Advances in Physics Theories and Applications, Vol. 23, pp. 19-25 (2013)

4/3 Problem for the Gravitational Field

Sergey G. Fedosin
PO box 614088, Sviazeva str. 22-79, Perm, Russia
E-mail: intelli@list.ru

Abstract

The gravitational field potentials outside and inside a uniform massive ball were determined using the superposition principle, the method of retarded potentials and Lorentz transformations. The gravitational field strength, the torsion field, the energy and the momentum of the field, as well as the effective masses associated with the field energy and its momentum were calculated. It was shown that 4/3 problem existed for the gravitational field as well as in the case of the electromagnetic field.

Keywords: energy, momentum, theory of relativity, gravitation, field potentials

1. Introduction

In field theory, there are a number of unsolved problems, which need deeper analysis and logical understanding. An example is the problem of choosing a universal form of the stress-energy tensor of the body, which would include the rest energy of the substance as well as the field energy and at the same time would provide an univocal connection with thermodynamic variables of the substance in the language of four-vectors and tensors. Another interesting problem is 4/3 problem, according to which the effective mass of the body field, which is calculated through the field momentum, and the effective mass of the field, found through the field energy, for some reason do not coincide with each other, with the ratio of the masses approximately equal to 4/3.

The problem of 4/3 is known for a long time for the mass of electromagnetic field of a moving charge. Joseph John Thomson, George Francis FitzGerald, Oliver Heaviside, George Frederick Charles Searle and many others write about it (Heaviside 1888/1894), (Searle 1897), (Hajra 1991). We also discuss this question with respect to the gravitational field of a moving ball (Fedosin 2008). Now we present a more accurate description of the problem, not limited to the approximation of small velocities.

2. Methods

In the calculation of the energy and the momentum of gravitational field of a uniform massive ball, we will use the superposition principle by means of summing up the field energies and momenta from all point particles forming the moving ball. This approach is reasonable in the case of a weak field, when the general theory of relativity changes to gravitomagnetism and the covariant theory of gravitation – to the Lorentz-invariant theory of gravitation (Fedosin 2009a). The field equations then become linear, allowing the use of the superposition principle. We will note that the gravitational field can be considered weak if the spacetime metric differs insignificantly from the Minkowski spacetime metric (the spacetime metric of the special theory of relativity). If the effects of gravitational time dilation and sizes contraction are significantly less than the similar effects due to the motion velocity of the reference frame under consideration, then this gravitational field can be considered weak.

3. Results and Discussions

3.1 The Gravitational Field Outside a Uniform Massive Ball

We will first define the gravitational field potentials for a ball moving at a constant velocity V along the axis OX of the reference frame K. We will proceed from the so-called Liénard-Wiechert potentials (Liénard 1898; Wiechert 1900) for any point particles that make up the ball. Popular presentation of the problem (for the electromagnetic field) can be found in Feynman's book (Feynman at all. 1964). Similarly to this, the differential scalar Liénard-Wiechert potential for the gravitational field from a point particle with mass dM has the following form:

$$d\psi = -\frac{\gamma\, dM}{r' - \boldsymbol{V} \cdot \boldsymbol{r}' / c_g}, \tag{1}$$

Where γ is the gravitational constant, c_g is the velocity of gravitation propagation, vector $\boldsymbol{r}'$ is the vector connecting the early position of the point particle at time t' and the position $\boldsymbol{r} = (x, y, z)$ at which the potential is determined at time t. In this case, the equation must hold:

$$t' = t - \frac{r'}{c_g}. \tag{2}$$

The meaning of equation (2) is that during the time period $t - t'$ the gravitational effect of the mass dM must cover the distance r' at velocity c_g up to the position $\boldsymbol{r} = (x, y, z)$ so that at this position the potential $d\psi$ would appear.

