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The Principle of Least Action in Covariant Theory of Gravitation

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Abstract

The integral equations for calculation of metric, equations of substance motion, as well as equations for gravitational and electromagnetic fields in covariant theory of gravitation are obtained by means of variation of action functional. In covariant form stress-energy tensor of gravitational field, strength tensor of gravitational field and 4-current of mass are determined. The meaning of the cosmological constant and its relation to the components of energy density in action functional are explained. The results obtained prove the validity of Mach's principle, assuming that gravitation effects are due to the flows of gravitons. The idea that metric can be entirely determined by variables describing fields' properties is substantiated.

Keywords: action; metric; cosmological constant; stress-energy tensor of gravitational field; equations of motion; field equations; covariant theory of gravitation.

1 Introduction

Covariant theory of gravitation (CTG) is one of the alternative gravitation theories in relation to the general relativity theory. The purpose of this article is to derive equations of CTG from the principle of least action. As the basis of our discussion we will use works of Einstein [1], Dirac [2], Pauli [3], Fock [4], Landau and Lifshitz [5].

We will use international system of units, basic coordinates in the form of coordinates with contravariant indices (x^0, x^1, x^2, x^3), metric tensor $g_{\mu\nu}$, metric signature (+, –, –, –). The presence of repeated indices in formulas implies Einstein summation convention, which is a separate summation for each repeated index.

2 The action function

In the case of continuously distributed throughout the volume of space matter, the action function for the matter in gravitational and electromagnetic fields in covariant theory of gravitation can be presented as follows:

$$S = \int L\,dt = \int \Bigg(k(R - 2\Lambda) - c\rho_0 - \frac{1}{c} D_\mu\, J^\mu + \frac{c}{16\pi\gamma} \Phi_{\mu\nu} \Phi^{\mu\nu} - \\ - \frac{1}{c} A_\mu\, j^\mu - \frac{c\varepsilon_0}{4} F_{\mu\nu} F^{\mu\nu} \Bigg) \sqrt{-g}\, d\Sigma, \qquad (1)$$

where L – Lagrange function or Lagrangian,

dt – differential of time in the used reference frame,

k – certain coefficient,

R – scalar curvature,

Λ – a constant, which characterizes the energy density of the system as a whole, and therefore is a function of the system,

c – speed of light, as a measure of velocity of electromagnetic and gravitational interactions propagation,

ρ_0 – density of substance mass in reference frame in which the substance is at rest,

$D_\mu = \left(\frac{\psi}{c}, -\boldsymbol{D} \right)$ – 4-potential of gravitational field which is described by scalar potential ψ and vector potential $\boldsymbol{D}$ of the field,

J^μ – 4-vector of mass current,

γ – gravitational constant,

$\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$ – gravitational tensor (tensor of gravitational field strength),

$\Phi^{\alpha\beta} = g^{\alpha\mu} g^{\nu\beta} \Phi_{\mu\nu}$ – determining the gravitational tensor with contravariant indices by means of the metric tensor $g^{\alpha\mu}$,

$A_\mu = \left(\frac{\varphi}{c}, -\boldsymbol{A} \right)$ – 4-potential of electromagnetic field, set by scalar potential φ and vector potential $\boldsymbol{A}$ of the field,

j^μ – 4-vector of electric current density,

ε_0 – electric constant (vacuum permittivity),

$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$ – electromagnetic tensor (field strength tensor),

$\sqrt{-g}\, d\Sigma = \sqrt{-g}\, c\, dt\, dx^1 dx^2 dx^3$ – invariant 4-volume, expressed through differential of time coordinate $dx^0 = c\, dt$, through product of differentials of spatial coordinates $dx^1 dx^2 dx^3$, and through the square root $\sqrt{-g}$ of determinant g of metric tensor, taken with the negative sign.

The symbol ∇_μ denotes covariant derivative with respect to coordinates (in this case the coordinates x^μ). Similarly, $\partial_\mu = \frac{\partial}{\partial x^\mu}$ is an operator of partial derivative with respect to coordinates or 4-gradient.

The integrand in (1) is Lagrangian function, consisting of six terms. The first term with the scalar curvature R depends on the metric tensor and its derivatives with respect to coordinates. In covariant theory of gravitation (CTG) metric is used to consider the impact of fundamental fields (which include electromagnetic and gravitational fields) of material bodies on the results of space-time measurements near the bodies. The field's effect on measurement results shows that under the field's action electromagnetic waves are deflected from rectilinear motion, electromagnetic clock changed its course, and measured distances changed its value. These effects can be described by introducing a curved space-time with metric tensor $g_{\mu\nu}$ instead of flat Minkowski space with its single metric tensor $\eta_{\mu\nu}$. In CTG gravitational field is an independent physical field and the metric tensor $g_{\mu\nu}$ has the geometric meaning and auxiliary function, unlike general relativity where the metric field completely replaces gravitational field.

In CTG second term in (1) is not simply related to rest energy substance density and its inertia with respect to the applied forces. According to [6], [7], the rest mass (and substance density at rest) is a consequence of strong gravitation and electromagnetic interactions operating at the level of elementary particles. But the first and the second terms in (1) are associated with microscopic fundamental fields, while other terms refer to action of macroscopic gravitational and electromagnetic fields. The division to microscopic and macroscopic fundamental fields follows from the theory of infinite nesting of matter in which its own gravitational field operates at every main level of matter. As a result, the usual gravitation is assumed as long-range component of strong gravitation.

The third term in (1) $-\frac{1}{c}D_{\mu}\, J^{\mu}$ is invariant with respect to different types of coordinate transformations that reflect the interaction of mass current density $J^{\mu}=\rho_0 u^{\mu}$ of arbitrary substance unit with gravitational field.

According to [8], the fourth term in (1) associated with energy field, is an invariant of gravitational field which does not change its form by changing the reference system. The fifth and sixth terms, for electromagnetic field, are similar in structure to the third and fourth terms for gravitational field. And 4-vector of electric current density j^{μ} can be determined by charge density ρ_{0q} of substance unit and 4-velocity: $j^{\mu} = \rho_{0q} u^{\mu}$.

In CTG 4-potentials D_{μ} and A_{μ} with covariant indices, and 4-currents J^{μ} and j^{μ} with contravariant indices were determined in [7] and [9] as initial concepts for construction of axiomatic theory. Hence we can make a conclusion that, for example, 4-vector $D^{\mu} = g^{\mu\nu} D_{\nu}$ can not be found in absence of information about metric in any frame of reference.

3 Variation of curvature invariant

To obtain equations for metric the variation of action function for the case when in Lagrangian the metric tensor $g_{\mu\nu}$ is variable should be set to zero. At the same time variation of metric tensor should be zero on the borders of four-dimensional volume for which in (1) integration is performed. For full variation of the action should be as follows:

$$\begin{aligned}\delta S = \delta \int \Big(& k(R-2\Lambda) - c\rho_0 - \frac{1}{c} D_{\mu} J^{\mu} + \frac{c}{16\pi\gamma} \Phi_{\mu\nu} \Phi^{\mu\nu} - \\ & - \frac{1}{c} A_{\mu} j^{\mu} - \frac{c\varepsilon_0}{4} F_{\mu\nu} F^{\mu\nu} \Big) \sqrt{-g}\, d\Sigma = 0.\end{aligned} \qquad (2)$$

Let's find the variation associated with the first term in (2). Using definition of scalar curvature R through Christoffel symbols, we obtain the same as in [2]:

$$\begin{aligned}\delta S_1 &= \delta \int k(R-2\Lambda)\sqrt{-g}\, d\Sigma = \\ &= k\delta \int R_1 \sqrt{-g}\, d\Sigma + k\delta \int R_2 \sqrt{-g}\, d\Sigma - 2\Lambda k \int \delta \sqrt{-g}\, d\Sigma,\end{aligned} \qquad (3)$$

where $R_1 = g^{\mu\nu} \left(\partial_{\kappa} \Gamma^{\kappa}_{\mu\nu} - \partial_{\nu} \Gamma^{\kappa}_{\mu\kappa} \right)$, $R_2 = g^{\mu\nu} \left(\Gamma^{\kappa}_{\mu\nu} \Gamma^{\sigma}_{\kappa\sigma} - \Gamma^{\kappa}_{\mu\sigma} \Gamma^{\sigma}_{\kappa\nu} \right)$, $R = R_1 + R_2$.

The expression for $R_1\sqrt{-g}$ can be obtained by means of differentiation by parts:

$$
\begin{aligned}
R_1\sqrt{-g} = \partial_\kappa\left(g^{\mu\nu}\Gamma^\kappa_{\mu\nu}\sqrt{-g}\right) - \partial_\nu\left(g^{\mu\nu}\Gamma^\kappa_{\mu\kappa}\sqrt{-g}\right) - \\
-\Gamma^\kappa_{\mu\nu}\partial_\kappa\left(g^{\mu\nu}\sqrt{-g}\right) + \Gamma^\kappa_{\mu\kappa}\partial_\nu\left(g^{\mu\nu}\sqrt{-g}\right).
\end{aligned} \tag{4}
$$

The first two terms on the right side of (4) are the total derivatives (divergence), and after substituting them in (3), the integrals of the divergence over the volume according to Gauss's theorem can be replaced by the integrals over the surface surrounding the volume for which the integration takes place. Since variation of metric tensor on the surface equals to zero, these terms will not contribute to the variation of action function, so that in (4) only the last two terms should be taken into account. Then we can use two relations:

$$
\partial_\kappa\left(g^{\mu\nu}\sqrt{-g}\right) = \sqrt{-g}\left(g^{\mu\nu}\Gamma^\alpha_{\kappa\alpha} - g^{\nu\beta}\Gamma^\mu_{\beta\kappa} - g^{\mu\alpha}\Gamma^\nu_{\alpha\kappa}\right), \tag{5}
$$

$$
\partial_\nu\left(g^{\mu\nu}\sqrt{-g}\right) = -\sqrt{-g}\, g^{\nu\beta}\Gamma^\mu_{\beta\nu}\,.
$$

