Initialization

Code

```
Quit
```

```
Print[$MachineName ]
Print[$Version]
```

```
pcl0517
```

```
10.0 for Linux x86 (64-bit) (December  4, 2014)
```

```
<<RBSFA`
Print[$RBSFAversion]
Print[$RBSFAcommit  ]
```

```
RB-SFA v2.1.3, Tue 28 Feb 2017 11:48:14
```

```
commit   3bf93b827ba584ae1cc1b7f265f38a5f0a32ea04
Author: Emilio  Pisanty <emilio .pisanty@icfo.eu>
Date:   Tue Feb 28 11:49:15 2017 +0100
    Improved  testing for previous
     versions of ReIm  to avoid error messages  on package reload.
```

```
$HistoryLength= 10;
LaunchKernels[8-$KernelCount];
ParallelEvaluate[{$MachineName , $RBSFAversion}] // Tally
```

```
{{{pcl0517, RB-SFA v2.1.3, Tue 28 Feb 2017 11:48:14}, 8}}
```

Formatting niceties

```
<<MaTeX`
SetOptions[MaTeX, "Preamble " → {"
\\usepackage{amssymb  ,upref}
\\usepackage{fourier}
\\usepackage{tgheros}
\\usepackage[T1]{fontenc}
\\usepackage{textcomp }
\\usepackage{microtype }
"}];
```

Formatting: label, tick and inset font sizes

```
lfs = 7;
tfs = 6;
ifs= 9;
```

```
$OutputDirectory= FileNameJoin [{NotebookDirectory[], "..", "Figures"}];
$MainDirectory=
    StringReplace[FileNameJoin [{NotebookDirectory[], ".."}], {" " → "\\ "}];
pdflatex[] := Run["cd "<>$MainDirectory<>
        " && pdflatex --output-directory=build Manuscript.tex"] /.
    {0 → "pdflatex successful"}
```

Definitions

Fields

```
bicircularA[t_] = F/ω {Cos[ω t], Sin[ω t], 0} + F/(2 ω) {Cos[2 ω t], -Sin[2 ω t], 0};
bicircularF[t_] = F {Sin[ω t], -Cos[ω t], 0} + F {Sin[2 ω t], Cos[2 ω t], 0};

bicircularArot[t_] = F/ω {3/2 Cos[3/2 ω t], 1/2 Sin[3/2 ω t], 0};
bicircularFrot[t_] = 2 F {Sin[3/2 ω t], 0, 0};

(* {{Cos[α t], -Sin[α t], 0}, {Sin[α t], Cos[α t], 0}, {0, 0, 1}} . ({{Cos[ω t]}, {Sin[ω t]}, {0}} + 1/2 {{Cos[2ω t]}, {-Sin[2ω t]}, {0}}) /. {α → ω/2} //FullSimplify
   {{Cos[α t], -Sin[α t], 0}, {Sin[α t], Cos[α t], 0}, {0, 0, 1}} . ({{Sin[ω t]}, {-Cos[ω t]}, {0}} + {{Sin[2ω t]}, {Cos[2ω t]}, {0}}) /. {α → ω/2} //FullSimplify *)

Rzα[t_] = {{Cos[ω/2 t], Sin[ω/2 t], 0}, {-Sin[ω/2 t], Cos[ω/2 t], 0}, {0, 0, 1}};

pi[{px_, py_, pz_}, t_, tt_] := {px, py, pz} + bicircularArot[t]
pi[{px_, py_, pz_}, t_] := pi[{px, py, pz}, t, 0]
```

Building the action

Saddle-point momentum

```
ps[t_, tt_] = -1/(t - tt) FullSimplify [Rzα[-tt].Integrate[bicircularA[τ], {τ, tt, t}]]
```

$$\left\{-\frac{1}{4\,(t-tt)\,\omega^2}F\left(-5\,\text{Sin}\left[\frac{3\,tt\,\omega}{2}\right]+4\,\text{Sin}\left[\frac{1}{2}(2\,t+tt)\,\omega\right]+\text{Sin}\left[2\,t\,\omega-\frac{tt\,\omega}{2}\right]\right),\right.$$
$$\left.-\frac{1}{4\,(t-tt)\,\omega^2}F\left(3\,\text{Cos}\left[\frac{3\,tt\,\omega}{2}\right]-4\,\text{Cos}\left[\frac{1}{2}(2\,t+tt)\,\omega\right]+\text{Cos}\left[2\,t\,\omega-\frac{tt\,\omega}{2}\right]\right),\ 0\right\}$$

Action

```
S[t_, tt_] = (Ip - m ω/2) (t - tt) + ((# /. {τ → t}) - (# /. {τ → tt})) &@Integrate[
    Total[1/2 (Rzα[tt - τ].ps[t, tt] + bicircularArot[τ])^2]
    , τ]
```

