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# Rough Neutrosophic TOPSIS for Multi-Attribute Group Decision Making

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**Abstract:** This paper is devoted to present Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method for multi-attribute group decision making under rough neutrosophic environment. The concept of rough neutrosophic set is a powerful mathematical tool to deal with uncertainty, indeterminacy and inconsistency. In this paper, a new approach for multi-attribute group decision making problems is proposed by extending the TOPSIS method under rough neutrosophic environment. Rough neutrosophic set is characterized by the upper and lower approximation operators and the pair of neutrosophic sets that are characterized by truth-membership degree, indeterminacy membership degree, and falsity membership degree. In the decision situation, ratings of alternatives with respect to each attribute are characterized by rough neutrosophic sets that reflect the decision makers' opinion. Rough neutrosophic weighted averaging operator has been used to aggregate the individual decision maker's opinion into group opinion for rating the importance of attributes and alternatives. Finally, a numerical example has been provided to demonstrate the applicability and effectiveness of the proposed approach.

**Keywords:** Multi-attribute group decision making; Neutrosophic set; Rough set; Rough neutrosophic set; TOPSIS

## 1 Introduction

Hwang and Yoon [1] put forward the concept of Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in 1981 to help select the best alternative with a finite number of criteria. Among numerous multi criteria decision making (MCDM) methods developed to solve real-world decision problems, (TOPSIS) continues to work satisfactorily in diverse application areas such as supply chain management and logistics [2, 3, 4, 5], design, engineering and manufacturing systems [6, 7], business and marketing management [8, 9], health, safety and environment management[10, 11], human resources management [12, 13, 14], energy management [15], chemical engineering [16], water resources management [17, 18], bi-level programming problem [19, 20], multi-level programming problem [21], medical diagnosis [22], military [23], education [24], others topics [25, 26, 27, 28, 29, 30], etc. Behzadian et al. [31] provided a state-of the-art literature survey on TOPSIS applications and methodologies. According to C. T. Chen [32], crisp data are inadequate to model real-life situations because human judgments including preferences are often vague. Preference information of alternatives provided by the decision makers may be poorly defined, partially known and incomplete. The concept of fuzzy set theory grounded by L. A. Zadeh [33] is capable of dealing with impreciseness in a mathematical form. Interval valued fuzzy set [34, 35, 36, 37] was proposed by several authors independently in 1975 as a generalization of fuzzy set. In 1986, K. T. Atanassov [38] introduced the concept of intuitionistic fuzzy set (IFS) by incorporating non-membership degree as independent entity to deal non-statistical impreciseness. In 2003, mathematical equivalence of intuitionistic fuzzy set (IFS) with interval-valued fuzzy sets was proved by Deschrijver and Kerre [39]. C. T. Chen [32] studied the TOPSIS method in fuzzy environment for solving multi-attribute decision making problems. Boran et al. [12] studied TOPSIS method in intuitionistic fuzzy environment and provided an illustrative example of personnel selection in a manufacturing company for a sales manager position. However, fuzzy sets and interval fuzzy sets are not capable of all types of uncertainties in different real physical problems involving indeterminate information.

In order to deal with indeterminate and inconsistent information, the concept of neutrosophic set [40, 41, 42, 43] is useful. In neutrosophic set each element of the universe is characterized by the truth membership degree, indeterminacy membership degree and falsity membership degree lying in the non-standard unit interval$]^-0, 1^+[$. However, it is difficult to apply directly the neutrosophic set in real engineering and scientific applications. Wang et al. [44] introduced single-valued neutrosophic set (SVNS) to face real scientific and engineering fields involving imprecise, incomplete, and inconsistent information. However, the idea was envisioned some years earlier by Smarandache [43] SVNS, a subclass of NS, can also represent each element of universe with the truth membership values, indeterminacy membership values and falsity membership values lying in the real unit interval [0, 1]. SVNS has caught much attention to the researchers on various topics such as, medical diagnosis [45], similarity measure [46, 47, 48, 49, 50], decision making [51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70], educational problems [71, 72], conflict resolution [73], social problem [74, 75], optimization [76, 77, 78, 79, 80, 81], etc.

Pawlak [82] proposed the notion of rough set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. It is a useful mathematical tool for dealing with uncertainty or incomplete information. Broumi et al. [83, 84] proposed new hybrid intelligent structure called rough neutrosophic set by combining the concepts of single valued neutrosophic set and rough set. The theory of rough neutrosophic set [83, 84] is also a powerful mathematical tool to deal with incompleteness. Rough neutrosophic set can be applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information existing in real scientific and engineering applications. In rough neutrosophic environment, Mondal and Pramanik [85] proposed rough neutrosophic multi-attribute decision-making based on grey relational analysis. Mondal and Pramanik [86] also proposed rough neutrosophic multi-attribute decision-making based on rough accuracy score function. Pramanik and Mondal [87] proposed cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Pramanik and Mondal [88] also proposed cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Pramanik and Mondal [88] also proposed some similarity measures namely, Dice and Jaccard similarity measures in rough neutrosophic environment and applied them for multi attribute decision making problem. Pramanik and Mondal [90] studied decision making in rough interval neutrosophic environment in 2015. Mondal and Pramanik [91] studied cosine, Dice and Jaccard similarity measures for interval rough neutrosophic sets and presented their applications in decision making problem. So decision making in rough neutrosophic environment appears to be a developing area of study. Mondal et al. [92] proposed rough trigonommetric Hamming similarity measures such as cosine, sine and cotangent rough similarity meaures and proved their basic properties. In the same study Mondal et al. [92] also provided a numerical example of selection of a smart phone for rough use based on the proposed methods.

The objective of the study is to extend the concept of TOPSIS method for multi-attribute group decision making (MAGDM) problems under single valued neutrosophic rough neutrosophic environment. All information provided by different domain experts in MAGDM problems about alternative and attribute values take the form of rough neutrosophic set. In a group decision making process, rough neutrosophic weighted averaging operator is used to aggregate all the decision makers' opinions into a single opinion to select best alternative.

The remaining part of the paper is organized as follows: section 2 presents some preliminaries relating to neutrosophic set, section 3 presents the concept of rough neutrosophic set. In section 4, basics of TOPSIS method are discussed. Section 5 is devoted to present TOPSIS method for MAGDM under rough neutrosophic environment. In section 6, a numerical example is provided to show the effectiveness of the proposed approach. Finally, section 7 presents the concluding remarks and scope of future research.

## 2 Neutrosophic sets and single valued neutrosophic set [43, 44]

### 2.1 Definition of Neutrosophic sets [40, 41, 42, 43]

**Definition** 2.1.1. [43]:

Assume that V be a space of points and v be a generic element in V. Then a neutrosophic set G in V is characterized by a truth membership function $T_G$, an indeterminacy membership function $I_G$ and a falsity membership function $F_G$. The functions $T_G$, $I_G$ and $F_G$ are real standard or non-standard subsets of $]^-0, 1^+[$ i.e. $T_G: V \to\ ]^-0, 1^+[$, $I_G: V \to\ ]^-0, 1^+[$, $F_G: V \to\ ]^-0, 1^+[$, and $^-0 \le T_G(v) + I_G(v) + F_G(v) \le 3^+$.

**2.1.2**.[43]:

The complement of a neutrosophic set G is denoted by $G^c$ and is defined by

$T_{G^c}(v) = \{1^+\} - T_G(v)$ ; $I_{G^c}(v) = \{1^+\} - I_G(v)$ ; $F_{G^c}(v) = \{1^+\} - I_G(v)$

**Definition 2.1.3.** [43]:

A neutrosophic set G is contained in another neutrosophic set H, $G \subseteq H$ iff the following conditions holds.

$\inf T_G(v) \le \inf T_H(v)$ $\sup T_G(v) \le \sup T_H(v)$

$\inf I_G(v) \ge \inf I_H(v)$, $\sup I_G(v) \ge \sup I_H(v)$

$\inf F_G(v) \ge \inf F_H(v)$, $\sup F_G(v) \ge \sup F_H(v)$

for all v in V.

**Definition 2.1.4.** [44]:

Assume that V be a universal space of points, and v be a generic element of V. A single-valued neutrosophic set P is characterized by a truth membership function $T_P(v)$, a falsity membership function $I_P(v)$, and an indeterminacy membership function $F_P(v)$. Here, $T_P(v)$, $I_P(v)$, $F_P(v) \in [0, 1]$. When V is continuous, a SVNS P can be written as

$$P = \int_V \langle T_P(v), F_P(v), I_P(v) \rangle / v, v \in V.$$

When V is discrete, a SVNS P can be written as

$$P = \sum \langle T_P(v), F_P(v), I_P(v) \rangle / v, \ \forall v \in V$$

It is obvious that for a SVNS P,

$$0 \le \sup T_P(v) + \sup F_P(v) + \sup I_P(v) \le 3, \ \forall v \in V$$

**Definition 2.1.5**. [44]:
The complement of a SVNS set P is denoted by $P^C$ and is defined as follows:

$$T_P^C(v) = F_P(v); \ I_P^C(v) = 1 - I_P(v); \ F_P^C(v) = T_P(v)$$

**Definition 2.1.6**. [44]:
A SVNS $P_G$ is contained in another SVNS $P_H$, denoted as $P_G \subseteq P_H$ if the following conditions hold.

$$T_{P_G}(v) \le T_{P_H}(v) \ ; \ I_{P_G}(v) \ge I_{P_H}(v) \ ; \ F_{P_G}(v) \ge F_{P_H}(v) \ , \ \forall v \in V.$$

**Definition 2.1.7**. [44]:
Two SVNSs $P_G$ and $P_H$ are equal, i.e., $P_G = P_H$, iff $P_G \subseteq P_H$ and $P_G \supseteq P_H$

**Definition 2.1.8**. [44]:
The union of two SVNSs $P_G$ and $P_H$ is a SVNS $P_Q$, written as $P_Q = P_G \cup P_H$.
Its truth, indeterminacy and falsity membership functions are as follows:

$$T_{P_Q}(v) = \max(T_{P_G}(v), T_{P_H}(v));$$

$$I_{P_Q}(v) = \min(I_{P_G}(v), I_{P_H}(v));$$

$$F_{P_Q}(v) = \min(F_{P_G}(v), F_{P_H}(v)), \ \forall v \in V.$$

**Definition 2.1.9.** [44]:
The intersection of two SVNSs $P_G$ and $P_H$ is a SVNS $P_C$ written as $P_C = P_G \cap P_H$. Its truth, indeterminacy and falsity membership functions are as follows:

$$T_{P_C}(v) = \min(T_{P_G}(v), T_{P_H}(v));$$

$$I_{P_C}(v) = \max(I_{P_G}(v), I_{P_H}(v));$$

$$F_{P_C}(v) = \max(F_{P_G}(v), F_{P_H}(v)), \ \forall v \in V.$$

**Definition 2.1.10**. [44]:
Wang et al. [44] defined the following operation for two SVNS $P_G$ and $P_H$ as follows:

$$P_G \otimes P_H = \left\langle \begin{matrix} T_{P_G}(v).T_{P_H}(v), \ I_{P_G}(v) + I_{P_H}(v) - I_{P_G}(v).I_{P_H}(v), \\ F_{P_G}(v) + F_{P_H}(v) - F_{P_G}(v).F_{P_H}(v) \end{matrix} \right\rangle,$$

$\forall v \in V$.