Substituting them in the last two terms in (4) and renaming some of indices on which the summation takes place, we obtain:

$$
\begin{aligned}
&-\Gamma^\kappa_{\mu\nu}\sqrt{-g}\left(g^{\mu\nu}\Gamma^\alpha_{\kappa\alpha} - g^{\nu\beta}\Gamma^\mu_{\beta\kappa} - g^{\mu\alpha}\Gamma^\nu_{\alpha\kappa}\right) - \Gamma^\kappa_{\mu\kappa}\sqrt{-g}\, g^{\nu\beta}\Gamma^\mu_{\beta\nu} = \\
&= -\sqrt{-g}\left(g^{\mu\nu}\Gamma^\alpha_{\kappa\alpha}\Gamma^\kappa_{\mu\nu} - g^{\nu\beta}\Gamma^\mu_{\beta\kappa}\Gamma^\kappa_{\mu\nu} - g^{\mu\alpha}\Gamma^\nu_{\alpha\kappa}\Gamma^\kappa_{\mu\nu} + g^{\nu\beta}\Gamma^\mu_{\beta\nu}\Gamma^\kappa_{\mu\kappa}\right) = \\
&= -2\sqrt{-g}\left(g^{\mu\nu}\Gamma^\alpha_{\kappa\alpha}\Gamma^\kappa_{\mu\nu} - g^{\nu\beta}\Gamma^\mu_{\beta\kappa}\Gamma^\kappa_{\mu\nu}\right) = -2\sqrt{-g}\, g^{\mu\nu}\left(\Gamma^\kappa_{\mu\nu}\Gamma^\sigma_{\kappa\sigma} - \Gamma^\kappa_{\mu\sigma}\Gamma^\sigma_{\kappa\nu}\right) = \\
&= -2R_2\sqrt{-g}\,.
\end{aligned}
$$

As a result, instead of (3) we can write:

$$
\delta S_1 = -k\int \delta\left(R_2\sqrt{-g}\right) d\Sigma - 2\Lambda k\int \delta\sqrt{-g}\, d\Sigma\,. \tag{6}
$$

The first variation in (6) will equal to:

$$\delta\left(R_2\sqrt{-g}\right)=\delta\left(\Gamma^{\kappa}_{\mu\nu}\Gamma^{\sigma}_{\kappa\sigma}\,g^{\mu\nu}\sqrt{-g}\right)-\delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\,g^{\mu\nu}\sqrt{-g}\right). \qquad (7)$$

Using the relation: $\Gamma^{\sigma}_{\kappa\sigma}=\frac{1}{\sqrt{-g}}\partial_{\kappa}\sqrt{-g}$, differentiation by parts, and using the second relation in (5), for the first part (7) we obtain:

$$\begin{aligned}&\delta\left(\Gamma^{\kappa}_{\mu\nu}\Gamma^{\sigma}_{\kappa\sigma}\,g^{\mu\nu}\sqrt{-g}\right)=\Gamma^{\kappa}_{\mu\nu}\,\delta\left(\Gamma^{\sigma}_{\kappa\sigma}\,g^{\mu\nu}\sqrt{-g}\right)+\Gamma^{\sigma}_{\kappa\sigma}\,g^{\mu\nu}\sqrt{-g}\,\delta\Gamma^{\kappa}_{\mu\nu}=\\&=\Gamma^{\kappa}_{\mu\nu}\,\delta\left(g^{\mu\nu}\partial_{\kappa}\sqrt{-g}\right)+\Gamma^{\sigma}_{\kappa\sigma}\,\delta\left(g^{\mu\nu}\Gamma^{\kappa}_{\mu\nu}\sqrt{-g}\right)-\Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\,\delta\left(g^{\mu\nu}\sqrt{-g}\right)=\\&=\Gamma^{\kappa}_{\mu\nu}\,\delta\left(g^{\mu\nu}\partial_{\kappa}\sqrt{-g}\right)-\Gamma^{\sigma}_{\kappa\sigma}\,\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right)-\Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\,\delta\left(g^{\mu\nu}\sqrt{-g}\right).\end{aligned}$$

(8)

The expression for derivative of metric tensor has the form: $\partial_{\alpha}g^{\nu\beta}=-g^{\beta\kappa}\Gamma^{\nu}_{\kappa\alpha}-g^{\nu\kappa}\Gamma^{\beta}_{\kappa\alpha}$. After multiplication by $\sqrt{-g}$, taking variation and another multiplication by $\Gamma^{\alpha}_{\nu\beta}$ we will obtain:

$$\begin{aligned}&\delta\left(\sqrt{-g}\,\partial_{\alpha}g^{\nu\beta}\right)\Gamma^{\alpha}_{\nu\beta}=-\delta\left(g^{\beta\kappa}\Gamma^{\nu}_{\kappa\alpha}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta}-\delta\left(g^{\nu\kappa}\Gamma^{\beta}_{\kappa\alpha}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta}=\\&=-2\delta\left(\Gamma^{\nu}_{\kappa\alpha}\,g^{\beta\kappa}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta}.\end{aligned}$$

We now transform the second part of (7), using replacing of indices, operation of differentiation by parts, and the previous expression:

$$
\begin{aligned}
&\delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\, g^{\mu\nu}\sqrt{-g}\right) = \delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\right) g^{\mu\nu}\sqrt{-g} + \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\,\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= 2\delta\left(\Gamma^{\kappa}_{\mu\sigma}\right)\Gamma^{\sigma}_{\kappa\nu}\, g^{\mu\nu}\sqrt{-g} + \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\,\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= 2\delta\left(\Gamma^{\kappa}_{\mu\sigma}\, g^{\mu\nu}\sqrt{-g}\right)\Gamma^{\sigma}_{\kappa\nu} - \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\,\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= -\delta\left(\sqrt{-g}\,\partial_{\sigma} g^{\kappa\nu}\right)\Gamma^{\sigma}_{\kappa\nu} - \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\,\delta\left(g^{\mu\nu}\sqrt{-g}\right).
\end{aligned}
\tag{9}
$$

Substitution of (8) and (9) in (7) gives the following result:

$$
\begin{aligned}
&\delta\left(R_2\sqrt{-g}\right) = \Gamma^{\kappa}_{\mu\nu}\,\delta\left(g^{\mu\nu}\partial_{\kappa}\sqrt{-g}\right) - \Gamma^{\sigma}_{\kappa\sigma}\,\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right) - \\
&-\Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\,\delta\left(g^{\mu\nu}\sqrt{-g}\right) + \delta\left(\sqrt{-g}\,\partial_{\sigma} g^{\kappa\nu}\right)\Gamma^{\sigma}_{\kappa\nu} + \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\,\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= \Gamma^{\kappa}_{\mu\nu}\,\delta\left(\partial_{\kappa}\left(g^{\mu\nu}\sqrt{-g}\right)\right) - \Gamma^{\sigma}_{\kappa\sigma}\,\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right) + \\
&+\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu} - \Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right).
\end{aligned}
\tag{10}
$$

The terms $\Gamma^{\kappa}_{\mu\nu}\,\delta\left(\partial_{\kappa}\left(g^{\mu\nu}\sqrt{-g}\right)\right)$ and $\Gamma^{\sigma}_{\kappa\sigma}\,\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right)$ in (10) can be transformed:

$$
\begin{aligned}
&\Gamma^{\kappa}_{\mu\nu}\,\delta\left(\partial_{\kappa}\left(g^{\mu\nu}\sqrt{-g}\right)\right) = \Gamma^{\kappa}_{\mu\nu}\,\partial_{\kappa}\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= \partial_{\kappa}\left(\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)\right) - \left(\partial_{\kappa}\Gamma^{\kappa}_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right).
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&\Gamma^{\sigma}_{\kappa\sigma}\,\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right) = \Gamma^{\sigma}_{\kappa\sigma}\,\partial_{\nu}\delta\left(g^{\kappa\nu}\sqrt{-g}\right) = \\
&= \partial_{\nu}\left(\Gamma^{\sigma}_{\kappa\sigma}\delta\left(g^{\kappa\nu}\sqrt{-g}\right)\right) - \left(\partial_{\nu}\Gamma^{\sigma}_{\kappa\sigma}\right)\delta\left(g^{\kappa\nu}\sqrt{-g}\right).
\end{aligned}
$$

In equations (11) there are such divergences as $\partial_\kappa\left(\Gamma^\kappa_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)\right)$, which, after substituting in (10) and then into (6) will be integrated over 4-volume and transformed into integrals over the surface, where variations are equal to zero. With this in mind, after substituting (10) and (11) into (6) we obtain:

$$\begin{aligned}
&-k\int \delta\left(R_2\sqrt{-g}\right)d\Sigma = \\
&= -k\int\left(\partial_\nu\Gamma^\sigma_{\mu\sigma} - \partial_\kappa\Gamma^\kappa_{\mu\nu} + \Gamma^\kappa_{\mu\sigma}\Gamma^\sigma_{\kappa\nu} - \Gamma^\sigma_{\kappa\sigma}\Gamma^\kappa_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma = \\
&= k\int R_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma,
\end{aligned} \tag{12}$$

where $R_{\mu\nu}$ is Ricci tensor.