$$(t-tt)\left(Ip-\frac{m\ \omega}{2}\right)+$$
$$\frac{1}{32(t-tt)^2\omega^4}F^2\Big(34\,tt+20\,t^3\omega^2-40\,t^2\,tt\,\omega^2+20\,t\,tt^2\,\omega^2+8\,t\,\text{Cos}[3\,t\,\omega]-16\,tt\,\text{Cos}[3\,t\,\omega]-$$
$$32\,t\,\text{Cos}[(t-tt)\,\omega]-2\,t\,\text{Cos}[2\,(t-tt)\,\omega]-8\,t\,\text{Cos}[3\,tt\,\omega]+32\,t\,\text{Cos}[(-t+tt)\,\omega]-$$
$$32\,tt\,\text{Cos}[(-t+tt)\,\omega]+2\,t\,\text{Cos}[2\,(-t+tt)\,\omega]-2\,tt\,\text{Cos}[2\,(-t+tt)\,\omega]+8\,tt\,\text{Cos}[(2\,t+tt)\,\omega]+$$
$$8\,tt\,\text{Cos}[(t+2\,tt)\,\omega]+\frac{16}{3}t^2\,\omega\,\text{Sin}[3\,t\,\omega]-\frac{32}{3}t\,tt\,\omega\,\text{Sin}[3\,t\,\omega]+\frac{16}{3}tt^2\,\omega\,\text{Sin}[3\,t\,\omega]\Big)-$$
$$\frac{1}{32(t-tt)^2\omega^4}F^2\Big(34\,t+20\,t^2\,tt\,\omega^2-40\,t\,tt^2\,\omega^2+20\,tt^3\,\omega^2-8\,tt\,\text{Cos}[3\,t\,\omega]-32\,t\,\text{Cos}[(t-tt)\,\omega]-$$
$$2\,t\,\text{Cos}[2\,(t-tt)\,\omega]-16\,t\,\text{Cos}[3\,tt\,\omega]+8\,tt\,\text{Cos}[3\,tt\,\omega]+8\,t\,\text{Cos}[(2\,t+tt)\,\omega]+$$
$$8\,t\,\text{Cos}[(t+2\,tt)\,\omega]+\frac{16}{3}t^2\,\omega\,\text{Sin}[3\,tt\,\omega]-\frac{32}{3}t\,tt\,\omega\,\text{Sin}[3\,tt\,\omega]+\frac{16}{3}tt^2\,\omega\,\text{Sin}[3\,tt\,\omega]\Big)$$

Calculation of the saddle points

Parameters

```
parameters = {getIonizationPotential["Neon", 0], √1.88 0.053, 0.057}; (*{Ip,F,ω}*)
```

Neon, $1.88\times10^{14}\ \text{W}/\text{cm}^2$, 912 nm

Getting the saddle points

Calculation

```
Block[{Ip, F, ω, γ},
  {Ip, F, ω} = parameters ; γ = Sqrt[2 Ip] ω / F;
  ΩRange = Range[7 ω, 75 ω, 1/40. ω];

  AbsoluteTiming [
    saddlePoints = Association[Table[
          m → GetSaddlePoints[
              ΩRange, S, Table[
                {{(0 - i 2 γ)/ω, (3 π + i 2 γ)/ω}, {55. ° + 0.35 i)/ω + 120 °k/ω, 100 ° + 1.1 i)/ω + 120 °k/ω}}
                , {k, 0, 0}]
              , IndependentVariables → {"τ", "tt"}
              , Tolerance → 10^-5/ω, Seeds → 150
              , Jacobian → FiniteDifference
            ]
          , {m , -1, 1}]]
  ]
]
```

```
{670.838335,
  ⟨|-1 → ⟨|0.399 → {{29.0471 + 3.10788 i, 5.43574 - 11.594 i}, {46.8644 - 11.4101 i,
                21.0255 - 24.3058 i}, {88.9384 + 7.95711 i, 67.6621 - 1.26354 i},
              {89.0879 - ··· 1 ···, ··· 1 ···}, ··· 1 ···, {128.907 - 6.81418 i,
                110.577 - 15.2968 i}, {163.553 - 6.39723 i, 143.516 - 14.6705 i},
              {163.629 + 6.58589 i, 143.629 - 1.83045 i}},
        ··· 2720 ···|>, 0 → ··· 1 ···, 1 → ⟨| ··· 1 ···|>|>}
```

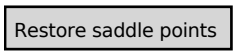
large output | show less | show more | show all | set size limit ...

```
ListPlot[Values@Map[Length, saddlePoints, {2}], PlotStyle → {Red, Blue, Green}]
```

Data handling

In-notebook save:

```
With[{data = Compress [saddlePoints]},
  Button["Restore saddle points", Set[saddlePoints, Uncompress [data]];
    saddlePoints;]
]
```



External export:

```
Save[NotebookDirectory[] <>
    "data - single-burst saddle points on the rotating frame .txt", saddlePoints]
```

Import from external export:

```
<< (NotebookDirectory[] <>
      "data - single-burst saddle points on the rotating frame .txt")
```

Check the import worked correctly:

```
saddlePoints // Dimensions
```

```
{3, 2721}
```

Initial map of the saddle points

```
Block[{Ip, F, ω, γ, saddles},
  {Ip, F, ω} = parameters ;
  γ = √(2 Ip ω) / F;
  Row[Table[
    saddles = saddlePoints[m ];

    Column [Table[
      Show[
        Graphics[
          ParallelTable[
            Map[
              Apply[Function[{t, τ},
                  Tooltip[Point[ReIm [ω (time /. {"tt" → t - τ, "t" → t, "τ" → τ})]],
                    {Ω/ω, ω{t - τ, t, τ}, Floor[ωRe[t - τ], 2π/3] / (2π/3)}]
                ]], saddles[Ω][[All]]]
            , {Ω, Keys[saddles][[1 ;; ;; 10]]}]
          ]
          , Frame → True, Axes → True
          , ImageSize → 500
          , FrameLabel → {"Re(ω" <> time <> ")", "Im (ω" <> time <> ")"}
          , Method → {"AxesInFront" → False}
        ]
        , {time , {"tt", "t", "τ"}}]]
    , {m , -1, 1}]]
]
```

```
Block[{Ip, F, ω, γ, saddles},
  {Ip, F, ω} = parameters ;
  γ = √(2 Ip ω) / F;
  Show[Table[
    saddles = saddlePoints[m ];

    Column [Table[
        Show[
          Graphics[
            ParallelTable[
              Map[
                Apply[Function[{t, τ},
                    Tooltip[Point[ReIm [ω (time /. {"tt" → t-τ, "t" → t, "τ" → τ})]],
                      {Ω/ω, ω {t-τ, t, τ}, Floor[ωRe[t-τ], 2π/3] / (2π/3)}]
                  ]], saddles[Ω][[All]]]
              , {Ω, Keys[saddles][[1 ;; ;; 10]]}]
            ]
            , Frame → True, Axes → True
            , ImageSize → 750
            , FrameLabel → {"Re(ω"<>time <>")", "Im (ω"<>time <>")"}
          ]
          , {time , {"tt"(*,"t","τ"*)}}]][[1, 1]]
      , {m , -1, 1}]]
]
```