**Definition 2.1.11**. [93]
Assume that

$$P_G = \left\{ \begin{matrix} \left(v_1 / \left(T_{P_G}(v_1), I_{P_G}(v_1), F_{P_G}(v_1)\right)\right), \cdots, \\ \left(v_n / \left(T_{P_G}(v_n), I_{P_G}(v_n), F_{P_G}(v_n)\right)\right) \end{matrix} \right\}$$

$$P_H = \left\{ \begin{matrix} \left(v_1 / \left(T_{P_H}(v_1), I_{P_H}(v_1), F_{P_H}(v_1)\right)\right), \cdots, \\ \left(v_n / \left(T_{P_H}(v_n), I_{P_H}(v_n), F_{P_H}(v_n)\right)\right) \end{matrix} \right\}$$

be two SVNSs in $v = \{v_1, v_2, v_3, \ldots, v_n\}$

Then the Hamming distance [93] between two SVNSs $P_G$ and $P_H$ is defined as follows:

$$d_P(P_G, P_H) = \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{P_G}(v) - T_{P_H}(v)| + |I_{P_G}(v) - I_{P_H}(v)| \\ + |F_{P_G}(v) - F_{P_H}(v)| \end{matrix} \right\rangle \quad (1)$$

and normalized Hamming distance [93] between two SVNSs $P_G$ and $P_H$ is defined as follow

$$^N d_P(P_G, P_H) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{P_G}(v) - T_{P_H}(v)| + |I_{P_G}(v) - I_{P_H}(v)| \\ + |F_{P_G}(v) - F_{P_H}(v)| \end{matrix} \right\rangle \quad (2)$$

with the following two properties

i. $0 \le d_P(P_G, P_H) \le 3$

ii. $0 \le {}^N d_P(P_G, P_H) \le 1$

Distance between two SVNSs:
Majumder and Samanta [93] studied similarity and entropy measure by incorporating Euclidean distances of SVNSs.

**Definition 2.1.12**. [93]: (Euclidean distance)

$$\text{Let } P_G = \left\{ \begin{matrix} \left(v_1 | \langle T_{P_G}(v_1), I_{P_G}(v_1), F_{P_G}(v_1) \rangle\right), \cdots, \\ \left(v_n | \langle T_{P_G}(v_n), I_{P_G}(v_n), F_{P_G}(v_n) \rangle\right) \end{matrix} \right\} \text{ and}$$

$$P_H = \left\{ \begin{matrix} \left(v_1 | \langle T_{P_H}(v_1), I_{P_H}(v_1), F_{P_H}(v_1) \rangle\right), \cdots, \\ \left(v_n | \langle T_{P_H}(v_n), I_{P_H}(v_n), F_{P_H}(v_n) \rangle\right) \end{matrix} \right\} \text{ be two}$$

SVNSs for $v_i \in V$, where i = 1, 2, . . . , n. Then the Euclidean distance between two SVNSs $P_G$ and $P_H$ can be defined as follows:

$$D_{euclid}(P_G, P_H) =$$

$$\left\langle \sum_{i=1}^{n} \left( \begin{matrix} \left(T_{P_G}(v_i) - T_{P_H}(v_i)\right)^2 + \left(I_{P_G}(v_i) - I_{P_H}(v_i)\right)^2 \\ + \left(F_{P_G}(v_i) - F_{P_H}(v_i)\right)^2 \end{matrix} \right) \right\rangle^{0.5} \quad (3)$$

and the normalized Euclidean distance [93] between two SVNSs $P_G$ and $P_H$ can be defined as follows:

$$D^N_{euclid}(P_G, P_H) =$$

$$\frac{1}{3n} \left\langle \sum_{i=1}^{n} \left( \begin{matrix} \left(T_{P_G}(v_i) - T_{P_H}(v_i)\right)^2 + \left(I_{P_G}(v_i) - I_{P_H}(v_i)\right)^2 \\ + \left(F_{P_G}(v_i) - F_{P_H}(v_i)\right)^2 \end{matrix} \right) \right\rangle^{0.5} \quad (4)$$

**Definition 2.1.13**. (Deneutrosophication of SVNS) [53]:
Deneutrosophication of SVNS $P_G$ can be defined as a process of mapping $P_G$ into a single crisp output $\theta^* \in V$ i.e. $f: P_G \to \theta^*$ for $v \in V$. If $P_G$ is discrete set then the vector $P_G = \{ v | \langle T_{P_G}(v), I_{P_G}(v), F_{P_G}(v) \rangle | v \in V \}$ is reduced to a single scalar quantity $\theta^* \in V$ by deneutrosophication. The obtained scalar quantity $\theta^* \in V$ best represents the aggregate distribution of three membership degrees of neutrosophic element $\langle T_{P_G}(v), I_{P_G}(v), F_{P_G}(v) \rangle$

## 3 Rough neutrosophic set [83, 84]

Rough set theory [82] has been developed based on two basic components. The components are crisp set and equivalence relation. The rough set logic is based on the approximation of sets by a couple of sets. These two are known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. Rough neutrosophic sets [83, 84] are the generalization of rough fuzzy sets [94, 95, 96] and rough intuitionistic fuzzy sets [97].

**Definition 3.1**. **Rough neutrosophic set [83,84]**

Assume that S be a non-null set and ρ be an equivalence relation on S. Assume that E be neutrosophic set in S with the membership function $T_E$, indeterminacy function $I_E$ and non-membership function $F_E$. The lower and the upper approximations of E in the approximation $(S, \rho)$ denoted by $\underline{L}(E)$ and $\overline{U}(E)$ are respectively defined as follows:

$$\underline{L}(E)=\left\langle < v, T_{\underline{L}(E)}(v), I_{\underline{L}(E)}(v), F_{\underline{L}(E)}(v) > / s \in [v]_\rho, v \in S \right\rangle \quad (5)$$

$$\overline{U}(E)=\left\langle < v, T_{\overline{U}(E)}(v), I_{\overline{U}(E)}(v), F_{\overline{U}(E)}(v) > / s \in [v]_\rho, v \in S \right\rangle \quad (6)$$

Here, $T_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho\, T_E(s)$, $I_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho\, I_E(s)$, $F_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho\, F_E(s)$, $T_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho\, T_E(s)$, $I_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho\, T_E(s)$, $F_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho\, I_E(s)$ .

So, $0 \le T_{\underline{L}(E)}(v) + I_{\underline{L}(E)}(v) + F_{\underline{L}(E)}(v) \le 3$

$0 \le T_{\overline{U}(E)}(v) + I_{\overline{U}(E)}(v) + F_{\overline{U}(E)}(v) \le 3$

The symbols $\vee$ and $\wedge$ indicate "max" and "min'' operators respectively. $T_E(s)$ , $I_E(s)$ and $F_E(s)$ represent the membership , indeterminacy and non-membership of S with respect to E. $\underline{L}(E)$ and $\overline{U}(E)$ are two neutrosophic sets in S.

Thus the mapping $\underline{L}$, $\overline{U}$ : N(S) $\rightarrow$ N(S) are, respectively, referred to as the lower and upper rough neutrosophic approximation operators, and the pair $(\underline{L}(E), \overline{U}(E))$ is called the rough neutrosophic set in $(S, \rho)$.

$\underline{L}(E)$ and $\overline{U}(E)$ have constant membership on the equivalence classes of ρ if $\underline{L}(E) = \overline{U}(E)$ ; i.e. $T_{\underline{L}(E)}(v) = T_{\overline{U}(E)}(v)$ , $I_{\underline{L}(E)}(v) = I_{\overline{U}(E)}(v)$ , $F_{\underline{L}(E)}(v) = F_{\overline{U}(E)}(v)$ for any v belongs to S.

E is said to be definable neutrosophic set in the approximation $(S, \rho)$. It is obvious that zero neutrosophic set ($0_N$) and unit neutrosophic sets ($1_N$) are definable neutrosophic sets.

**Definition 3.2 [83, 84].**

If N(E) = ( $\underline{L}(E), \overline{U}(E)$ ) be a rough neutrosophic set in (S, ρ), the complement of N(E) is the rough neutrosophic set and is denoted as $\sim N(E) = (\underline{L}(E)^C, \overline{U}(E)^C)$ ,where $\underline{L}(E)^C, \overline{U}(E)^C$ are the complements of neutrosophic sets of $\underline{L}(E), \overline{U}(E)$ respectively.