For variations of metric tensor $g^{\mu\nu}$ and $\sqrt{-g}$ we can write down:

$$\delta g^{\mu\nu} = -g^{\mu\alpha}g^{\nu\beta}\delta g_{\alpha\beta}, \qquad \delta\sqrt{-g} = \frac{\sqrt{-g}}{2}g^{\alpha\beta}\delta g_{\alpha\beta}. \tag{13}$$

Using (13) in (12), we obtain:

$$\begin{aligned}
&-k\int \delta\left(R_2\sqrt{-g}\right)d\Sigma = k\int R_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma = \\
&= k\int R_{\mu\nu}\left(\sqrt{-g}\,\delta g^{\mu\nu} + g^{\mu\nu}\delta\sqrt{-g}\right)d\Sigma = \\
&= k\int R_{\mu\nu}\left(-g^{\mu\alpha}g^{\nu\beta} + g^{\mu\nu}\frac{1}{2}g^{\alpha\beta}\right)\sqrt{-g}\,\delta g_{\alpha\beta}d\Sigma = \\
&= k\int\left(-R^{\alpha\beta} + \frac{1}{2}g^{\alpha\beta}R\right)\sqrt{-g}\,\delta g_{\alpha\beta}d\Sigma.
\end{aligned}$$

With this result and the expression $\delta\sqrt{-g}$ from (13), we obtain:

$$\delta S_1 = k\int \left(-R^{\alpha\beta} + \frac{1}{2} g^{\alpha\beta} R - \Lambda g^{\alpha\beta} \right) \sqrt{-g}\, \delta g_{\alpha\beta}\, d\Sigma\,. \qquad (14)$$

4 Variation of invariant mass density

The second term in (2) is an invariant associated with the 4-current of mass J^μ, so we can write down:

$$c\rho_0 = \sqrt{g_{\mu\nu} J^\mu J^\nu} = \sqrt{J_\nu J^\nu}\,. \qquad (15)$$

4-vector J^μ can be determined through 4-velosity $u^\mu = \frac{dx^\mu}{d\tau}$, where $dx^\mu = (dx^0, dx^1, dx^2, dx^3)$ is displacement 4-vector, $d\tau$ – differential of proper time, as follows: $J^\mu = \rho_0 u^\mu = \frac{u^\mu \sqrt{g_{\alpha\beta} J^\alpha J^\beta}}{c}$, and $u_\beta u^\beta = c^2$. In elementary particle physics instead of the values of mass and velocity of particles their energies E and momentums $\boldsymbol{p}$ are used as the quantities directly found from experiments. These quantities are part of 4-momentum of a particle: $p^\alpha = \left(\frac{E}{c}, \boldsymbol{p}\right) = m u^\alpha$, and the invariant mass m become a secondary concept, which may be found from the relation $\sqrt{p_\beta p^\beta} = mc = \frac{\sqrt{E^2 - p^2 c^2}}{c}$. Accordingly, to calculate the velocity of particles in special relativity theory is applied relation: $\boldsymbol{v} = \frac{c^2}{E}\boldsymbol{p}$.

The variation of the second term in (2) with (15) has the following form:

$$\delta S_2 = -\int \delta\left(\sqrt{g_{\mu\nu} J^\mu J^\nu} \sqrt{-g} \right) d\Sigma\,. \qquad (16)$$

We determine the variation in (16) with the help of (13):

$$\delta\left(\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}\sqrt{-g}\right)=\sqrt{-g}\,\delta\left(\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}\right)+\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}\,\delta\sqrt{-g}=$$

$$=\frac{J^{\mu}J^{\nu}\sqrt{-g}\,\delta g_{\mu\nu}}{2\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}}+\frac{g_{\mu\nu}J^{\mu}\sqrt{-g}\,\delta J^{\nu}}{\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}}+\frac{1}{2}\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}\sqrt{-g}\,g^{\alpha\beta}\,\delta g_{\alpha\beta}= \qquad (17)$$

$$=\frac{1}{2}\left(\frac{J^{\alpha}J^{\beta}}{\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}}+g^{\alpha\beta}\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}\right)\sqrt{-g}\,\delta g_{\alpha\beta}+\frac{g_{\alpha\beta}J^{\alpha}}{\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}}\sqrt{-g}\,\delta J^{\beta}.$$

In (17) variation δJ^{β} is contained which according to [2], [4] can be found using displacement 4-vector ξ^{μ}. The displacement ξ^{μ} is variation of coordinates, which gives variation of mass 4-current J^{β}:

$$\delta J^{\beta}=\nabla_{\sigma}\left(J^{\sigma}\xi^{\beta}-J^{\beta}\xi^{\sigma}\right)=\frac{1}{\sqrt{-g}}\partial_{\sigma}\left[\sqrt{-g}\left(J^{\sigma}\xi^{\beta}-J^{\beta}\xi^{\sigma}\right)\right]. \qquad (18)$$

Equation (18) was obtained based on the condition that mass of substance unit in variation of coordinates remains constant despite the change in density and its volume. With the help of (15) and (18) the last term in (17) can be transformed through 4-velosity u_{β}:

$$\frac{g_{\alpha\beta}J^{\alpha}}{\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}}\sqrt{-g}\,\delta J^{\beta}=\frac{1}{c}u_{\beta}\,\partial_{\sigma}\left[\sqrt{-g}\left(J^{\sigma}\xi^{\beta}-J^{\beta}\xi^{\sigma}\right)\right]=$$

$$=\frac{1}{c}\partial_{\sigma}\left[u_{\beta}\sqrt{-g}\left(J^{\sigma}\xi^{\beta}-J^{\beta}\xi^{\sigma}\right)\right]-\frac{1}{c}\partial_{\sigma}u_{\beta}\sqrt{-g}\left(J^{\sigma}\xi^{\beta}-J^{\beta}\xi^{\sigma}\right).$$

The term with complete divergence in integration over 4-volume in the action function will not make any contribution. The remaining term in the previous equation can be transformed further:

$$-\frac{1}{c}\partial_\sigma u_\beta \sqrt{-g}\left(J^\sigma \xi^\beta - J^\beta \xi^\sigma\right) = -\frac{1}{c}(\partial_\sigma u_\beta - \partial_\beta u_\sigma) J^\sigma \xi^\beta \sqrt{-g} =$$
$$= -\frac{1}{c}(\nabla_\sigma u_\beta - \nabla_\beta u_\sigma) J^\sigma \xi^\beta \sqrt{-g} = -\frac{1}{c} J^\sigma \nabla_\sigma u_\beta \xi^\beta \sqrt{-g}\,. \qquad (19)$$

Here the condition $J^\sigma \nabla_\beta u_\sigma = \rho_0 u^\sigma \nabla_\beta u_\sigma = 0$ was used, since it follows from the equation $u^\sigma u_\sigma = c^2$, to which the covariant derivative ∇_β is applied.

The symmetrical stress-energy tensor of substance is:

$$\phi^{\alpha\beta} = \frac{c\,J^\alpha J^\beta}{\sqrt{g_{\mu\nu} J^\mu J^\nu}}\,. \qquad (20)$$

Substituting (20) in (17) and using (19) instead of the last term in (17), we obtain the variation δS_2 in (16):

$$\delta S_2 = \int\left(-\frac{1}{2c}\phi^{\alpha\beta}\,\delta g_{\alpha\beta} - \frac{1}{2} g^{\alpha\beta}\sqrt{g_{\mu\nu} J^\mu J^\nu}\,\delta g_{\alpha\beta} + \frac{1}{c} J^\sigma \nabla_\sigma u_\beta \xi^\beta\right)\sqrt{-g}\,d\Sigma\,. \qquad (21)$$

5 Variation of Lagrangian of gravitational field and its sources

The effect of macroscopic gravitational field is revealed in the third and fourth terms in (2), which gives the following:

$$\delta S_3 = -\int \frac{1}{c}\,\delta\left(D_\mu J^\mu \sqrt{-g}\right) d\Sigma\,, \qquad \delta S_4 = \int \frac{c}{16\pi\gamma}\,\delta\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\sqrt{-g}\right) d\Sigma\,. \qquad (22)$$

We first consider variation for δS_3 in (22), using $\delta\sqrt{-g}$ by (13) and then (18) for δJ^μ :

$$\delta\left(D_\mu J^\mu\sqrt{-g}\right)=D_\mu\delta\left(J^\mu\sqrt{-g}\right)+J^\mu\sqrt{-g}\,\delta D_\mu=$$
$$=D_\mu\,\partial_\sigma\left[\sqrt{-g}\left(J^\sigma\xi^\mu-J^\mu\xi^\sigma\right)\right]+\frac{1}{2}D_\mu J^\mu g^{\alpha\beta}\sqrt{-g}\,\delta g_{\alpha\beta}+J^\mu\sqrt{-g}\,\delta D_\mu .$$

Transformation of the first term:

$$D_\mu\,\partial_\sigma\left[\sqrt{-g}\left(J^\sigma\xi^\mu-J^\mu\xi^\sigma\right)\right]=$$
$$=\partial_\sigma\left[D_\mu\sqrt{-g}\left(J^\sigma\xi^\mu-J^\mu\xi^\sigma\right)\right]-\partial_\sigma D_\mu\sqrt{-g}\left(J^\sigma\xi^\mu-J^\mu\xi^\sigma\right).$$

Neglecting the term with total derivative, we consider the following:

$$-\partial_\sigma D_\mu\sqrt{-g}\left(J^\sigma\xi^\mu-J^\mu\xi^\sigma\right)=-(\partial_\sigma D_\mu-\partial_\mu D_\sigma)\,J^\sigma\xi^\mu\sqrt{-g}=\Phi_{\mu\sigma}J^\sigma\xi^\mu\sqrt{-g}\,.$$

Substituting these results into (22), we obtain:

$$\delta S_3=\int\left(-\frac{1}{c}\Phi_{\beta\sigma}J^\sigma\xi^\beta-\frac{1}{2c}D_\mu J^\mu g^{\alpha\beta}\,\delta g_{\alpha\beta}-\frac{1}{c}J^\beta\delta D_\beta\right)\sqrt{-g}\,d\Sigma. \qquad (23)$$

Variation for δS_4 in (22) with (13) is equal to:

$$\delta\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\sqrt{-g}\right)=\delta\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\right)\sqrt{-g}+\Phi_{\mu\nu}\Phi^{\mu\nu}\,\delta\sqrt{-g}=$$
$$=\Phi_{\mu\nu}\delta\Phi^{\mu\nu}\sqrt{-g}+\Phi^{\mu\nu}\delta\Phi_{\mu\nu}\sqrt{-g}+\frac{1}{2}\Phi_{\mu\nu}\Phi^{\mu\nu}g^{\alpha\beta}\sqrt{-g}\;\delta g_{\alpha\beta}\,. \qquad (24)$$