Classifying the saddle points

```
Block[{Ip, F, ω, γ, saddles, classifierFunction, sortingFunction, keyColour},
  {Ip, F, ω} = parameters ;
  γ = √(2 Ip ω) / F;

  classifierFunction = Function[{t, τ, Ω}, Which @@ Flatten[{Table[{
      {And[Re[ωτ] < 0.9, Floor[ωRe[t-τ], 2π/3] / (2π/3) == k-1], "N"<>ToString[k]},
      {And[0.9 < Re[ωτ], Re[e^(-iπ/4) (ωt-5)] < 0,
          Floor[ωRe[t-τ], 2π/3] / (2π/3) == k-1], "A"<>ToString[k]},
      {And[Re[e^(-iπ/4) (ωt-5)] > 0, Re[e^(+iπ/4) (ωt-7.4)] < 0,
```

```
                    Floor[ω Re[t - τ], 2 π / 3] / (2 π / 3) == k - 1], "B" <> ToString[k]},
                {And[Re[e^(+i π/4) (ω t - 7.4)] > 0, Re[e^(-i π/4) (ω t - 9.3)] < 0,
                    Floor[ω Re[t - τ], 2 π / 3] / (2 π / 3) == k - 1], "C" <> ToString[k]},
                {And[Re[e^(-i π/4) (ω t - 9.3)] > 0, Floor[ω Re[t - τ], 2 π / 3] / (2 π / 3) == k - 1],
                  "D" <> ToString[k]}
              }, {k, 1, 1}], {True, "Discard"}}]];
sortingFunction =
  Function[list, SortBy[list, Function[Re[ω #[[1]] - Floor[ω Re[#[[1]] - #[[2]]], 2 π]]]]];
keyColour = <|"A1" → <|1 → Black, 2 → Blue|>, "B1" → <|1 → Darker[Green], 2 → Orange|>,
    "N1" → <|1 → Lighter[Purple, 0.1]|>, "C1" → <|1 → Darker[Red], 2 → Magenta|>,
    "D1" → <|1 → Darker[Cyan], 2 → Pink|>|>;

selection = Association[ParallelTable[
      m → KeySort[
          ClassifyQuantumOrbits[saddlePoints[m][[1 ;; ;; 10]], classifierFunction,
            sortingFunction, DiscardedLabels → {(*"Discard"*)}]]
      , {m, -1, 1}]];

Table[
  Column[Join[
      Table[
        Show[
          Graphics[
            Table[Table[{
                  KeyValueMap[
                    Function[{n, t, τ},
                      {keyColour[index, n] /. Missing[__] → Gray,
                        Tooltip[Point[
                          ReIm[ω time /. {"tt" → t - τ, "t" → t,
                            "τ" → τ + 0.1 i 1/(π ω) Floor[ω Re[τ - t], π]}]
                          ], {Ω / ω, index, n, ω {t - τ, t, τ},
                          1/π Floor[ω Re[t - τ], π]}]}
                    ]@*Apply[Sequence]@*Flatten@*List
                  , selection[m, index, Ω]]
                }, {Ω, Keys[selection[m, index]]}],
              {index, Keys[selection[m]]}]
          ]
          , Frame → True, Axes → True
```

```
            , ImageSize → {{550}, {550}}
            , FrameLabel → {"Re(ω"<>time <>")", "Im (ω"<>time <>")"}
            , Method → {"AxesInFront" → False}
          ]
          , {time , {"tt", "t", "τ"}}]]]
    , {m , -1, 1}]
]

Block[{Ip, F, ω, γ, saddles, classifierFunction, sortingFunction, keyColour},
  {Ip, F, ω} = parameters ;
  γ = √(2 Ip) ω / F;

  classifierFunction = Function[{t, τ, Ω}, Which @@ Flatten[{Table[{
                {And[Re[ω τ] < 0.9, Floor[ω Re[t - τ], 2π/3] / (2π/3) == k - 1], "N" <> ToString[k]},
                {And[0.9 < Re[ω τ], Re[e^(-i π/4) (ω t - 5)] < 0,
                     Floor[ω Re[t - τ], 2π/3] / (2π/3) == k - 1], "A" <> ToString[k]},
                {And[Re[e^(-i π/4) (ω t - 5)] > 0, Re[e^(+i π/4) (ω t - 7.4)] < 0,
                     Floor[ω Re[t - τ], 2π/3] / (2π/3) == k - 1], "B" <> ToString[k]},
                {And[Re[e^(+i π/4) (ω t - 7.4)] > 0, Re[e^(-i π/4) (ω t - 9.3)] < 0,
                     Floor[ω Re[t - τ], 2π/3] / (2π/3) == k - 1], "C" <> ToString[k]},
                {And[Re[e^(-i π/4) (ω t - 9.3)] > 0, Floor[ω Re[t - τ], 2π/3] / (2π/3) == k - 1],
                  "D" <> ToString[k]}
              }, {k, 1, 1}], {True, "Discard"}}]];
  sortingFunction =
    Function[list, SortBy[list, Function[Re[ω #[[1]] - Floor[ω Re[#[[1]] - #[[2]], 2π]]]]];
  keyColour = <|"A1" → <|1 → Black, 2 → Blue|>, "B1" → <|1 → Darker[Green], 2 → Orange|>,
      "N1" → <|1 → Lighter[Purple, 0.1]|>, "C1" → <|1 → Darker[Red], 2 → Magenta|>,
      "D1" → <|1 → Darker[Cyan], 2 → Pink|>|>;

  selection = Association[ParallelTable[
        m → KeySort[
            ClassifyQuantumOrbits [saddlePoints[m ][[1 ;; ;; 10]], classifierFunction,
              sortingFunction, DiscardedLabels → {(*"Discard"*)}]]
        , {m , -1, 1}]];

  Column [Table[
      Show[Table[
          Show[
            Graphics[
```

```
Table[Table[{
      KeyValueMap[
        Function[{n, t, τ},
          {keyColour[index, n] /. Missing[__] → Gray,
            Tooltip[Point[
              ReIm [ωtime /. {"tt" → t-τ, "t" → t,
                "τ" → τ+ 0.1 i 1/(πω) Floor[ωRe[τ-t], π]}]
            ], {m , Ω/ω, index, n, ω{t-τ, t, τ},
              1/π Floor[ωRe[t-τ], π]}]}
        ]@*Apply[Sequence]@*Flatten@*List
      , selection[m , index, Ω]]
    }, {Ω, Keys[selection[m , index]]}],
  {index, Keys[selection[m ]]}]
]
, Frame → True, Axes → True
, ImageSize → {{800}, {800}}
, FrameLabel → {"Re(ω"<>time <>")", "Im (ω"<>time <>")"}
, Method → {"AxesInFront" → False}
]
, {m , -1, 1}]]
, {time , {"tt", "t", "τ"}}]]
]
```

