Shortest Path Problem under Trapezoidal Neutrosophic Information

Said Broumi
Laboratory of Information Processing, Faculty of Science Ben M'sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco
broumisaid78@gmail.com

Assia Bakali
Ecole Royale Navale, Boulevard Sour Jdid, B.P 16303 Casablanca, Morocco
assiabakali@yahoo.fr

Mohamed Talea
Laboratory of Information Processing, Faculty of Science Ben M'sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco
taleamohamed@yahoo.fr

Florentin Smarandache
Department of Mathematic, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA
fsmarandache@gmail.com

***Abstract*—In this research paper, a new approach is proposed for computing the shortest path length from source node to destination node in a neutrosophic environment. The edges of the network are assigned by trapezoidal fuzzy neutrosophic numbers. A numerical example is provided to show the performance of the proposed approach.**

Keywords—neutrosophic sets; trapezoidal neutrosophic sets; shortest path problem; score function

I. Introduction

Smarandache [1] proposed the concept of neutrosophic sets (in short NSs) as a means of expressing the inconsistencies and indeterminacies that exists in most real-life problems. The proposed concept generalized fuzzy sets and intuitionistic fuzzy set theory [3, 4]. The notion of NSs is described with three functions: truth, an indeterminacy and a falsity, where the functions are totally independent, the three functions are inside the unit interval]−0, 1+[. To practice NSs in real life situations efficiently. A new version of NSs named Single valued Neutrosophic Sets (in short SVNSs) was defined by Smarandache in [1]. Subsequently Wang et al. [5] defined various operations and operators for the SVNS model. Additional literature on single valued neutrosophic sets can be found in [6-14, 16]. Also later on, Smarandache extended the neutrosophic set to neutrosophic overset, underset, and offset [15]. Ye [17] presented the concept of trapezoidal fuzzy neutrosophic set (in short TrFNSs) and studied some interesting results with proofs and examples. In TrFNSs, the truth, the indeterminate and the false membership degrees are expressed with Trapezoidal Fuzzy Numbers (TrFN) instead of real numbers. Smarandache and Kandasamy [25, 28-29] introduced the concept of neutrosophic graph based on the indeterminacy component (I). Later on, in [18-23, 26-27] Broumi et al. introduced different types of neutrosophic graph based on the neutrosophic values (T, I, F) including single valued neutrosophic graphs, interval valued neutrosophic graphs and bipolar neutrosophic graphs. In graph theory, the shortest path problems (in short SPP) is one of the known famous problems studied in the numerous discipline including operation research, computer science, communication network and so on. In the literature, many research papers have been focused seriously on fuzzy shortest path problems and their extensions [30-39]. Till now, few research papers deal with shortest path problems in neutrosophic environment. In [40-44], Broumi et al. presented some algorithms for solving the shortest path problems in neutrosophic environment. All these algorithms are based on the score functions. In this paper, the addition operation and the order relation have been given by Ye [17]. In this research paper, our main objective is to solving the shortest path problems in a network, where the edges weight are represented by trapezoidal fuzzy neutrosophic numbers.

This paper is constructed as follows: In Section 2, some basic definitions of neutrosophic sets, SVN-sets and trapezoidal fuzzy neutrosophic sets are introduced. In section 3, a new proposed algorithm for computing the trapezoidal fuzzy neutrosophic shortest path problem on a network is presented. In Section 4, a numerical example is given for computing the shortest path and shortest distance from the source node to destination node. We conclude the paper in Section 5.

II. Preliminaries

In this section, some definitions related to the concept of neutrosophic sets, single valued neutrosophic and trapezoidal fuzzy neutrosophic sets are taken from [2, 5, 17]

Definition 2.1 [2] Let ζ be a universal set. The neutrosophic set A on the universal set ζ categorized into three membership functions called the true $T_A(x)$, indeterminate $I_A(x)$ and false $F_A(x)$ contained in real standard or non–standard subset of $]^-0, 1^+[$ respectively and denoted as following

$$A= \{<x: T_A(x), I_A(x), F_A(x)> x\in \zeta \} \quad (1)$$

978-1-5090-5443-5/17/$31.00 ©2017 IEEE

Definition 2.2 [5] Let ζ be a universal set. The single valued neutrosophic sets (in short SVNS) A on the universal is denoted as following:

$$A= \{<x: T_A(x), I_A(x), F_A(x)>, x \in \zeta \} \quad (2)$$

The function $T_A(x) \in$ [0, 1], $I_A(x) \in$ [0, 1] and $I_A(x) \in$ [0, 1] are named "degree of truth, indeterminacy and falsity membership of x in A", satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(\mathrm{x}) \le 3 \quad (3)$$