Suppose there is continuous distribution of point particles and at $t = 0$ these particles are described by the coordinates (x_0, y_0, z_0) and the center of distribution of point particles coincides with the origin of the reference frame. Then at time t the distribution center of the point particles would move along the axis OX to the position $x = Vt$, and the radius vector of an arbitrary particle of distribution would equal $\boldsymbol{r}_2 = (x_0 + Vt, y_0, z_0)$. At the early time t' the position of this point particle is specified by the vector $\boldsymbol{r}_1 = (x_0 + Vt', y_0, z_0)$. Since $\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{r}_1$ and $r' = c_g (t - t')$ according to (2), then for the square r'^2 we can write down:

$$r'^2 = (x - x_0 - Vt')^2 + (y - y_0)^2 + (z - z_0)^2 = c_g^2 (t - t')^2. \tag{3}$$

The right side of (3) is a quadratic equation for the time t'. After we find t' from (3), we can then find r' from (2). If we consider that in (1) the product of vectors is $\boldsymbol{V} \cdot \boldsymbol{r}' = V (x - x_0 - Vt')$, then substituting r' also in (1), we obtain the following expression (Fedosin 2009b):

$$d\psi = -\frac{\gamma\, dM}{\sqrt{1-V^2/c_g^2}\sqrt{\frac{(x-x_0-Vt)^2}{1-V^2/c_g^2}+(y-y_0)^2+(z-z_0)^2}}. \qquad (4)$$

According to (4), the differential gravitational potential $d\psi$ of the point mass dM at the time t during its motion along the axis OX depends on the initial position (x_0, y_0, z_0) of this mass at $t=0$. If we use the extended Lorentz transformations for the spatial coordinates in (4):

$$x^* = \frac{x-x_0-Vt}{\sqrt{1-V^2/c_g^2}}, \qquad y^* = y-y_0, \qquad z^* = z-z_0, \qquad (5)$$

and then let the velocity V tend to zero, we obtain the formula for the potential in the reference frame K^* the origin of which coincides with the point mass dM :

$$d\psi^* = -\frac{\gamma\, dM}{\sqrt{x^{*2}+y^{*2}+z^{*2}}}. \qquad (6)$$

In (6) in the reference frame K^* the vector $\boldsymbol{r}^* = (x^*, y^*, z^*)$ at the proper time t^* specifies the same point in space as the vector $\boldsymbol{r} = (x, y, z)$ in the reference frame K at the time t. If we introduce the gravitational four-potential $D_\mu = \left(\frac{\psi}{c_g}, -\boldsymbol{D}\right)$, including the scalar potential ψ and the vector potential $\boldsymbol{D}$ (Fedosin 1999), then the relation between the scalar potential (6) in the reference frame K^* and the scalar potential (4) in the reference frame K can be considered as the consequence of extended Lorentz transformations in four-dimensional formalism, which are applied to the differential four-potential of a single point particle. These transformations are carried out by multiplying the corresponding transformation matrix by the four-potential, which gives the four-potential in a different reference frame with its own coordinates and time.

Since in the reference frame K^* the point mass is at rest, its vector potential is $d\boldsymbol{D}^* = 0$, and the four-potential has the form: $dD_\mu^* = \left(\frac{d\psi^*}{c_g}, 0\right)$. In order to move to the reference frame K, in which the reference frame K^* is moving at the constant velocity V along the axis OX, we must use the matrix of inverse partial Lorentz transformation (Fedosin 2009a):

$$L_k^{\mu} = \begin{pmatrix} \frac{1}{\sqrt{1-V^2/c_g^2}} & -\frac{V}{c_g\sqrt{1-V^2/c_g^2}} & 0 & 0 \\ -\frac{V}{c_g\sqrt{1-V^2/c_g^2}} & \frac{1}{\sqrt{1-V^2/c_g^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$dD_k = L_k^{\mu} dD_{\mu}^* = \left(\frac{d\psi^*}{c_g\sqrt{1-V^2/c_g^2}}, -\frac{V\,d\psi^*}{c_g^2\sqrt{1-V^2/c_g^2}}, 0, 0 \right) = \left(\frac{d\psi}{c_g}, -dD_x, 0, 0 \right). \qquad (7)$$

From (7) taking into account (6) and (5) we obtain the following relations:

$$d\psi = \frac{d\psi^*}{\sqrt{1-V^2/c_g^2}} = -\frac{\gamma\, dM}{\sqrt{1-V^2/c_g^2}\sqrt{\frac{(x-x_0-Vt)^2}{1-V^2/c_g^2}+(y-y_0)^2+(z-z_0)^2}},$$

$$dD_x = \frac{d\psi\ V}{c_g^2}, \qquad dD_x = 0, \qquad dD_z = 0. \qquad (8)$$

The first equation in (8) coincides with (4) and the differential vector potential of the point mass is directed along its motion velocity.