Figure

Making the data

Calculation

```
Block[{Ip, F, ω, γ, saddles, classifierFunction, sortingFunction, keyColour},
  {Ip, F, ω} = parameters ;
  γ = √(2 Ip) ω / F;

  classifierFunction = Function[{t, τ, Ω}, Which@@Flatten[{Table[{
      {And[Re[ωτ] < 0.9, Floor[ωRe[t-τ], 2π/3] / (2π/3) == k-1], "N"<>ToString[k]},
      {And[0.9 < Re[ωτ], Re[e^(-i π/4) (ωt-5)] < 0,
```

```
                    Floor[ω Re[t-τ], 2π/3]/(2π/3) == k-1], "A"<>ToString[k]},
                {And[Re[e^(-iπ/4) (ωt-5)] > 0, Re[e^(+iπ/4) (ωt-7.4)] < 0,
                    Floor[ω Re[t-τ], 2π/3]/(2π/3) == k-1], "B"<>ToString[k]},
                {And[Re[e^(+iπ/4) (ωt-7.4)] > 0, Re[e^(-iπ/4) (ωt-9.3)] < 0,
                    Floor[ω Re[t-τ], 2π/3]/(2π/3) == k-1], "C"<>ToString[k]}(*,
                {And[Re[e^(-i π/4)(ω t-9.3)]>0, Floor[ω Re[t-τ],2π/3]/(2π/3)==k-1],
                  "D"<>ToString[k]}*)
              }, {k, 1, 1}], {True, "Discard"}}]];
sortingFunction=
  Function[list, SortBy[list, Function[Re[ω#[[1]] - Floor[ωRe[#[[1]] - #[[2]], 2π]]]]];

keyColour = <|"A1" → <|1 → Black, 2 → Blue|>,
    "B1" → <|1 → Darker[Green, 0.2], 2 → Darker[Red, 0.05]|>,
    "C1" → <|1 → Magenta, 2 → Darker[Cyan, 0.1]|>,
    "N1" → <|1 → GrayLevel[0.6]|>(*,"D1"→<|1→Darker[Cyan],2→Pink|>*)|>;

selection= Association[ParallelTable[
      m → KeySort[ClassifyQuantumOrbits [saddlePoints[m ][[1 ;; ;; 1]],
            classifierFunction, sortingFunction, DiscardedLabels→ {"Discard"}]]
      , {m , -1, 1}]];
Print[AbsoluteTiming [
    data = Association[Table[
            m → Association[ParallelTable[
                  index→ Association[Table[
                        n→ Association[Table[
                              Ω→ Function[{t, τ},
                                    ⟨|"HO" → Ω/ω, "τ" → τ,
                                    "int" → Log10[Abs[1/τ^(3/2) e^(-iS[t,t-τ]+iΩt)]^2]|⟩
                              ]@@AssociationTranspose[
                                selection[m , index]][n, Ω]
                              , {Ω, Keys[selection[m , index]][[1 ;; ;; 10]]}]]
                        , {n, 1, Length[Keys[
                              AssociationTranspose[selection[m , index]]]]}]]
                  , {index, Keys[selection[m ]][[1 ;; -1]]}]]
            , {m , -1, 1}]];
    ]];
]
```

```
{335.164672, Null}
```

Data handling

In-notebook save:

```
With[{dataa = Compress [data]},
  Button["Restore data", Set[data, Uncompress [dataa]]; data;]
]
With[{selectionn= Compress [selection]},
  Button["Restore selection", Set[selection, Uncompress [selectionn]];
    selection;]
]
```

