№ 6
27.09.2023

RESEARCH AND IMPLEMENTATION

# TADQIQ VA TATBIQ

ILMIY-USLUBIY JURNALI

zenodo Google scholar OpenAIRE CERTIFICATE № 078777 INDEX COPERNICUS INTERNATIONAL

- ANIQ FANLAR
- TABIIY FANLAR
- SIYOSIY FANLAR
- IJTIMOIY FANLAR
- IQTISOD FANLARI
- TEXNIKA FANLARI
- TIBBIYOT FANLARI
- PEDAGOGIKA FANLARI

fer-teach.uz @ferteach_uz

Jurnal oyda bir marta nashr etiladi.

**Nashr tillari**:
O'zbek, rus, ingliz va qoraqalpoq

*O'zbekiston Respublikasi Prezidenti Administratsiyasi huzuridagi Axborot va ommaviy kommunikatsiya agentligida 2023-yil 3-mayda №078777 raqam bilan ro'yxatga olingan*

**Murojaat uchun**

@ferteach_uz
+99893-343-13-14
https://fer-teach.uz

# ARALASH PARABOLIK TENGLAMA UCHUN INTEGRAL SHARTLI MASALA.

**_M.I.Saidov_**
*TATU Farg'ona filiali assisitenti.*

***Annotatsiya:*** *Ushbu maqolada aralash parabolik tenglama uchun integral shartli masala va uning yechimi yoritilgan.*

***Kalit so'zlar:*** *parabolik tenglama, aralash parabolik tenglama, differensial tenglama, funksiya, soha, integral tenglama.*

---

# INTEGRAL CONDITIONAL PROBLEM FOR A MIXED PARABOLIC EQUATION.

**_Saidov M.I._**
*assistant of the Fergana branch of TUIT*

***Abstract:*** *The article discusses an integral conditional problem and its solution for a mixed parabolic equation.*

***Key words:*** *parabolic equation, mixed parabolic equation, differential equation, function, domain of definition, integral equation.*

---

$D$ orqali $y=0$, $y=1$ va $x=-T$ to'g'ri chiziqlar bilan chegaralangan yarim polosani belgilaylik, bu erda $T=const>0$. Bu sohada quyidagi operatorni qaraylik:

$$Lu \equiv \begin{cases} L_1 u \equiv u_{xx} - u_y, & (x,y) \in D_1 = D \cap (x>0), \\ L_2 u \equiv u_{yy} + u_x, & (x,y) \in D_2 = D \cap (x<0). \end{cases}$$

$Lu=0$ – $D$ sohada aralash parabolik tenglama bo'lib, $D_1$ sohada to'g'ri parabolik, $D_2$ sohada esa teskari parabolikdir. Bu maqolada biz $L[u]=0$ aralash parabolik tenglama uchun $D$ sohada integral shartli quyidagi masalani o'rganamiz.

1–**masala.** $D$ sohaning yopig'ida aniqlangan, uzluksiz va chegaralangan shunday $u(x,y)$ funksiya topilsinki, u $D_1$ va $D_2$ sohalarda mos ravishda $L_1 u=0$ va $L_2 u=0$ tenglamalarni hamda quyidagi shartlarni qanoatlantirsin:

$$\lim_{x\to -0} u_x(x,y) = a_1(y) \lim_{x\to +0} u_x(x,y) + a_2(y) D_{0y}^{\alpha}\left[b_2(y) u(0,y)\right] +$$
$$+ a_3(y) D_{y1}^{\beta}\left[b_3(y) u(0,y)\right] + b_1(y), \qquad 0<y<1;$$