Definition 2.3 [17]. Let ζ be a universal set and ψ [0, 1] be *the sets of all trapezoidal fuzzy numbers on [0, 1]*. The trapezoidal fuzzy neutrosophic sets (in short TrFNSs) $\breve{A}$ on the universal is denoted as following:

$$\breve{A} = \{<x: \breve{T}_A(x), \breve{I}_A(x), \breve{F}_A(x)>, x \in \zeta \} \quad (4)$$

Where $\breve{T}_A(\mathrm{x}): \zeta \to \psi[0,1]$, $\breve{I}_A(\mathrm{x}): \zeta \to \psi[0,1]$ and $\breve{F}_A(\mathrm{x}): \zeta \to \psi[0,1]$. The trapezoidal fuzzy numbers

$$\breve{T}_A(\mathrm{x}) = (\mathrm{T}_A^1(x), \mathrm{T}_A^2(x), \mathrm{T}_A^3(x), \mathrm{T}_A^4(x)), \quad (5)$$

$$\breve{I}_A(x) = (I_A^1(\mathrm{x}), I_A^2(\mathrm{x}), I_A^3(\mathrm{x}), I_A^4(\mathrm{x})) \text{ and} \quad (6)$$

$\breve{F}_A(x) = (F_A^1(\mathrm{x}), F_A^2(\mathrm{x}), F_A^3(\mathrm{x}), F_A^4(\mathrm{x}))$, respectively denotes degree of truth, indeterminacy and falsity membership of x in $\breve{A}$ $\forall x \in \zeta$.

$$0 \le \mathrm{T}_A^4(x) + I_A^4(x) + F_A^4(x) \le 3. \quad (7)$$

For notational convenience, the trapezoidal fuzzy neutrosophic value (TrFNV) $\tilde{A}$ is denoted by $\breve{A} = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ where,

$$(\mathrm{T}_A^1(\mathrm{x}), \mathrm{T}_A^2(\mathrm{x}), \mathrm{T}_A^3(\mathrm{x}), \mathrm{T}_A^4(\mathrm{x})) = (t_1, t_2, t_3, t_4), \quad (8)$$

$$(I_A^1(\mathrm{x}), I_A^2(\mathrm{x}), I_A^3(\mathrm{x}), I_A^4(\mathrm{x})) = (i_1, i_2, i_3, i_4), \text{ and} \quad (9)$$

$$(F_A^1(\mathrm{x}), F_A^2(\mathrm{x}), F_A^3(\mathrm{x}), F_A^4(\mathrm{x})) = (f_1, f_2, f_3, f_4) \quad (10)$$

with $t_1 \le t_2 \le t_3 \le t_4$, $i_1 \le i_2 \le i_3 \le i_4$ and $f_1 \le f_2 \le f_3 \le f_4$

where, the truth membership function is given as bellow:

$$\breve{T}_A(x) = \begin{cases} \dfrac{x-t_1}{t_2-t_1} & t_1 \le x \le t_2 \\ 1 & t_2 \le x \le t_3 \\ \dfrac{x-t_1}{t_2-t_1} & t_3 \le x \le t_4 \\ 0 & otherwise \end{cases} \quad (11)$$

The indeterminacy membership is given as below:

$$\breve{I}_A(\mathrm{x}) = \begin{cases} \dfrac{x-i_1}{i_2-i_1} & i_1 \le x \le i_2 \\ 1 & i_2 \le x \le i_3 \\ \dfrac{i_4-x}{i_4-i_3} & i_3 \le x \le i_4 \\ 0 & otherwise \end{cases} \quad (12)$$

And the falsity membership function is given as below:

$$\breve{F}_A(x) = \begin{cases} \dfrac{x-f_1}{f_2-f_1} & f_1 \le x \le f_2 \\ 1 & f_2 \le x \le f_3 \\ \dfrac{f_4-x}{f_4-f_3} & f_3 \le x \le f_4 \\ 0 & otherwise \end{cases} \quad (13)$$

Definition 2.4 [17]. The trapezoidal fuzzy neutrosophic number $\breve{A} = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ is said to be trapezoidal fuzzy neutrosophic zero if and only if