Restore data

Restore selection

External export:

```
Save[NotebookDirectory[] <>
    "data - single-burst saddle points on the rotating frame .txt", saddlePoints]
Save[NotebookDirectory[] <>
    "data - single-burst saddle-point selection on the rotating frame .txt",
  saddlePoints]
```

Import from external export:

```
<< (NotebookDirectory[] <>
      "data - single-burst saddle points on the rotating frame .txt")
<< (NotebookDirectory[] <>
      "data - single-burst saddle-point selection on the rotating frame .txt")
```

Check the import worked correctly:

```
data // Dimensions
selection// Dimensions
```

```
{3, 4}
```

```
{3, 4, 2721}
```

Figure

```
Block[{Ip, F, ω, γ, saddles, classifierFunction, sortingFunction, keyColour},
  {Ip, F, ω} = parameters ;
  γ = √(2 Ip ω) / F;

  classifierFunction = Function[{t, τ, Ω}, Which @@ Flatten[{Table[{
                {And[Re[ω τ] < 1, Floor[ω Re[t - τ], 2 π/3] / (2 π/3) == k - 1], "N" <> ToString[k]},
                {And[1 < Re[ω τ], Re[e^(-i π/4) (ω t - 5)] < 0,
                    Floor[ω Re[t - τ], 2 π/3] / (2 π/3) == k - 1], "A" <> ToString[k]},
```

```
                {And[Re[e^(-i π/4) (ω t-5)] > 0, Re[e^(+i π/4) (ω t-7.4)] < 0,
                    Floor[ω Re[t-τ], 2π/3]/(2π/3) == k-1], "B"<>ToString[k]},
                {And[Re[e^(+i π/4) (ω t-7.4)] > 0, Re[e^(-i π/4) (ω t-9.3)] < 0,
                    Floor[ω Re[t-τ], 2π/3]/(2π/3) == k-1], "C"<>ToString[k]}(*,
                {And[Re[e^(-i π/4)(ω t-9.3)]>0, Floor[ω Re[t-τ],2π/3]/(2π/3)==k-1],
                  "D"<>ToString[k]}*)
              }, {k, 1, 1}], {True, "Discard"}}]];
sortingFunction=
  Function[list, SortBy[list, Function[Re[ω#[[1]]-Floor[ωRe[#[[1]]-#[[2]], 2π]]]]];

keyColour = <|"A1" → <|1 → Black, 2 → Blue|>,
    "B1" → <|1 → Darker[Green, 0.2], 2 → Darker[Red, 0.05]|>,
    "C1" → <|1 → Magenta, 2 → Darker[Cyan, 0.1]|>,
    "N1" → <|1 → GrayLevel[0.6]|>(*,"D1"→<|1→Darker[Cyan],2→Pink|>*)|>;

(*selection=Association[ParallelTable[
      m →KeySort[ClassifyQuantumOrbits [saddlePoints[m ][[1;;;;1]],
            classifierFunction,sortingFunction,DiscardedLabels→{"Discard"}]]
      ,{m ,-1,1}]];
Print[AbsoluteTiming [
    data=Association[Table[
            m →Association[ParallelTable[
                  index→Association[Table[
                        n→Association[Table[
                              Ω→Function[{t,τ},
                                    ⟨|"HO"→Ω/ω,"τ"→τ,
                                    "int"→Log10[Abs[1/τ^(3/2) e^(-i S[t,t-τ]+i Ω t)]^2]|⟩
                                ]@@AssociationTranspose[
                                  selection[m ,index]][n,Ω]
                              ,{Ω,Keys[selection[m ,index]][[1;;;;10]]}]]
                        ,{n,1,Length[Keys[
                              AssociationTranspose[selection[m ,index]]]]}]]
                  ,{index,Keys[selection[m ]][[1;;-1]]}]]
            ,{m ,-1,1}]];
  ]];*)

Column [{
    figureCa= Show[{
          Graphics[
            Table[
              Table[
```

