

Integrating Computational Thinking with Mathematical Problem Solving*

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Abstract. The Erasmus+ project Computational Thinking and Mathematical Problem Solving, an Analytics Based Learning Environment (CT&MathABLE) provides comprehensive learning analytics driven support for developing Computational and Algebraic Thinking in K-12 schools. Through the deployment of digital technology the project provides educators with new approaches to skills development that builds on well supported learning pathways and is individually tailored to the learner. This is achieved through a novel learning systems architecture which supports individualized development paths and integration of Computational Thinking and mathematical conceptual development with tailored problem solving and assessment frameworks.

Keywords: Computational Thinking · CT · Algebraic Thinking · AT · Mathematics Education · Curriculum · Learning Pathways.

1 Introduction

As part of curricula reforms, many European countries have already included elements of Computational Thinking (CT) skills in compulsory schooling [1]. CT is a type of analytical thinking that employs mathematical and engineering thinking to understand and solve complex problems within the constraints of the real world [3]. Algebraic Thinking is defined as the ability to generalize, represent, justify, and reason with abstract mathematical structures and relationships [2]. One of the most attractive ways to do this is by integrating AT

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and CT education into computer science or similar courses. For instance, the Ministry of Culture and Education in Finland highlights new literacy competencies, which include ICT skills, media literacy and programming. On the other hand, Lithuania on the other hand has introduced compulsory CT education from early grades through to the end of compulsory schooling. To address this variation in curricula, an in depth analysis of six European national curricula has been conducted to expose CT and AT content in mathematics and other subjects, initially focusing on students aged 9 to 14 years. This analysis forms the foundation for developing individualized learning pathways. Our analysis reveals considerable similarity, allowing for the development of core learning paths, however, there is considerable regional variation.

2 Method

Establishing learning pathways of broad relevance requires an in-depth understanding of the relationships between conceptual development in the domain and the linkage of this understanding to conceptual progression within each topic or skill area. We define the ability to think computationally as a combination of higher-order cognitive skills: a) abstraction, b) algorithmic thinking, c) analytical thinking and decomposition, d) data collection, analyses and representation, e) evaluation and adjustment, and f) transferability (generalization). Algebraic Thinking lies at the core of Mathematics and serves as an integral component of the broader construct Mathematical Thinking. Our approach builds upon a review of related literature, which establishes a research-informed classification of Algebraic Thinking skills and competencies. This classification structure was used to derive an initial set of codes that enable us to annotate the curricula of six European countries (Finland, Hungary, Lithuania, Spain, Sweden, and Türkiye). Following this comprehensive analysis and classification of curricula a final coding structure was developed that captured the conceptual content of CT and Algebraic Thinking as evident in the analyzed curricula.

3 Analysis and Results

As one might expect the six national curricula differed both structurally and in terms of content and order of introduction of concepts. A curriculum typically consists of a series of topics, and within each topic area a list of its detailed learning material and outcomes is specified. The curricula forming the empirical data for our study therefore need to be consolidated during analysis. Appendix 1 contains the number of detailed learning statements in each curriculum ordered by country. One reason for the richness of the Hungarian curriculum is that it contains two kinds of details. One is the preparation for the knowledge, and the other is the real learning outcome. Some topic details are divided into 2-3 parts in a country, while it is in only one row in the others. These differences were reduced during the steps of consolidation. Duplication, associated with cognitive

progression in key topics was an important aspect of the analysis, since these sequences need to be incorporated into the CT&MathABLE learning pathways.

After eliminating duplicates, the Hungarian curriculum was selected as a reference point, since it is the most detailed. Each row of the other curricula was assigned to the corresponding topic in the Hungarian curriculum, if it existed, or a new topic (and perhaps code) was created where needed. The most important outcome of the current study is the identification of topic areas and concentrations within the curricula of our sample of six countries. The countries we studied exhibit a strong correlation, with commonality between curricula of over 47% between any four countries and over 80% with another three. Each country has its own national focus, with greater coverage of certain topics compared to the other countries studied. Spain places emphasis on problem-solving and pattern recognition; Finland prioritizes equations and operations; Hungary focuses on comparison, sorting, and equations; Lithuania emphasizes measurements and problem-solving; Sweden places considerable focus on problem-solving and ratios; and for Türkiye highlights measurements and equations as particularly significant. Further details are provided in Appendix 1.

