.412213013	0.014131214	-0.09710	0.0000061253	-268.3902544	219.9339521	-7975
1	16	900	0.2204	-23471	-1	0.87	190424.05	0.0000136116	7.931361281	0.015896611	-0.03244	0.0000006911	-268.3902544	38.01860534	-23701
stopping criterion $\Sigma=0$; Σ 0.000456029738												Σ 0.0000068165			
2	21	400	0.3526	7926	1	1.00	352408.69	0.0000085082	8.412213013	0.014131214	0.09710	0.0000061253	-219.9339521	268.3902544	7975
2	23	500	0.2468	-11216	-1	0.86	210776.56	0.0000121556	8.011466049	0.015580307	-0.04033	0.0000017978	-219.9339521	119.8158634	-11316
2	25	900	0.2468	-21407	-1	0.89	218832.29	0.0000121556	8.039962097	0.01547006	-0.04167	0.0000009734	-219.9339521		-21627
stopping criterion $\Sigma=0$; Σ 0.000097270189												Σ 0.0000088966			
3	32	500	0.2468	11216	1	0.86	210776.56	0.0000121556	8.011466049	0.015580307	0.04033	0.0000017978	-119.8158634	219.9339521	11316
3	34	800	0.2204	-18501	-1	0.81	177600.88	0.0000136116	7.878429262	0.016110934	-0.03035	0.0000008202	-119.8158634	-76.10732644	-18697
stopping criterion $\Sigma=0$; Σ -0.000019305963												Σ 0.0000026180			
4	43	800	0.2204	18501	1	0.81	177600.88	0.0000136116	7.878429262	0.016110934	0.03035	0.0000008202	76.10732644	119.8158634	18697
4	45	400	0.1102	8310	1	0.47	51695.38	0.0000272232	6.927596042	0.020836985	0.00447	0.0000002690	76.10732644		8386
stopping criterion $\Sigma=0$; Σ -0.000181163269												Σ 0.0000010892			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	23471	1	0.87	190424.05	0.0000136116	7.931361281	0.015896611	0.03244	0.0000006911	-38.01860534	268.3902544	23701
6	65	400	0.0968	-5862	-1	0.36	34418.18	0.0000309917	6.617849084	0.022833174	-0.00259	0.0000002213	-38.01860534		-5900
stopping criterion $\Sigma=0$; Σ -0.000150806063												Σ 0.0000009124			

Jacobian				
0.0000068165	-0.0000061253	0	0	-0.0000006911
-0.0000061253	0.0000088966	-0.0000017978	0	0
0	-0.0000017978	0.0000026180	-0.0000008202	0
0	0	-0.0000008202	0.0000010892	0
-0.0000006911	0	0	0	0.0000009124

inverse Jacobian				
877418.3539	738171.4792	663420.3159	499563.1751	664629.9133
738171.4792	758370.6529	681574.014	513233.1199	559152.7052
663420.3159	681574.014	1112454.086	837690.8	502529.9335
499563.1751	513233.1199	837690.8	1548913.28	378410.8553
664629.9133	559152.7052	502529.9335	378410.8553	1599440.45

[F]
0.0004560297
0.0000972702
-0.0000193060
-0.0001811633
-0.0001508061

[Δx]
268.3902544
219.9339521
119.8158634
-76.10732644
38.01860534

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

Dejan Brkic, Strumicka 88, 11050 Belgrade, Serbia; dejanrgf@tesla.rcub.bg.ac.rs, dejanbrkic0611@gmail.com

Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 7

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-7975	-1	0.99	349233.64	0.0000085082	8.40532649	0.014154379	-0.09732	0.0000061017	-103.2760659	90.00523078	-7988
1	16	900	0.2204	-23701	-1	0.85	186676.91	0.0000136116	7.916278277	0.015957244	-0.03254	0.0000006865	-103.2760659	13.54338157	-23791
stopping criterion $\Sigma=0$; Σ											0.000142572813				
Σ											0.0000067882				
2	21	400	0.3526	7975	1	0.99	349233.64	0.0000085082	8.40532649	0.014154379	0.09732	0.0000061017	-90.00523078	103.2760659	7988
2	23	500	0.2468	-11316	-1	0.84	207225.07	0.0000121556	7.998549009	0.015630669	-0.04044	0.0000017870	-90.00523078	53.19904573	-11353
2	25	900	0.2468	-21627	-1	0.87	214144.15	0.0000121556	8.023511147	0.015533563	-0.04180	0.0000009665	-90.00523078		-21717
stopping criterion $\Sigma=0$; Σ											0.000071784087				
Σ											0.0000088551				
3	32	500	0.2468	11316	1	0.84	207225.07	0.0000121556	7.998549009	0.015630669	0.04044	0.0000017870	-53.19904573	90.00523078	11353
3	34	800	0.2204	-18697	-1	0.80	174618.70	0.0000136116	7.865562181	0.016163688	-0.03046	0.0000008145	-53.19904573	-8.565709334	-18759
stopping criterion $\Sigma=0$; Σ											-0.000015462794				
Σ											0.0000026015				
4	43	800	0.2204	18697	1	0.80	174618.70	0.0000136116	7.865562181	0.016163688	0.03046	0.0000008145	8.565709334	53.19904573	18759
4	45	400	0.1102	8386	1	0.47	51453.11	0.0000272232	6.924039497	0.020858397	0.00449	0.0000002677	8.565709334		8395
stopping criterion $\Sigma=0$; Σ											-0.000052601500				
Σ											0.0000010822				
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	23701	1	0.85	186676.91	0.0000136116	7.916278277	0.015957244	0.03254	0.0000006865	-13.54338157	103.2760659	23791
6	65	400	0.0968	-5900	-1	0.35	33985.86	0.0000309917	6.608290265	0.022899277	-0.00260	0.0000002202	-13.54338157		-5913
stopping criterion $\Sigma=0$; Σ											-0.000058615355				
Σ											0.0000009067				

Jacobian

0.0000067882	-0.0000061017	0	0	-0.0000006865
-0.0000061017	0.0000088551	-0.0000017870	0	0
0	-0.0000017870	0.0000026015	-0.0000008145	0
0	0	-0.0000008145	0.0000010822	0
-0.0000006865	0	0	0	0.0000009067

inverse Jacobian

882915.2641	743154.417	667862.6001	502680.877	668451.4312
743154.417	763463.0485	686113.6864	516417.9422	562639.0819
667862.6001	686113.6864	1119510.719	842623.363	505635.9642
502680.877	516417.9422	842623.363	1558281.129	380577.5767
668451.4312	562639.0819	505635.9642	380577.5767	1608975.255

[F]

0.0001425728
0.0000717841
-0.0000154628
-0.0000526015
-0.0000586154

[Δx]

103.2760659
90.00523078
53.19904573
-8.565709334
13.54338157

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

Dejan Brkic, Strumicka 88, 11050 Belgrade, Serbia; dejanrgf@tesla.rcub.bg.ac.rs, dejanbrkic0611@gmail.com

Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 8

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)	
1	12	400	0.3526	-7988	-1	1.00	350012.76	0.0000085082	8.407022272	0.01414867	-0.09742	0.0000060979	15.64678921	-12.4224294	-7985	
1	16	900	0.2204	-23791	-1	0.85	187234.06	0.0000136116	7.918540357	0.015948129	-0.03261	0.0000006854	15.64678921	-2.204883833	-23778	
stopping criterion $\Sigma=0$; Σ -0.000028874273												Σ 0.0000067832				
2	21	400	0.3526	7988	1	1.00	350012.76	0.0000085082	8.407022272	0.01414867	0.09742	0.0000060979	12.4224294	-15.64678921	7985	
2	23	500	0.2468	-11353	-1	0.85	207812.31	0.0000121556	8.000700322	0.015622265	-0.04052	0.0000017846	12.4224294	-6.280508448	-11347	
2	25	900	0.2468	-21717	-1	0.87	214800.99	0.0000121556	8.025838021	0.015524557	-0.04190	0.0000009647	12.4224294		-21704	
stopping criterion $\Sigma=0$; Σ -0.000003283335												Σ 0.0000088472				
3	32	500	0.2468	11353	1	0.85	207812.31	0.0000121556	8.000700322	0.015622265	0.04052	0.0000017846	6.280508448	-12.4224294	11347	
3	34	800	0.2204	-18759	-1	0.80	175254.17	0.0000136116	7.868322633	0.016152349	-0.03052	0.0000008135	6.280508448	6.976695969	-18745	
stopping criterion $\Sigma=0$; Σ 0.000000176288												Σ 0.0000025980				
4	43	800	0.2204	18759	1	0.80	175254.17	0.0000136116	7.868322633	0.016152349	0.03052	0.0000008135	-6.976695969	-6.280508448	18745	
4	45	400	0.1102	8395	1	0.47	51661.65	0.0000272232	6.927101861	0.020839959	0.00449	0.0000002676	-6.976695969		8388	
stopping criterion $\Sigma=0$; Σ 0.000012651510												Σ 0.0000010811				
5	omitted to preserve linear independency of Jacobian															
6	61	900	0.2204	23791	1	0.85	187234.06	0.0000136116	7.918540357	0.015948129	0.03261	0.0000006854	2.204883833	-15.64678921	23778	
6	65	400	0.0968	-5913	-1	0.35	34046.65	0.0000309917	6.609641599	0.022889915	-0.00260	0.0000002200	2.204883833		-5911	
stopping criterion $\Sigma=0$; Σ 0.000008727435												Σ 0.0000009054				