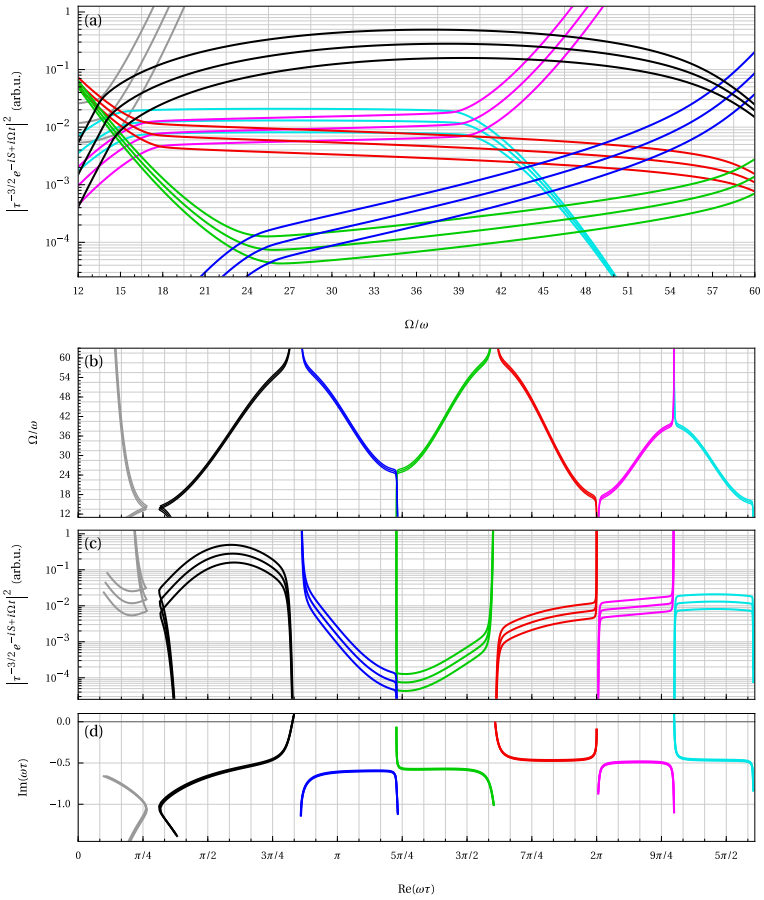
```
                Table[
                  If[index== "N1" &&n == 2, ##&[], {
                      keyColour[index, n] /. Missing[__] → Gray,
                      Thickness[0.003],
                      Line[
                        Table[
                          Values[data[m , index, n, Ω]][[{"HO", "int"}]]
                          , {Ω, Keys[data[m , index, n]][[1 ;; ;; 1]]}]
                      ]
                    }]
                  , {m , -1, 1}]
                , {n, {2, 1}}]
            , {index, Reverse@Keys[selection[1]][[1 ;; -1]]}]
        ],
        Graphics[{White, {Rectangle[{11.5, -13.6}, {12, -8.9}],
              Rectangle[{60, -13.6}, {60.5, -8.9}]}, {Rectangle[{12, -13.7},
                {60, -13.6}], Rectangle[{12, -8.9}, {60, -8.5}]}}]
      }
      , PlotRange → {{12, 60}, {-13.6, -8.9}(*{-15,-10.55}*)}
      , Frame → True
      , AspectRatio→ 1 / 2.5
      , ImageSize → 400
      , FrameLabel → {MaTeX["\\Omega /\\omega ", FontSize→ lfs],
          MaTeX["\\left|\\tau^{-3/2}e^{-iS+i\\Omega  t}\\right|^2\\text{
                (arb.u.)}", FontSize→ lfs]}
      , PlotRangeClipping→ True
      , Axes → False
      , FrameTicks → {{Join[
              {#, MaTeX["10^{" <> ToString[#+9] <> "}" /. {"10^{0}" → "1"}, FontSize→
                    {0.01, 0}} &/@Range[-15, -9],
              {#, "", {0.005, 0}} &/@Flatten[Outer[Plus,
                    Log10[Range[2., 9.]], Range[-15, -10]]]
            ], None}, {Join[
              {#, MaTeX[ToString[#], FontSize→ tfs], {0.0075, 0}} &/@
                Range[0, 60, 3],
              {#, "", {0.00375, 0}} &/@Range[1, 60, 1]
            ], None}}
      , GridLines→ {Range[3/2, 60, 3], Join[
            Range[-15, -10],
            Flatten[Outer[Plus, Log10[Range[2., 9.]], Range[-15, -10]]]
          ]}
      , GridLinesStyle→ Directive[GrayLevel[0.8]]
      , ImagePadding →
        {{Scaled[0.095], Scaled[0.007]}, {Scaled[0.065], Scaled[0.002]}}
      , ImagePadding → {{Scaled[0.035], Scaled[0.001]},
          {Scaled[0.025], Scaled[0.001]}}
      , Epilog→ {Inset[MaTeX["\\text{(a)}", FontSize→ ifs],
            Scaled[{0.005, 0.99}], Scaled[{0, 1}]]}
    ],
  figureCb= Show[{
```

```
      Graphics[
        Table[
          Table[
            Table[
              If[index == "N1" && n == 2, ## &[], {
                  keyColour[index, n] /. Missing[__] → Gray,
                  Thickness[0.002],
                  Line[
                    Table[
                      {Re[ω data[m , index, n, Ω][["τ"]]],
                        data[m , index, n, Ω][["HO"]]}
                      , {Ω, Keys[data[m , index, n]][[1 ;; ;; 1]]}]
                  ]
                }]
              , {m , -1, 1}]
            , {n, {2, 1}}]
          , {index, Reverse@Keys[selection[1]][[1 ;; -1]]}]
        ],
        Graphics[
          {White, {Rectangle[{0, 10}, {8.2, 11}], Rectangle[{0, 63}, {8.2, 64}]}}]
      }
      , PlotRange → {{0, 8.2}, {11, 63}}
      , Frame → True
      , AspectRatio → 1 / 4
      (*, ImageSize → 950*)
      , ImageSize → 400
      , FrameLabel → {"", MaTeX["\\Omega /\\omega ", FontSize → lfs]}
      , PlotRangeClipping → True
      , Axes → False
      , FrameTicks → {{Join[
              {#, MaTeX[ToString[#], FontSize → tfs], {0.0075, 0}} & /@ Range[0, 70, 6]
              {#, "", {0.00375, 0}} & /@ Range[1, 70, 1]
            ], None}, {Join[
              {#π, "", {0.008, 0}} & /@ Range[0, 3, 1 / 4],
              {#π, "", {0.004, 0}} & /@ Range[0, 3, 1 / 12]
            ], None}}
      , GridLines → {πRange[0, 3, 1 / 12], Range[3/2, 70, 3]}
      , GridLinesStyle → Directive[GrayLevel[0.8]]
      , ImagePadding →
        {{Scaled[0.095], Scaled[0.007]}, {Scaled[0.0005], Scaled[0.0005]}}
      , ImagePadding → {{Scaled[0.035], Scaled[0.001]},
          {Scaled[0.001], Scaled[0.001]}}
      , Epilog → {Inset[MaTeX["\\text{(b)}", FontSize → ifs],
            Scaled[{0.005, 0.99}], Scaled[{0, 1}]]}
    ],
  figureCc = Show[{
        Graphics[
          Table[
            Table[
              Table[
                If[index == "N1" && n == 2, ## &[], {
```

```
                    keyColour[index, n] /. Missing[__] → Gray,
                    Thickness[0.003],
                    Line[
                      Table[
                        {Re[ωdata[m , index, n, Ω][["τ"]]],
                          data[m , index, n, Ω][["int"]]}
                        , {Ω, Keys[data[m , index, n]][[1 ;; ;; 1]]}]