4 Conclusion

We have comprehensively analysed mathematics education literature to formulate a precise definition of the cognitive development areas encompassed by Algebraic Thinking. This definition, along with a higher-order CT definition, have been applied to coding of statements within the mathematics curricula of six European nations. We observe substantial similarity, as well as intriguing differences, in terms of the frequency with which certain codes are referenced. Content analysis reinforces the assertion that the core of the curricula shares significant similarity, with a congruence of nearly 50%. A general learning pathway is also evident, commencing with the exploration of simple objects through classification and categorization, as learners delve into their properties and relationships. Drawing from their experiences, learners identify patterns and acquire the ability to generalize. Their mathematical vocabulary expands and matures, culminating in the integration of advanced concepts and definitions, which, combined with their arithmetical skills, equip them for problem-solving. This overarching learning trajectory encompasses the key elements of Algebraic Thinking and will serve as a foundation for the subsequent phase of the project, facilitating the creation of tasks necessary to support individual learning paths within each trajectory. This personalisation will be achieved by leveraging learning analytics, aiding each learner in defining a unique trajectory along the path, based on their previous task performance and demonstrated accomplishments.

References

1. Bocconi, S., Chiocciariello, A., Kamylylis, P., Dagienė, V., Wastiau, P., Engelhardt, K., Earp, J., Horvath, M., Jasutė, E., Malagoli, C., Masiulionytė-Dagienė,

- V., Stupurienė, G., Giannoutsou, N., Inamorato dos Santos, A., Punie, Y., Cachia, R.: Reviewing computational thinking in compulsory education: state of play and practices from computing education. Joint Research Centre (European Commission), Publications Office of the European Union, LU (2022), <https://data.europa.eu/doi/10.2760/126955>
2. Bråting, K., Kilhamn, C.: Exploring the intersection of algebraic and computational thinking. *Mathematical Thinking and Learning* **23**(2), 170–185 (Apr 2021). <https://doi.org/10.1080/10986065.2020.1779012>
 3. Denning, P.J., Tedre, M.: Computational Thinking: A Disciplinary Perspective. *Informatics in Education* **20**(3), 361–390 (Jul 2021). <https://doi.org/10.15388/infedu.2021.21>, <https://infedu.vu.lt/journal/INFEDU/article/701>, publisher: Vilnius University Institute of Data Science and Digital Technologies

Appendix 1

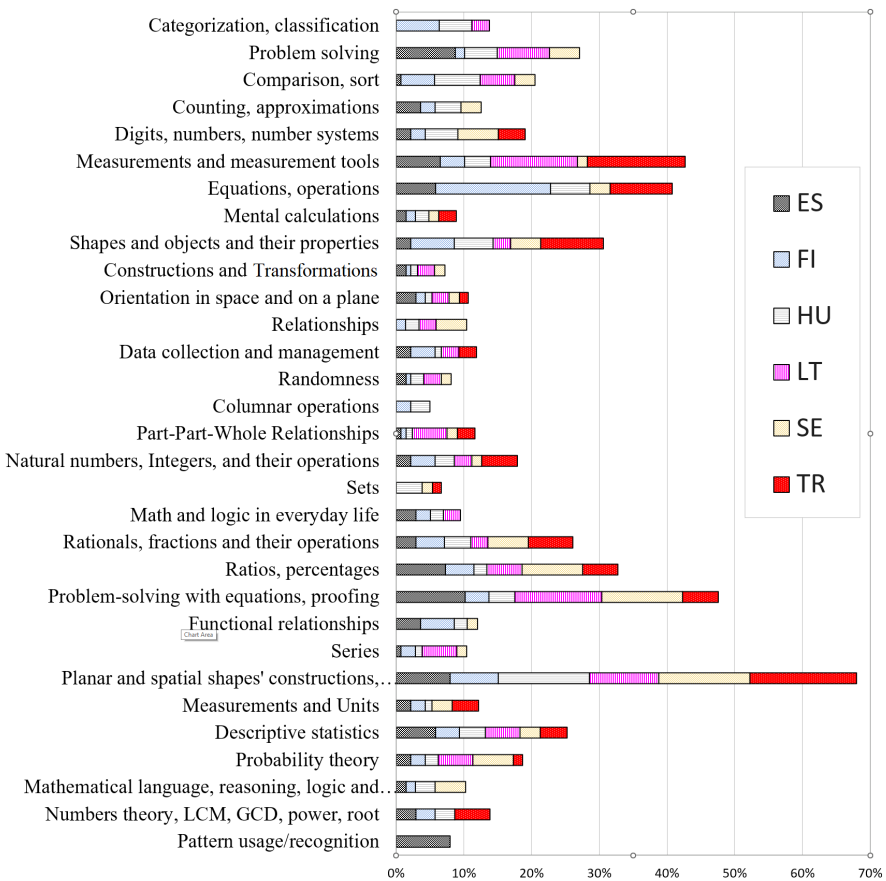


Fig. 1. Curricula comparison