$\underline{L}(E)^c = \left\langle < v, T_{\underline{L}(E)}(v), 1 - I_{\underline{L}(E)}(v), F_{\underline{L}(E)}(v) > / \; v \in S \right\rangle$ and

$\overline{U}(E)^c = \left\langle < v, T_{\overline{U}(E)}(v), 1 - I_{\overline{U}(E)}(v), F_{\overline{U}(E)}(v) > / \; v \in S \right\rangle$

**Definition 3. 3 [83, 84]**

If $N(E_1)$ and $N(E_2)$ be two rough neutrosophic sets in S, then the following definitions hold:

$N(E_1) = N(E_2) \Leftrightarrow \underline{L}(E_1) = \underline{L}(E_2) \wedge \overline{U}(E_1) = \overline{U}(E_2)$

$N(E_1) \subseteq N(E_2) \Leftrightarrow \underline{L}(E_1) \subseteq \underline{L}(E_2) \wedge \overline{U}(E_1) \subseteq \overline{U}(E_2)$

$N(E_1) \bigcup N(E_2) = < \underline{L}(E_1) \bigcup \underline{L}(E_2), \overline{U}(E_1) \bigcup \overline{U}(E_2) >$

$N(E_1) \bigcap N(E_2) = < \underline{L}(E_1) \bigcap \underline{L}(E_2), \overline{U}(E_1) \bigcap \overline{U}(E_2) >$

$N(E_1) + N(E_2) = < \underline{L}(E_1) + \underline{L}(E_2), \overline{U}(E_1) + \overline{U}(E_2) >$

$N(E_1) \,.\, N(E_2) = < \underline{L}(E_1) \,.\, \underline{L}(E_2), \overline{U}(E_1) \,.\overline{U}(E_2) >$

If α, β, γ be rough neutrosophic sets in (S, ρ), then the following properties are satisfied.

**Properties I:**

1. $\sim(\sim \alpha) = \alpha$
2. $\alpha \cup \beta = \beta \cup \alpha$, $\beta \cup \alpha = \alpha \cup \beta$
3. $(\gamma \cup \beta) \cup \alpha = \gamma \cup (\beta \cup \alpha)$, $(\gamma \cap \beta) \cap \alpha = \gamma \cap (\beta \cap \alpha)$
4. $(\gamma \cup \beta) \cap \alpha = (\gamma \cup \beta) \cap (\gamma \cup \alpha)$, $(\gamma \cap \beta) \cup \alpha = (\gamma \cap \beta) \cup (\gamma \cap \alpha)$

**Proof.** For proofs of the properies, see [83,84].

**Properties II**:

De Morgan's Laws are satisfied for rough neutrosophic sets

1. $\sim (N(E_1) \cup N(E_2)) = (\sim N(E_1)) \cap (\sim N(E_2))$
2. $\sim (N(E_1) \cap N(E_2)) = (\sim N(E_1)) \cup (\sim N(E_2))$

**Proof.** For proofs of the properies, see [83,84].

**Properties III:**

If $E_1$ and $E_2$ are two neutrosophic sets of universal collection (U) such that $E_1 \subseteq E_2$, *then* 1. $N(E_1) \subseteq N(E_2)$

2. $N(E_1 \bigcap E_2) \subseteq N(E_2) \bigcap N(E_2)$
3. $N(E_1 \bigcup E_2) \supseteq N(E_2) \bigcup N(E_2)$

**Proof.** For proofs of the properies, see [83,84].

**Properties IV:**

1. $\underline{L}(E) = \sim \overline{U}(\sim E)$
2. $\overline{U}(E) = \sim \underline{L}(\sim E)$
3. $\underline{L}(E) \subseteq \overline{U}(E)$

**Proof.** For proofs of the properies, see [83,84].

## 4 TOPSIS

The TOPSIS is used to determine the best alternative from the compromise solutions. The best compromise solution should have the shortest Euclidean distance from the positive ideal solution (PIS) and the farthest Euclidean distance from the negative ideal solution (NIS). The TOPSIS method can be described as follows. Assume that K = {$K_1$, $K_2$, …,$K_m$} be the set of alternatives, L = {$L_1$, $L_2$, …, $L_n$} be the set of criteria and $p_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$ is the rating of the alternative $K_i$ with respect to the criterion $L_j$, $w_j$ is the weight of the j- th criterion $L_j$.

The procedure of TOPSIS method is presented using the following steps:

**Step 1. Normalization the decision matrix**

Calculation of the normalized value $[\vartheta]_{ij}^{N}$ is as follows:

For benefit criterion, $\vartheta_{ij} = (\vartheta_{ij} - \vartheta_j^-)/(\vartheta_j^+ - \vartheta_j^-)$, where $\vartheta_j^+ = \max_i (v_{ij})$ and $\vartheta_j^- = \min_i (v_{ij})$ or setting $\vartheta_j^+$ is the desired level and $\vartheta_j^-$ is the worst level.

For cost criterion, $\vartheta_{ij} = (\vartheta_j^- - \vartheta_{ij})/(\vartheta_j^- - \vartheta_j^+)$

**Step 2. Weighted normalized decision matrix**

In the weighted normalized decision matrix, the upgraded ratings are calculated as follows:

$\eta_{ij} = w_j \times \eta_{ij}$ for i = 1, 2, . . . , m and j = 1, 2, . . . , n. Here $w_j$ is the weight of the j-th criterion such that $w_j \geq 0$ for j = 1, 2, . . . , n and $\sum_{j=1}^{n} w_j = 1$

**Step 3. The positive and the negative ideal solutions**

The positive ideal solution (PIS) and the negative ideal solution (NIS) are calculated as follows:

$$PIS = M^+ = \left\langle \eta_1^+, \eta_2^+, \cdots \eta_n^+ \right\rangle = \left\langle \left(\max_j \eta_{ij} / j \in C_1\right), \left(\min_j \eta_{ij} / j \in C_2\right) : j = 1, 2, \cdots, n \right\rangle$$ and

$$NIS = M^- = \left\langle \eta_1^-, \eta_2^-, \cdots \eta_n^- \right\rangle = \left\langle \left(\min_j \eta_{ij} / j \in C_1\right), \left(\max_j \eta_{ij} / j \in C_2\right) : j = 1, 2, \cdots, n \right\rangle$$

where $C_1$and $C_2$ are the benefit and cost type criteria respectively.

**Step 4. Calculation of the separation measures for each alternative from the PIS and the NIS**

The separation values for the PIS and the separation values for the NIS can be determined by using the n-dimensional Euclidean distance as follows:

$\delta_i^+ = \left\langle \sum_{j=1}^{n} \left(\eta_{ij} - \eta_j^+\right)^2 \right\rangle^{0.5}$ for i = 1, 2, . . . , m.

$\delta_i^- = \left\langle \sum_{j=1}^{n} \left(\eta_{ij} - \eta_j^-\right)^2 \right\rangle^{0.5}$ for i = 1, 2, . . . , m.

**Step 5. Calculation of the relative closeness coefficient to the PIS**

The relative closeness coefficient for the alternative $K_i$ with respect to $M^+$ is

$\chi_i = \dfrac{\delta_i^-}{(\delta_i^+ + \delta_i^-)}$ for i = 1, 2, . . . m.

Obviously, $0 \leq \chi_i \leq 1$ . According to relative closeness coefficient to the ideal alternative, larger value of $\chi_i$ indicates the better alternative $K_i$.

**Step 6. Ranking the alternatives**

Rank the alternatives according to the descending order of the relative-closeness coefficients to the PIS.

## 5 Topsis method for multi-attribute decision making under rough neutrosophic environment

Assume that a multi-attribute decision-making problem be characterized by m alternatives and n attributes. Assume that K = ($K_1$, $K_2$,..., $K_m$) be the set of alternatives, and L = ($L_1$, $L_2$, ..., $L_n$) be the set of attributes. The rating measured by the decision maker describes the performance of the alternative $K_i$ against the attribute $L_j$. Assume that W = {$w_1$, $w_2$ . . . , $w_n$} be the weight vector assigned for the attributes $L_1$, $L_2$, ..., $L_n$ by the decision makers. The values associated with the alternatives for multi-attribute decision-making problem (MADM) with respect to the attributes can be presented in rough neutrosophic decision matrix (see Table 1).

**Table1**: Rough neutrosophic decision matrix

$D = \left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle_{m \times n} =$

| | $L_1$ | $L_2$ | $\cdots$ | $L_n$ |
|---|---|---|---|---|
| $K_1$ | $\langle \underline{d}_{11}, \overline{d}_{11} \rangle$ | $\langle \underline{d}_{12}, \overline{d}_{12} \rangle$ | ... | $\langle \underline{d}_{1n}, \overline{d}_{1n} \rangle$ |
| $K_2$ | $\langle \underline{d}_{21}, \overline{d}_{21} \rangle$ | $\langle \underline{d}_{22}, \overline{d}_{22} \rangle$ | ... | $\langle \underline{d}_{2n}, \overline{d}_{2n} \rangle$ |
| . | ... | ... | ... | ... |
| . | ... | ... | ... | ... |
| $K_m$ | $\langle \underline{d}_{m1}, \overline{d}_{m1} \rangle$ | $\langle \underline{d}_{m2}, \overline{d}_{m2} \rangle$ | ... | $\langle \underline{d}_{mn}, \overline{d}_{mn} \rangle$ |

(7)

Here $\left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle$ is the rough neutrosophic number according to the i-th alternative and the j-th attribute.

In decision-making situation, there exist many attributes of alternatives. Some of them are important and others may be less important. So it is important to select proper weights of attributes for decision-making situation.

**Definition 5.1**. Accumulated geometric operator (AGO) [85]

Assume a rough neutrosophic number in the form: $\left\langle \underline{L}_{ij}(\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}), \overline{U}_{ij}(\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij}) \right\rangle$. We transform the rough neutrosophic number into SVNNs using the accumulated geometric operator (AGO). The operator is expressed as follows.

$$N_{ij}\left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle = \left\langle \underline{L}_{ij} . \overline{U}_{ij} \right\rangle^{0.5} = N_{ij}\left\langle (\underline{T}_{ij} \overline{T}_{ij})^{0.5}, (\underline{I}_{ij} \overline{I}_{ij})^{0.5}, (\underline{F}_{ij} \overline{F}_{ij})^{0.5} \right\rangle \quad (8)$$

Using AGO operator [85], the rating of each alternative with respect to each attribute is transformed into SVNN for MADM problem. The rough neutrosophic values (transformed as SVNN) associated with the alternatives for

MADM problems can be represented in decision matrix ( see Table 2).