                    ]
                  }]
                , {m , -1, 1}]
              , {n, {2, 1}}]
            , {index, Reverse@Keys[selection[1]][[1 ;; -1]]}]
          ],
          Graphics[{White, {Rectangle[{0, -14}, {8.2, -13.6}],
                Rectangle[{0, -8.9}, {8.2, -8.8}]}}]
        }
        , PlotRange → {{0, 8.2}, {-13.6, -8.9}}
        , Frame → True
        , AspectRatio→ 1 / 4
        , ImageSize → 400
        , FrameLabel →
          {"", MaTeX["\\left|\\tau^{-3/2}e^{-iS+i\\Omega  t}\\right|^2\\text{
                  (arb.u.)}", FontSize→ 1fs]}
        , Method → {"AxesInFront" → False}
        , PlotRangeClipping→ True
        , Axes → None
        , FrameTicks → {{Join[
                {#, MaTeX["10^{"<>ToString[#+9]<>"}" /. {"10^{0}" → "1"}, FontSize→
                      {0.01, 0}} & /@Range[-15, -9],
                {#, "", {0.005, 0}} & /@Flatten[Outer[Plus,
                      Log10[Range[2., 9.]], Range[-15, -10]]]
              ], None}, {Join[
                {#π, "", {0.008, 0}} & /@Range[0, 3, 1 / 4],
                {#π, "", {0.004, 0}} & /@Range[0, 3, 1 / 12]
              ], None}}
        , GridLines→ {πRange[0, 3, 1 / 12], Join[
              Range[-15, -10],
              Flatten[Outer[Plus, Log10[Range[2., 9.]], Range[-15, -10]]]
            ]}
        , GridLinesStyle→ Directive[GrayLevel[0.8]]
        , ImagePadding →
          {{Scaled[0.095], Scaled[0.007]}, {Scaled[0.001], Scaled[0.002]}}
        , ImagePadding → {{Scaled[0.035], Scaled[0.001]},
            {Scaled[0.001], Scaled[0.001]}}
        , Epilog→ {Inset[MaTeX["\\text{(c)}", FontSize→ ifs],
              Scaled[{0.005, 0.99}], Scaled[{0, 1}]]}
      ],
    figureCd= Show[{
          Graphics[{GrayLevel[0.4], Thickness[0.001], Line[{{0, 0}, {8.2, 0}}]}],
          Graphics[
            Table[
              Table[
                Table[
```

```
                    If[index == "N1" && n == 2, ##&[], {
                        keyColour[index, n] /. Missing[__] → Gray,
                        Thickness[0.003],
                        Line[
                          Select[
                            Table[
                              ReIm [ωdata[m , index, n, Ω][["τ"]]]
                              , {Ω, Keys[data[m , index, n]][[1 ;; ;; 1]]}]
                            , #[[2]] > -1.45 &]
                        ]
                      }]
                    , {m , -1, 1}]
                  , {n, {2, 1}}]
              , {index, Reverse@Keys[selection[1]][[1 ;; -1]]}]
    ],
    Graphics[{White, {Rectangle[{0, -1.6}, {8.2, -1.45}],
          Rectangle[{0, 0.1}, {8.2, 0.2}]}}]
  }
  , PlotRange → {{0, 8.2}, {-1.45, 0.1}}
  , Frame → True
  , AspectRatio → Automatic
  , ImageSize → 400
  , FrameLabel → {MaTeX["\\mathrm {Re}(\\omega \\tau)", FontSize → lfs],
      MaTeX["\\mathrm {Im }(\\omega \\tau)", FontSize → lfs]}
  , Method → {"AxesInFront" → False}
  , PlotRangeClipping → True
  , Axes → True
  , AxesOrigin → {0, 0}
  , FrameTicks → {{Join[
          {#, MaTeX[ToString[PaddedForm [#, {2, 1}]], FontSize → tfs]} &/@
            Range[-1.5, 0., 0.5]
        ], None}, {Join[
          {#π, MaTeX[StringReplace[ToString[Numerator [#]] <>
                      "\\pi/" <> ToString[Denominator [#]],
                    {"/1" → "", "0\\pi" → "0", "1\\pi" → "\\pi"}],
                  FontSize → tfs], {0.008, 0}} &/@Range[0, 3, 1/4],
          {#π, "", {0.004, 0}} &/@Range[0, 3, 1/12]
        ], None}}
  , GridLines → {πRange[0, 3, 1/12], Range[-1.5, 0., 0.5]}
  , GridLinesStyle → Directive[GrayLevel[0.8]]
  , ImagePadding →
    {{Scaled[0.095], Scaled[0.007]}, {Scaled[0.065], Scaled[0.002]}}
  , ImagePadding → {{Scaled[0.035], Scaled[0.001]},
      {Scaled[0.025], Scaled[0.001]}}
  , Epilog → {Inset[MaTeX["\\text{(d)}", FontSize → ifs],
        Scaled[{0.005, 0.95}], Scaled[{0, 1}]]}
 ]
}]

]
```




```
FileByteCount[Export[FileNameJoin[{$OutputDirectory, "figureCa-intensity-HO.pdf"}],
    figureCa, ImageSize → 380]]
FileByteCount[Export[FileNameJoin[{$OutputDirectory, "figureCb-tau-HO.pdf"}],
    figureCb, ImageSize → 380]]
FileByteCount[Export[FileNameJoin[{$OutputDirectory, "figureCc-int-tau.pdf"}],
    figureCc, ImageSize → 380]]
FileByteCount[Export[FileNameJoin[{$OutputDirectory, "figureCd-re-im -tau.pdf"}],
    figureCd, ImageSize → 380]]
```

```
48170
```

```
30058
```

```
41268
```

```
36522
```

```
pdflatex[]
```

```
pdflatex successful
```