After integration of (8) over all point masses inside the ball on the basis of the principle of superposition, the standard formulas are obtained for the potentials of gravitational field around the moving ball, with retardation of the gravitational interaction taken into account:

$$\psi = -\frac{\gamma M}{\sqrt{(x-Vt)^2+(1-V^2/c_g^2)(y^2+z^2)}}, \qquad \boldsymbol{D} = \frac{\psi \boldsymbol{V}}{c_g^2}, \qquad (9)$$

Where ψ – the scalar potential of the moving ball, M – the mass of the ball, (x, y, z) – the coordinates of the point at which the potential is determined at the time t (on the condition that the center of the ball at $t = 0$ was in the origin of coordinate system), $\boldsymbol{D}$ – the vector potential of the ball.

In (9) it is assumed that the ball is moving along the axis OX at a constant speed V, so that $D_x = \frac{\psi V}{c_g^2}$, $D_y = 0$, $D_z = 0$. With the help of the field potentials we can calculate the field strengths around the ball by the formulas (Fedosin 1999):

$$\boldsymbol{G} = -\nabla\psi - \frac{\partial \boldsymbol{D}}{\partial t}, \qquad \boldsymbol{\Omega} = \nabla \times \boldsymbol{D}, \qquad (10)$$

Where $\boldsymbol{G}$ is the gravitational field strength, $\boldsymbol{\Omega}$ – the gravitational torsion in Lorentz-invariant theory of gravitation (gravitomagnetic field in gravitomagnetism).
In view of (9) and (10) we find:

$$G_x = -\frac{\gamma M \left(x - Vt\right)(1 - V^2/c_g^2)}{\sqrt{[\left(x - Vt\right)^2 + (1 - V^2/c_g^2)(y^2 + z^2)]^3}} ,$$

$$G_y = -\frac{\gamma M \, y (1 - V^2/c_g^2)}{\sqrt{[\left(x - Vt\right)^2 + (1 - V^2/c_g^2)(y^2 + z^2)]^3}} ,$$

$$G_z = -\frac{\gamma M \, z (1 - V^2/c_g^2)}{\sqrt{[\left(x - Vt\right)^2 + (1 - V^2/c_g^2)(y^2 + z^2)]^3}} , \qquad \Omega_x = 0 , \qquad (11)$$

$$\Omega_y = \frac{\gamma M \, z V (1 - V^2/c_g^2)}{c_g^2 \sqrt{[\left(x - Vt\right)^2 + (1 - V^2/c_g^2)(y^2 + z^2)]^3}} ,$$

$$\Omega_z = -\frac{\gamma M \, y V (1 - V^2/c_g^2)}{c_g^2 \sqrt{[\left(x - Vt\right)^2 + (1 - V^2/c_g^2)(y^2 + z^2)]^3}} .$$

The energy density of the gravitational field is determined by the formula (Fedosin 1999):

$$u = -\frac{1}{8\pi\gamma}\left(G^2 + c_g^2 \Omega^2\right) = -\frac{\gamma M^2 (1 - V^2/c_g^2)[\left(x - Vt\right)^2 + (1 + V^2/c_g^2)(y^2 + z^2)]}{8\pi \, [\left(x - Vt\right)^2 + (1 - V^2/c_g^2)(y^2 + z^2)]^3} . \qquad (12)$$

The total energy of the field outside the ball at a constant velocity should not depend on time. So it is possible to integrate the energy density of the field (12) over the external space volume at $t = 0$. For this purpose we shall introduce new coordinates:

$$x = \sqrt{1 - V^2/c_g^2} \; r \cos\theta , \qquad y = r \sin\theta \cos\varphi , \qquad z = r \sin\theta \sin\varphi . \qquad (13)$$

The volume element is determined by the formula $d\Upsilon = J \, dr \, d\theta \, d\varphi$, where J is determinant of Jacobian matrix:

$$J = \frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \varphi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \varphi} \end{vmatrix} .$$

It follows that $d\Upsilon = r^2 \sin\theta \sqrt{1 - V^2/c_g^2} \, dr \, d\theta \, d\varphi$. The integral over the space of the energy density (12) will equal:

$$U_b = \int u\, d\Upsilon = -\frac{\gamma M^2}{8\pi c_g^2 \sqrt{1-V^2/c_g^2}} \int \frac{[c_g^2 + V^2(\sin^2\theta - \cos^2\theta)]\, \sin\theta\, dr\, d\theta\, d\varphi}{r^2}. \qquad (14)$$