**Table 2**. Tranformed rough neutrosiphic decision matrix

$D=\langle d\rangle_{m\times n}=\langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}=$

| | $L_1$ | $L_2$ | ... | $L_n$ |
|---|---|---|---|---|
| $K_1$ | $\langle T_{11}, I_{11}, F_{11}\rangle$ | $\langle T_{12}, I_{12}, F_{12}\rangle$ | ... | $\langle T_{1n}, I_{1n}, F_{1n}\rangle$ |
| $K_2$ | $\langle T_{21}, I_{21}, F_{21}\rangle$ | $\langle T_{22}, I_{22}, F_{22}\rangle$ | ... | $\langle T_{2n}, I_{2n}, F_{2n}\rangle$ |
| . | ... | ... | ... | ... |
| . | ... | ... | ... | ... |
| $K_m$ | $\langle T_{m1}, I_{m1}, F_{m1}\rangle$ | $\langle T_{m2}, I_{m2}, F_{m2}\rangle$ | ... | $\langle T_{mn}, I_{mn}, F_{mn}\rangle$ |

(9)

In the matrix $\langle d\rangle_{m\times n}=\langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}$, $T_{ij}$, $I_{ij}$ and $F_{ij}$ (i = 1, 2,..., n and j = 1, 2,..., m) denote the degree of truth membership value, indeterminacy membership value and falsity membership value of alternative $K_i$ with respect to attribute $L_j$.

The ratings of each alternative with respect to the attributes can be explained by the neutrosophic cube [98] proposed by Dezert. The vertices of neutrosophic cube are (0, 0, 0), (1, 0, 0), (1, 0, 1), (0, 0, 1), (0, 1, 0), (1, 1, 0), (1, 1, 1) and (0, 1, 1). The acceptance ratings [53, 99] in neutrosophic cube are classified in three types namely,

I. Highly acceptable neutrosophic ratings,

II. Manageable neutrosophic rating

III. Unacceptable neutrosophic ratings.

Definition 5.2. (Highly acceptable neutrosophic ratings) [99]

In decision making process, the sub cube ( $\Theta$ ) of a neutrosophic cube ($\Omega$) (i.e. $\Theta\subset\Omega$) reflects the field of highly acceptable neutrosophic ratings ( $\Psi$ ). Vertices of $\Lambda$ are defined with the eight points (0.5, 0, 0),(1, 0, 0),(1, 0, 0.5), (0.5, 0, 0.5), (0.5, 0, 0.5), (1, 0, 0.5), (1, 0.5, 0.5) and (0.5, 0.5, 0.5). U includes all the ratings of alternative considered with the above average truth membership degree, below average indeterminacy degree and below average falsity membership degree for multi-attribute decision making. So, $\Psi$ has a great role in decision making process and can be defined as follows:

$\Psi = \left\langle (\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5}\right\rangle$ where $0.5 < (\underline{T}_{ij}\overline{T}_{ij})^{0.5} < 1$, $0 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} < 0.5$ and $0 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} < 0.5$, for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

Definition 5.3. (Unacceptable neutrosophic ratings) [99]

The field $\Sigma$ of unacceptable neutrosophic ratings $\Lambda$ is defined by the ratings which are characterized by 0% membership degree, 100% indeterminacy degree and 100% falsity membership degree. Hence, the set of unacceptable ratings $\Lambda$ can be considered as the set of all ratings whose truth membership value is zero.

$\Lambda = \left\langle (\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5}\right\rangle$ where $(\underline{T}_{ij}\overline{T}_{ij})^{0.5} = 0$, $0 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} \le 1$ and $0 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} \le 1$, for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

In decision making situation, consideration of $\Lambda$ should be avoided.

Definition 5.4. (Manageable neutrosophic ratings) [99]

Excluding the field of high acceptable ratings and unacceptable ratings from a neutrosophic cube, tolerable neutrosophic rating field $\Phi$ $(=\Omega\cap\neg\Theta\cap\neg\Sigma)$ is determined. The tolerable neutrosophic rating ( $\Delta$ ) considered membership degree is taken in decision making process. $\Delta$ can be defined by the expression as follows:

$\Delta = \left\langle (\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5}\right\rangle$ where $0 < (\underline{T}_{ij}\overline{T}_{ij})^{0.5} < 0.5$, $0.5 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} < 1$ and $0.5 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} < 1$.

for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

Definition 5.5 [53].

Fuzzification of transformed rough neutrosophic set $N=\langle T_N(v), I_N(v), F_N(v)\rangle$ for any v ∈ V can be defined as a process of mapping N into fuzzy set F = $\{v/\mu_F(v)/v\in V\}$ i.e. f: N $\rightarrow$ F for v ∈ V. The representative fuzzy membership degree $\mu_F(v)\in[0,1]$ of the vector $\{v/\langle T_N(v), I_N(v), F_N(v)\rangle, v\in V\}$ is defined from the concept of neutrosophic cube. It can be obtained by determining the root mean square of 1-$T_N$(v), $I_N$(v), and $F_N$(v) for all v ∈ V. Therefore the equivalent fuzzy membership degree is defined as follows:

$$\mu_F(v)=\begin{cases}1-\left\langle\left\{(1-T_N(v))^2+(I_N(v))^2+(F_N(v))^2\right\}/3\right\rangle^{0.5} & \forall v\in\psi\cup\Delta\\ 0 & \forall v\in\Lambda\end{cases}\quad(10)$$

Now the steps of decision making using TOPSIS method under rough neutrosophic environment are stated as follows.

**Step 1. Determination of the weights of decision makers**

Assume that a group of k decision makers having their own decision weights involved in the decision making. The importance of the decision makers in a group may not be equal. Assume that the importance of each decision maker is considered with linguistic variables and expressed it by rough neutrosophic numbers.

Assume that $\left\langle \underline{N}_k(\underline{T}_k, \underline{I}_k, \underline{F}_k), \overline{N}_k(\overline{T}_k, \overline{I}_k, \overline{F}_k)\right\rangle$ be a rough neutrosophic number for the rating of k−th decision maker. Using AGO operator, we obtain $E_k$ = $\langle T_k, I_k, F_k\rangle$ as a single valued neutrosophic number for the rating of k−th decision maker. Then, according to equation (10) the weight of the k−th decision maker can be written as:

$$\xi_k=\frac{1-\left\langle\left\{(1-T_k(v))^2+(I_k(v))^2+(F_k(v))^2\right\}/3\right\rangle^{0.5}}{\sum_{k=1}^{r}\left\langle 1-\left\{(1-T_k(v))^2+(I_k(v))^2+(F_k(v))^2\right\}/3\right\rangle^{0.5}}\quad(11)$$

and $\Sigma_{k=1}^{r}\xi_k = 1$

**Step 2. Construction of the aggregated rough neutrosophic decision matrix based on the assessments of decision makers**

Assume that $D^k = \left\langle \underline{d}^{(k)}_{ij}, \overline{d}^{(k)}_{ij} \right\rangle_{m\times n}$ be the rough neutrosophic decision matrix of the k−th decision maker. According to equation (11), $D^k = \left(d^{(k)}_{ij}\right)_{m\times n}$ be the single-valued neutrosophic decision matrix corresponding to the rough neutrosophic decision matrix and $\xi = (\xi_1, \xi_2, ..., \xi_k)^T$ be the weight vector of decision maker such that each $\xi_k \in [0, 1]$. In the group decision making process, all the individual assessments need to be accumulated into a group opinion to make an aggregated single valued neutrosophic decision matrix. This aggregated matrix can be obtained by using rough neutrosophic aggregation operator as follows:

$D = (d_{ij})_{m\times n}$ where,

$$(d_{ij})_{m\times n} = RNWA_{\xi}\left(d^1_{ij}, d^2_{ij}, \cdots, d^r_{ij}\right) = \xi_1 d^1_{ij} \oplus \xi_2 d^2_{ij} \oplus \cdots \oplus \xi_r d^r_{ij}$$

$$= \left\langle 1 - \prod_{k=1}^{r}(1 - T^{(r)}_{ij})^{\xi_k}, \prod_{k=1}^{r}(I^{(r)}_{ij})^{\xi_k}, \prod_{k=1}^{r}(F^{(r)}_{ij})^{\xi_k}, \right\rangle \tag{12}$$

Here, $d^r_{ij} = \left\langle \underline{d}^r_{ij}.\overline{d}^r_{ij} \right\rangle^{0.5}$

Now the aggregated rough neutrosophic decision matrix is defined as follows:

$$(d_{ij})_{m\times n} = \left\langle (\underline{T}_{ij}.\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}.\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}.\overline{F}_{ij})^{0.5} \right\rangle_{m\times n}$$

| | $L_1$ | $L_2$ | ... | $L_n$ |
|---|---|---|---|---|
| $K_1$ | $\langle T_{11}, I_{11}, F_{11}\rangle$ | $\langle T_{12}, I_{12}, F_{12}\rangle$ | ... | $\langle T_{1n}, I_{1n}, F_{1n}\rangle$ |
| $K_2$ | $\langle T_{21}, I_{21}, F_{21}\rangle$ | $\langle T_{22}, I_{22}, F_{22}\rangle$ | ... | $\langle T_{2n}, I_{2n}, F_{2n}\rangle$ |
| . | ... | ... | ... | ... |
| . | ... | ... | ... | ... |
| $K_m$ | $\langle T_{m1}, I_{m1}, F_{m1}\rangle$ | $\langle T_{m2}, I_{m2}, F_{m2}\rangle$ | ... | $\langle T_{mn}, I_{mn}, F_{mn}\rangle$ |

(13)

Here, $d_{ij} = \langle T_{ij}, I_{ij}, F_{ij}\rangle = \left\langle (\underline{T}_{ij}.\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}.\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}.\overline{F}_{ij})^{0.5} \right\rangle$ is the aggregated element of rough neutrosophic decision matrix D for i = 1, 2, . . . m and j = 1, 2, . .. n.