Since $\Phi^{\mu\nu}=g^{\mu\alpha}g^{\beta\nu}\Phi_{\alpha\beta}$, the tensor $\Phi_{\alpha\beta}$ is antisymmetrical, then using $\delta g^{\beta\nu}$ by (13), we obtain:

$$\Phi_{\mu\nu}\delta\Phi^{\mu\nu}\sqrt{-g} = \Phi_{\mu\nu}\delta\left(g^{\mu\alpha}g^{\beta\nu}\Phi_{\alpha\beta}\right)\sqrt{-g} =$$
$$= \Phi_{\mu\nu}\left[g^{\mu\alpha}g^{\beta\nu}\delta\Phi_{\alpha\beta} + g^{\mu\alpha}\Phi_{\alpha\beta}\delta g^{\beta\nu} + g^{\beta\nu}\Phi_{\alpha\beta}\delta g^{\mu\alpha}\right]\sqrt{-g} =$$
$$= \Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g} + 2\Phi_{\mu\nu}g^{\mu\alpha}\Phi_{\alpha\beta}\sqrt{-g}\,\delta g^{\beta\nu} =$$
$$= \Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g} - 2g^{\nu\beta}\Phi_{\kappa\nu}\Phi^{\kappa\alpha}\sqrt{-g}\,\delta g_{\alpha\beta}.$$

Substitution of this expression into (24) gives the following result:

$$\delta\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\sqrt{-g}\right) = \Phi_{\mu\nu}\delta\Phi^{\mu\nu}\sqrt{-g} + \Phi^{\mu\nu}\delta\Phi_{\mu\nu}\sqrt{-g} +$$
$$+\frac{1}{2}\Phi_{\mu\nu}\Phi^{\mu\nu}g^{\alpha\beta}\sqrt{-g}\,\delta g_{\alpha\beta} = 2\Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g} - 2g^{\nu\beta}\Phi_{\kappa\nu}\Phi^{\kappa\alpha}\sqrt{-g}\,\delta g_{\alpha\beta} +$$
$$+\frac{1}{2}\Phi_{\mu\nu}\Phi^{\mu\nu}g^{\alpha\beta}\sqrt{-g}\,\delta g_{\alpha\beta}. \tag{25}$$

We denote $U^{\alpha\beta}$ as the stress-energy tensor of gravitational field:

$$U^{\alpha\beta} = \frac{c^2}{4\pi\gamma}\left(g^{\alpha\nu}\Phi_{\kappa\nu}\Phi^{\kappa\beta} - \frac{1}{4}g^{\alpha\beta}\Phi_{\mu\nu}\Phi^{\mu\nu}\right) = -\frac{c^2}{4\pi\gamma}\left(\Phi^{\alpha}{}_{\kappa}\Phi^{\kappa\beta} + \frac{1}{4}g^{\alpha\beta}\Phi_{\mu\nu}\Phi^{\mu\nu}\right). \tag{26}$$

Remembering that $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$, using differentiation by parts, as well as equality for an antisymmetrical tensor: $\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\right) = \sqrt{-g}\,\nabla_\alpha\Phi^{\alpha\beta}$, for the term $2\Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g}$ in (25) we obtain:

$$2\Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g} = 2\Phi^{\alpha\beta}\delta\left(\partial_\alpha D_\beta - \partial_\beta D_\alpha\right)\sqrt{-g} = 2\Phi^{\alpha\beta}\left(\partial_\alpha\delta D_\beta - \partial_\beta\delta D_\alpha\right)\sqrt{-g} =$$
$$= 4\Phi^{\alpha\beta}\sqrt{-g}\,\partial_\alpha\delta D_\beta = 4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right) - 4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\right)\delta D_\beta =$$
$$= 4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right) - 4\,\nabla_\alpha\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta. \tag{27}$$

The term $4\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right)$ in the last expression is divergence and can be neglected for variation of action function. Substituting (26) and (27) in (25), and the result in (22), we obtain:

$$\delta S_4 = \int\left(-\frac{c}{4\pi\gamma}\nabla_\alpha \Phi^{\alpha\beta}\,\delta D_\beta - \frac{1}{2c}U^{\alpha\beta}\,\delta g_{\alpha\beta}\right)\sqrt{-g}\,d\Sigma \ . \qquad (28)$$

6 Variation of Lagrangian function of electromagnetic field and its sources

Variation in (2) for electromagnetic field is the same as for gravitational field in the previous section. For the fifth and sixth terms in (2) we can write down:

$$\delta S_5 = -\int\frac{1}{c}\,\delta\left(A_\mu\, j^\mu\sqrt{-g}\right)d\Sigma\,, \qquad \delta S_6 = -\int\frac{c\varepsilon_0}{4}\,\delta\left(F_{\mu\nu}F^{\mu\nu}\sqrt{-g}\right)d\Sigma\,. \qquad (29)$$

Replacing in (22) D_μ with A_μ, J^μ with j^μ, $\Phi_{\beta\sigma}$ with $F_{\beta\sigma}$, instead of (23) we obtain:

$$\delta S_5 = \int\left(-\frac{1}{c}F_{\beta\sigma}\, j^\sigma\,\xi^\beta - \frac{1}{2c}A_\mu\, j^\mu\, g^{\alpha\beta}\,\delta g_{\alpha\beta} - \frac{1}{c}\, j^\beta\,\delta A_\beta\right)\sqrt{-g}\,d\Sigma. \qquad (30)$$

In deriving (30) was used the expression for variation of electromagnetic 4-current, similar to (18):

$$\delta j^\beta = \nabla_\sigma\left(j^\sigma\,\xi^\beta - j^\beta\,\xi^\sigma\right) = \frac{1}{\sqrt{-g}}\,\partial_\sigma\left[\sqrt{-g}\left(j^\sigma\,\xi^\beta - j^\beta\,\xi^\sigma\right)\right]. \qquad (31)$$

The stress-energy tensor of electromagnetic field is:

$$W^{\alpha\beta} = \varepsilon_0 c^2\left(-g^{\alpha\nu}F_{\kappa\nu}F^{\kappa\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right) = \varepsilon_0 c^2\left(F^{\alpha}{}_{\kappa}F^{\kappa\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right).$$

(32)

With the help of this tensor the variation δS_6 will equal as in (28) to:

$$\delta S_6 = \int \left(c\varepsilon_0 \nabla_\alpha F^{\alpha\beta}\, \delta A_\beta - \frac{1}{2c} W^{\alpha\beta} \delta g_{\alpha\beta} \right) \sqrt{-g}\, d\Sigma \ . \qquad (33)$$

7 The equations for metric

Let us put together and substitute in (2) all terms in (14), (21), (23), (28), (30) and (33) containing the variation $\delta g_{\alpha\beta}$ of metric tensor. Due to the arbitrariness of the variation the sum of all these terms should equal to zero. The result is the following:

$$k\left(-R^{\alpha\beta} + \frac{1}{2} g^{\alpha\beta} R - \Lambda g^{\alpha\beta} \right) - \frac{1}{2c}\phi^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} \sqrt{g_{\mu\nu} J^\mu J^\nu} - \frac{1}{2c} D_\mu J^\mu g^{\alpha\beta} -$$
$$- \frac{1}{2c} U^{\alpha\beta} - \frac{1}{2c} A_\mu j^\mu g^{\alpha\beta} - \frac{1}{2c} W^{\alpha\beta} = 0.$$

Let us rewrite this equation with $k = -\frac{c^3}{16\pi\gamma\beta}$, where β – the coefficient of order 1, as an equation for determining metric tensor $g^{\alpha\beta}$ with the help of known sources of energy-momentum. Here, instead of Λ, we introduce a new constant χ, according to the relation: $\Lambda = \frac{8\pi\gamma\beta\chi}{c^4}$. As well as Λ, the constant χ determines the properties of system as a whole. It has the following result:

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = \frac{8\pi\gamma\beta}{c^4} \Big(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta} + c\sqrt{g_{\mu\nu} J^\mu J^\nu}\, g^{\alpha\beta} +$$
$$+ D_\mu J^\mu g^{\alpha\beta} + A_\mu j^\mu g^{\alpha\beta} - \chi g^{\alpha\beta} \Big). \qquad (34)$$

In case when such a large system, like our universe is considered Λ has a special name – the cosmological constant. It is estimated as 10^{-52} m^{-2}. Hence the value χ is of the order $\frac{c^4\Lambda}{8\pi\gamma\beta} \approx 5\cdot 10^{-10}$ J/m^3, having dimension of energy density. For other systems, which can roughly be considered as systems with the matter continuously distributed throughout the volume of space, the constants Λ and χ can have other values.

The equation (34) was obtained based on the condition that either the variations of coordinates ξ^{β} and 4-potentials δD_{β} and δA_{β} in function of action (2) equal to zero, or the sums of all the terms with these variations are always equal to zero. In the first case (34) is an equation for the metric of a system, in which the motion of charged and gravitating substance by the definite trajectories were originally determined and the calibrated values of field potentials (that means, the constants appearing in potentials were determined) were determined as well. In the second case variations of coordinates (trajectories of substance) and variations of potentials are allowed due to their mutual influence on each other. However, it is assumed that each time the combinations of terms in function of action defining the relationship between the substance and the field, including generation of fields by substance and field influence on substance are of such kind that they equal to zero and do not affect the function of action and the metric. In the second case the initial distribution of substance in space and its initial velocity and initial values of the potentials are arbitrarily, so the laws of connection between the subsequent motion of substance and fields due to some reasons lead to the equation (34). Obviously, the validity of the second case requires additional evidence, or should be postulated, whereas in the former case it is not required.

Outside the substance, where gravitational and electromagnetic 4-currents J^{μ} and j^{μ} tend to zero, the contribution to the metric according to (34) is made only by the stress-energy tensor of gravitational field $U^{\alpha\beta}$ (26) and the stress-energy tensor of electromagnetic field $W^{\alpha\beta}$ (32). If the metric is determined within the substance, the contribution to the metric depends on all terms in (34).

We should note that the right side of (34) contains additional terms that in general relativity theory usually are not considered. In particular, (34) includes all invariant scalar values from the function (1), including terms $\Phi_{\mu\nu}\Phi^{\mu\nu}$ and $F_{\mu\nu}F^{\mu\nu}$, which became components of the tensors $U^{\alpha\beta}$ and $W^{\alpha\beta}$, respectively.