$$u(x,0)=\varphi_1(x), \qquad 0\le x<+\infty; \tag{1}$$

$$u(x,0)=\varphi_2(x), \qquad -T\le x\le 0; \tag{2}$$

$$\int_0^1 u(x,y)dy=\varphi_3(x), \quad -T\le x\le 0, \tag{3}$$

bu erda $a_j(y), b_j(y), \varphi_j(x), j=\overline{1,3}$ –berilgan funksiyalar bo‘lib, $a_j(y)\in C^1[0,1]$, $a_1(y)\neq 0$, $y\in[0,1]$; $b_j(y)\in C[0,1]$; $\varphi_1(0)=\varphi_2(0)=0$ va $\varphi_1(x)$ funksiya $[0,+\infty)$ oraliqda uzluksiz va chegaralangan; $\varphi_2(x)$, $\varphi_3(x)\in C^1[-T,0]$; $\alpha$ va $\beta$ lar esa $(0,1)$ segmentdan olingan ixtiyoriy haqiqiy sonlar; $D_{0y}^{\alpha}$ va $D_{y1}^{\beta}$ lar kasr tartibli differensial operatorlar bo‘lib,

$$D_{0y}^{\alpha}g(y)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dy}\int_0^y (y-t)^{-\alpha}g(t)dt,$$

$$D_{y1}^{\beta}g(y)=-\frac{1}{\Gamma(1-\beta)}\frac{d}{dy}\int_y^1 (t-y)^{-\beta}g(t)dt$$

tengliklar bilan aniqlanadi; $\Gamma(z)$ – Eylerning gamma funksiyasi [1].

Qo‘yilgan masalaning echimi mavjud va yagonaligini isbotlaymiz. Faraz qilaylik, $u(x,y)-I$ masalaning yechimi bo‘lsin. Masala shartlariga asoslanib,

$$u(-0,y)=u(+0,y)=\tau(y), \qquad 0\le y\le 1; \tag{4}$$

$$\lim_{x\to+0}u_x(x,y)=\nu(y), \qquad 0<y<1 \tag{5}$$

belgilashlarni kiritaylik.

Ma’lumki, $L_1u=0$ tenglamaning $D_1$ sohaning yopig‘ida aniqlangan, uzluksiz, chegaralangan hamda (1) va (5) shartlarni qanoatlantiruvchi yechimi

$$u(x,y)=\int_0^{+\infty}\frac{\sqrt{x\xi}}{2y}I_{-1/2}\left(\frac{x\xi}{2y}\right)e^{\frac{-(x^2+\xi^2)}{4y}}\varphi_1(\xi)d\xi-\frac{1}{\sqrt{\pi}}\int_0^y \nu(t)(y-t)^{-1/2}e^{\frac{-x^2}{4(y-t)}}dt, \tag{6}$$

ko‘rinishda aniqlanadi [2], bu erda $I_{-1/2}(z)$ – mavhum argumentli Bessel funksiyasi [3].

(4) belgilashni e’tiborga olib, (6) formulada $x\to+0$da limitga o‘tamiz. Natijada noma’lum $\tau(y)$ va $\nu(y)$ funksiyalar orasidagi

$$\tau(y)=-\frac{1}{\sqrt{\pi}}\int_0^y \nu(t)(y-t)^{-1/2}dt+\Phi(y), \qquad 0\le y\le 1 \tag{7}$$

munosabatga ega bo‘lamiz, bu erda $\Phi(y)=(\pi y)^{-1/2}\int_0^{+\infty}\varphi_1(\xi)e^{-\xi^2/4y}d\xi$.

(7) – $\nu(y)$ noma’lum funksiyaga nisbatan Abel integral tenglamasi [4] bo‘lib, uning yagona yechimi quyidagi formula bilan aniqlanadi:

$$\nu(y) = -\frac{1}{\sqrt{\pi}}\frac{d}{dy}\int_0^y (y-t)^{-1/2}\left[\tau(t)-\Phi(t)\right]dt, \quad 0<y<1. \tag{8}$$