**Step 3. Determination of the attribute weights**

In the decision-making process, all attributes may not have equal importance. So, every decision maker may have their own opinion regarding attribute weights. To obtain the group opinion of the chosen attributes, all the decision makers' opinions need to be aggregated. Assume that $\left\langle \underline{w}^j_{(k)}, \overline{w}^j_{(k)} \right\rangle$ be rough neutrosophic number (RNN) assigned to the attribute $L_j$ by the k−th decision maker. According to equation (8) $w^j_k$ be the neutrosophic number assigned to the attribute $L_j$ by the k−th decision maker. Then the combined weight W = ($w_1$, $w_2$ . . . , $w_n$) of the attribute can be determined by using rough neutrosophic weighted aggregation (RNWA) operator

$$w_j = RNWA_{\xi}(w^{(1)}_j, w^{(2)}_j, \cdots, w^{(r)}_j) = \xi_1 w^{(1)}_j \oplus \xi_2 w^{(2)}_j \oplus \cdots \oplus \xi_r w^{(r)}_j$$

$$= \left\langle 1 - \prod_{k=1}^{r}(1 - T^{(r)}_j)^{\xi_k}, \prod_{k=1}^{r}(I^{(r)}_j)^{\xi_k}, \prod_{k=1}^{r}(F^{(r)}_j)^{\xi_k} \right\rangle \tag{14}$$

Here, $\xi^r_{ij} = \left\langle \underline{d}_{ij}.\overline{d}_{ij} \right\rangle$; $w_j = \left\langle T^{(r)}_j, I^{(r)}_j, F^{(r)}_j \right\rangle =$

$\left\langle (\underline{T}^{(r)}_j.\overline{T}^{(r)}_j)^{0.5}, (\underline{I}^{(r)}_j.\overline{I}^{(r)}_j)^{0.5}, (\underline{F}^{(r)}_j.\overline{F}^{(r)}_j)^{0.5} \right\rangle$ for j = 1, 2, . . . n.

W = ($w_1$, $w_2$ . . . , $w_n$) (15)

**Step 4. Aggregation of the weighted rough neutrosophic decision matrix**

In this section, the obtained weights of attribute and aggregated rough neutrosophic decision matrix need to be further fused to make the aggregated weighted rough neutrosophic decision matrix. Then, the aggregated weighted rough neutrosophic decision matrix can be defined by using the multiplication properties between two neutrosophic sets as follows:

$$D \otimes W = D^W = \left\langle d^{w_j}_{ij} \right\rangle_{m\times n} = \left\langle T^{w_j}_{ij}, I^{w_j}_{ij}, F^{w_j}_{ij} \right\rangle_{m\times n} =$$

| | $L_1$ | $L_2$ | ... | $L_n$ |
|---|---|---|---|---|
| $K_1$ | $\langle T^{w_1}_{11}, I^{w_1}_{11}, F^{w_1}_{11}\rangle$ | $\langle T^{w_2}_{12}, I^{w_2}_{12}, F^{w_2}_{12}\rangle$ | ... | $\langle T^{w_n}_{1n}, I^{w_n}_{1n}, F^{w_n}_{1n}\rangle$ |
| $K_2$ | $\langle T^{w_1}_{21}, I^{w_1}_{21}, F^{w_1}_{21}\rangle$ | $\langle T^{w_2}_{22}, I^{w_2}_{22}, F^{w_2}_{22}\rangle$ | ... | $\langle T^{w_n}_{2n}, I^{w_n}_{2n}, F^{w_n}_{2n}\rangle$ |
| . | ... | ... | ... | ... |
| . | ... | ... | ... | ... |
| $K_m$ | $\langle T^{w_1}_{m1}, I^{w_1}_{m1}, F^{w_1}_{m1}\rangle$ | $\langle T^{w_2}_{m2}, I^{w_2}_{m2}, F^{w_2}_{m2}\rangle$ | ... | $\langle T^{w_n}_{mn}, I^{w_n}_{mn}, F^{w_n}_{mn}\rangle$ |

(16)

Here, $d^{w_j}_{ij} = \left\langle T^{w_j}_{ij}, I^{w_j}_{ij}, F^{w_j}_{ij} \right\rangle$ is an element of the aggregated weighted rough neutrosophic decision matrix $D^W$ for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

**Step 5. Determination of the rough relative positive ideal solution (RRPIS) and the rough relative negative ideal solution (RRNIS)**

After transferring RNS decision matrix, assume $D_N = \left\langle d^W_{ij} \right\rangle_{m\times n} = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m\times n}$ be a SVNS based decision matrix, where, $T_{ij}$, $I_{ij}$ and $F_{ij}$ are the membership degree, indeterminacy degree and non-membership degree of evaluation for the attribute $L_j$ with respect to the alternative $K_i$. In practical siuation, two types of attributes namely, benefit type attribute and cost type attribute are considered in multi-attribute decision making problems.

**Definition 5.6.**

Assume that $C_1$and $C_2$ be the benefit type attribute and cost type attribute respectively. Suppose that $G^+_N$ is the relative rough neutrosophic positive ideal solution (RRNPIS) and $G^-_N$ is the relative rough neutrosophic negative ideal solution (RRNNIS).

Then $G^+_N$ can be defined as follows:

$$G^+_N = \left\langle d^{w+}_1, d^{w+}_2, \cdots, d^{w+}_n \right\rangle \tag{17}$$

Here $d^{w+}_j = \left\langle T^{w+}_j, I^{w+}_j, F^{w+}_j \right\rangle$ for j = 1, 2, . . . , n.

$$T^{w+}_j = \{(\max_i\{T^{w_j}_{ij}\} / j \in C_1), (\min_i(T^{w_j}_{ij}) / j \in C_2)\}$$

$$I^{w+}_j = \{(\min_i\{I^{w_j}_{ij}\} / j \in C_1), (\max_i(I^{w_j}_{ij}) / j \in C_2)\}$$

$F_j^{w+} = \{(\min_i \{F_{ij}^{w} j\} / j \in C_1), (\max_i (F_{ij}^{w} j) / j \in C_2)\}$

Then $G_N^-$ can be defined as follows:

$$G_N^- = \langle d_1^{w-}, d_2^{w-}, \cdots, d_n^{w-} \rangle \quad (18)$$

Here $d_j^{w-} = \langle T_j^{w-}, I_j^{w-}, F_j^{w-} \rangle$ for j = 1, 2, . . . , n.

$T_j^{w-} = \{(\min_i \{T_{ij}^{w} j\} / j \in C_1), (\max_i (T_{ij}^{w} j) / j \in C_2)\}$

$I_j^{w-} = \{(\max_i \{I_{ij}^{w} j\} / j \in C_1), (\min_i (I_{ij}^{w} j) / j \in C_2)\}$

$F_j^{w-} = \{(\max_i \{F_{ij}^{w} j\} / j \in C_1), (\min_i (F_{ij}^{w} j) / j \in C_2)\}$

**Step 6. Determination of the distance measure of each alternative from the RRNPIS and the RRNNIS**

The normalized Euclidean distance measure of all alternative $\langle T_{ij}^{w} j, I_{ij}^{w} j, F_{ij}^{w} j \rangle$ from the RRNPIS $\langle d_1^{w+}, d_2^{w+}, ..., d_n^{w+} \rangle$ for i = 1, 2, …, m and j = 1, 2, …, n can be written as follows:

$$\delta_{euclid}^{i+}(d_{ij}^{w} j, d_{ij}^{w+}) = \frac{1}{3n} \left\langle \sum_{j=1}^{n} \begin{pmatrix} (T_{ij}^{w} j(v_j) - T_j^{w+}(v_j))^2 + (I_{ij}^{w} j(v_j) - I_j^{w+}(v_j))^2 \\ + (F_{ij}^{w} j(v_j) - F_j^{w+}(v_j))^2 \end{pmatrix} \right\rangle^{0.5} \quad (19)$$

The normalized Euclidean distance measure of all alternative $\langle T_{ij}^{w} j, I_{ij}^{w} j, F_{ij}^{w} j \rangle$ from the RRNPIS $\langle d_1^{w-}, d_2^{w-}, ..., d_n^{w-} \rangle$ for i = 1, 2, …, m and j = 1, 2, …, n can be written as follows:

$$\delta_{euclid}^{i-}(d_{ij}^{w} j, d_{ij}^{w-}) = \frac{1}{3n} \left\langle \sum_{j=1}^{n} \begin{pmatrix} (T_{ij}^{w} j(v_j) - T_j^{w-}(v_j))^2 + (I_{ij}^{w} j(v_j) - I_j^{w-}(v_j))^2 \\ + (F_{ij}^{w} j(v_j) - F_j^{w-}(v_j))^2 \end{pmatrix} \right\rangle^{0.5} \quad (20)$$

**Step 7. Determination of the relative closeness co-efficient to the rough neutrosophic ideal solution for rough neutrosophic sets**

The relative closeness coefficient of each alternative $K_i$ with respect to the neutrosophic positive ideal solution $G_N^+$ is defined as follows:

$$\chi_i^* = \frac{\langle \delta_{euclid}^{i-}(d_{ij}^{w} j, d_{ij}^{w-}) \rangle}{\langle \delta_{euclid}^{i-}(d_{ij}^{w} j, d_{ij}^{w-}) + \delta_{euclid}^{i+}(d_{ij}^{w} j, d_{ij}^{w+}) \rangle} \quad (21)$$

Here $0 \le \chi_i^* \le 1$ . According to the relative closeness coefficient values larger the values of $\chi_i^*$ reflects the better alternative $K_i$ for i = 1, 2, …, n.

**Step 8. Ranking the alternatives**

Rank the alternatives according to the descending order of the relative-closeness coefficients to the RRNPIS.

**6 Numerical example**

In order to demonstrate the proposed method, logistic center location selection problem is described here. Suppose that a new modern logistic center is required in a town. There are three locations $K_1$, $K_2$, $K_3$. A committee of three decision makers or experts $D_1$, $D_2$, $D_3$ has been formed to select the most appropriate location on the basis of six parameters obtained from expert opinions, namely, cost ($L_1$), distance to suppliers ($L_2$), distance to customers ($L_3$), conformance to government and law ($L_4$), quality of service ($L_5$), and environmental impact ($L_6$).