Jacobian

0.0000067832	-0.0000060979	0	0	-0.0000006854
-0.0000060979	0.0000088472	-0.0000017846	0	0
0	-0.0000017846	0.0000025980	-0.0000008135	0
0	0	-0.0000008135	0.0000010811	0
-0.0000006854	0	0	0	0.0000009054

inverse Jacobian

884135.2837	744293.124	668819.3565	503247.8269	669267.4931
744293.124	764623.2528	687087.9435	516993.8804	563410.6029
668819.3565	687087.9435	1120956.442	843454.795	506278.9172
503247.8269	516993.8804	843454.795	1559632.545	380945.5608
669267.4931	563410.6029	506278.9172	380945.5608	1611104.18

[F]

-0.0000288743
-0.0000032833
0.0000001763
0.0000126515
0.0000087274

[Δx]

-15.64678921
-12.4224294
-6.280508448
6.976695969
-2.204883833

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

Dejan Brkic, Strumicka 88, 11050 Belgrade, Serbia; dejanrgf@tesla.rcub.bg.ac.rs, dejanbrkic0611@gmail.com

Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 9

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)	
1	12	400	0.3526	-7985	-1	1.00	350374.55	0.0000085082	8.407808403	0.014146024	-0.09741	0.0000060997	11.27034377	-9.764773828	-7983	
1	16	900	0.2204	-23778	-1	0.85	187641.76	0.0000136116	7.920191301	0.015941481	-0.03261	0.0000006857	11.27034377	-1.473030063	-23768	
											0.13000					
											stopping criterion $\Sigma=0; \Sigma$		-0.000015901479			
											Σ	0.0000067854				
2	21	400	0.3526	7985	1	1.00	350374.55	0.0000085082	8.407808403	0.014146024	0.09741	0.0000060997	9.764773828	-11.27034377	7983	
2	23	500	0.2468	-11347	-1	0.85	208205.99	0.0000121556	8.00213906	0.015616648	-0.04052	0.0000017854	9.764773828	-5.665890179	-11343	
2	25	900	0.2468	-21704	-1	0.88	215309.93	0.0000121556	8.027635962	0.015517604	-0.04190	0.0000009652	9.764773828		-21694	
											-0.01500					
											stopping criterion $\Sigma=0; \Sigma$		-0.000007559800			
											Σ	0.0000088502				
3	32	500	0.2468	11347	1	0.85	208205.99	0.0000121556	8.00213906	0.015616648	0.04052	0.0000017854	5.665890179	-9.764773828	11343	
3	34	800	0.2204	-18745	-1	0.80	175605.01	0.0000136116	7.86984233	0.016146111	-0.03051	0.0000008139	5.665890179	1.614375179	-18738	
											-0.01000					
											stopping criterion $\Sigma=0; \Sigma$		0.000001392479			
											Σ	0.0000025993				
4	43	800	0.2204	18745	1	0.80	175605.01	0.0000136116	7.86984233	0.016146111	0.03051	0.0000008139	-1.614375179	-5.665890179	18738	
4	45	400	0.1102	8388	1	0.47	51710.89	0.0000272232	6.927823091	0.02083562	0.00449	0.0000002678	-1.614375179		8386	
											-0.03500					
											stopping criterion $\Sigma=0; \Sigma$		0.000006357783			
											Σ	0.0000010817				
5	omitted to preserve linear independency of Jacobian															
6	61	900	0.2204	23778	1	0.85	187641.76	1.36116E-05	7.920191301	0.015941481	0.03261	0.0000006857	1.473030063	-11.27034377	23768	
6	65	400	0.0968	-5911	-1	0.35	34092.67	3.09917E-05	6.610663188	0.022882841	-0.00260	0.0000002201	1.473030063		-5910	
											-0.03000					
											stopping criterion $\Sigma=0; \Sigma$		0.000006393787			
											Σ	0.0000009058				