$(t_1, t_2, t_3, t_4) =$ (0, 0, 0, 0), (i_1, i_2, i_3, i_4) = (1, 1, 1, 1) and (f_1, f_2, f_3, f_4) =(1, 1, 1, 1) (14)

Definition 2.5 [17]. Let $\breve{A}_1$ and $\breve{A}_2$ two TrFNVs defined on the set of real numbers, denoted as :

$\breve{A}_1 = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (\mathrm{c}_1, c_2, c_3, c_4) \rangle$ and

$\breve{A}_2 = \langle (e_1, e_2, e_3, e_4), (f_1, f_2, f_3, f_4), (g_1, g_2, g_3, g_4) \rangle$ and $\eta > 0$.

Hence , the operations rules are defined as following:

(i) $$\breve{A}_1 \oplus \breve{A}_2 = \left\langle \begin{matrix} \begin{pmatrix} (a_1 + e_1 - a_1 e_1), (a_2 + e_2 - a_2 e_2), \\ (a_3 + e_3 - a_3 e_3), (a_4 + e_4 - a_4 e_4) \end{pmatrix}, \\ ((\mathrm{b}_1 f_1), (\mathrm{b}_2 f_2), (\mathrm{b}_3 f_3), (\mathrm{b}_4 f_4)), \\ (c_1 g_1), (c_2 g_2), (c_3 g_3), (c_4 g_4)) \end{matrix} \right\rangle \quad (15)$$

978-1-5090-5443-5/17/$31.00 ©2017 IEEE

(ii) $$\breve{A}_1 \otimes \breve{A}_2 = \left\langle \begin{matrix} (a_1e_1, a_2e_2, a_3e_3, a_4e_4), \\ \begin{pmatrix} (b_1+f_1-b_1f)_1, (b_2+f_2-b_2f_2), \\ (b_3+f_3-b_3f_3), (b_4+f_4-b_4f_4) \end{pmatrix}, \\ \begin{pmatrix} (c_1+g_1-c_1g_1), (c_2+g_2-c_2g_2), \\ (c_3+g_3-c_3g_3), (c_4+g_4-c_4g_4) \end{pmatrix} \end{matrix} \right\rangle \quad (16)$$

(iii) $$\eta\breve{A} = \left\langle \begin{matrix} \begin{pmatrix} (1-(1-a_1)^{\eta}), (1-(1-a_2)^{\eta}), \\ (1-(1-a_3)^{\eta}), (1-(1-a_4)^{\eta}) \end{pmatrix} \\ (b_1^{\eta}, b_2^{\eta}, b_3^{\eta}, b_4^{\eta}), (c_1^{\eta}, c_2^{\eta}, c_3^{\eta}, c_4^{\eta}) \end{matrix} \right\rangle \quad (17)$$

(iv) $$\breve{A}_1^{\eta} = \left\langle \begin{matrix} (a_1^{\eta}, a_2^{\eta}, a_3^{\eta}, a_4^{\eta}), \\ \left((1-(1-b_1)^{\eta}), (1-(1-b_2)^{\eta}), (1-(1-b_3)^{\eta}), 1-(1-b_4)^{\eta})\right), \\ \left((1-(1-c_1)^{\eta}), (1-(1-c_2)^{\eta}), (1-(1-c_3)^{\eta})), (1-(1-c_4)^{\eta})\right) \end{matrix} \right\rangle$$

where $\eta > 0$ (18)

Ye [17] gave the definition of score function $s(\breve{A}_1)$ and accuracy function $H(\breve{A}_1)$ to compare the grades of TrFNS. These functions shows that greater is the value, the greater is the TrFNS and by using these concept paths can be ranked.

Definition 2.6. Let $\breve{A}_1$ be a TrFNV denoted as

$\breve{A}_1 = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ Hence , the score function and the accuracy function of TrFNV are denoted as below:

(i) $$s(\breve{A}_1) = \frac{1}{12}\begin{bmatrix} 8 + (t_1+t_2+t_3+t_4) - (i_1+i_2+i_3+i_4) \\ -(f_1+f_2+f_3+f_4) \end{bmatrix} \quad (19)$$

(ii) $$H(\breve{A}_1) = \frac{1}{4}\left[(t_1+t_2+t_3+t_4) - (f_1+f_2+f_3+f_4)\right] \quad (20)$$

In order to make a comparisons between two TrFNV, Ye **[17]**, presented the order relations between two TrFNVs.