Based on the proposed approach the considered problem is solved using the following steps:

**Step 1. Determination of the weights of decision makers**

The importance of three decision makers in a selection committee may be different based on their own status. Their decision values are considered as linguistic terms (seeTable-3). The importance of each decision maker expressed by linguistic term with its corresponding rough neutrosophic values shown in Table-4. The weights of decision makers are determined with the help of equation (11) as:

$\xi_1 = 0.398$, $\xi_2 = 0.359$, $\xi_3 = 0.243$.

We transform rough neutrosophic number (RNN) to neutrosophic number (NN) with the help of AGO operator [85] in Table 3, Table 4 and Table 5.

**Step 2. Construction of the aggregated rough neutrosophic decision matrix based on the assessments of decision makers**

The linguistic terms along with RNNs are defined in Table-5 to rate each alternative with respect to each attribute. The assessment values of each alternative $K_i$ (i = 1, 2, 3) with respect to each attribute $L_j$ (j = 1, 2, 3, 4, 5, 6) provided by three decision makers are listed in Table-6. Then the aggregated neutrosophic decision matrix can be obtained by fusing all the decision maker opinions with the help of aggregation operator (equation 12) (see Table 7).

**Step 3. Determination of the weights of attributes**

The linguistic terms shown in Table-3 are used to evaluate each attribute. The importance of each attribute for every decision maker is rated with linguistic terms shown in Table-6. Three decision makers' opinions need to be aggregated to final opinion.

The fused attribute weight vector is determined by using equation (14) as follows:

$$W = \begin{Bmatrix} \langle 0.761, 0.205, 0.195 \rangle, \langle 0.800, 0.181, 0.159 \rangle, \langle 0.737, 0.241, 0.196 \rangle, \\ \langle 0.761, 0.223, 0.169 \rangle, \langle 0.774, 0.203, 0.172 \rangle, \langle 0.804, 0.184, 0.172 \rangle \end{Bmatrix} \quad (23)$$

**Step 4. Construction of the aggregated weighted rough neutrosophic decision matrix**

Using equation (16) and clculating the combined weights of the attributes and the ratings of the alternatives, the aggregated weighted rough neutrosophic decision matrix is obtained (see Table-8).

**Step 5. Determination of the rough neutrosophic relative positive ideal solution and the rough neutrosophic relative negative ideal solution**

The RNRPIS can be calculated from the aggregated weighted decision matrix on the basis of attribute types i.e. benefit type or cost type by using equation (17) as

$$G_N^{+}= \begin{bmatrix} \langle 0.670, 0.289, 0.274\rangle, \langle 0.694, 0.284, 0.252\rangle, \langle 0.588, 0.388, 0.309\rangle, \\ \langle 0.607, 0.374, 0.286\rangle, \langle 0.642, 0.331, 0.303\rangle, \langle 0.708, 0.270, 0.253\rangle \end{bmatrix} \quad (25)$$

Here $d_1^{w+}=\langle T_1^{w+}, I_1^{w+}, F_1^{w+}\rangle$ is calculated as:

$T_1^{w+}=$ max [0.670, 0.485, 0.454] = 0.670, $I_1^{w+}=$ min [0.289, 0.449, 0.471] = 0.289,

$F_1^{w+}=$ min [0.274, 0.377, 0.463]= 0.274.

Similarly, other RNRPISs are calculated.

Using equation (18), the RNRNIS are calculated from aggregated weighted decision matrix based on attribute types i.e. benefit type or cost type.

$$G_N^{-}= \begin{bmatrix} \langle 0.454, 0.471, 0.463\rangle, \langle 0.588, 0.377, 0.353\rangle, \langle 0.469, 0.480, 0.309\rangle, \\ \langle 0.522, 0.441, 0.358\rangle, \langle 0.524, 0.429, 0.372\rangle, \langle 0.512, 0.435, 0.414\rangle \end{bmatrix} \quad (26)$$

Here, $d_1^{w-}=\langle T_1^{w-}, I_1^{w-}, F_1^{w-}\rangle$ is calculated as

$T_1^{w}$ = min [0.670, 0.485, 0.454] = 0.454, $I_1^{w-}=$ max [0.289, 0.449, 0.471] = 0.471,

$F_1^{w-}=$ max [0.274, 0.377, 0.463] = 0.463.

Other RNRNISs are calculated in similar way.

**Step 6. Determination of the distance measure of each alternative from the RRNPIS and the RRNNIS and relative closeness co-efficient**

Normalized Euclidean distance measures defined in equation (19) and equation (20) are used to determine the distances of each alternative from the RRNPIS and the RNNIS.

**Step 7. Determination of the relative closeness co-efficient to the rough neutrosophic ideal solution for rough neutrosophic sets**

Using equation (21) and distances, relative closeness coefficient of each alternative $K_1$ , $K_2$ , $K_3$ with respect to the rough neutrosophic positive ideal solution $G_N^{+}$ is calculated (see Table 9).

**Table 9. Distance measure and relative closeness co-efficient**

| $Alternatives(K_i)$ | $\delta_{euclid}^{i+}$ | $\delta_{euclid}^{i-}$ | $\chi_i^{*}$ |
|---|---|---|---|
| $K_1$ | 0.0078 | 0.1248 | 0.9411 |
| $K_2$ | 0.1192 | 0.0682 | 0.3639 |
| $K_3$ | 0.1025 | 0.0534 | 0.3425 |

(27)

**Step 9. Ranking the alternatives**

According to the values of relative closeness coefficient of each alternative (see Table 9), the ranking order of three alternatives is obtained as follows:

$K_1 \succ K_2 \succ K_3$.

Thus $K_1$ is the best the logistic center.

## 7 Conclusion

In general, realistic MAGDM problems adhere to uncertain, imprecise, incomplete, and inconsistent data and rough neutrosophic set theory is adequate to deal with it. In this paper, we have proposed rough neutrosophic TOPSIS method for MAGDM. We have also proposed rough neutrosophic aggregate operator and rough neutrosophic weighted aggregate operator. In the decision-making situation, the ratings of each alternative with respect to each attribute are presented as linguistic variables characterized by rough neutrosophic numbers. Rough neutrosophic aggregation operator has been used to aggregate all the opinions of decision makers. Rough neutrosophic positive ideal and rough neutrosophic negative ideal solution have been defined to form aggregated weighted decision matrix. Euclidean distance measure has been used to calculate the distances of each alternative from positive as well as negative ideal solutions for relative closeness co-efficient of each alternative. The proposed rough neutrosophic TOPSIS approach can be applied in pattern recognition, artificial intelligence, and medical diagnosis in rough neutrosophic environment.

## References


[1] C. L. Hwang and K. Yoon. Multiple attribute decision making: methods and applications. Springer, New York, 1981.

[2] C. T. Chen, C. T. Lin, and S. F. Huang. A fuzzy approach for supplier evaluation and selection in supply chain management. International Journal of Production Economics, 102 (2006), 289–301.

[3] C. Kahraman, O. Engin, O. Kabak, and I. Kaya. Information systems outsourcing decisions using a group decision-making approach. Engineering Applications of Artificial Intelligence, 22 (2009), 832–841.

[4] S. K. Patil and R. Kant. A fuzzy AHP-TOPSIS framework for ranking the solutions of knowledge management adoption in supply chain to overcome its barriers. Expert System Applications, 41(2) (2014), 679–693.

[5] W. P. Wang. Toward developing agility evaluation of mass customization systems using 2-tuple linguistic computing. Expert System Applications, 36 (2009), 3439–3447.

[6] H. S. Shih. Incremental analysis for MCDM with an application to group TOPSIS. European Journal of Operational Research, 186 (2008), 720–734.

[7] S. P. Wan, F. Wang, L. L. Lin, and J. Y. Dong. An intuitionistic fuzzy linear programming method for logistics outsourcing provider selection. Knowledge Based Systems, 82 (2015), 80-94.

[8] E. K. Aydogan. Performance measurement model for Turkish aviation firms using the rough-AHP and TOPSIS methods under fuzzy environment. Expert System Applications, 38 (2011), 3992–3998.

[9] Y. Peng, G. Wang, G. Kou, and Y. Shi. An empirical study of classification algorithm evaluation for finan-

cial risk prediction. Applied Soft Computing, 11 (2011), 2906 -2915.

[10] R. A. Krohling and V. C. Campanharo. Fuzzy TOPSIS for group decision making: A case study for accidents with oil spill in the sea. Expert System with Applications, 38 (2011), 4190–4197.

[11] S. P. Sivapirakasam, J. Mathew, and M. Surianarayanan. Multi-attribute decision making for green electrical discharge machining. Expert System with Applications, 38 (2011), 8370–8374.

[12] F. E. Boran, S. Genc, M. Kurt, and D. Akay. Personnel selection based on intuitionistic fuzzy sets. Human Factors and Ergonomics in Manufacturing & Service Industries, 21 (2011), 493-503.

[13] A. Kelemenis, K. Ergazakis, and D. Askounis. Support managers' selection using an extension of fuzzy TOPSIS. Expert System Applications, 38 (2011), 2774–2782.

[14] X. Sang, X. Liu, and J. Qin. An analytical solution to fuzzy TOPSIS and its application in personnel selection for knowledge-intensive enterprise. Applied Soft Computing, 30 (2015), 190–204.

[15] T. Kaya, and C. Kahraman. Multi criteria decision making in energy planning using a modified fuzzy TOPSIS methodology. Expert System with Applications, 38 (2011), 6577–6585.

[16] Y. F. Sun, Z. S. Liang, C. J. Shan, H. Viernstein, and F. Unger. Comprehensive evaluation of natural antioxidants and antioxidant potentials in Ziziphus jujube Mill, Var. spinosa (Bunge) Hu ex H. F. Chou fruits based on geographical origin by TOPSIS method. Food. Chemistry, 124 (2011), 1612–1619.

