

Initialization

```
<<MaTeX`
SetOptions[MaTeX, "Preamble " -> {
  \usepackage{amssymb ,upref}
  \usepackage{fourier}
  \usepackage{tgheros}
  \usepackage[T1]{fontenc}
  \usepackage{textcomp }
  \usepackage{microtype }
  \usepackage{xcolor}
  " }];

lfs = 10;
tfs = 9;

$OutputDirectory= FileNameJoin [{NotebookDirectory[], "..", "Figures"}];
$MainDirectory=
  StringReplace[FileNameJoin [{NotebookDirectory[], ".."}], {" " -> "\\ " }];
pdflatex[] := Run["cd "<>$MainDirectory<
  " && pdflatex --output-directory=build Manuscript.tex"] /.
  {0 -> "pdflatex successful"}
```

Preliminaries

$$AL[t_] = \frac{F1}{\omega_1} \{ \cos[\omega_1 t], \sin[\omega_1 t] \} + \frac{F2}{\omega_2} \{ \cos[\omega_2 t], -\sin[\omega_2 t] \}$$

$$\left\{ \frac{F1 \cos[t \omega_1]}{\omega_1} + \frac{F2 \cos[t \omega_2]}{\omega_2}, \frac{F1 \sin[t \omega_1]}{\omega_1} - \frac{F2 \sin[t \omega_2]}{\omega_2} \right\}$$

$$(*AL[t_] =$$

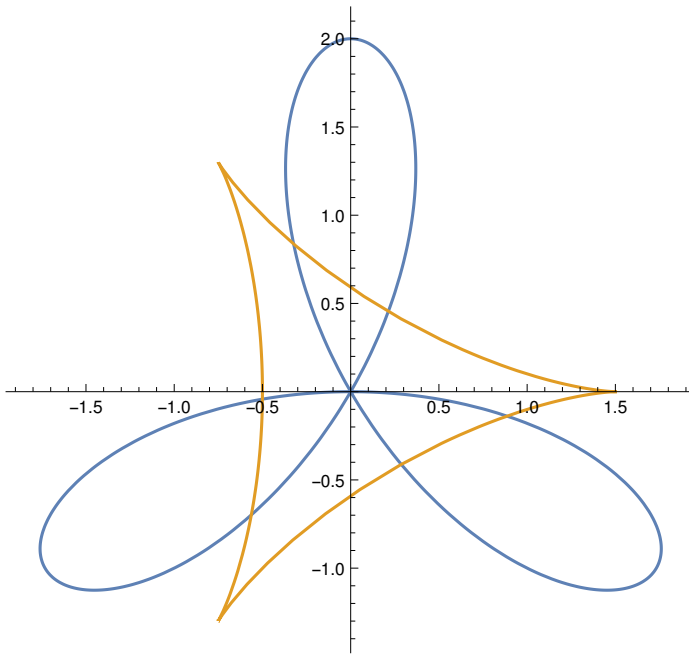
$$\text{Re} \left[\text{ExpToTrig} \left[\frac{F1 - (1, i)}{i \omega_1 \sqrt{2}} \text{Exp}[-i \omega_1 t] + \frac{F2 (1, -i)}{i \omega_2 \sqrt{2}} \text{Exp}[-i \omega_2 t] \right] \right] /. \{ \text{Re} \rightarrow (\# \&), \text{Im} \rightarrow (0 \&) \} *)$$

$$\left\{ \frac{F1 \sin[t \omega_1]}{\sqrt{2} \omega_1} - \frac{F2 \sin[t \omega_2]}{\sqrt{2} \omega_2}, -\frac{F1 \cos[t \omega_1]}{\sqrt{2} \omega_1} - \frac{F2 \cos[t \omega_2]}{\sqrt{2} \omega_2} \right\}$$

$$FL[t_] = -D[AL[t], t]$$

$$\{ F1 \sin[t \omega_1] + F2 \sin[t \omega_2], -F1 \cos[t \omega_1] + F2 \cos[t \omega_2] \}$$

```
Block[{F1 = 1, F2 = 1, ω1 = 1, ω2 = 2},
  ParametricPlot [
    {FL[t], AL[t]}
    , {t, 0,  $\frac{2\pi}{\omega_1}$ }
  ]
]
```