Definition 2.7 Let $\breve{A}_1$ and $\breve{A}_2$ be two TrFNV defined on the set of real numbers , denoted as

$\breve{A}_1 = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ and $\breve{A}_2 = \langle (p_1, p_2, p_3, p_4), (q_1, q_2, q_3, q_4), (r_1, r_2, r_3, r_4) \rangle$. Hence , the ranking method is defined as follows:

1) If $s(\breve{A}_1) \succ s(\breve{A}_2)$, then $\breve{A}_1$ is greater than $\breve{A}_2$, that is, $\breve{A}_1$ is superior to $\breve{A}_2$, denoted by $\breve{A}_1 \succ \breve{A}_2$

2) If $s(\breve{A}_1) = s(\breve{A}_2)$, and $H(\breve{A}_1) \succ H(\breve{A}_2)$ then $\breve{A}_1$ is greater than $\breve{A}_2$, that is, $\breve{A}_1$ is superior to $\breve{A}_2$, denoted by $\breve{A}_1 \succ \breve{A}_2$.

III. TrFN- Shortest Path Problem

In this section, the edge length in a network is considered to be trapezoidal fuzzy neutrosophic number. To find the shortest path in a network , where the edges are characterized by trapezoidal fuzzy neutrosophic number. We present the following procedure:

Step 1 Suppose $\tilde{d}_1$ = <(0, 0, 0, 0) (1, 1, 1, 1), (1, 1, 1, 1)> and label the source node1 as [$\tilde{d}_1$ = < (0, 0, 0, 0),(1, 1, 1,1), (1, 1, 1, 1)>, -]

Let n is the destination node.

Step 2: Select $\tilde{d}_j$ = min { $\tilde{d}_i \oplus \tilde{d}_{ij}$ } for all j= 2,3,…,n.

Step 3: If the minimum provided correspond to one value of i then label node j as [$\tilde{d}_j$, i]. If the minimum provided correspond to several values of i, then it indicate that there exist more than one TrFN-path between the source node and the node j. Hence the TrFN-distance along path is $\tilde{d}_j$, so select any value of i.

Step 4: Set the destination node n be labeled as [$\tilde{d}_n$,*l*], then the TrFN-shortest path distance from source node to destination node is $\tilde{d}_n$.

Step 5: Since the destination node n is labeled [$\tilde{d}_n$, *l*]. In order to find the TrFN-shortest path connecting the source node and the destination node, identify the label of the node *l*. Set it as [$\tilde{d}_l$,*p*], Repeat step 2 and step 3 until the node 1 is obtained.

Step 6: To obtain the TrFN-shortest path, we should joining all the nodes provided by the step 5.

IV. Illustrative Example

Consider a small network shown in the following figure 1 in which each edge length is represented by a trapezoidal fuzzy neutrosophic number (see table 1). This network includes 6 nodes and 8 directed edges. This problem is to compute the shortest path between source node and destination node in the given network.

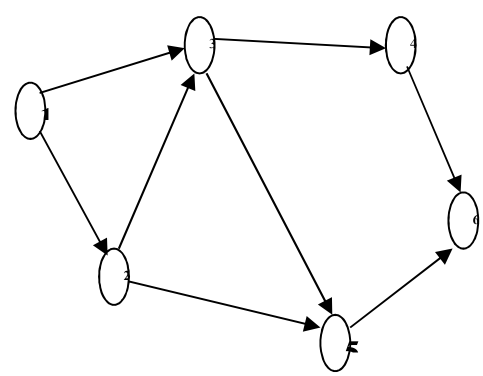

Fig. 1. Trapezoidal fuzzy neutrosophic network

The edges weight of the trapezoidal fuzzy neutrosophic network are represented by trapezoidal fuzzy neutrosophic numbers.