[17] A. Afshar, M. A. Marino, M. Saadatpour, and A. Afshar. Fuzzy TOPSIS multi-criteria decision analysis applied to Karun reservoirs system. Water Resource Management, 25(2) (2011), 545–563.

[18] J. Dai, J. J. Qi, J. Chi, and B. Chen. Integrated water resource security evaluation of Beijing based on GRA and TOPSIS. Frontiers of Earth Science in China, 4(3) (2010), 357–362.

[19] I. A. Baky and M. A. Abo-Sinna. TOPSIS for bi-level MODM problems. Applied Mathematical Modelling, 37(3) (2013), 1004–1015.

[20] P. P. Dey, S. Pramanik, and B. C. Giri. TOPSIS approach to linear fractional bi-level MODM problem based on fuzzy goal programming. Journal of Industrial and Engineering International, 10(4) (2014), 173-184.

[21] S. Pramanik, D. Banerjee, and B. C. Giri. TOPSIS approach to chance constrained multi-objective multilevel quadratic programming problem. Global Journal of Engineering Science and Research Management, (2016). DOI:10.5281/zenodo.55308.

[22] S. Rahimi, L. Gandy, and N. Mogharreban. A web-based high-performance multi criterion decision support system for medical diagnosis. International Journal of Intelligent Systems, 22 (2007), 1083–1099.

[23] M. Da˘gdeviren, Yavuz, and N. Kılınc. Weapon selection using the AHP and TOPSIS methods under fuzzy environment. Expert System with Applications, 36(4) (2009), 8143–8151.

[24] J. Min and K. H. Peng. Ranking emotional intelligence training needs in tour leaders: an entropy-based TOPSIS approach. Current Issues in Tourism, 15(6) (2012), 563-576.

[25] P. P. Dey, S. Pramanik, and B. C. Giri. TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. Neutrosophic Sets and Systems, 10 (2015), 88-95.

[26] P. P. Dey, S. Pramanik, and B.C. Giri. TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications, Brussells, Pons Editions, 2016, 65-77.

[27] Y. H. He, L. B. Wang, Z. Z. He, and M. Xie. A fuzzy topsis and rough set based approach for mechanism analysis of product infant failure. Eng, Appl. Artif. Intel. http://dx.doi.org/10.1016/j.engappai.2015.06.002, 2015.

[28] R. Lourenzutti and R. A. Krohling. A generalized TOPSIS method for group decision making with heterogeneous information in a dynamic environment. Information Sciences, 330 (2016), 1–18.

[29] Z. Pei. A note on the TOPSIS method in MADM problems with linguistic evaluations. Applied Soft Computing, 36 (2015), 24-35.

[30] S. Pramanik, D. Banerjee, and B. C. Giri. TOPSIS approach for multi attribute group decision making in refined neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications, Brussells, Pons Editions, 2016, 79-91.

[31] M. Behzadian, K. Otaghsara, and J. Ignatius. A state-of the-art survey of TOPSIS applications. Expert Systems with Applications, 39(17) (2012), 13051–13069.

[32] C. T. Chen. Extensions of the TOPSIS for group decision making under fuzzy environment. Fuzzy Sets and Systems, 114(1) (2000), 1-9.

[33] L. A. Zadeh. Fuzzy sets. Information and Control, 8(3) (1965), 338–353.

[34] I. Grattan-Guiness. Fuzzy membership mapped onto interval and many-valued quantities. Z. Math. Logik. Grundladen Math, 22 (1975), 149-160.

[35] K. U. Jahn. Intervall-wertige mengen. Math. Nach, 68 (1975), 115-132.

[36] R. Sambuc. Fonctions Φ-floues. Application `a l'aide au diagnostic en pathologie thyroidienne. Ph.D. Thesis, Univ. Marseille, France, 1975.

[37] L. A. Zadeh. The concept of a linguistic variable and its application to approximate reasoning-1. Information Sciences, 8(1975), 199–249.

[38] K. T. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1) (1986), 87–96.

[39] G. Deschrijver and E. E. Kerre. On the relationship between some extensions of fuzzy set theory. Fuzzy Sets and Systems, 133(2) (2003), 227-235.

[40] F. Smarandache. Neutrosophic set–a generalization of intuitionistic fuzzy set. Journal of Defense Resources Management, 1(1) (2010), 107-116.
[41] F. Smarandache. Neutrosophic set–a generalization of intuitionistic fuzzy sets. International Journal of Pure and Applied Mathematics, 24(3) (2005), 287-297.
[42] F. Smarandache. Linguistic paradoxes and tautologies. Libertas Mathematica, University of Texas at Arlington, IX (1999), 143-154.
[43] F. Smarandache. A unifying field of logics. Neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, (1998).
[44] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman. Single valued neutrosophic sets. Multispace Multistructure, 4 (2010), 410–413.
[45] J. Ye. Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. Artificial Intelligence in Medicine, 63 (2015), 171–179.
[46] P. Biswas, S. Pramanik, and B. C. Giri. Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Neutrosophic Sets and Systems, 8 (2015), 47-57.
[47] S. Broumi and F. Smarandache. Several similarity measures of neutrosophic sets. Neutrosophic Sets and Systems, 1(2013), 54–62.
[48] K. Mondal and S. Pramanik. Neutrosophic tangent similarity measure and its application to multiple attribute decision making. Neutrosophic Sets and Systems, 9 (2015), 85-92.
[49] K. Mondal and S. Pramanik. Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. Journal of New Theory, 8 (2015), 41-50.
[50] J. Ye. Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. International Journal of Fuzzy Systems, 16 (2014), 204–215.
[51] P. Biswas, S. Pramanik, and B. C. Giri. Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2(2014), 102–110.
[52] P. Biswas, S. Pramanik, and B. C. Giri. A new methodology for neutrosophic multi-attribute decision making with unknown weight information. Neutrosophic Sets and Systems, 3(2014), 42–52.
[53] P. Biswas, S. Pramanik, and B. C. Giri. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. Neural Computing and Applications, 27(3), 727-737.
[54] P. Biswas, S. Pramanik, and B. C. Giri. Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. Neutrosophic Sets and Systems, 12 (2016), 20-40.
[55] P. Biswas, S. Pramanik, and B. C. Giri. Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets and Systems, 12 (2016), 127-138.
[56] P. Chi and P. Liu. An extended TOPSIS method for the multi-attribute decision making problems on interval neutrosophic set. Neutrosophic Sets and Systems, 1 (2013), 63–70.
[57] A. Kharal. A neutrosophic multi-criteria decision making method. New. Math. Nat. Comput, 10 (2014), 143–162.
[58] P. Liu, Y. Chu, Y. Li, and Y. Chen. Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making. International Journal of Fuzzy Systems, 16(2) (2014), 242–255.
[59] P. Liu and Y. Wang. Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Computing and Applications, 25(7) (2014), 2001–2010.
[60] J. J. Peng, J. Q. Wang, J. Wang, H. Y. Zhang, and X. H. Chen. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. International Journal of Systems Science, 47 (10) (2016), 2342-2358.
[61] S. Pramanik, P. Biswas, and B. C. Giri. Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications, (2015), doi:10.1007/s00521-015-2125-3.
[62] S. Broumi, J. Ye, and F. Smarandache. An extended TOPSIS method for multiple attribute decision making based on interval neutrosophic uncertain linguistic variables. Neutrosophic Sets and Systems, 8 (2015), 22-31.
[63] R. Sahin and P. Liu. Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. Neural Computing and Applications, (2015), doi: 10.1007/s00521-015-1995-8.
[64] J. Ye. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems, 42 (2013), 386–394.
[65] J. Ye. Single valued neutrosophic cross-entropy for multicriteria decision making problems. Applied Mathematical Modelling, 38 (3) (2013),1170–1175.
[66] J. Ye. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of Intelligent and Fuzzy Systems, 26 (2014), 2459–2466.
[67] J. Ye. Trapezoidal neutrosophic set and its application to multiple attribute decision-making. Neural Computing and Applications, 26 (2015),1157–1166.
[68] J. Ye. Bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. Neural Computing and Applications, (2015), Doi: 10.1007/s00521-015-2123-5.
[69] S. Pramanik, S. Dalapati, and T. K. Roy. Logistics center location selection approach based on neutrosophic multi-criteria decision making. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications, Brussells, Pons Editions, 2016, 161-174.
[70] S. Pramanik, D. Banerjee, and B.C. Giri. Multi–criteria group decision making model in neutrosophic refined

set and its application. Global Journal of Engineering Science and Research Management, 3(6) (2016), 12-18.

[71] K. Mondal and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified Neutrosophic environment. Neutrosophic Sets and Systems, 6 (2014), 28-34.

[72] K. Mondal and S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7 (2015), 62-68.

[73] S. Pramanik and T. K. Roy. Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. Neutrosophic Sets and Systems, 2 (2014), 82-101.

[74] K. Mondal and S. Pramanik. A study on problems of Hijras in West Bengal based on neutrosophic cognitive maps. Neutrosophic Sets and Systems, 5(2014), 21-26.

[75] S. Pramanik and S. Chakrabarti. A study on problems of construction workers in West Bengal based on neutrosophic cognitive maps. International Journal of Innovative Research in Science Engineering and Technology, 2(11) (2013), 6387-6394.

[76] M. Abdel-Baset, M. I. M. Hezam, and F. Smarandache. Neutrosophic goal programming. Neutrosophic Sets and Systems, 11 (2016), 112-118.

[77] P. Das and T. K. Roy. Multi-objective non-linear programming problem based on neutrosophic optimization technique and its application in riser design problem. Neutrosophic Sets and Systems, 9 (2015), 88-95.

[78] I. M. Hezam, M. Abdel-Baset, and F. Smarandache. Taylor series approximation to solve neutrosophic multiobjective programming problem. Neutrosophic Sets and Systems, 10 (2015), 39-45.

[79] S. Pramanik. Neutrosophic multi-objective linear programming. Global Journal of Engineering Science and Research Management, 3(8) (2016), 36-46.

