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PCR5 and Neutrosophic Probability in Target Identification (revisited)

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Abstract. In this paper, we use PCR5 in order to fusion the information of two sources providing subjective probabilities of an event A to occur in the following form: chance that A occurs, indeterminate chance of occurrence of A, chance that A does not occur.

Keywords. Target Identification, PCR5, neutrosophic measure, neutrosophic probability, normalized neutrosophic probability.

I. INTRODUCTION

Neutrosophic Probability [1] was defined in 1995 and published in 1998, together with neutrosophic set, neutrosophic logic, and neutrosophic probability.

The words "neutrosophy" and "neutrosophic" were introduced by F. Smarandache in his 1998 book. Etymologically, "neutrosophy" (noun) [French neutre < Latin neuter, neutral, and Greek sophia, skill/wisdom] means knowledge of neutral thought. While "neutrosophic" (adjective), means having the nature of, or having the characteristic of Neutrosophy.

Neutrosophy is a new branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

Zadeh introduced the degree of membership/truth (t) in 1965 and defined the fuzzy set.

Atanassov introduced the degree of nonmembership/ falsehood (f) in 1986 and defined the intuitionistic fuzzy set.

Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set. He has coined the words "neutrosophy" and "neutrosophic". In 2013 he refined/split the neutrosophic set to n components: t_1, t_2, …t_j; i_1, i_2, …, i_k; f_1, f_2, …, f_l, with j+k+l = n > 3. And, as particular cases of refined neutrosophic set, he split the fuzzy set truth into t_1, t_2, …; and the intuitionistic fuzzy set into t_1, t_2, … and f_1, f_2, … .

See: http://fs.gallup.unm.edu/neutrosophy.htm.

For single valued neutrosophic logic, the sum of the components is:

$0 \le t+i+f \le 3$ when all three components are independent;

$0 \le t+i+f \le 2$ when two components are dependent, while the third one is independent from them;

$0 \le t+i+f \le 1$ when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1).

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

II. DEFINITION OF NEUTROSOPHIC MEASURE

A neutrosophic space is a set which has some indeterminacy with respect to its elements.

Let X be a neutrosophic space, and Σ a σ-neutrosophic algebra over X. A *neutrosophic measure* ν is defined by for neutrosophic set $A \in \Sigma$ by

$$\nu : X \to R^3,$$
$$\nu(A) = (m(A), m(neutA), m(antiA)), \quad (1)$$

with *antiA* = the opposite of A, and *neutA* = the neutral (indeterminacy), neither A nor anti A (as defined above); for any $A \subseteq X$ and $A \in \Sigma$,

m(A) means *measure of the determinate part of A;*
m(neutA) means measure of indeterminate part of A;

and *m(antiA)* means *measure of the determinate part of antiA*; where ν is a function that satisfies the following two properties:

a) Null empty set: $\nu(\Phi) = (0,0,0)$.
b) Countable additivity (or σ-additivity): For all countable collections $\{A_n\}_{n\in L}$ of disjoint neutrosophic sets in Σ, one has:

$$\nu\left(\bigcup_{n\in L} A_n\right) = \left(\sum_{n\in L} m(A_n), \sum_{n\in L} m(neutA_n), \sum_{n\in L} m(antiA_n) - (n-1)m(X)\right) \quad (2)$$

where *X* is the whole neutrosophic space, and

$$\sum_{n\in L} m(antiA_n) - (n-1)m(X) = m(X) - \sum_{n\in L} m(A_n) = m(\bigcap_{n\in L} antiA_n). \quad (3)$$

A neutrosophic measure space is a triplet (X, Σ, ν).

III. NORMALIZED NEUTROSOPHIC MEASURE

A neutrosophic measure is called normalized if

$$\nu(X) = (m(X), m(neutX), m(antiX)) = (x_1, x_2, x_3), \quad (4)$$

with $x_1 + x_2 + x_3 = 1$, and $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$, where, of course, *X* is the whole neutrosophic measure space.

As a particular case of neutrosophic measure ν is the neutrosophic probability measure, i.e. a neutrosophic measure that measures probable/possible propositions

$$0 \leq \nu(X) \leq 3,$$

where *X* is the whole neutrosophic probability sample space.

For single valued neutrosophic logic, the sum of the components is:

$0 \leq x_1+x_2+x_3 \leq 3$ when all three components are independent;

$0 \leq x_1+x_2+x_3 \leq 2$ when two components are dependent, while the third one is independent from them;

$0 \leq x_1+x_2+x_3 \leq 1$ when all three components are dependent.

When three or two of the components x_1, x_2, x_3 are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1).

If all three components x_1, x_2, x_3 are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

IV. NORMALIZED PROBABILITY

We consider the case when the sum of the components m(A) + m(neutA) + m(antiA) = 1.