Endi masala shartlarini va (4), (5) belgilashlarni hisobga olib, $L_2u=0$ tenglama va (2), (3) shartlarda $x$ ni nolga intiltiramiz:

$$\tau''(y)+\nu(y)=0, \quad 0<y<1; \qquad \tau(0)=0, \quad \int_0^1 \tau(y)dy=\varphi_3(0). \tag{9}$$

(8) tenglikni e'tiborga olsak, (9) tengliklardan $\tau(y)$ noma'lum funksiyaga nisbatan

$$\begin{aligned}&\tau''(y)-a_1(y)D_{0y}^{1/2}\tau(y)+a_2(y)D_{0y}^{\alpha}\left[b_2(y)\tau(y)\right]+\\ &+a_3(y)D_{y1}^{\beta}\left[b_3(y)\tau(y)\right]=-b_1(y)-a_1(y)\Phi_1(y), \; 0<y<1\end{aligned} \tag{10}$$

ko'rinishdagi integro–differensial tenglama va ushbu

$$\tau(0)=0, \quad \int_0^1 \tau(y)dy=\varphi_3(0) \tag{11}$$

chegaraviy shartlar kelib chiqadi, bu erda $\Phi_1(y)=\frac{1}{\sqrt{\pi}}\frac{d}{dy}\int_0^y (y-t)^{-1/2}\Phi(t)dt$.

{(10), (11)} masala yechimining mavjudligi va yagonaligini isbotlaymiz.

Avval bir jinsli masalani, ya'ni quyidagi masalani qaraylik.

$$\tau''(y)-a_1(y)D_{0y}^{1/2}\tau(y)+a_2(y)D_{0y}^{\alpha}\left[b_2(y)\tau(y)\right]+a_3(y)D_{y1}^{\beta}\left[b_3(y)\tau(y)\right]=0, \; 0<y<1 \tag{12}$$

$$\tau(0)=0, \qquad \int_0^1 \tau(y)dy=0. \tag{13}$$

**Lemma.** Agar $[0,1]$ oraliqda $a_1(y)>0$, $a_2(y)\le 0$, $a_3(y)\le 0$ va $b_2(y)>0$, $b_3(y)>0$ tengsizliklar bajarilib, $b_2(y)$–kamaymaydigan, $b_3(y)$–o'smaydigan funksiya bo'lsa, {(12), (13)} masala faqat trivial yechimga ega.

Bu lemma kasr tartibli differensial operatorlar uchun ekstremum prinsipi [1] va aniq integral uchun o'rta qiymat haqidagi teoremadan foydalanib isbotlanadi.

Yuqoridagi lemmadan quyidagi teorema kelib chiqadi:

**Teorema.** Agar lemma shartlari bajarilgan bo'lsa, {(10), (11)} masala bittadan ortiq yechimga ega bo'lmaydi.

Endi {(10), (11)} masala yechimining mavjudligini ko'rsatamiz. Shu maqsadda (10) tenglamada $y$ ni $z$ bilan almashtirib, so'ngra $\tau(0)=0$ ekanligini e'tiborga olib va $\tau'(0)=C$ belgilash kiritib, $[0,y]$ oraliqda $z$ bo'yicha ikki marta ketma-ket integrallaymiz:

$$\begin{aligned}&\tau(y)-Cy-\int_0^y (y-s)\left\{a_1(s)D_{0s}^{1/2}\tau(s)-a_2(s)D_{0s}^{\alpha}\left[b_2(s)\tau(s)\right]-\right.\\ &\left.-a_3(s)D_{s1}^{\beta}\left[b_3(s)\tau(s)\right]\right\}ds+\int_0^y (y-s)\left\{b_1(s)+a_1(s)\Phi_1(s)\right\}ds=0, \; 0<y<1.\end{aligned} \tag{14}$$