978-1-5090-5443-5/17/$31.00 ©2017 IEEE

TABLE I. THE EDGES WEIGHT OF THE TRAPEZOIDAL FUZZY NEUTROSOPHIC GRAPHS

Edges	Trapezoidal fuzzy neutrosophic distance
1-2	<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>
1-3	<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>
2-3	<(0.3, 0.4, 0.6, 0.7), (0.1, 0.2, 0.3, 0.5), (0.3, 0.5, 0.7, 0.9)>
2-5	<(0.1, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.7), (0.2, 0.3, 0.6, 0.7)>
3-4	<(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>
3-5	<(0.3, 0.6, 0.7, 0.8), (0.1, 0.2, 0.3, 0.4), (0.1, 0.4, 0.5, 0.6)>
4-6	<(0.4, 0.6, 0.8, 0.9), (0.2, 0.4, 0.5, 0.6), (0.1, 0.3, 0.4, 0.5)>
5-6	<(0.2, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.6), (0.1, 0,3, 0.5, 0.6)>

Using the algorithm proposed in section 2, we can determine the shortest path between any two nodes. Let node 1 is the source node and node 6 is the destination node.

Suppose $\tilde{d}_1$ = <(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)> and label the source node 1 as [<(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)> ,-], the value $\tilde{d}_2$, $\tilde{d}_3$, $\tilde{d}_4$, $\tilde{d}_5$ and $\tilde{d}_5$ can be computed following the iterations described below:

Iteration1: The node 2 has on predecessor, which is node 2. Following the step 2 in the proposed algorithm, we put i=1and j=2, hence the value of $\tilde{d}_2$ can be computed as follows:

$\tilde{d}_2$= min{ $\tilde{d}_1 \oplus \tilde{d}_{12}$ } = min{<(0, 0, 0), (1, 1, 1), (1, 1, 1)> ⊕ <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>= <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>

So, the minimum provided correspond to the node 1.Hence, the node 2 is labeled as

[<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>, 1]

$\tilde{d}_2$= <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>

Iteration 2: The node 3 has two predecessors, which are node 1 and node 2. Following the step 2 in the proposed algorithm, we put i=1, 2and j=3, hence the value of $\tilde{d}_3$ can be computed as follows:

$\tilde{d}_3$=min { $\tilde{d}_1 \oplus \tilde{d}_{13}, \tilde{d}_2 \oplus \tilde{d}_{23}$ }= min {<((0, 0, 0, 0), (1, 1, 1), (1, 1, 1, 1)> ⊕ <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>⊕ <(0.3, 0.4, 0.6, 0.7), (0.1, 0.2, 0.3, 0.5), (0.3, 0.5, 0.7, 0.9)>} = min{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15, 0.3), (0.12, 0.25, 0.42, 0.72)>}

S (<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) using Eq.19, we have

$$s(\tilde{A}_1) = \frac{1}{12}\begin{bmatrix} 8+(t_1+t_2+t_3+t_4)-(i_1+i_2+i_3+i_4) \\ -(f_1+f_2+f_3+f_4) \end{bmatrix} = 0.54$$

S (<(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15,0.3), (0.12, 0.25, 0.42, 0.72)>) = 0.70

Since S (<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) < S (<(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15,0.3), (0.12, 0.25, 0.42, 0.72)>)

So, min{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15, 0.3), (0.12, 0.25, 0.42, 0.72)>}

$\tilde{d}_3$ = <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>

So, the minimum provided correspond to the node 1.Hence, the node 3 is labeled as [(<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) , 1]

$\tilde{d}_3$= <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>

Iteration 3: The node 4 has one predecessor, which is node 3. Following the step 2 in the proposed algorithm, we put i=1and j=4, hence the value of $\tilde{d}_4$ can be computed as follows:

$\tilde{d}_4$= min{ $\tilde{d}_3 \oplus \tilde{d}_{34}$ }= min {<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3,0.4)> ⊕ <(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>} = <(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>

So min{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3,0.4)> ⊕ <(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>}= <(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>

So, the minimum provided correspond to the node 3.Hence, the node 4 is labeled as [<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)> ,3]

$\tilde{d}_4$ =<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>

Iteration 4: The node 5 has two predecessors, which are node 2 and node 3. Following the step 2 in the proposed algorithm, we put i=2, 3 and j=5, hence the value of $\tilde{d}_5$ can be computed as follows:

$\tilde{d}_5$= min{ $\tilde{d}_2 \oplus \tilde{d}_{25}, \tilde{d}_3 \oplus \tilde{d}_{35}$ } = min{<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>⊕ <(0.1, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.7), (0.2, 0.3, 0.6, 0.7)>, <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> ⊕ <(0.3, 0.6, 0.7, 0.8), (0.1, 0.2, 0.3, 0.4), (0.1, 0.4, 0.5, 0.6)>} =

min{<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> , <(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>}