In general relativity, there is no stress-energy tensor of gravitational field $U^{\alpha\beta}$ in the form in which we have determined it. This follows from the fact that in general relativity, gravitational field reduces to the metric field, where the components $g_{\mu\nu}$ of metric tensor are considered as potentials describing the gravitational field. In this case, the presence of $U^{\alpha\beta}$ in the right side of equation for the metric (34) would mean that gravitational field is the source itself. In the absence of substance that would lead to a vicious circle where a metric gravitational field generates itself, the field gives the metric and the metric gives the field. As opposed to it, in covariant theory of gravitation (CTG), the metric is only an auxiliary geometric field induced by gravitation and electromagnetic field, taken in all their forms at different scale levels of matter.

In CTG uses the metric theory of relativity [7], the essence of which is dependence of metric on not only the properties of motion of system, but also on the type of test bodies, which can be both substance particles and quanta of field. Test bodies are needed to determine the metric of system in nature, for measurement procedure of scale and time, and have different properties due to the difference in equations of motion. As a result, the coefficient β in (34) may be different for different systems and should be found separately for complete definition of metric tensor. In particular, β was found in several situations, such as calculating deviation of test body's motion under the action of gravitation, and calculations of perihelion shift. Notes on the last four terms in (34) and the term with constant χ will be made later in the section "Tensors of energy".

8 The equations of motion of substance and field

To obtain the equations of motion of substance it is necessary to select in the full variation of action (2) those terms which contain variations of coordinates ξ^{β}. Due to the arbitrariness of ξ^{β} the sum of all such terms should be equal to zero. From the sum of (21), (23) and (30) we obtain:

$$\frac{1}{c} J^{\sigma} \nabla_{\sigma} u_{\beta} - \frac{1}{c} \Phi_{\beta\sigma} J^{\sigma} - \frac{1}{c} F_{\beta\sigma} j^{\sigma} = 0 .$$

Taking into account that $J^{\sigma}=\rho_0 u^{\sigma}$, $j^{\sigma}=\rho_{0q} u^{\sigma}$, and using the operator of proper-time-derivative [7]: $u^{\sigma}\nabla_{\sigma}=\frac{D}{D\tau}$, where D denotes covariant differential, τ is proper time, the last equation can be written as follows:

$$\rho_0 \frac{Du_{\beta}}{D\tau}=\rho_0 a_{\beta}=\Phi_{\beta\sigma}\rho_0 u^{\sigma}+F_{\beta\sigma}\rho_{0q} u^{\sigma}, \tag{35}$$

where a_{β} – 4-acceleration with covariant index, the first term on the right is the density of gravitational force, and the last term sets electromagnetic Lorentz force for charge density ρ_{0q}.

According to (35), the contribution to 4-acceleration of substance unit makes the gravitational acceleration $\Phi_{\beta\sigma} u^{\sigma}$ and 4-acceleration in electromagnetic field $\frac{\rho_{0q}}{\rho_0} F_{\beta\sigma} u^{\sigma}$. The physical meaning of (35) consists in the fact that it determines the impact on the substance of fields when the metric tensor of the system (this means that $\delta g_{\alpha\beta}=0$ in action function), and field potentials ($\delta D_{\beta}=0$, $\delta A_{\beta}=0$ in action function) are preset.

The relation connecting the tensor of gravitational field $\Phi^{\mu\nu}$ with its source in the form of 4-vector J^{μ}, follows from (23) and (28) as the consequence of variation δD_{β} for gravitational 4-potential. Taking into account the antisymmetry of the tensor $\Phi^{\alpha\beta}=-\Phi^{\beta\alpha}$ we obtain:

$$\nabla_{\alpha}\Phi^{\alpha\beta}=-\frac{4\pi\gamma}{c^2}J^{\beta}, \qquad \text{or} \qquad \nabla_{\nu}\Phi^{\mu\nu}=\frac{4\pi\gamma}{c^2}J^{\mu}. \tag{36}$$

A similar relation for electromagnetic field, from expressions for variation δA_{β} of electromagnetic 4-potential in (30) and (33) has the form:

$$\nabla_\alpha F^{\alpha\beta} = \frac{1}{c^2 \varepsilon_0} j^\beta , \qquad \text{or} \qquad \nabla_\nu F^{\mu\nu} = -\frac{1}{c^2 \varepsilon_0} j^\mu = -\mu_0 j^\mu , \qquad (37)$$

where $\mu_0 = \frac{1}{c^2 \varepsilon_0}$ – vacuum permeability.

Equations (36) and (37), as it is evident after obtaining them from variation of action function, are valid in case when the variation of coordinates of substance and the variation of metric are equal to zero, that is $\xi^\beta = 0$, $\delta g_{\alpha\beta} = 0$. This means that if the motion of substance and metric of system are preset, we can calculate how the substance generates the field strengths.

If we consider the definition of gravitational tensor: $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$, and take the covariant derivative of the tensor, followed by a cyclic permutation of indices, then the following equation is performed identically:

$$\nabla_\sigma \Phi_{\mu\nu} + \nabla_\nu \Phi_{\sigma\mu} + \nabla_\mu \Phi_{\nu\sigma} = 0 . \qquad (38)$$

Another form of (38) is:

$$\varepsilon^{\alpha\beta\gamma\delta} \nabla_\gamma \Phi_{\alpha\beta} = 0 ,$$

where $\varepsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol or totally antisymmetrical unit tensor.

Equation (38) sets the gravitational field equations without sources, so that the set of equations (36) and (38) completely determines the properties of the gravitational field.

For electromagnetic field we have the similar formula as in (38):

$$\nabla_\sigma F_{\mu\nu} + \nabla_\nu F_{\sigma\mu} + \nabla_\mu F_{\nu\sigma} = 0 \qquad \text{or} \qquad \varepsilon^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta} = 0 . \qquad (39)$$

Equations (37) and (39) are the Maxwell equations, written in four-dimensional notation.

Relation (36) can be written in another way: $\nabla^{\nu}\Phi_{\mu\nu}=\frac{4\pi\gamma}{c^{2}}J_{\mu}$. If we take on both sides of this equation the contravariant derivative ∇^{μ}, and use the definition $\Phi_{\mu\nu}=\nabla_{\mu}D_{\nu}-\nabla_{\nu}D_{\mu}$, then due to the symmetry and changing of the order of differentiation the left side will equal to zero. This leads to the continuity equation (mass conservation), which imposes certain conditions on 4-velocity and the density of substance:

$$\nabla^{\mu}J_{\mu}=\nabla_{\mu}J^{\mu}=\nabla_{\mu}\left(\rho_{0}u^{\mu}\right)=0\,.$$

For conservation of the electric charge, we have a similar relation:

$$\nabla^{\mu}j_{\mu}=\nabla_{\mu}j^{\mu}=\nabla_{\mu}\left(\rho_{0q}u^{\mu}\right)=0\,.$$

If we specify a condition for 4-vector of gravitational potential D_{μ}, or for 4-vector of electromagnetic potential A_{μ}, then it provides definite relation between scalar and vector potentials. The standard approach is Lorenz gauge, which gives the following conditions:

$$\nabla^{\mu}D_{\mu}=\nabla_{\mu}D^{\mu}=0\,,\qquad\qquad \nabla^{\mu}A_{\mu}=\nabla_{\mu}A^{\mu}=0\,. \tag{40}$$

Substituting (40) into (36) and (37) and using $\Phi^{\mu\nu}=\nabla^{\mu}D^{\nu}-\nabla^{\nu}D^{\mu}$, $F^{\mu\nu}=\nabla^{\mu}A^{\nu}-\nabla^{\nu}A^{\mu}$, while the expression $\nabla_{\nu}\nabla^{\nu}=\Box$ is D'Alembert operator, we obtain wave equations for 4-potentials in the Lorenz gauge:

$$\Box D^{\mu}=-\frac{4\pi\gamma}{c^{2}}J^{\mu}\,,\qquad\qquad \Box A^{\mu}=\mu_{0}\,j^{\mu}\,.$$

9 About applicability of equations of motion in general case

As we mentioned above, the equation for the metric (34) is correct in case when the motion of substance and field potentials are completely specified. However, in most cases only the initial state of motion and initial potentials are known, later the motion of substance is determined by field and is set indirectly.

How the equation (34) should be used in this case, and under what conditions? To answer this question, we assume that the motion of substance and field potentials at a short interval of time after initial time point remain unchanged. Then we can find metric tensor from (34). After that, assuming immutability of metric and the motion of substance in the second time interval, using (36) and (37) we can calculate the derivatives of tensors $\Phi^{\alpha\beta}$ and $F^{\alpha\beta}$ by coordinates. After integration of these derivatives the field strengths can be found which are part of these tensors. Since now correct tensors $\Phi^{\alpha\beta}$ and $F^{\alpha\beta}$ are known, with the help of them on the third time interval in the equation of motion (35) the acceleration of substance and its motion can be estimated, and the motion can be adjusted. In the fourth time interval the data about motion of substance from the third interval and about fields from the second interval can be used in order to evaluate the change of metric. Further calculations are repeated in the specified order. Thus the actual motion of substance in gravitational and electromagnetic fields, and space-time metric can be found approximately by an iterative procedure by means of the above-mentioned equations for metric, motion of substance and fields.

10 Tensors of energy

Let's return to the equation for metric (34). It is known that the covariant derivative of left side of (34) is equal to zero, which is a characteristic of the Hilbert-Einstein tensor located there. Consequently, the covariant derivative of the right side of (34) must also equal to zero:

$$\nabla_\beta \left(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta} + c\sqrt{g_{\mu\nu} J^\mu J^\nu}\, g^{\alpha\beta} + D_\mu J^\mu g^{\alpha\beta} + A_\mu j^\mu g^{\alpha\beta} - \chi g^{\alpha\beta} \right) = 0 . \tag{41}$$

Taking into account the definition of stress-energy tensor of substance (20), relations $\sqrt{g_{\mu\nu} J^\mu J^\nu} = c\rho_0$, $J^\alpha = \rho_0 u^\alpha$, and using operator of the proper-time-derivative: $u^\sigma \nabla_\sigma = \frac{D}{D\tau}$, we can write down:

$$\nabla_\beta \phi^{\alpha\beta} = \nabla_\beta \left(\frac{c\, J^\alpha J^\beta}{\sqrt{g_{\mu\nu} J^\mu J^\nu}} \right) = \frac{1}{\rho_0} \nabla_\beta \left(J^\alpha J^\beta \right) + J^\alpha J^\beta \nabla_\beta \left(\frac{1}{\rho_0} \right) =$$
$$= u^\beta \nabla_\beta J^\alpha - u^\alpha u^\beta \nabla_\beta \rho_0 = \frac{DJ^\alpha}{D\tau} - u^\alpha \frac{D\rho_0}{D\tau} = \rho_0 \frac{Du^\alpha}{D\tau} = \rho_0 a^\alpha \,, \qquad (42)$$

where a^α – 4-acceleration.