$C$ noma'lum sonni topish maqsadida, (11) ning ikkinchi shartidan foydalanamiz, ya'ni (14) tenglikni $y$ bo'yicha $[0,1]$ oraliqda integrallab, uni $\varphi_3(0)$ ga tenglaymiz. Natijada hosil bo'lgan tenglikdan $C$ noma'lum son bir qiymatli topiladi:

$$C = 2\varphi_3(0) - \int_0^1 (1-s)^2 \left[ a_1(s) D_{0s}^{1/2}\tau(s) - a_2(s) D_{0s}^{\alpha}\left[b_2(s)\tau(s)\right] - \right.$$
$$\left. - a_3(s) D_{s1}^{\beta}\left[b_3(s)\tau(s)\right]\right] + \int_0^1 (1-s)^2 \left[b_1(s) + a_1(s)\Phi_1(s)\right] ds.$$

$C$ ning bu ifodasini (14) ga qo'yib, kasr tartibli hosilalar yoyilmasidan, bo'laklab integrallash qoidasidan va takroriy integrallarda integrallash tartibini o'zgartirish qoidasidan foydalanib, {(10), (11)} masalaga ekvivalent bo'lgan quyidagi ikkinchi tur Fredgolm integral tenglamasiga kelamiz:

$$\tau(y) + \int_0^1 \tau(t) K(y,t) dt = f(y), \qquad (15)$$

bu erda agar $y > t$ bo'lsa,

$$f(y) = 2\varphi_3(0) y - y\int_0^1 (1-s)^2 \left[b_1(s) + a_1(s)\Phi_1(s)\right] ds - \int_0^y (y-s)\left[b_1(s) + a_1(s)\Phi_1(s)\right] ds,$$

$$K(y,t) = \frac{y}{\sqrt{\pi}} \int_0^1 (s-t)^{-1/2} \left[2(1-s)a_1(s) + (h-s)a'_1(s)\right] ds -$$
$$- \frac{y \cdot b_2(t)}{\Gamma(1-\alpha)} \int_t^1 (s-t)^{-\alpha} \left[2(1-s)a_2(s) - (1-s)^2 a'_2(s)\right] ds -$$
$$- \frac{y \cdot b_3(t)}{\Gamma(1-\beta)} \int_0^t (t-s)^{-\beta} \left[2(1-s)a_3(s) - (1-s)^2 a'_3(s)\right] ds -$$
$$- \frac{1}{\sqrt{\pi}} \int_t^y (s-t)^{-1/2} \left[a_1(s) - (y-s)a'_1(s)\right] ds +$$
$$+ \frac{b_2(t)}{\Gamma(1-\alpha)} \int_t^y (s-t)^{-\alpha} \left[a_2(s) - (y-s)\cdot a'_2(s)\right] ds +$$
$$+ \frac{b_3(t)}{\Gamma(1-\beta)} \int_0^t (t-s)^{-\beta} \left[a_3(s) - (y-s)a'_3(s)\right] ds$$

agar $y < t$ bo'lsa,

$$K(y,t) = \frac{y}{\sqrt{\pi}} \int_t^1 (s-t)^{-1/2} \left[2(1-s)a_1(t) - (1-s)^2 a'_1(s)\right] ds -$$
$$- \frac{y b_2(t)}{\Gamma(1-\alpha)} \int_t^y (s-t)^{-\alpha} \left[2(1-s)a_2(s) - (1-s)^2 a'_2(s)\right] ds -$$
$$- \frac{y b_3(t)}{\Gamma(1-\beta)} \int_0^t (t-s)^{-\beta} \left[2(1-s)a_3(s) - (1-s)^2 a'_3(s)\right] ds$$

Berilgan funksiyalarga qo‘yilgan shartlardan foydalanib ko‘rsatish mumkinki, $f(y)\in C[0,1]$, $K(y,t)$ funksiya esa $\{[0,1]\times[0,1]\}$ kvadratda $y\neq t$ bo‘lganda uzluksiz, $y=t$ da esa birinchi tur sakrashga ega.