$\text{RotationMatrix}[\theta] \cdot \{ax, ay\}$

$\{ax \cos[\theta] - ay \sin[\theta], ay \cos[\theta] + ax \sin[\theta]\}$

$\text{AR}[t_] = \text{RotationMatrix}[\alpha t] \cdot \text{AL}[t]$

$$\left\{ \cos[t \alpha] \left(\frac{F1 \cos[t \omega_1]}{\omega_1} + \frac{F2 \cos[t \omega_2]}{\omega_2} \right) - \sin[t \alpha] \left(\frac{F1 \sin[t \omega_1]}{\omega_1} - \frac{F2 \sin[t \omega_2]}{\omega_2} \right), \right. \\ \left. \left(\frac{F1 \cos[t \omega_1]}{\omega_1} + \frac{F2 \cos[t \omega_2]}{\omega_2} \right) \sin[t \alpha] + \cos[t \alpha] \left(\frac{F1 \sin[t \omega_1]}{\omega_1} - \frac{F2 \sin[t \omega_2]}{\omega_2} \right) \right\}$$

$\text{FR}[t_] = \text{RotationMatrix}[\alpha t] \cdot \text{FL}[t]$

$$\{-(-F1 \cos[t \omega_1] + F2 \cos[t \omega_2]) \sin[t \alpha] + \cos[t \alpha] (F1 \sin[t \omega_1] + F2 \sin[t \omega_2]), \\ \cos[t \alpha] (-F1 \cos[t \omega_1] + F2 \cos[t \omega_2]) + \sin[t \alpha] (F1 \sin[t \omega_1] + F2 \sin[t \omega_2])\}$$

$$dAR[t_] = -D[AR[t], t]$$

$$\left\{ (F1 \cos[t \omega_1] - F2 \cos[t \omega_2]) \sin[t \alpha] + \alpha \left(\frac{F1 \cos[t \omega_1]}{\omega_1} + \frac{F2 \cos[t \omega_2]}{\omega_2} \right) \sin[t \alpha] - \right.$$

$$\left. \cos[t \alpha] (-F1 \sin[t \omega_1] - F2 \sin[t \omega_2]) + \alpha \cos[t \alpha] \left(\frac{F1 \sin[t \omega_1]}{\omega_1} - \frac{F2 \sin[t \omega_2]}{\omega_2} \right), \right.$$

$$\left. -\cos[t \alpha] (F1 \cos[t \omega_1] - F2 \cos[t \omega_2]) - \alpha \cos[t \alpha] \left(\frac{F1 \cos[t \omega_1]}{\omega_1} + \frac{F2 \cos[t \omega_2]}{\omega_2} \right) - \right.$$

$$\left. \sin[t \alpha] (-F1 \sin[t \omega_1] - F2 \sin[t \omega_2]) + \alpha \sin[t \alpha] \left(\frac{F1 \sin[t \omega_1]}{\omega_1} - \frac{F2 \sin[t \omega_2]}{\omega_2} \right) \right\}$$

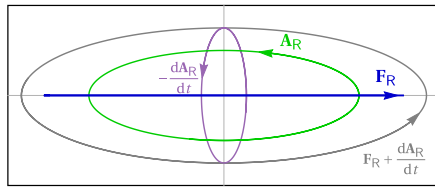
Fields in the rotating frame

```

Block[{F1 = 1, F2 = 1, ω1 = 1, ω2 = 2, α =  $\frac{\omega_2 - \omega_1}{2}$ , Rx = 2.4, Ry = 1., t0 = +50.°},

figureAb = Show[{
  Graphics[{
    AbsoluteThickness[0.65],
    Line[{{-Rx, -Ry}, {-Rx, Ry}, {Rx, Ry}, {Rx, -Ry}, {-Rx, -Ry}}],
    GrayLevel[0.7], AbsoluteThickness[0.5],
    Line[{{{-Rx, 0}, {Rx, 0}}, {{0, -Ry}, {0, Ry}}}]
  ]},
  ParametricPlot [
    {-dAR[t], -(FR[t] - dAR[t]), -AR[t], -FR[t]}
    , {t, t0, t0 + 2π}
    , PlotStyle →
      {Gray, RGBColor[0.6, 0.4, 0.7], Darker[Green, 0.2], Darker[Blue, 0.2]}
    , Frame → False
    , Axes → False
  ] /.
  {Line[pts_] := {Arrowheads[Small ], AbsoluteThickness[0.75], Arrow[pts]}},
  Graphics[{
    Inset[MaTeX["(\\mathrm {b})", FontSize→10], {0, -1.35}],
    Inset[MaTeX["\\color[rgb]{0,0,0.8}{\\mathbf {F}_\\mathsf {R}}",
      FontSize→10], {1.8, 0.2}],
    Inset[MaTeX["\\color[rgb]{0,0.8,0}{\\mathbf {A}_\\mathsf {R}}",
      FontSize→10], {0.75, 0.6}],
    Inset[MaTeX["\\color[rgb]{0.6,0.4,0.7}{-\\frac{\\mathrm
      d\\mathbf {A}_\\mathsf {R}}{dt}}", FontSize→8], {-0.5, 0.2}],
    Inset [
      MaTeX [
        "\\color[gray]{0.5}{\\mathbf {F}_\\mathsf {R}+\\frac{\\mathrm
          d\\mathbf {A}_\\mathsf {R}}{dt}}",
        FontSize→8], {1.9, -0.7}]
  ]
}
(*, PlotRange→All*)
, PlotRange → {1.05 Rx {-1, 1}, Ry  $\frac{-3.15, 2.95}{2.2}$  (*{-1.35, 1.3}*)}
, PlotRangePadding→None
, ImageSize → {{1500}, {130}}
]
]
FileByteCount[
  Export[FileNameJoin [{"$OutputDirectory", "figureAb-fields.pdf"}], figureAb]
pdflatex[]