We shall take into account that due to the Lorentz contraction during the motion along the axis OX the ball must be as Heaviside ellipsoid, the surface equation of which at $t=0$ is the following:

$$\frac{x^2}{1-V^2/c_g^2} + y^2 + z^2 = R^2. \qquad (15)$$

After substituting (13) in (15), it becomes apparent that the radius r at the integration in (14) must change from R to ∞, and the angles θ and φ change the same way as in spherical coordinates (from 0 to π for the angle θ, and from 0 to 2π for the angle φ). For the energy of the gravitational field outside the moving ball we find:

$$U_b = -\frac{\gamma M^2 (1+V^2/3c_g^2)}{2R\sqrt{1-V^2/c_g^2}} = \frac{U_{b0}\,(1+V^2/3c_g^2)}{\sqrt{1-V^2/c_g^2}}, \qquad (16)$$

Where $U_{b0} = -\frac{\gamma M^2}{2R}$ is the field energy around the stationary ball.

We can introduce the effective relativistic mass of the field related to the energy of moving ball:

$$m_{gb} = \frac{U_b\sqrt{1-V^2/c_g^2}}{c_g^2} = \frac{U_{b0}\,(1+V^2/3c_g^2)}{c_g^2}. \qquad (17)$$

We shall now consider the momentum density of the gravitational field:

$$\boldsymbol{g} = \frac{\boldsymbol{H}}{c_g^2}, \qquad (18)$$

Where $\boldsymbol{H} = -\frac{c_g^2}{4\pi\gamma}[\boldsymbol{G}\times\boldsymbol{\Omega}]$ is the vector of energy flux density of the gravitational field (Heaviside vector) (Fedosin 1999).

Substituting in (18) the components of the field (11), we find:

$$g_x = -\frac{\gamma M^2 (1-V^2/c_g^2)^2\, (y^2+z^2)V}{4\pi c_g^2\, [(x-Vt)^2 + (1-V^2/c_g^2)(y^2+z^2)]^3}, \qquad (19)$$

$$g_y = \frac{\gamma M^2 (1-V^2/c_g^2)^2\, (x-Vt)\, y\, V}{4\pi c_g^2\, [(x-Vt)^2 + (1-V^2/c_g^2)(y^2+z^2)]^3},$$

$$g_z = \frac{\gamma M^2 (1-V^2/c_g^2)^2 (x-Vt)\, z\, V}{4\pi c_g^2 \, [(x-Vt)^2 + (1-V^2/c_g^2)(y^2+z^2)]^3} .$$

We can see that the components of the momentum density of gravitational field (19) look the same as if a liquid flowed around the ball from the axis OX, carrying similar density of the momentum – liquid spreads out to the sides when meeting with the ball and merges once again on the opposite side of the ball. Integrating the components of the momentum density of the gravitational field (19) by volume outside the moving ball at $t=0$ as in (14), we obtain:

$$P_x = \int g_x \, d\Upsilon = -\frac{\gamma M^2 V}{4\pi c_g^2 \sqrt{1-V^2/c_g^2}} \int \frac{\sin^3\theta \, dr\, d\theta\, d\varphi}{r^2} = -\frac{2\gamma M^2 V}{3R c_g^2 \sqrt{1-V^2/c_g^2}} . \qquad (20)$$

$$P_y = \int g_y \, d\Upsilon = 0 , \qquad\qquad P_z = \int g_z \, d\Upsilon = 0 .$$

In (20) the total momentum of the field has only the component along the axis OX. By analogy with the formula for relativistic momentum the coefficient before the velocity V in (20) can be interpreted as the effective mass of the external gravitational field moving with the ball:

$$m_{pb} = \frac{P_x \sqrt{1-V^2/c_g^2}}{V} = -\frac{2\gamma M^2}{3R c_g^2} = \frac{4U_{b0}}{3c_g^2} , \qquad (21)$$

Where $U_{b0} = -\frac{\gamma M^2}{2R}$ is the energy of the external static field of the ball at rest.

Comparing (21) and (17) gives:

$$m_{gb} = \frac{3(1+V^2/3c_g^2)\, m_{pb}}{4} . \qquad (22)$$

The discrepancy between the masses m_{gb} and m_{pb} in (22) shows the existence of the problem of 4/3 for gravitational field in the Lorentz-invariant theory of gravitation.