Jacobian

0.0000067854	-0.0000060997	0	0	-0.0000006857
-0.0000060997	0.0000088502	-0.0000017854	0	0
0	-0.0000017854	0.0000025993	-0.0000008139	0
0	0	-0.0000008139	0.0000010817	0
-0.0000006857	0	0	0	0.0000009058

inverse Jacobian

883726.3969	743923.4822	668482.0413	502997.3527	668982.3671
743923.4822	764245.1827	686742.9138	516737.6927	563151.3258
668482.0413	686742.9138	1120411.02	843049.9879	506042.0283
502997.3527	516737.6927	843049.9879	1558832.824	380769.841
668982.3671	563151.3258	506042.0283	380769.841	1610404.839

[F]

-0.0000159015
-0.0000075598
0.0000013925
0.0000063578
0.0000063938

[Δx]

-11.27034377
-9.764773829
-5.665890179
1.614375178
-1.473030064

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

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Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 10

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-7983	-1	1.00	350336.58	0.0000085082	8.407725945	0.014146301	-0.09740	0.0000061002	-0.401422549	0.24529831	-7983
1	16	900	0.2204	-23768	-1	0.85	187627.86	0.0000136116	7.920135052	0.015941707	-0.03260	0.0000006858	-0.401422549	0.058059084	-23768
stopping criterion $\Sigma=0$; Σ 0.000001187876												Σ 0.0000067860			
2	21	400	0.3526	7983	1	1.00	350336.58	0.0000085082	8.407725945	0.014146301	0.09740	0.0000061002	-0.24529831	0.401422549	7983
2	23	500	0.2468	-11343	-1	0.85	208187.09	0.0000121556	8.002070061	0.015616917	-0.04051	0.0000017857	-0.24529831	0.045310755	-11343
2	25	900	0.2468	-21694	-1	0.88	215296.56	0.0000121556	8.027588783	0.015517786	-0.04189	0.0000009654	-0.24529831		-21695
stopping criterion $\Sigma=0$; Σ -0.000000358452												Σ 0.0000088513			
3	32	500	0.2468	11343	1	0.85	208187.09	0.0000121556	8.002070061	0.015616917	0.04051	0.0000017857	-0.045310755	0.24529831	11343
3	34	800	0.2204	-18738	-1	0.80	175576.86	0.0000136116	7.869720474	0.016146611	-0.03051	0.0000008141	-0.045310755	-0.544487718	-18739
stopping criterion $\Sigma=0$; Σ 0.000000123019												Σ 0.0000025997			
4	43	800	0.2204	18738	1	0.80	175576.86	0.0000136116	7.869720474	0.016146611	0.03051	0.0000008141	0.544487718	0.045310755	18739
4	45	400	0.1102	8386	1	0.47	51694.78	0.0000272232	6.927587182	0.020837039	0.00449	0.0000002678	0.544487718		8387
stopping criterion $\Sigma=0$; Σ -0.000000625939												Σ 0.0000010818			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	23768	1	0.85	187627.86	0.0000136116	7.920135052	0.015941707	0.03260	0.0000006858	-0.058059084	0.401422549	23768
6	65	400	0.0968	-5910	-1	0.35	34091.59	0.0000309917	6.610639052	0.022883008	-0.00260	0.0000002201	-0.058059084		-5910
stopping criterion $\Sigma=0$; Σ -0.000000222710												Σ 0.0000009060			

Jacobian

0.0000067860	-0.0000061002	0	0	-0.000006858
-0.0000061002	0.0000088513	-0.0000017857	0	0
0	-0.0000017857	0.0000025997	-0.0000008141	0
0	0	-0.0000008141	0.0000010818	0
-0.000006858	0	0	0	0.0000009060

inverse Jacobian

883571.1344	743779.1659	668357.8493	502918.4147	668877.5103
743779.1659	764098.0505	686616.3414	516657.3599	563052.7496
668357.8493	686616.3414	1120219.906	842930.5044	505957.6041
502918.4147	516657.3599	842930.5044	1558622.659	380717.3005
668877.5103	563052.7496	505957.6041	380717.3005	1610136.352

[F]

0.0000011879
-0.0000003585
0.0000001230
-0.0000006259
-0.0000002227

[\Delta x]