S (<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>) = 0.69

S (<(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>) = 0.81

978-1-5090-5443-5/17/$31.00 ©2017 IEEE

Since S (<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>) S (<(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>)

min{<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> , <(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>}

= <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>

So, the minimum provided correspond to the node 2.Hence, the node 5 is labeled as [<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>, 2]

$\tilde{d}_5$ = <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>

Iteration 5: The node 6 has two predecessors, which are node 4 and node 5. Following the step 2 in the proposed algorithm, we put i=4, 5 and j=6, hence the value of $\tilde{d}_6$ can be computed as follows:

$\tilde{d}_6$ = min{ $\tilde{d}_4 \oplus \tilde{d}_{46}, \tilde{d}_5 \oplus \tilde{d}_{56}$ }= min{<(0.36, 0.58, 0.75,0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>$\oplus$ <(0.4, 0.6, 0.8, 0.9), (0.2, 0.4, 0.5, 0.6), (0.1, 0.3, 0.4, 0.5)>, <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> $\oplus$ <(0.2, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.6), (0.1, 0.5,0.3, 0.6)>} = min{<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>, <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)> }

S (<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>) = 0.87

S (<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>) = 0.81

Since S (<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>) < S (<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>)

min{<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>, <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)> }= <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>

$\tilde{d}_6$ = <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>

So, the minimum provided correspond to the node 5.Hence, the node 6 is labeled as [<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>, 5]

Since the destination node of the proposed network is the node 6. Hence, the TrFN- shortest distance between source node 1 and destination node is <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>

So, the TrFN-shortest path between the source node 1 and the destination node 6 can be determined using the following method:

The node 6 takes the label [<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>, 5], which indicate that we are moving from node 5. The node 5 takes the label [<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>, 2] , which indicate that we are moving from node 2. The node 2 takes the label [<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>, 1] which indicate that we are moving from node 1.So, joining all the provided nodes, we get the TrFN-shortest path between the source node 1 and the destination node 6. Hence the TrFN-shortest path is given as follows: $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$

Following the algorithm described in section 2, the computational results for finding the TrFN-shortest path from source node 1 to destination node 6 are summarized in table 2.

TABLE II. SUMMARIZE OF TRAPEZOIDAL FUZZY NEUTROSOPHIC DISTANCE AND SHORTEST PATH

Node	$\tilde{d}_i$	shortest path between the i-th and 1st node
2	<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>	$1 \rightarrow 2$
3	<(0.2, 0.4, 0.5), (0.3, 0.5, 0.6), (0.1, 0.2, 0.3)>	$1 \rightarrow 3$
4	<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>	$1 \rightarrow 3 \rightarrow 4$
5	<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>	$1 \rightarrow 2 \rightarrow 5$
6	<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$

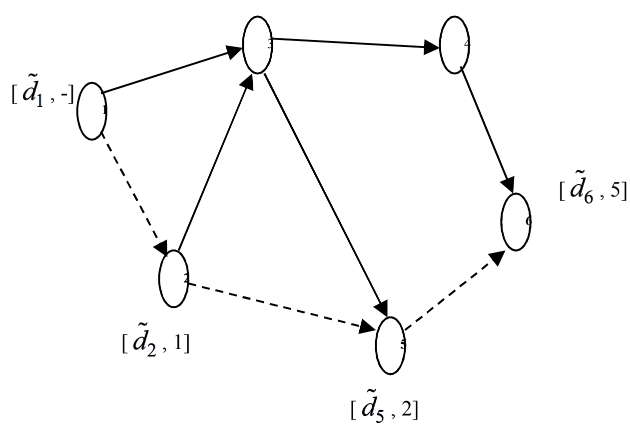

Fig. 2. TrFN-shortest path from source node 1 to destination node 6

V. CONCLUSION

In this research paper, a new algorithm based on trapezoidal fuzzy neutrosophic numbers is presented for finding the shortest path problem in a network where the edges weight are represented by TrFNN. A numerical example is introduced to show the efficacy of the proposed algorithm. So in the next work, we plan to implement this approach practically.

978-1-5090-5443-5/17/$31.00 ©2017 IEEE

ACKNOWLEDGMENT

The authors are very grateful to the chief editor and reviewers for their comments and suggestions, which is helpful in improving the paper.

REFERENCES

[1] F. Smarandache, Neutrosophy. Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998.