3.2 The Gravitational Field Inside a Moving Ball

For a homogeneous ball with the density of substance ρ_0 (measured in the comoving frame), which is moving along the axis OX, the potentials inside the ball (denoted by subscript i) depend on time and are as follows (Fedosin 2009b):

$$\psi_i = -\frac{2\pi\gamma\rho_0}{\sqrt{1-V^2/c_g^2}} \left[R^2 - \frac{1}{3}\left(\frac{(x-Vt)^2}{1-V^2/c_g^2} + y^2 + z^2 \right) \right] , \qquad \boldsymbol{D}_i = \frac{\psi_i \boldsymbol{V}}{c_g^2} . \qquad (23)$$

In view of (10) we can calculate the internal field strength and torsion field:

$$G_{xi}=-\frac{4\pi\gamma\rho_0\left(x-Vt\right)}{3\sqrt{1-V^2/c_g^2}}, \qquad G_{yi}=-\frac{4\pi\gamma\rho_0 y}{3\sqrt{1-V^2/c_g^2}}, \qquad G_{zi}=-\frac{4\pi\gamma\rho_0 z}{3\sqrt{1-V^2/c_g^2}},$$

$$\Omega_{xi}=0, \qquad \Omega_{yi}=\frac{4\pi\gamma\rho_0 zV}{3c_g^2\sqrt{1-V^2/c_g^2}}, \qquad \Omega_{zi}=-\frac{4\pi\gamma\rho_0 yV}{3c_g^2\sqrt{1-V^2/c_g^2}}. \qquad (24)$$

Similarly to (12) for the energy density of the field we find:

$$u_i=-\frac{1}{8\pi\gamma}\left(G_i^2+c_g^2\Omega_i^2\right)=-\frac{2\pi\gamma\rho_0^2[\left(x-Vt\right)^2+(1+V^2/c_g^2)(y^2+z^2)]}{9(1-V^2/c_g^2)}. \qquad (25)$$

According to (25) the minimum energy density inside a moving ball is achieved on its surface, and in the center at $t=0$ it is zero.

The integral of (25) by volume of the ball at $t=0$ in coordinates (13) with the volume element $d\Upsilon=r^2\sin\theta\sqrt{1-V^2/c_g^2}\,dr\,d\theta\,d\varphi$ equals:

$$U_i=\int u_i\,d\Upsilon=-\frac{2\pi\gamma\rho_0^2}{9c_g^2\sqrt{1-V^2/c_g^2}}\int[c_g^2+V^2(\sin^2\theta-\cos^2\theta)]r^4\sin\theta\,dr\,d\theta\,d\varphi. \qquad (26)$$

According to the theory of relativity the moving ball looks like Heaviside ellipsoid with equation of the surface (15) at $t=0$, and in the coordinates (13) the radius in the integration in (26) varies from 0 to R. With this in mind for the energy of the gravitational field inside the moving ball, we have:

$$U_i=-\frac{\gamma M^2(1+V^2/3c_g^2)}{10R\sqrt{1-V^2/c_g^2}}=\frac{U_{i0}(1+V^2/3c_g^2)}{\sqrt{1-V^2/c_g^2}}, \qquad (27)$$

Where $U_{i0}=-\frac{\gamma M^2}{10R}$ is the field energy inside a stationary ball with radius R.

The effective mass of the field associated with energy (27) is:

$$m_{gi}=\frac{U_i\sqrt{1-V^2/c_g^2}}{c_g^2}=\frac{U_{i0}\,(1+V^2/3c_g^2)}{c_g^2}. \qquad (28)$$

Substituting in (18) the components of the field strengths (24), we find the components of the vector of momentum density of gravitational field:

$$g_{xi}=-\frac{4\pi\gamma\rho_0^2(y^2+z^2)V}{9c_g^2\,(1-V^2/c_g^2)}, \qquad g_{yi}=\frac{4\pi\gamma\rho_0^2(x-Vt)\,yV}{9c_g^2\,(1-V^2/c_g^2)},$$

$$g_{zi} = \frac{4\pi\gamma\rho_0^2 (x - Vt)\, zV}{9c_g^2 (1 - V^2/c_g^2)} . \qquad (29)$$

The vector connecting the origin of coordinate system and center of the ball depends on the time and has the components $(Vt, 0, 0)$. From this in the point, coinciding with the center of the ball, the components of the vector of the momentum density of the gravitational field are always zero. At $t = 0$ the center of the ball passes through the origin of the coordinate system, and at the time from (29) it follows that the maximum density of the field momentum $g_{\max} = -\frac{4\pi\gamma\rho_0^2 R^2 V}{9c_g^2 (1 - V^2/c_g^2)} = -\frac{\gamma M^2 V}{4\pi R^4 c_g^2 (1 - V^2/c_g^2)}$ is achieved on the surface of the ball on the circle of radius R in the plane YOZ, which is perpendicular to the line OX of the ball's motion. The same follows from (19).