[80] S. Pramanik. Neutrosophic linear goal programming. Global Journal of Engineering Science and Research Management, 3(7) (2016), 01-11.

[81] R. Roy and P. Das. A multi-objective production planning roblem based on neutrosophic linear rogramming approach. Internal Journal of Fuzzy Mathematical Archive, 8(2) (2015), 81-91.

[82] Z. Pawlak. Rough sets. International Journal of Information and Computer Sciences, 11(5) (1982), 341-356.

[83] S. Broumi, F. Smarandache, and M. Dhar. Rough neutrosophic sets. Italian Journal of Pure and Applied Mathematics, 32 (2014), 493-502.

[84] S. Broumi, F. Smarandache, and M. Dhar. Rough neutrosophic sets. Neutrosophic Sets and Systems, 3(2014), 60-66.

[85] K. Mondal and S. Pramanik. Rough neutrosophic multi-Attribute decision-making based on grey relational analysis. Neutrosophic Sets and Systems, 7 (2014), 8-17.

[86] K. Mondal and S. Pramanik. Rough neutrosophic multi-attribute decision-making based on rough accuracy score function. Neutrosophic Sets and Systems, 8 (2015), 16-22.

[87] S. Pramanik and K. Mondal. Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Journal of New Theory, 4 (2015), 90-102.

[88] S. Pramanik and K. Mondal. Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Global Journal of Advanced Research 2(1) (2015), 212-220.

[89] S. Pramanik and K. Mondal. Some rough neutrosophic similarity measure and their application to multi attribute decision making. Global Journal of Engineering Science and Research Management, 2(7) (2015), 61-74.

[90] S. Pramanik and K. Mondal. Interval neutrosophic multi-Attribute decision-making based on grey relational analysis. Neutrosophic Sets and Systems, 9 (2015), 13-22.

[91] K. Mondal and S. Pramanik. Decision making based on some similarity measures under interval rough neutrosophic environment. Neutrosophic Sets and Systems, 10 (2015), 46-57.

[92] K. Mondal, S. Pramanik, and F. Smarandache. Several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in decision making. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications, Brussells, Pons Editions, 2016, 93-103.

[93] P. Majumder and S. K. Samanta. On similarity and entropy of neutrosophic sets. Journal of Intelligent and Fuzzy Systems, 26(2014), 1245–1252.

[94] R. Biswas. On rough sets and fuzzy rough sets. Bulletin of the Polish Academy of Sciences. Mathematics 42 (4) (1994), 343-349.

[95] A. Nakamura. Fuzzy rough sets. Note. Multiple Valued Logic, Japan, 9 (1998), 1–8.

[96] S. Nanda and S. Majumdar. Fuzzy rough sets. Fuzzy Sets and Systems, 45 (1992), 157–160.

[97] M. D. Cornelis and E. E. Cock. Intuitionistic fuzzy rough sets: at the crossroads of imperfect knowledge. Expert System, 20(5) (2003), 260-270.

[98] J. Dezert. Open questions in neutrosophic inferences. Multiple-Valued Logic: An International Journal, 8 (2002), 439-472.

[99] M. Sodenkamp. Models, methods and applications of group multiple-criteria decision analysis in complex and uncertain systems. Dissertation, University of Paderborn, Germany, 2013.

**Table 3**. Linguistic terms for rating attributes

| Linguistic Terms | Rough neutrosophic numbers | Neutrosophic numbers |
|---|---|---|
| Very good / Very important (VG/VI) | $\langle(0.85, 0.05, 0.05), (0.95, 0.15, 0.15)\rangle$ | $\langle 0.899, 0.087, 0.087\rangle$ |
| Good / Important(G /I) | $\langle(0.75, 0.15, 0.10), (0.85, 0.25, 0.20)\rangle$ | $\langle 0.798, 0.194, 0.141\rangle$ |
| Fair / Medium(F/M) | $\langle(0.45, 0.35, 0.35), (0.55, 0.45, 0.55)\rangle$ | $\langle 0.497, 0.397, 0.439\rangle$ |
| Bad / Unimportant (B / UI) | $\langle(0.25, 0.55, 0.65), (0.45, 0.65, 0.75)\rangle$ | $\langle 0.335, 0.598, 0.698\rangle$ |
| Very bad/Very Unimportant (VB/VUI) | $\langle(0.05, 0.75, 0.85), (0.15, 0.85, 0.95)\rangle$ | $\langle 0.087, 0.798, 0.899\rangle$ |

**Table 4.** Importance of decision makers expressed in terms of rough neutrosophic numbers

| DM | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|
| LT | VI | I | M |
| RNN | $\langle(0.85, 0.05, 0.05), (0.95, 0.15, 0.15)\rangle$ | $\langle(0.75, 0.15, 0.10), (0.85, 0.25, 0.20)\rangle$ | $\langle(0.45, 0.35, 0.35), (0.55, 0.45, 0.55)\rangle$ |
| NN | $\langle 0.899, 0.087, 0.087\rangle$ | $\langle 0.798, 0.194, 0.141\rangle$ | $\langle 0.497, 0.397, 0.439\rangle$ |

**Table 5**. Linguistic terms for rating the candidates innterms of rough neutrosophic numbers and neutrosophic numbers

| Linguistic terms | RNNs | NNs |
|---|---|---|
| Extremely Good/High (EG/EH) | $\langle(1.00,0.00,0.00),(1.00,0.00,0.00)\rangle$ | $\langle 1.000, 0.000, 0.000\rangle$ |
| Very Good/High (VG/VH) | $\langle(0.85,0.05,0.05),(0.95,0.15,0.15)\rangle$ | $\langle 0.899, 0.087, 0.087\rangle$ |
| Good/High (G/H) | $\langle(0.75,0.15,0.10),(0.85,0.25,0.20)\rangle$ | $\langle 0.798, 0.194, 0.141\rangle$ |
| Medium Good/High (MG/MH) | $\langle(0.55,0.30,0.25),(0.65,0.40,0.35)\rangle$ | $\langle 0.598, 0.346, 0.296\rangle$ |
| Medium/Fair (M/F) | $\langle(0.45,0.45,0.35),(0.55,0.55,0.55)\rangle$ | $\langle 0.497, 0.497, 0.439\rangle$ |
| MediumBad/MediumLaw(MB/ML) | $\langle(0.30,0.60,0.55),(0.40,0.70,0.65)\rangle$ | $\langle 0.346, 0.648, 0.598\rangle$ |
| Bad/Law (G/L) | $\langle(0.15,0.70,0.75),(0.25,0.80,0.85)\rangle$ | $\langle 0.194, 0.748, 0.798\rangle$ |
| Very Bad/Low (VB/VL) | $\langle(0.05,0.80,0.85),(0.15,0.90,0.95)\rangle$ | $\langle 0.087, 0.849, 0.899\rangle$ |
| VeryVeryBad/low(VVB/VVL) | $\langle(0.05,0.95,0.95),(0.05,0.85,0.95)\rangle$ | $\langle 0.050, 0.899, 0.950\rangle$ |

**Table 6.** Assessments of alternatives and attribute in terms of linguisterm terms given by three decision makers

| Alternatives ($K_i$) | Decision Makers | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ |
|---|---|---|---|---|---|---|---|
| $K_1$ | $D_1$ | VG | G | G | G | G | VG |
| | $D_2$ | VG | VG | G | G | G | VG |
| | $D_3$ | G | VG | G | G | VG | G |
| $K_2$ | $D_1$ | M | G | M | G | G | M |
| | $D_2$ | G | MG | G | G | MG | G |
| | $D_3$ | M | G | M | MG | M | M |
| $K_3$ | $D_1$ | M | VG | G | MG | VG | M |
| | $D_2$ | M | M | G | G | M | G |
| | $D_3$ | G | M | M | MG | G | VG |

**Table 7**. Aggregated transformed rough neutrosophic decision matrix

| | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ |
|---|---|---|---|---|---|---|
| $K_1$ | ⟨0.881, 0.106, 0.098⟩ | ⟨0.867, 0.126, 0.111⟩ | ⟨0.798, 0.194, 0.141⟩ | ⟨0.798, 0.194, 0.141⟩ | ⟨0.830, 0.160, 0.125⟩ | ⟨0.880, 0.106, 0.098⟩ |
| $K_2$ | ⟨0.637, 0.307, 0.292⟩ | ⟨0.741, 0.239, 0.184⟩ | ⟨0.637, 0.315, 0.292⟩ | ⟨0.761, 0.223, 0.169⟩ | ⟨0.677, 0.284, 0.242⟩ | ⟨0.637, 0.307, 0.292⟩ |
| $K_3$ | ⟨0.597, 0.334, 0.333⟩ | ⟨0.735, 0.217, 0.231⟩ | ⟨0.748, 0.231, 0.186⟩ | ⟨0.686, 0.281, 0.227⟩ | ⟨0.787, 0.182, 0.175⟩ | ⟨0.755, 0.212, 0.197⟩ |

**Table 8.** Aggregated weighted rough neutrosophic decision matrix

| | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ |
|---|---|---|---|---|---|---|
| $K_1$ | ⟨0.670, 0.289, 0.274⟩ | ⟨0.694, 0.284, 0.252⟩ | ⟨0.588, 0.388, 0.309⟩ | ⟨0.607, 0.374, 0.286⟩ | ⟨0.642, 0.331, 0.303⟩ | ⟨0.708, 0.270, 0.253⟩ |
| $K_2$ | ⟨0.485, 0.449, 0.377⟩ | ⟨0.593, 0.377, 0.344⟩ | ⟨0.469, 0.480, 0.431⟩ | ⟨0.579, 0.396, 0.309⟩ | ⟨0.524, 0.429, 0.372⟩ | ⟨0.512, 0.435, 0.414⟩ |
| $K_3$ | ⟨0.454, 0.471, 0.463⟩ | ⟨0.588, 0.359, 0.353⟩ | ⟨0.551, 0.416, 0.346⟩ | ⟨0.522, 0.441, 0.358⟩ | ⟨0.609, 0.348, 0.317⟩ | ⟨0.607, 0.357, 0.335⟩ |

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