[2] F. Smarandache, A geometric interpretation of the neutrosophic set — A generalization of the intuitionistic fuzzy set, Granular Computing (GrC), 2011 IEEE International Conference, 2011, pp.602– 606 .

[3] L. Zadeh, Fuzzy sets. Inform and Control, 8, 1965, pp.338-353

[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20, 1986, pp. 87-96.

[5] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, Single valued Neutrosophic Sets, Multisspace and Multistructure 4, 2010, pp. 410-413.

[6] H.L. Yang, Z. L. Guo, Y. She and XLiao, On single valued neutrosophic relations, Journal of Intelligent & Fuzzy Systems 30, 2016, pp. 1045–1056

[7] M. Ali, and F. Smarandache, Complex Neutrosophic Set, Neural Computing and Applications, Vol. 25, 2016, pp.1-18.

[8] J. Ye, Single-Valued Neutrosophic Minimum Spanning Tree and Its Clustering Method, Journal of Intelligent Systems 23(3), 2014, pp. 311–324.

[9] Deli, S. Yusuf, F. Smarandache and M. Ali, Interval valued bipolar neutrosophic sets and their application in pattern recognition, IEEE World Congress on Computational Intelligence 2016.

[10] C. Liu, Interval neutrosophic fuzzy Stochastic multi-criteria decision making method based on MYCIN certainty factor and prospect theory, Rev. Téc.Ing.Univ.Zulia. Vol.39,N 10, 2016, pp.52-58.

[11] K. Mandal and K. Basu, Improved similarity measure in neutrosophic environment and its application in finding minimum spanning tree, Journal of Intelligent & fuzzy Systems 31,2016, pp.1721-1730.

[12] Deli and Y. Subas, A Ranking methods of single valued neutrosophic numbers and its application to multi-attribute decision making problems, International Journal of Machine Learning and Cybernetics, 2016, 1-14.

[13] J. Ye, Single-Valued Neutrosophic similarity measures for multiple attribute decision making, Neutrosophic Sets and Systems, Vol1, 2014,pp.

[14] P. Biswas, S. Pramanik and B. C. Giri, Cosine Similarity Measure Based Multi-attribute Decision-Making with Trapezoidal fuzzy Neutrosophic numbers, Neutrosophic sets and systems, 8, 2014, pp.47-57.

[15] F. Smarandache, Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off- Logic, Probability, and Statistics, 168 p., Pons Editions, Bruxelles, Belgique, 2016

[16] P. Biswas, S. Pramanik and B. C. Giri, Aggregation of Triangular Fuzzy Neutrosophic Set Information and its Application to Multiattribute Decision Making, Neutrosophic sets and systems, 12, 2016, pp.20-40.

[17] J. Ye. Trapezoidal fuzzy neutrosophic set and its application to multiple attribute decision making. Neural Computing and Applications, 2014. DOI 10.1007/s00521-014-1787-6.

[18] S. Broumi, M. Talea, A. Bakali, F. Smarandache, Single Valued Neutrosophic Graphs, Journal of New Theory, N 10, 2016, pp. 86-101.

[19] S. Broumi, M. Talea, A. Bakali, F. Smarandache, On Bipolar Single Valued Neutrosophic Graphs, Journal of New Theory, N11, 2016, pp.84-102.

[20] S. Broumi, M. Talea, A. Bakali, F. Smarandache, Interval Valued Neutrosophic Graphs, Critical Review, XII, 2016. pp.5-33.

[21] S. Broumi, A. Bakali, M, Talea, and F, Smarandache, Isolated Single Valued Neutrosophic Graphs. Neutrosophic Sets and Systems, Vol. 11, 2016, pp.74-78

[22] S. Broumi, F. Smarandache, M. Talea and A. Bakali, An Introduction to Bipolar Single Valued Neutrosophic Graph Theory. Applied Mechanics and Materials, vol.841,2016, 184-191.

[23] S. Broumi, M. Talea, F. Smarandache and A. Bakali, Single Valued Neutrosophic Graphs: Degree, Order and Size. IEEE International Conference on Fuzzy Systems (FUZZ), 2016, pp. 2444-2451.

[24] F. Smarandache, Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies, Neutrosophic Sets and Systems, Vol. 9, 2015, pp.58.63.

[25] F. Smarandache, Types of Neutrosophic Graphs and neutrosophic AlgebraicStructures together with their Applications in Technology," seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania 06 June 2015.