We can integrate the components of the momentum density of gravitational field (29) over the volume inside the moving ball at $t = 0$ in the coordinates (13) similar to (20):

$$P_{xi} = \int g_{xi}\, d\Upsilon = -\frac{4\pi\gamma\rho_0^2 V}{9c_g^2 \sqrt{1 - V^2/c_g^2}} \int r^4 \sin^3\theta\, dr\, d\theta\, d\varphi = -\frac{2\gamma M^2 V}{15Rc_g^2 \sqrt{1 - V^2/c_g^2}} . \qquad (30)$$

$$P_{yi} = \int g_{yi}\, d\Upsilon = 0 , \qquad P_{zi} = \int g_{zi}\, d\Upsilon = 0 .$$

As in (20), the total momentum of the field (30) has only the component along the axis OX. By analogy with (21) the coefficient before the velocity V in (30) is interpreted as the effective mass of the gravitational field inside the ball:

$$m_{pi} = \frac{P_{xi}\sqrt{1 - V^2/c_g^2}}{V} = -\frac{2\gamma M^2}{15Rc_g^2} = \frac{4U_{i0}}{3c_g^2} , \qquad (31)$$

Where $U_{i0} = -\frac{\gamma M^2}{10R}$ is the field energy inside a stationary ball.

Comparing (28) and (31) gives:

$$m_{gi} = \frac{3(1 + V^2/3c_g^2)\, m_{pi}}{4} . \qquad (32)$$

Connection (32) between the masses of the field inside the ball is the same as in (22) for the masses of the external field, so the problem of 4/3 exists inside the ball too.

4. Conclusion

A characteristic feature of the fundamental fields, which include the gravitational and electromagnetic fields, is the similarity of their equations for the potentials and the field strengths. As it was shown above, the external potentials (9) of the gravitational (and similarly, the electromagnetic) field of the moving ball are similar by their form to the potentials of the point mass (point charge) (8), and can be obtained both using the superposition principle of potentials of the point masses inside the ball, and using the Lorentz transformation. We also presented the exact field potentials (23) inside the moving ball, for which both the superposition principle and the Lorentz transformation are satisfied.

From the stated above we saw that the 4/3 problem was common for both the electromagnetic and the gravitational field. It also followed from this that considering the contribution of the energy and the momentum of both fields into the mass of the moving body were to be done in the same way, taking into account the negative values of the energy and the momentum of gravitational field and the positive values of the energy and the momentum of electromagnetic field.

References

Fedosin, S.G. (1999), "Fizika i filosofiia podobiia: ot preonov do metagalaktik", Perm: Style-MG.

Fedosin, S.G. (2008), "Mass, Momentum and Energy of Gravitational Field", *Journal of Vectorial Relativity* **3** (3), 30-35.

Fedosin, S.G. (2009a), "Fizicheskie teorii i beskonechnaia vlozhennost' materii", Perm.

Fedosin, S.G. (2009b), Comments to the book: "Fizicheskie teorii i beskonechnaia vlozhennost' materii", Perm.

Feynman, R.P., Leighton, R., & Sands M. (1964), "The Feynman lectures on physics". Massachussets: Addison-Wesley. (Vol. 2).

Hajra, S. (1991), "Classical Electrodynamics – Reexamined", *Indian Journal of Theoretical Physics* **42** (2), 164.

Heaviside, O. (1888/1894), "Electromagnetic waves, the propagation of potential, and the electromagnetic effects of a moving charge", *Electrical papers* **2**, 490-499.

Liénard, A.M. (1898), "Champ électrique et Magnétique", *L'éclairage électrique* **16** (27-29). 5-14, 53-59, 106-112.

Searle, G.F.C. (1897), "On the steady motion of an electrified ellipsoid", *The Philosophical Magazine* Series 5, **44** (269), 329-341.

Wiechert, E. (1900), "Elektrodynamische Elementargesetze", *Archives Néerlandaises* **5**, 549-573.