Now we find the covariant derivative of stress-energy tensor of gravitational field. Since the metric tensor under covariant differentiation acts as a constant, using (36), we obtain from (26):

$$\frac{4\pi\gamma}{c^2} \nabla_\beta U^{\alpha\beta} = \nabla_\beta \left(g^{\alpha\nu} \Phi_{\kappa\nu} \Phi^{\kappa\beta} - \frac{1}{4} g^{\alpha\beta} \Phi_{\mu\nu} \Phi^{\mu\nu} \right) =$$
$$= g^{\alpha\nu} \Phi^{\kappa\beta} \nabla_\beta \Phi_{\kappa\nu} + g^{\alpha\nu} \Phi_{\kappa\nu} \nabla_\beta \Phi^{\kappa\beta} - \frac{1}{4} g^{\alpha\beta} \Phi^{\mu\nu} \nabla_\beta \Phi_{\mu\nu} - \frac{1}{4} g^{\alpha\beta} \Phi_{\mu\nu} \nabla_\beta \Phi^{\mu\nu} =$$
$$= -\Phi^{\kappa\beta} \nabla_\beta \Phi^\alpha{}_\kappa + g^{\alpha\nu} \Phi_{\kappa\nu} \frac{4\pi\gamma}{c^2} J^\kappa - \frac{1}{2} g^{\alpha\beta} \Phi^{\mu\nu} \nabla_\beta \Phi_{\mu\nu} \,.$$
$$(43)$$

We apply (38) to the last term in (43), on condition that tensor $\Phi_{\alpha\beta}$ is antisymmetrical:

$$-\frac{1}{2} g^{\alpha\beta} \Phi^{\mu\nu} \nabla_\beta \Phi_{\mu\nu} = -\frac{1}{2} g^{\alpha\beta} \Phi^{\mu\nu} \left(-\nabla_\nu \Phi_{\beta\mu} - \nabla_\mu \Phi_{\nu\beta} \right) =$$
$$= \frac{1}{2} g^{\alpha\beta} \Phi^{\mu\nu} \nabla_\nu \Phi_{\beta\mu} + \frac{1}{2} g^{\alpha\beta} \Phi^{\mu\nu} \nabla_\mu \Phi_{\nu\beta} = \frac{1}{2} \Phi^{\mu\nu} \nabla_\nu \Phi^\alpha{}_\mu - \frac{1}{2} g^{\alpha\beta} \Phi^{\mu\nu} \nabla_\mu \Phi^\alpha{}_\nu =$$
$$= \Phi^{\mu\nu} \nabla_\nu \Phi^\alpha{}_\mu \,.$$

Substituting this into (43) we obtain the relation between the covariant derivative of stress-energy tensor of gravitational field and 4-vector density of gravitational force:

$$\nabla_\beta U^{\alpha\beta} = -\Phi^\alpha{}_\kappa J^\kappa . \qquad (44)$$

We obtain a similar expression for the covariant derivative of stress-energy tensor of electromagnetic field and 4-vector density of electromagnetic force (Lorentz force density) with (32) and (39):

$$\nabla_\beta W^{\alpha\beta} = -F^\alpha{}_\kappa j^\kappa . \qquad (45)$$

Substituting (42), (44) and (45) into (41) we obtain:

$$\nabla_\beta \left(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta}\right) = \rho_0 a^\alpha - \Phi^\alpha{}_\kappa J^\kappa - F^\alpha{}_\kappa j^\kappa = 0 . \qquad (46)$$

The zero right side of (46) follows from the equations of substance motion in gravitational and electromagnetic fields (35). Consequently, the covariant derivative for the remaining terms in (41) must also equal to zero:

$$\begin{aligned} &\nabla_\beta\, g^{\alpha\beta} \left(c\sqrt{g_{\mu\nu} J^\mu J^\nu} + D_\mu J^\mu + A_\mu j^\mu - \chi \right) = \\ &= \nabla^\alpha \left(c\sqrt{g_{\mu\nu} J^\mu J^\nu} + D_\mu J^\mu + A_\mu j^\mu - \chi \right) = 0 . \end{aligned} \qquad (47)$$

In brackets in (47) there is a scalar quantity, in this case the covariant derivative ∇^α is equal to partial derivative ∂^α (that is 4-gradient). Relation (47) is automatically satisfied if we assume that the constant in brackets is set equal to zero. This gives the relation:

$$c\sqrt{g_{\mu\nu} J^\mu J^\nu} + D_\mu J^\mu + A_\mu j^\mu = \chi = const . \qquad (48)$$

Equality (48) is necessary to perform in (34) the limit relations for tensors at infinity, where there is neither substance nor fields. As it is indicated in [4], at infinity the right side of (34) with energy tensors is equal to zero, and the space-time becomes flat, bringing to zero the left side of Hilbert-Einstein tensor. Taking into account (48) the equations for metric obtain the simplest form:

$$R^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}R=\frac{8\pi\gamma\beta}{c^4}\left(\phi^{\alpha\beta}+U^{\alpha\beta}+W^{\alpha\beta}\right). \qquad (49)$$

Let's use (15) and reveal in (48) the scalar product of 4-vectors, with the help of $D_\mu=\left(\frac{\psi}{c},-\boldsymbol{D}\right)$, $A_\mu=\left(\frac{\varphi}{c},-\boldsymbol{A}\right)$, $J^\mu=\rho_0 u^\mu$, $j^\mu=\rho_{0q}u^\mu$:

$$\rho_0 c^2+\frac{\rho_0\psi u^0}{c}-\rho_0\boldsymbol{D}\cdot\boldsymbol{u}+\frac{\rho_{0q}\varphi u^0}{c}-\rho_{0q}\boldsymbol{A}\cdot\boldsymbol{u}=\chi\,. \qquad (50)$$

Here $\boldsymbol{u}$ is a 3-vector, which is part of 4-velocity u^μ. In uncurved space-time according to special relativity theory $u^\mu=\left(\frac{c}{\sqrt{1-v^2/c^2}},\quad \frac{\boldsymbol{v}}{\sqrt{1-v^2/c^2}}\right)$, then $u^0=\frac{c}{\sqrt{1-v^2/c^2}}$, $\boldsymbol{u}=\frac{\boldsymbol{v}}{\sqrt{1-v^2/c^2}}$, where $\boldsymbol{v}$ is the velocity of substance motion. This shows that $\frac{\rho_0\psi u^0}{c}$ is the energy density for substance in gravitational field with scalar potential ψ. The vector potential $\boldsymbol{D}$ of gravitational field is also associated with energy, but its value $-\rho_0\boldsymbol{D}\cdot\boldsymbol{u}$ can have different sign depending on the direction of vector $\boldsymbol{u}$, which is proportional to speed $\boldsymbol{v}$, and the direction of vector $\boldsymbol{D}$. The same is true in respect of the density of electromagnetic energy – it depends on the charge density ρ_{0q}, the scalar electric potential φ and vector potential $\boldsymbol{A}$ of electromagnetic field.

Now suppose that in (50) the macroscopic gravitational and electromagnetic fields are off and their potentials are equal to zero. In this case, the density of substance must reach a certain value ρ'_0, which depends only on fundamental microscopic fields acting at the level of elementary particles. Then we will obtain $\chi=\rho'_0 c^2$, and (50) can be rewritten as follows:

$$\rho_0 c^2=\rho'_0 c^2-\frac{\rho_0\psi u^0}{c}+\rho_0\boldsymbol{D}\cdot\boldsymbol{u}-\frac{\rho_{0q}\varphi u^0}{c}+\rho_{0q}\boldsymbol{A}\cdot\boldsymbol{u}=\rho'_0 c^2-\varepsilon_g-\varepsilon_e\,, \qquad (51)$$

where ε_g and ε_e denote the energy density of substance in gravitational and electromagnetic fields, respectively.

As ε_g for substance is usually negative (due to the negative gravitational potential ψ), then from (51) it follows that density of substance ρ_0 in gravitational field becomes greater than density of substance ρ'_0 in the absence of field (when the substance of a body is divided into parts and separated to infinity). The same thing can be said about mass – in gravitational field it is expected to increase due to the contribution of gravitational mass-energy of substance in the field. Thus we have obtained the result similar to that which we have proved in [6] and [10], but in relation to contribution of mass-energy of field to the total mass of system of substance and field. Then we found that the mass of a spherical body grows due to its field, and with constant volume, this means an increase in the effective density of the substance.

We can integrate (51) over the volume of substance of a spherical uncharged body in static position, when the body is at rest and does not rotate. If the substance is infinitely slowly superimposed on the body by parts in the form of thin spherical shells with the same density of substance, we can assume that in (49) $u^0 = c$, as well as:

$$\int \rho_0 c^2 dx^1 dx^2 dx^3 = mc^2 , \qquad \int \rho'_0 c^2 dx^1 dx^2 dx^3 = m' c^2 ,$$

$$\int \rho_0 \psi \, dx^1 dx^2 dx^3 = -\int \frac{\gamma \rho_0 m(r)}{r} dx^1 dx^2 dx^3 ,$$

where m – the observed mass of the body with its radius R,

m' – the mass of substance of the body without taking into account the energy of gravitation,

$m(r) = \frac{4\pi r^3 \rho_0}{3}$ – the mass inside the radius r, increasing from 0 to the radius of the body R with increasing of mass.

As a result, (51) becomes equality for the masses:

$$mc^2 = m'c^2 + \int \frac{\gamma \rho_0 m(r)}{r} dx^1 dx^2 dx^3 = m'c^2 + \frac{3\gamma m^2}{5R}, \qquad (52)$$

where γ is gravitational constant.