We may denote the normalized neutrosophic probability of an event A as $NP(\mathcal{A}) = (t, i, f)$, where t is the chance that $\mathcal{A}$ occurs, i is indeterminate chance of occurrence of $\mathcal{A}$, and f is the chance that $\mathcal{A}$ does not occur.

V. THE PCR5 FORMULA

Let the frame of discernment $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}, n \geq 2$. Let $G = (\Theta, \cup, \cap, C)$ be the super-power set, which is Θ closed under union, intersection, and respectively complement.

Let's consider two masses provided by 2 sources:

$m_1, m_2 : G \rightarrow [0, 1]$.

The conjunctive rule is defined as

$$m_{12}(X) = \sum_{X_1, X_2 \in G} m_1(X_1) m_2(X_2). \quad (5)$$

Then the Proportional Conflict Redistribution Rule (PCR) #5 formula for 2 sources of information is defined as follows:

$$\forall X \in G \setminus \{\phi\},$$

$$m_{PCR5}(X) = m_{12}(X) + \sum_{Y \in G \setminus \{X\}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_1(X) + m_2(Y)}\right] \quad (6)$$

where all denominators are different from zero.

If a denominator is zero, that fraction is discarded.

VI. APPLICATION IN INFORMATION FUSION

Suppose an airplane $\mathcal{A}$ is detected by the radar. What is the chance that $\mathcal{A}$ is friendly, neutrally, or enemy?

Let's have two sources that provide the following information:

$$NP_1^{(\mathcal{A})}(t_1, i_1, f_1), \text{ and } NP_2^{(\mathcal{A})}(t_2, i_2, f_2).$$

Then:

$$[NP_1 \oplus NP_2](t) = t_1 t_2 + \left(\frac{t_1^2 i_2}{t_1+i_2} + \frac{t_2^2 i_1}{t_2+i_1}\right) + \left(\frac{t_1^2 f_2}{t_1+f_2} + \frac{t_2^2 f_1}{t_2+f_1}\right) \quad (7)$$

Because: $t_1 i_2$ is redistributed back to the truth (t) and indeterminacy proportionally with respect to t_1 and respectively i_2:

$$\frac{x_1}{t_1} = \frac{y_1}{i_2} = \frac{t_1 i_2}{t_1+i_2}, \quad (8)$$

whence $x_1 = \frac{t_1^2 i_2}{t_1+i_2}$, $y_1 = \frac{t_1 i_2^2}{t_1+i_2}$. (9)

Similarly, $t_2 i_1$ is redistributed back to t and i proportionally with respect to t_2 and respectively i_1:

$$\frac{x_2}{t_2} = \frac{y_2}{i_1} = \frac{t_2 i_1}{t_2+i_1}, \quad (10)$$

whence $x_2 = \frac{t_2^2 i_1}{t_2+i_1}$, $y_2 = \frac{t_2 i_1^2}{t_2+i_1}$. (11)

Similarly, $t_1 f_2$ is redistributed back to t and f (falsehood) proportionally with respect to t_1 and respectively f_2:

$$\frac{x_3}{t_1} = \frac{z_1}{f_2} = \frac{t_1 f_2}{t_1+f_2}, \quad (12)$$

whence $x_3 = \frac{t_1^2 f_2}{t_1+f_2}$, $z_1 = \frac{t_1 f_2^2}{t_1+f_2}$. (13)

Again, similarly $t_2 f_1$ is redistributed back to t and f proportionally with respect to t_2 and respectively f_1:

$$\frac{x_4}{t_2} = \frac{z_2}{f_1} = \frac{t_2 f_1}{t_2+f_1}, \quad (14)$$

whence $x_4 = \frac{t_2^2 f_1}{t_2+f_1}$, $z_2 = \frac{t_2 f_1^2}{t_2+f_1}$. (15)

In the same way, $i_1 f_2$ is redistributed back to i and f proportionally with respect to i_1 and respectively f_2:

$$\frac{y_3}{i_1} = \frac{z_3}{f_2} = \frac{i_1 f_2}{i_1+f_2}, \quad (16)$$

whence $y_3 = \frac{i_1^2 f_2}{i_1+f_2}$, $z_3 = \frac{i_1 f_2^2}{i_1+f_2}$. (17)

While $i_2 f_1$ is redistributed back to i and t proportionally with respect to i_2 and respectively f_1:

$$\frac{y_4}{i_2} = \frac{z_4}{f_1} = \frac{i_2 f_1}{i_2+f_1}, \quad (18)$$

whence $y_4 = \frac{i_2^2 f_1}{i_2+f_1}$, $z_4 = \frac{i_2 f_1^2}{i_2+f_1}$. (19)

