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SOLUTIONS OF SOME LINEAR DIOPHANTINE EQUATIONS

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Abstract:
This paper concerns with the problem of obtaining solutions of some linear Diophantine equations.

Key Words: Diophantine Equation & Euclidean Algorithm

1. Introduction:

The linear Diophantine equations and their solutions are one of the well-known results in number theory. Study of these equations can be found in many works such as Dickson [2], Gallian [3], and so on. A Linear Diophantine equation with two variables x and y has the form

$$ax + by = c \qquad \text{-------- (1)}$$

Where a, b and c are all integers. We are interested in integer solutions, that is integers x and y that satisfy equation (1). In Dickson it is stated that if a and b are relatively prime and if (u,v) is an integer solution of (1), then the whole set of integer solutions of (1) can be expressed as

$$\left.\begin{array}{l} x = u + bw \\ y = v - aw \end{array}\right\} \qquad \text{--------- (2)}$$

Where w is an arbitrary integer. In other words, the whole set of integer solutions of (1) is

$$S = \{(u + bw, v - aw) |\, w = 0, \pm 1, \pm 2 \ \ldots.\}$$

2. Preliminaries:

Theorem 2.1:

The equation $a_1 x_1 + \cdots + a_n x_n = C$ is solvable if and only if $\gcd(a_1, \ldots, a_n)\,|c$, In case of solvability, one can choose $n - 1$ solutions such that each solution is an integer linear combination of those $n - 1$ solutions.

Proof:

Let $d = \gcd(a_1, \ldots, a_n)$. If C is not divisible by d, then the equation $a_1 x_1 + \cdots + a_n x_n = C$ is not solvable, since for any integers $x_1, \ldots, x_n$, the left hand side is divisible by d and the right – hand side is not.

Actually, we need to prove that $\gcd(x_1, x_2 \ldots, x_n)$ is a linear combination with integer coefficients of $x_1, x_2 \ldots, x_n$. Since,

$$\gcd(x_1, \ldots, x_n) = \gcd(\gcd(x_1, \ldots, x_{n-1}), x_n).$$

$\gcd(x_1, \ldots, x_n)$ is a linear combination of x_n and $\gcd(x_1, \ldots, x_{n-1})$. Then inductively $\gcd(x_1, \ldots, x_n)$ is a linear combination of $x_1, \ldots, x_{n-1}, x_n$

3. Results:

Example 3.1: Find the solution of the linear Diophantine Equation $60x + 33y = 9$.

Solution: Let $a = 60$, $b = 33$, $c = 9$ and $(60, 33) = 3$, we can see 3/9.

By using Euclidean Algorithm

$$60 = 1.33 + 27$$
$$33 = 1.27 + 6$$
$$27 = 4.6 + 3$$
$$6 = 2.3 + 0$$

The last non zero remainder is 3. So $(60,33) = 3$

$$\begin{aligned} 3 &= 27 - 4.6 \\ &= 27 - 4.(33 - 27) \\ &= 5.27 - 4.33 \\ &= 5(60 - 33) - 4.33 \\ &= 5.60 - 9.33 \end{aligned}$$

We take $u = 5$ and $v = -9$. One solution is then

$$x_0 = 5.9/3 = 15$$
$$y_0 = -9.9/3 = -27$$

All the solutions are given by

$$x = 15 + \frac{33.n}{3}$$
$$\Rightarrow x = 15 + 11n$$
$$y = -27 - \frac{60n}{3}$$
$$\Rightarrow y = -27 - 20n \ \text{ Where } \ n \sum z$$

Example 3.2: Find the solution of the linear Diophantine Equation $442m + 255n = 17$.

Solution: Let $a = 442$, $b = 255$, $c = 17$ and $(442, 255) = 17$.

By using Euclidean Algorithm

$$442 = 1.255 + 187$$
$$255 = 1.187 + 68$$
$$187 = 2.68 + 51$$
$$68 = 1.51 + 17$$
$$51 = 3.17 + 0$$

The last non zero remainder is 17. So

$$(442, 255) = 17.$$

$$17 = 68 - (51 \times 1)$$
$$= 68 - [(187 - (68 \times 2)) \times 1]$$
$$= (68 \times 3) - 187$$
$$= [(255 - (187 \times 1)) \times 3] - 187$$
$$= (255 \times 3) - (187 \times 4)$$
$$= (255 \times 3) - [(442 - (255 \times 1)) \times 4]$$
$$= (255 \times 7) - (442 \times 4)$$

We take $u = 7$ and $v = -4$ one solution is then

$$x_0 = 7.{}^{17}/_{17} = 7 \text{ and } y_0 = -4.{}^{17}/_{17} = -4$$

All the solutions are given by

$$x = 7 + \frac{442}{17}t = 7 + 26t$$

$y = -4 - \frac{255}{17}t = -4 - 15t$ Where $t \sum z$

Conclusion:

In this paper, we have obtained solutions of some Linear Diophantine equation one may attempt to find integer solutions to other choices of multivariate linear Diophantine equation.

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