In (52) the module of mass-energy of gravitational field is added to the mass-energy of the body. In reality, during the formation of space objects in gravitational field the virial theorem applies according to which approximately half of gravitational field energy leaves the system in the form of radiation, and the other half heats the substance. This reduces by half the additive to mass-energy in (52).

For the main objects of stellar level of matter the contribution to (51) of energy density of substance in electromagnetic field ε_e is little in comparison to ε_g. In particular, for neutron stars, gravitational energy is equal to $E_\gamma = -\frac{k\gamma M_s^2}{R_s} \approx -2.4 \cdot 10^{46}$ J, here $k \approx 0.6$ in approximation of uniform density of substance, γ – gravitational constant, $M_s = 2.7 \cdot 10^{30}$ kg, $R_s = 12$ km – mass and radius of a typical neutron star. The electromagnetic energy reaches a maximum in magnetars at magnetic pole of which the magnetic field can be about $B_m = 1.9 \cdot 10^{11}$ T. Since magnetic energy density is given in form $\frac{B^2}{2\mu_0}$, then the integral over the entire volume inside the star and beyond it gives the magnitude of the magnetic energy of about 10^{41} J, which is considerably less than the modulus of gravitational energy.

The similar situation exists at the level of elementary particles, where according to the theory of infinite nesting of matter [8], an analog of a neutron star is a nucleon. The energy of a proton in its own field of strong gravitation is estimated with the formula $E_\Gamma = -\frac{k\Gamma M_p^2}{R_p}$, where M_p and R_p denote mass and radius of a proton, $\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514 \cdot 10^{29}$ m$^3\cdot$kg$^{-1}\cdot$s^{-2} – strong gravitational constant, e – elementary charge, ε_0 – vacuum permittivity, M_e –

electron mass. The expression for electric energy of a proton in the case of uniform charge distribution is as follows: $E_e = \frac{ke^2}{4\pi\varepsilon_0 R_p}$. Consequently, for a proton the ratio of module energy of strong gravitation to electrical energy is equal to the ratio of a proton to an electron mass, and the energy of strong gravitation dominates.

The global dominance of gravitational forces over electromagnetic forces leads to possibility of formation of elementary particles of substance, massive bodies and other objects found in space. For the observable universe cosmological constant Λ is estimated by 10^{-52} m^{-2}, and constant χ reaches $\chi \approx 5 \cdot 10^{-10}$ J/m^3. We consider that $\chi = \rho_0' c^2$ characterizes the visible universe as a whole, setting the rest energy density of substance distributed in space, without taking into account the energy fields. We further assume that gravitational fields are a consequence of flows of gravitons, which are produced by tiny particles of all the substance that exists in the universe. The more substance is in the universe, the greater is the density of substance and the more is the density of gravitons' flows. Then relation (50) supports the Einstein's idea that the inertia of a body must increase near other gravitational masses [11], which is in turn the development of Mach's principle of the impact of distant masses to acceleration of bodies.

We should remind that in the usual interpretation the cosmological constant is proportional to vacuum energy density, and still it is not known exactly what particles or fields are responsible for this energy. There are also suggestions that the cosmological constant depends on the time, on the scalar curvature of space-time (in f(R) gravity) or on the energy-momentum tensor [12].

We will further evaluate the scalar curvature R for our universe. To find R we multiply (49) by $g_{\alpha\beta}$ and taking into account the fact that $g_{\alpha\beta} R^{\alpha\beta} = R$, $g_{\alpha\beta} g^{\alpha\beta} = 4$, while for stress-energy tensors (26) and (32) there are equations: $g_{\alpha\beta} U^{\alpha\beta} = 0$, $g_{\alpha\beta} W^{\alpha\beta} = 0$. Using the expression (20), we obtain:

$$R = -\frac{8\pi\gamma\beta\rho_0}{c^2} .$$

If we assume that the density of substance can be calculated as follows $\rho_0 \approx \rho_0' = \frac{\chi}{c^2} = \frac{\Lambda c^2}{8\pi\gamma\beta}$, the curvature of cosmological constant accurate to the sign equals to: $R = -\Lambda$. At the first approximation the observed universe can be regarded as the space-time of constant negative curvature, for which the equation is $R = -\frac{12}{r^2}$. Hence the radius of such a world would be $r = \frac{\sqrt{3}\,c}{\sqrt{2\pi\gamma\beta\rho_0}} = \sqrt{\frac{12}{\Lambda}} \approx 3.4 \cdot 10^{26}$ m. On the other hand, if the observable universe is close to the state of Schwarzschild's black hole, its radius should equal to: $r_{bh} = \frac{2\gamma M}{c^2} = \frac{\sqrt{3}\,c}{\sqrt{8\pi\gamma\rho_0}} = \frac{r}{2}$. In both cases, in determination of space-time curvature, and in evaluating the radius of the black hole, speed of light is used as a measure of speed and means for measuring.

11 Analysis of equation of motion in relation to mass

According to (46) the equation of substance motion in gravitational and electromagnetic fields has the form:

$$\nabla_\beta \left(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta} \right) = 0 . \tag{53}$$

Let's consider (53) in weak field limit, where the special relativity theory is valid and the covariant derivative becomes the 4-gradient. In this case we obtain:

$$J^\mu = \rho_0 u^\mu = \rho_0 \left(\frac{c}{\sqrt{1 - v^2/c^2}}, \quad \frac{\boldsymbol{v}}{\sqrt{1 - v^2/c^2}} \right),$$

and for the scalar components of stress-energy tensor of substance (20) we can write down:

$$\phi^{00} = \frac{\rho_0 c^2}{1 - v^2/c^2} , \qquad \phi^{0i} = \frac{K^i}{c} = \frac{\rho_0 c v^i}{1 - v^2/c^2} , \tag{54}$$

where the index $i = 1, 2, 3$, so that in the Cartesian coordinate system $v^1 = v_x$, $v^2 = v_y$, $v^3 = v_z$, and vector $\boldsymbol{K} = \frac{\rho_0 c^2 \boldsymbol{v}}{1 - v^2/c^2}$ is 3-vector of mechanical energy flow density or relativistic Umov's vector.

The gravitational tensor $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$ is defined by the 4-potential $D_\mu = \left(\frac{\psi}{c}, -\boldsymbol{D} \right)$ of the gravitational field, which depends on the scalar potential ψ and vector potential $\boldsymbol{D}$ of the gravitational field. We shall introduce gravitational acceleration $\boldsymbol{G}$ and torsion field $\boldsymbol{\Omega}$ (gravitomagnetic field) as follows:

$$\boldsymbol{G} = -\nabla \psi - \frac{\partial \boldsymbol{D}}{\partial t}, \qquad \boldsymbol{\Omega} = \nabla \times \boldsymbol{D}.$$

With the help of the quantities $\boldsymbol{G}$ and $\boldsymbol{\Omega}$ the scalar components of the stress-energy tensor of gravitational field (26) can be represented as follows:

$$U^{00} = -\frac{1}{8\pi\gamma}(G^2 + c^2 \Omega^2), \qquad U^{0i} = \frac{H^i}{c}, \tag{55}$$

where H^i denotes the components of 3-vector energy flow density of gravitational field (Heaviside vector) $\boldsymbol{H} = -\frac{c^2}{4\pi\gamma}[\boldsymbol{G} \times \boldsymbol{\Omega}]$.

For the stress-energy tensor of electromagnetic field (32) in substance which is not magnetized and not polarized in the limit of special relativity we obtain similarly:

$$W^{00} = \frac{\varepsilon_0}{2}(E^2 + c^2 B^2), \qquad W^{0i} = \frac{P^i}{c}, \tag{56}$$

where P^i denote the components of 3-vector of electromagnetic energy flow density (Poynting vector) $\boldsymbol{P}=\varepsilon_0 c^2 \left[\boldsymbol{E}\times\boldsymbol{B}\right]$.

Substituting (54), (55) and (56) into (53) for $\alpha=0$ and for the replacement ∇_β on ∂_β in Minkowski space makes an expression of Poynting's theorem:

$$\partial_\beta\left(\phi^{0\beta}+U^{0\beta}+W^{0\beta}\right)=0\,, \qquad \text{or} \qquad \frac{\partial}{\partial t}\left(\phi^{00}+U^{00}+W^{00}\right)=-\nabla\cdot(\boldsymbol{K}+\boldsymbol{H}+\boldsymbol{P})\,. \tag{57}$$

The values ϕ^{00}, U^{00} and W^{00} set the energy density of substance, gravitational and electromagnetic fields respectively.

Let's integrate (57) over the entire volume of space occupied by substance and field. We shall consider the quasi-stationary case, when all energy flows are closed so that the volume integral of divergence of the right side of (57) becomes close to zero. This means that the amount of energy flows, extending forever into infinity is little. Then taking into account (54), (55) and (56) we should obtain:

$$\int\left[\frac{\rho_0 c^2}{1-v^2/c^2}-\frac{1}{8\pi\gamma}(G^2+c^2\Omega^2)+\frac{\varepsilon_0}{2}(E^2+c^2B^2)\right]dV=const=m'c^2\,, \tag{58}$$

$$\int\frac{\rho_0 c^2}{1-v^2/c^2}dV=\int\frac{\rho_0 c^2}{1-v^2/c^2}\sqrt{1-v^2/c^2}\,dV_0=\int\frac{c^2 dm}{\sqrt{1-v^2/c^2}}=mc^2+E_k\,,$$

$$mc^2=m'c^2-E_k+\int\left[\frac{1}{8\pi\gamma}(G^2+c^2\Omega^2)-\frac{\varepsilon_0}{2}(E^2+c^2B^2)\right]dV\,.$$

We denoted by m' the mass of substance when the substance is separated to infinity and is in rest there. In this case all the fields G, Ω, E and B will tend to zero. Equation (58) shows that the mass of substance m in relation to the mass of substance m' increases due to the contribution of total mass-energy of gravitational field and decreases due to the contribution of mass-energy of electromagnetic field and the kinetic energy of motion of substance E_k. Actually (58) confirms the relation (52) from the previous section.