Then

$$[NP_1 \oplus NP_2](i) = i_1 i_2 + \left(\frac{i_1^2 t_2}{i_{11}+t_2} + \frac{i_2^2 t_1}{i_2+t_1}\right) + \left(\frac{i_1^2 f_2}{i_1+f_2} + \frac{i_2^2 f_1}{i_2+f_1}\right), \quad (20)$$

and

$$[NP_1 \oplus NP_2](f) = f_1 f_2 + \left(\frac{f_1^2 t_2}{f_1+t_2} + \frac{f_2^2 t_1}{f_2+t_1}\right) + \left(\frac{f_1^2 i_2}{f_1+i_2} + \frac{f_2^2 i_1}{f_2+i_1}\right). \quad (21)$$

VII. EXAMPLE

Let's compute: $(0.6, 0.1, 0.3) \wedge_N (0.2, 0.3, 0.5)$.

$t_1 = 0.6, i_1 = 0.1, f_1 = 0.3$, and

$t_2 = 0.2, i_2 = 0.3, f_2 = 0.5$,

are replaced into the three previous neutrosophic logic formulas:

- *(using PCR5 rule)*

$$[NP_1 \oplus nm_2](t) = 0.6(0.2) + \left(\frac{0.6^2(0.3)}{0.6+0.3} + \frac{0.2^2(0.1)}{0.2+0.1}\right) + \left(\frac{0.6^2(0.5)}{0.6+0.5} + \frac{0.2^2(0.3)}{0.2+0.3}\right) \simeq 0.44097$$

.

$$[NP_1 \oplus NP_2](i) = 0.1(0.3) + \left(\frac{0.1^2(0.2)}{0.1+0.2} + \frac{0.3^2(0.6)}{0.3+0.6}\right) + \left(\frac{0.1^2(0.5)}{0.1+0.5} + \frac{0.3^2(0.3)}{0.3+0.3}\right) \simeq 0.15000$$

.

$$[NP_1 \oplus NP_2](f) = 0.3(0.5) + \left(\frac{0.3^2(0.2)}{0.3+0.2} + \frac{0.5^2(0.6)}{0.5+0.6}\right) + \left(\frac{0.3^2(0.3)}{0.3+0.3} + \frac{0.5^2(0.1)}{0.5+0.1}\right) \simeq 0.40903$$

.

- *(using Dempster's rule)*

Conj. rule:

0.12	0.03	0.15

Dempster's rule:

0.40	0.10	0.50

This is actually a PCR5 formula for a frame of discernment $\Omega = \{\theta_1, \theta_2, \theta_3\}$ whose all intersections are empty.

We can design a PCR6 formula too for the same frame.

Another method will be to use the neutrosophic $N - norm$, which is a generalization of fuzzy $T - norm$.

If we have two neutrosophic probabilities

	Friend	Neutral	Enemy
NP_1	t_1	i_1	f_1
NP_2	t_2	i_2	f_2

then

$$NP_1 \oplus NP_2 = (t_1 + i_1 + f_1) \cdot (t_2 + i_2 + f_2) =$$

$$t_1 t_2 + t_1 i_2 + t_2 i_1 + i_1 i_2 + t_1 f_1 + t_1 f_2 + t_2 f_1 + i_1 f_2 + i_2 f_1 + f_1 f_2$$

Of course, the quantity of $t_1 t_2$ will go to Friend, quantity of $i_1 i_2$ will go to Neutral, and quantity of $f_1 f_2$ will go to Enemy.

The other quantities will go depending on the pessimistic or optimistic way:

a) In the pessimistic way (lower bound) $t_1 i_2 + t_2 i_1$ will go to Neutral, and $t_1 f_2 + t_2 f_1 + i_1 f_2 + i_2 f_1$ to Enemy.

b) In the optimistic way (upper bound) $t_1 i_2 + t_2 i_1$ will go to Friend, and $t_1 f_2 + t_2 f_1 + i_1 f_2 + i_2 f_1$ to Neutral. About $t_1 f_2 + t_2 f_1$, we can split it half-half to Friend and respectively Enemy.

We afterwards put together the pessimistic and optimistic ways as an interval neutrosophic probability.

c) Of course, the reader or expert can use different transfers of intermediate mixed quantities $t_1 i_2 + t_2 i_1$, and respectively $t_1 f_2 + t_2 f_1 + i_1 f_2 + i_2 f_1$ to Friend, Neutral, and Enemy.

CONCLUSION

We have introduced the application of neutrosophic probability into information fusion, using the combination of information provided by two sources using the PCR5.

Other approaches can be done, for example the combination of the information using the N-norm and N-conorm, which are generalizations of the T-norm and T-conorm from the fuzzy theory to the neutrosophic theory.

More research is needed in this direction.

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