```



(b)

21570

pdflatex successful

Lab frame & frame transformation

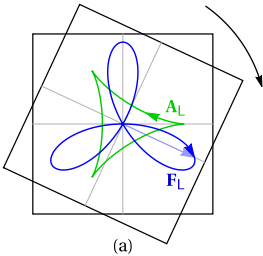
```
Block[{F1 = 1, F2 = 1, ω1 = 1, ω2 = 2, R = 2.2, t0 = 50. °},
```

```
figureAa = Show[{
  Graphics[{
    AbsoluteThickness[0.65],
    Line[R{{-1, -1}, {-1, 1}, {1, 1}, {1, -1}, {-1, -1}}],
    GrayLevel[0.7], AbsoluteThickness[0.5],
    Line[{{{-R, 0}, {R, 0}}, {{0, -R}, {0, R}}]}],
  Graphics[Rotate[{
    AbsoluteThickness[0.65],
    Line[R{{-1, -1}, {-1, 1}, {1, 1}, {1, -1}, {-1, -1}}],
    GrayLevel[0.7], AbsoluteThickness[0.5],
    Line[{{{-R, 0}, {R, 0}}, {{0, -R}, {0, R}}]}], Arg[Complex @@FL[t0]]],
  Graphics[{
    Lighter[Blue, 0.6],
    Thickness[0.004],
    Arrowheads[Small],
    Arrow[{{0, 0}, FL[t0]}]
  }],
  Graphics[{
    Thickness[0.005],
    Arrow[Table[1.6 R {Cos[θ], -Sin[θ]}, {θ, -55°, -15°, 1°}]]
  }],
  ParametricPlot [
    AL[t]
    , {t, t0, t0 + 2π}
    , PlotStyle → Darker[Green, 0.2]
    , Frame → False
    , Axes → False
  ] /.
  {Line[pts_] → {Arrowheads[Small], AbsoluteThickness[0.75], Arrow[pts]}},
  ParametricPlot [
    FL[t]
    , {t, t0, t0 + 2π}
    , PlotStyle → Blue
    , Frame → False
    , Axes → False
```

```

] /.
{Line[pts_] -> {Arrowheads[Small ], AbsoluteThickness[0.75], Arrow[pts]}}},
Graphics[{
  Inset[MaTeX["(\mathrm {a})", FontSize->10], {0, -3.}],
  Inset[MaTeX["\color[rgb]{0,0,0.8}{\mathbf {F}}_\mathsf {L}]",
    FontSize->10], {1.3, -1.4}],
  Inset[MaTeX["\color[rgb]{0,0.8,0}{\mathbf {A}}_\mathsf {L}]",
    FontSize->10], {1.3, 0.4}]
}]
}, PlotRange -> {{-3., 3.5}, {-3.15, 2.95}}
, PlotRangePadding -> None
, ImageSize -> {{1500}, {130}}
]
]
FileByteCount[
  Export[FileNameJoin[{$OutputDirectory, "figureAa-frames .pdf"}], figureAa]
pdflatex[]

```



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