3D version

```
Block[{Ip, F, ω, γ, saddles, classifierFunction, sortingFunction, keyColour},
  {Ip, F, ω} = parameters ;
```

```
γ = Sqrt[2 Ip] ω / F;

classifierFunction = Function[{t, τ, Ω}, Which @@ Flatten[{Table[{
    {And[Re[ω τ] < 1, Floor[ω Re[t - τ], 2 π/3] / (2 π/3) == k - 1], "N" <> ToString[k]},
    {And[1 < Re[ω τ], Re[E^(-I π/4) (ω t - 5)] < 0,
      Floor[ω Re[t - τ], 2 π/3] / (2 π/3) == k - 1], "A" <> ToString[k]},
    {And[Re[E^(-I π/4) (ω t - 5)] > 0, Re[E^(+I π/4) (ω t - 7.4)] < 0,
      Floor[ω Re[t - τ], 2 π/3] / (2 π/3) == k - 1], "B" <> ToString[k]},
    {And[Re[E^(+I π/4) (ω t - 7.4)] > 0, Re[E^(-I π/4) (ω t - 9.3)] < 0,
      Floor[ω Re[t - τ], 2 π/3] / (2 π/3) == k - 1], "C" <> ToString[k]} (*,
    {And[Re[E^(-I π/4) (ω t - 9.3)] > 0, Floor[ω Re[t - τ], 2 π/3] / (2 π/3) == k - 1],
     "D" <> ToString[k]} *)
  }, {k, 1, 1}], {True, "Discard"}}]];
sortingFunction =
  Function[list, SortBy[list, Function[Re[ω #[[1]] - Floor[ω Re[#[[1]] - #[[2]], 2 π]]]]];

keyColour = <|"A1" → <|1 → Black, 2 → Blue|>,
    "B1" → <|1 → Darker[Green, 0.2], 2 → Darker[Red, 0.05]|>,
    "C1" → <|1 → Magenta, 2 → Darker[Cyan, 0.1]|>,
    "N1" → <|1 → GrayLevel[0.6]|> (*, "D1" → <|1 → Darker[Cyan], 2 → Pink|> *)|>;

(*selection = Association[ParallelTable[
      m → KeySort[ClassifyQuantumOrbits[saddlePoints[m][[1;;;;1]],
            classifierFunction, sortingFunction, DiscardedLabels → {"Discard"}]]
      , {m, -1, 1}]];
Print[AbsoluteTiming[
    data = Association[Table[
            m → Association[ParallelTable[
                  index → Association[Table[
                        n → Association[Table[
                              Ω → Function[{t, τ},
                                    <|"HO" → Ω/ω, "τ" → τ,
                                    "int" → Log10[Abs[1/τ^(3/2) E^(-I S[t, t-τ] + I Ω t)]^2]|>
                                  ] @@ AssociationTranspose[
                                    selection[m, index]][n, Ω]
                              , {Ω, Keys[selection[m, index]][[1;;;;10]]}]]
                        , {n, 1, Length[Keys[
                              AssociationTranspose[selection[m, index]]]]}]]}]]
```

```
                                                                                --
                    ,{index,Keys[selection[m ]][[1;;-1]]}]]
              ,{m ,-1,1}]];
  ]];*)

Show[{
    Graphics3D[
      Table[
        Table[
          Table[
            If[index== "N1" && n == 2, ## &[], {
                keyColour[index, n] /. Missing[__] → Gray,
                Thickness[0.003],
                Line[
                  Select[
                    Table[
                      {Re[ω data[m , index, n, Ω][["τ"]]],
                        data[m , index, n, Ω][["HO"]], data[m , index, n, Ω][["int"]]}
                      , {Ω, Keys[data[m , index, n]][[1 ;; ;; 1]]}]
                    , #[[2]] > -1.45 &]
                ]
              }]
            , {m , -1, 1}]
          , {n, {2, 1}}]
        , {index, Reverse@Keys[selection[1]][[1 ;; -1]]}]
    ]
  }
  , BoxRatios→ {2, 1, 1 / 1.5}
  , PlotRange → {{0, 8.2}, {12, 60}, {-15.5, -10.55}}
  , ImageSize → 800
  , Axes → True
  , AxesLabel → {"Re(ωt)", "Ω/ω", "|τ^-3/2 e^-iS+iΩt|^2 (arb.u.)"}
  , Axes → True
  , Ticks→ {
      {#π, StringReplace[ToString[Numerator [#]] <> "π/" <> ToString[Denominator [#]],
              {"/1" → "", "0π" → "0", "1π" → "π"}]} & /@Range[0, 3, 1 / 4],
      Automatic ,
      Join[
        {#, Superscript["10", ToString[#+11]] /. {Superscript["10", "0"] → "1"},
              {0.01, 0}} & /@Range[-15, -10],
        {#, "", {0.005, 0}} & /@Flatten[Outer[Plus,
              Log10[Range[2., 9.]], Range[-15, -10]]]
      ]}
]

]
```