[26] S. Broumi, F. Smarandache, M. Talea and A. Bakali, Decision-Making Method Based On the Interval Valued Neutrosophic Graph, Future Technologie, 2016, IEEE, pp. 44-50.

[27] S. Broumi, F. Smarandache, M. Talea and A. Bakali, Operations on Interval Valued Neutrosophic Graphs, chapter in book- New Trends in Neutrosophic Theory and Applications- Florentin Smarandache and Surpati Pramanik (Editors), 2016, pp. 231-254. ISBN 978-1-59973-498-9

[28] F. Smarandache: Symbolic Neutrosophic Theory (Europanova asbl, Brussels, 195 p., Belgium 2015.

[29] W. B. Vasantha Kandasamy, K. Ilanthenral and F.Smarandache: Neutrosophic Graphs: A New Dimension to Graph Theory Kindle Edition, 2015.

[30] Ngoor and M. M. Jabarulla, Multiple Labeling Approach For Finding shortest Path with Intuitionstic Fuzzy Arc Length, International Journal of Scientific and Engineering Research,V3,Issue 11,pp.102-106,2012.

[31] P.K. De and Amita Bhinchar. Computation of Shortest Path in a fuzzy network. International journal computer applications. 11(2), 2010, pp. 0975-8887.

[32] D. Chandrasekaran, S. Balamuralitharan and K. Ganesan, A Sortest Path Lenght on A Fuzzy Network with Triangular Intuitionistic Fuzzy Number, ARPN Journal of Engineering and Applied Sciences, Vol 11, N 11, 2016, pp.6882-6885.

[33] Anuuya and R.Sathya, Type -2 fuzzy shortest path, International Journal of Fuzzy Mathematical Archive , vol 2, 2013, pp.36-42.

[34] Kumar, and M. Kaur, Solution of fuzzy maximal flow problems using fuzzy linear programming. World Academy of Science and Technology. 87: 28-31, (2011).

[35] P. Jayagowri and G. G. Ramani, Using Trapezoidal Intuitionistic Fuzzy Number to Find Optimized Path in a Network, Volume 2014, Advances in Fuzzy Systems, 2014, 6 pages.

[36] Kumar and M. Kaur, A New Algorithm for Solving Shortest Path Problem on a Network with Imprecise Edge Weight, Applications and Applied Mathematics, Vol. 6, Issue 2, 2011, pp. 602 – 619.

[37] G. Kumar, R. K. Bajaj and N .Gandotra, "Algorithm for shortest path problem in a network with interval valued intuitionistic trapezoidal fuzzy number, Procedia Computer Science 70, 2015, pp.123-129.

[38] V. Anuuya and R.Sathya, Shortest path with complement of type -2 fuzzy number, Malya Journal of Matematik , S(1), 2013, pp.71-76.

[39] S. Majumdar and A. Pal, Shortest Path Problem on Intuitionistic Fuzzy Network, Annals of Pure and Applied Mathematics, Vol. 5, No. 1, November 2013, pp. 26-36.

[40] S. Broumi, A. Bakali, M. Talea and F. Smarandache and P.K, Kishore Kumar, Shortest Path Problem on Single Valued Neutrosophic Graphs, 2017 International Symposium on Networks, Computers and Communications (ISNCC) , in press

[41] S. Broumi, A. Bakali, T. Mohamed, F. Smarandache and L. Vladareanu, Shortest Path Problem Under Triangular Fuzzy Neutrosophic Information, 2016 10th International Conference on Software, Knowledge, Information Management & Applications (SKIMA),2016,pp.169-174.

[42] S. Broumi, A. Bakali, M. Talea, F. Smarandache and M. Ali, Shortest Path Problem under Bipolar Neutrosphic Setting, Applied Mechanics and Materials, Vol. 859, 2016, pp 59-66.

978-1-5090-5443-5/17/$31.00 ©2017 IEEE

[43] S. Broumi, A. Bakali, M. Talea, F. Smarandache and L. Vladareanu, Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, 2016,pp.417-422.

[44] S. Broumi, A. Bakali, M. Talea, F. Smarandache and L. Vladareanu, Applying Dijkstra Algorithm for Solving Neutrosophic Shortest Path Problem, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, November 30 - December 3, 2016,pp.412-416.

978-1-5090-5443-5/17/$31.00 ©2017 IEEE