0.40142255
0.24529831
0.045310757
-0.544487717
0.058059085

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

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Standard procedure (pressure drop independent Jacobian); Re=Re(Q)

Iteration 11

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{-0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	pressure drop (Pa)
1	12	400	0.3526	-7983	-1	1.00	350300.12	0.0000085082	8.407646735	0.014146568	-0.09740	0.0000061001	-1.102568369	0.950998724	-7983
1	16	900	0.2204	-23768	-1	0.85	187587.87	0.0000136116	7.91997326	0.015942358	-0.03260	0.0000006858	-1.102568369	0.141673816	-23769
											0.13000				
											stopping criterion $\Sigma=0$; Σ	0.000001583582			
											Σ	0.0000067859			
2	21	400	0.3526	7983	1	1.00	350300.12	0.0000085082	8.407646735	0.014146568	0.09740	0.0000061001	-0.950998724	1.102568369	7983
2	23	500	0.2468	-11343	-1	0.85	208147.69	0.0000121556	8.001926176	0.015617479	-0.04051	0.0000017856	-0.950998724	0.540744448	-11343
2	25	900	0.2468	-21695	-1	0.88	215246.85	0.0000121556	8.02741338	0.015518464	-0.04189	0.0000009654	-0.950998724		-21696
											-0.01500				
											stopping criterion $\Sigma=0$; Σ	0.000000726086			
											Σ	0.0000088511			
3	32	500	0.2468	11343	1	0.85	208147.69	0.0000121556	8.001926176	0.015617479	0.04051	0.0000017856	-0.540744448	0.950998724	11343
3	34	800	0.2204	-18739	-1	0.80	175540.04	0.0000136116	7.869561121	0.016147265	-0.03051	0.0000008140	-0.540744448	-0.223278803	-18739
											-0.01000				
											stopping criterion $\Sigma=0$; Σ	-0.000000110621			
											Σ	0.0000025996			
4	43	800	0.2204	18739	1	0.80	175540.04	0.0000136116	7.869561121	0.016147265	0.03051	0.0000008140	0.223278803	0.540744448	18739
4	45	400	0.1102	8387	1	0.47	51688.04	0.0000272232	6.927488535	0.020837632	0.00449	0.0000002678	0.223278803		8387
											-0.03500				
											stopping criterion $\Sigma=0$; Σ	-0.000000681726			
											Σ	0.0000010818			
5	omitted to preserve linear independency of Jacobian														
6	61	900	0.2204	23768	1	0.85	1.9E+05	1.36116E-05	7.91997326	0.015942358	0.03260	0.0000006858	-0.141673816	1.102568369	23769
6	65	400	0.0968	-5910	-1	0.35	3.4E+04	3.09917E-05	6.610542057	0.022883679	-0.00260	0.0000002201	-0.141673816		-5910
											-0.03000				
											stopping criterion $\Sigma=0$; Σ	-0.000000627811			
											Σ	0.0000009059			

Jacobian

0.0000067859	-0.0000061001	0	0	-0.0000006858
-0.0000061001	0.0000088511	-0.0000017856	0	0
0	-0.0000017856	0.0000025996	-0.0000008140	0
0	0	-0.0000008140	0.0000010818	0
-0.0000006858	0	0	0	0.0000009059

inverse Jacobian

883595.1532	743800.6368	668378.3123	502935.1464	668894.3818
743800.6368	764120.0603	686637.3206	516674.5165	563067.8998
668378.3123	686637.3206	1120254.202	842958.5474	505972.1032
502935.1464	516674.5165	842958.5474	1558681.159	380729.2204
668894.3818	563067.8998	505972.1032	380729.2204	1610176.49

[F]

0.0000015836
0.0000007261
-0.0000001106
-0.0000006817
-0.0000006278

[Δx]

1.102568372
0.950998727
0.540744451
-0.223278801
0.141673818

*from the previous iteration

**velocity calculated using calculated flow, Q from the previous iteration

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Standard procedure (pressure drop independent Jacobian)