12 Additional notes

In our view, the equation of motion of substance (35) should look a little differently:

$$\frac{DJ_\beta}{D\tau}=\frac{D(\rho_0 u_\beta)}{D\tau}=\Phi_{\beta\sigma}\rho_0 u^\sigma+F_{\beta\sigma}\rho_{0q}u^\sigma. \qquad (59)$$

In (59) the substance density ρ_0 has been included under the sign of total derivative with respect to proper time. This allows us to describe cases where the substance density changes and thus creates an additional acceleration of substance. Meanwhile, equation (35) was obtained from the variation of coordinates described in [2] and [4], with constant mass in the variation. This led to the fact that (35) differs from (59), as in case $\rho_0=const$, and therefore ρ_0 can be outside of the total differential.

It is interesting that we can choose the stress-energy tensor of substance so that its covariant derivative just gives the rate of change of substance's 4-current. This tensor has an unusual form in terms of indices, but formal covariant derivative gives the correct result. Instead of (20) we shall write down: $\phi^{\alpha\beta}=\frac{2c\,J^\alpha J^\beta}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}$. Taking into account the continuity equation $\nabla_\beta J^\beta=0$ we obtain:

$$\nabla_\beta\phi^{\alpha\beta}=\nabla_\beta\left(\frac{2c\,J^\alpha J^\beta}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}\right)=\frac{2c\,J^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}+2c\,J^\alpha J^\beta\nabla_\beta\left(\frac{1}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}\right)=$$

$$=\frac{2c\,J^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}-\frac{c\,J^\alpha J^\beta}{(g_{\alpha\beta}J^\alpha J^\beta)^{1,5}}\nabla_\beta\left(g_{\alpha\beta}J^\alpha J^\beta\right)=\frac{c\,J^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}=u^\beta\nabla_\beta J^\alpha=\frac{DJ^\alpha}{D\tau}.$$

In fact, we have found the tensor $\phi^{\alpha\beta} = \frac{2c\,J^{\alpha}\,J^{\beta}}{\sqrt{g_{\alpha\beta}\,J^{\alpha}\,J^{\beta}}}$ in (17). The reason that we have not used it as it is described here, is that instead of the exact form of variation δJ^{β} as the function of coordinate's variations, the simplified form (18) was used in derivation of motion equations. This leads to (35), but not to (59).

From variation (48) it follows that the total variation of mass 4-current δJ^{β} is associated with variation of metric tensor $\delta g_{\mu\nu}$, variation of electromagnetic 4-current δj^{μ} and variations of 4-potentials:

$$\delta\left(c\sqrt{g_{\mu\nu}\,J^{\mu}\,J^{\nu}} + D_{\mu}\,J^{\mu} + A_{\mu}\,j^{\mu} - \chi\right) = 0\,, \tag{60}$$

$$\frac{1}{2}u^{\mu}\,J^{\nu}\delta g_{\mu\nu} + u_{\nu}\,\delta J^{\nu} + D_{\mu}\,\delta J^{\mu} + J^{\mu}\delta D_{\mu} + A_{\mu}\,\delta\,j^{\mu} + j^{\mu}\,\delta A_{\mu} = 0\,.$$

The relationship of variations in (60) is connected through variation of coordinates ξ^{β} in such a way that the mass of any local volume does not change during variation. However, we can admit a situation when the mass-energy is converted into radiation energy, or substance density changes due to inflow or outflow of mass. Then, some results presented in this paper will require a corresponding change.

Let's now substitute (48) into (2), and with equations $\sqrt{g_{\mu\nu}\,J^{\mu}\,J^{\nu}} = c\rho_0$, $-2k\Lambda - \frac{\chi}{c} = 0$, we obtain:

$$\delta S = \delta\int\left(kR + \frac{c}{16\pi\gamma}\Phi_{\mu\nu}\Phi^{\mu\nu} - \frac{c\varepsilon_0}{4}F_{\mu\nu}\,F^{\mu\nu}\right)\sqrt{-g}\,d\Sigma = 0\,.$$

After implementation of variation in this equation we will obtain the equations for metric (49), but without the tensor $\phi^{\alpha\beta}$:

$$R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = \frac{8\pi\gamma\beta}{c^4}\left(U^{\alpha\beta} + W^{\alpha\beta}\right). \tag{61}$$

The metric obtained in (61) is as it should be outside the substance, and this metric depends directly on the magnitude of existing fields and indirectly on the distribution of substance in this field.

In CTG there is a concept of geodesic line, which coincides with the expression of general relativity theory, but only for field quanta. The equation of motion (35), taking into account $D\tau = d\tau$, can be written as:

$$\frac{D}{d\tau}\left(\frac{dx_\beta}{d\tau}\right) = \frac{Ddx_\beta}{d\tau d\tau} = \Phi_{\beta\sigma}\frac{dx^\sigma}{d\tau} + \frac{\rho_{0q}}{\rho_0}F_{\beta\sigma}\frac{dx^\sigma}{d\tau}. \qquad (62)$$

For field quanta we obtain $ds = cd\tau = 0$. Multiplying (62) by $d\tau d\tau$, we obtain the equality to zero of the right side: $Ddx_\beta = 0$. Dividing this by the square differential of time coordinate λ, which measures time along the trajectory of quantum, and recalling the definition of operator of proper-time-derivative, we obtain the geodesic equation in covariant indices:

$$\frac{Ddx_\beta}{d\lambda\, d\lambda} = \frac{D}{d\lambda}\left(\frac{dx_\beta}{d\lambda}\right) = \frac{dx^\sigma}{d\lambda}\nabla_\sigma\left(\frac{dx_\beta}{d\lambda}\right) = \frac{dx^\sigma}{d\lambda}\partial_\sigma\left(\frac{dx_\beta}{d\lambda}\right) - \Gamma^\gamma_{\sigma\beta}\frac{dx^\sigma}{d\lambda}\frac{dx_\gamma}{d\lambda} =$$

$$= \frac{d}{d\lambda}\left(\frac{dx_\beta}{d\lambda}\right) - \Gamma^\gamma_{\sigma\beta}\frac{dx^\sigma}{d\lambda}\frac{dx_\gamma}{d\lambda} = 0. \qquad (63)$$

While the field quanta are distributed outside the substance in a given field (gravitational and electromagnetic), their movement takes place in accordance with the equation of motion (63), and the space-time metric is determined from (61). What can change while the field quanta pass through the substance? If the substance is rare and does not interact with quanta, the quanta move between the particles of substance. Then in principle, equation (61) should be valid for the metric with the amendment that it is now necessary to take into account the stress-energy tensor of field of strong gravitation acting at the level of elementary particles. This new tensor should look as a supplement to the stress-energy tensor $U^{\alpha\beta}$ of normal gravitation (26), replacing the constant of gravitation γ by the constant of strong gravitation Γ , and with a factor of proportionality. In practice instead of using this new tensor, it is replaced by the stress-energy tensor of substance $\phi^{\alpha\beta}$, and one say that "the substance alters the space-time metric inside, and affects the metric beyond its borders".

This leads to the equation for the metric in the form of (49). But another interpretation is possible – a field always affects the metric, whereas the role of substance is reduced only to creation of field. In this case it is necessary to impose the condition on the properties of test objects with the help of which we study metric and find metric tensor components – these test bodies must interact with the substance at a distance and only through the fields, without mechanical contact randomly changing the movement.

The above interpretation of relationship between substance, metric and field is difficult in general relativity, in which gravitation is hiding in the shadow of geometrical metric field and losing its physical essence. Metric gravitational field (metric of space-time) in general relativity depends on the substance and electromagnetic field and is fully determined by them. But how does the substance change physically the metric field even if it is far away from it? What is the relationship mechanism between the substance and the field? All this remains a mystery.

In covariant theory of gravitation the Fatio-Le Sage's gravitation theory is considered as the basic idea of generating gravitational field, which allows to describe in the same way strong gravitation at the level of elementary particles and usual macroscopic gravitation [13], as well as electromagnetic interaction between bodies [7]. Quanta of gravitation, which are formed by relativistic objects at the lower levels of matter presumably in the form of electromagnetic radiation and neutrinos, become gravitons for the objects of higher levels of matter and create there gravitational interaction. The gradients of energy density of gravitons' flows may be considered as gravitational field strengths. Then the gravitational potential is the difference between the energy density of gravitons' flows near or inside the bodies, and the energy density of gravitons' flows at infinity in the absence of bodies. These flows of gravitons are responsible for deviation of test particles and field quanta near the massive bodies. In this picture the field quanta of lower levels of matter generate macroscopic fields and form macroscopic metric, and the substance (regarded as an aggregation of objects from different levels of matter, distinguishing by their characteristic sizes and masses) interacts with the field quanta and generates them.

References

[1] A. Einstein, Die Grundlage der allgemeinen Relativitätstheorie. Annalen der Physik, 354 (7) (1916), 769-822.

[2] P.A.M. Dirac, *General theory of relativity*. Florida State University, John Wiley & Sons, Inc., New York (1975).

[3] W. Pauli. *Theory of Relativity*. Pergamon Press, Inc., New York (1958).

[4] V. A. Fock, *The Theory of Space, Time and Gravitation*. Macmillan, New York (1964).

[5] L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, Vol. 2 (4th ed.). Butterworth-Heinemann, Oxford (1975).

[6] S.G. Fedosin, Energy, Momentum, Mass and Velocity of Moving Body. vixra.org, 13 Jun 2011.

[7] S.G. Fedosin, *Fizicheskie teorii i beskonechnaia vlozhennost' materii*, Perm (2009).

[8] S.G. Fedosin, *Fizika i filosofiia podobiia: ot preonov do metagalaktik*, Style-MG, Perm (1999).

[9] S.G. Fedosin, The General Theory of Relativity, Metric Theory of Relativity and Covariant Theory of Gravitation: Axiomatization and Critical Analysis. vixra.org, 26 Mar 2011.

[10] S.G. Fedosin, The Principle of Proportionality of Mass and Energy: New Version. vixra.org, 13 Jul 2011.

[11] A. Einstein, *The Meaning of Relativity*, Fifth Edition, Princeton University Press, Princeton (1955).

[12] N. J. Poplawski, A Lagrangian description of interacting energy. arXiv:gr-qc/0608031v2 .

[13] S.G. Fedosin, Model of Gravitational Interaction in the Concept of Gravitons. Journal of Vectorial Relativity, 4 (1) (2009), 1–24.