Iteration 12

Node	Pipe	Length (m)	Diameter (m)	Pressure drop (Pa)*	Sign	Velocity (m/s)	Re	ϵ/D	Colebrook: $\lambda^{0.5}$	λ	Q (m ³ /s)	$ \partial Q/\partial(\Delta p) $	correction 1 (Pa)	correction 2 (Pa)	Pipe	Final pressure drop (Pa)	Final flow distribution (dm ³ /s)
1	12	400	0.3526	-7983	-1	1.00	3.5E+05	8.50822E-06	8.407647007	0.014146567	-0.09740	0.0000061000	-0.067492331	0.067847344	12	-7983	-97.4
1	16	900	0.2204	-23769	-1	0.85	1.9E+05	1.36116E-05	7.919963238	0.015942399	-0.03260	0.0000006858	-0.067492331	0.008751709	16	-23769	-32.6
											0.13000						
											stopping criterion $\Sigma=0$; Σ	0.000000038119					
											Σ	0.0000067858					
2	21	400	0.3526	7983	1	1.00	3.5E+05	8.50822E-06	8.407647007	0.014146567	0.09740	0.0000061000	-0.067847344	0.067492331	21	7983	97.4
2	23	500	0.2468	-11343	-1	0.85	2.1E+05	1.21556E-05	8.001919208	0.015617506	-0.04051	0.0000017856	-0.067847344	0.048257143	23	-11343	-40.5
2	25	900	0.2468	-21696	-1	0.88	2.2E+05	1.21556E-05	8.027401075	0.015518512	-0.04189	0.0000009654	-0.067847344		25	-21696	-41.9
											-0.01500						
											stopping criterion $\Sigma=0$; Σ	0.000000102645					
											Σ	0.0000088510					
3	32	500	0.2468	11343	1	0.85	2.1E+05	1.21556E-05	8.001919208	0.015617506	0.04051	0.0000017856	-0.048257143	0.067847344	32	11343	40.5
3	34	800	0.2204	-18739	-1	0.80	1.8E+05	1.36116E-05	7.869557693	0.016147279	-0.03051	0.0000008140	-0.048257143	0.032289467	34	-18740	-30.5
											-0.01000						
											stopping criterion $\Sigma=0$; Σ	-0.000000021982					
											Σ	0.0000025996					
4	43	800	0.2204	18739	1	0.80	1.8E+05	1.36116E-05	7.869557693	0.016147279	0.03051	0.0000008140	-0.032289467	0.048257143	43	18740	30.5
4	45	400	0.1102	8387	1	0.47	5.2E+04	2.72232E-05	6.927502331	0.020837549	0.00449	0.0000002678	-0.032289467		45	8387	4.5
											-0.03500						
											stopping criterion $\Sigma=0$; Σ	-0.000000004352					
											Σ	0.0000010818					
5	omitted to preserve linear independency of Jacobian																
6	61	900	0.2204	23769	1	0.85	1.9E+05	1.36116E-05	7.919963238	0.015942399	0.03260	0.0000006858	-0.008751709	0.067492331	61	23769	32.6
6	65	400	0.0968	-5910	-1	0.35	3.4E+04	3.09917E-05	6.610534676	0.022883731	-0.00260	0.0000002201	-0.008751709		65	-5910	-2.6
											-0.03000						
											stopping criterion $\Sigma=0$; Σ	-0.000000038358					
											Σ	0.0000009059					

Jacobian

0.0000067858	-0.0000061000	0	0	-0.000006858
-0.0000061000	0.0000088510	-0.0000017856	0	0
0	-0.0000017856	0.0000025996	-0.0000008140	0
0	0	-0.0000008140	0.0000010818	0
-0.000006858	0	0	0	0.0000009059

inverse Jacobian

883612.1988	743816.4602	668392.1731	502944.3933	668905.9242
743816.4602	764136.2005	686651.4563	516683.938	563078.7323
668392.1731	686651.4563	1120275.844	842972.8491	505981.5662
502944.3933	516683.938	842972.8491	1558707.502	380735.4455
668905.9242	563078.7323	505981.5662	380735.4455	1610205.659

[F]

0.0000000381
0.0000001026
-0.0000000220
-0.0000000044
-0.0000000384

[Δx]

0.067492339
0.067847351
0.048257149
0.032289472
0.008751715

*from the previous iteration
 **velocity calculated using calculated flow, Q from the previous iteration
 Standard procedure (pressure drop independent Jacobian); Re=Re(Q)
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Figure caption list

Figure 1. Convergence properties of the proposed method for the solution of the problem of water distribution analysis expressed in the nodal form (both with the independent Jacobian)

Etable 1.

Etable 2.

(Etable 1 and Etable 2 are submitted as Electronic Annexes to the discussion; do not embed them into the text, instead provide a link to this material)



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