(15) ga mos bir jinsli integral tenglamaga {(12), (13)} bir jinsli masala mos keladi. {(12), (13)} bir jinsli masala faqat trivial yechimga ega bo‘lgani uchun (15) ga mos bir jinsli tenglama ham faqat trivial yechimga ega bo‘ladi. U holda, Fredgolm alternativasiga [3] asosan, (15) integral tenglamaning yechimi mavjud va yagona.

$\tau(y)$ funksiya (15) dan topilgandan so‘ng $\nu(y)$ funksiya (8) tenglik bilan aniqlanadi. Shundan so‘ng $I$ masalaning yechimi $D_1$ sohada (6) formula bilan topiladi, $D_2$ sohada esa $L_2u=0$ tenglamaning (2), (3), (4) shartlarni qanoatlantiruvchi yechimi sifatida aniqlanadi. Oxirgi masalani $I_0$ deb belgilaymiz va bir qiymatli yechilishini isbotlaymiz.

Faraz qilaylik, $u(x,y)-I_0$ masalaning yechimi bo‘lsin. $u(x,1)=\varphi(x), -T\leq x\leq 0$ belgilash kiritaylik. U holda, $u(x,y)$ funksiyani $D_2$ sohada $L_2u=0$ tenglama uchun birinchi chegaraviy masalaning yechimi sifatida

$$u(x,y)=\int_0^1\tau(\eta)G(0,\eta;x,y)d\eta+\int_x^0\varphi_2(\xi)G_\eta(\xi,0;x,y)d\xi-\int_x^0\varphi(\xi)G_\eta(\xi,1;x,y)d\xi \tag{16}$$

ko‘rinishda yozish mumkin bo‘ladi [5] , bu erda

$$G(\xi,\eta;x,y)=\frac{1}{2\sqrt{\pi(\xi-x)}}\sum_{n=-\infty}^{\infty}\left\{\exp\left[-\frac{(y-\eta-2n)^2}{4(\xi-x)}\right]-\exp\left[-\frac{(y+\eta-2n)^2}{4(\xi-x)}\right]\right\},\xi>x.$$

(16) funksiyani (3) shartga bo‘ysundirib va $u(x,1)=\varphi(x)$, $x\in[-T,0]$ belgilashni e’tiborga olib, ba’zi bir shakl almashtirishlardan so‘ng

$$\varphi(x)+\int_x^0\varphi(\xi)K_1(x,\xi)d\xi=f_1(x),\qquad -T\leq x\leq 0 \tag{17}$$

integral tenglamaga ega bo‘lamiz, bu erda,

$$K_1(x,\xi)=\int_x^\xi\frac{1}{\sqrt{\pi(\xi-z)}}\frac{\partial}{\partial z}K_2(z,\xi)dz,$$

$$K_2(x,\xi)=\frac{2}{\sqrt{\pi(\xi-x)}}\sum_{n=1}^{+\infty}\left\{\exp\left[-\frac{(2n-1)^2}{\xi-x}\right]-\exp\left[-\frac{n^2}{\xi-x}\right]\right\},\ \ \xi>x,$$

$$f_1(x)=\frac{\varphi_3(0)}{\sqrt{-\pi x}}+\frac{1}{\sqrt{\pi}}\int_x^0\frac{1}{\sqrt{\xi-x}}\left[\Phi_2'(\xi)-\varphi_3'(\xi)\right]d\xi,$$

$$\Phi_2(x)=\int_0^1\left[\int_0^1\tau(\eta)G(0,\eta;x,y)d\eta+\int_x^0\varphi_2(\xi)G_\eta(\xi,0;x,y)d\xi\right]dy.$$

Berilgan funksiyalarga qo‘yilgan shartlarga asoslanib ko‘rsatish mumkinki, $f_1(x)\in C[-T,0]$, $K_1(x,\xi)\in C([-T,0]\times[-T,0])$. Buni va (17) – Volterning ikkinchi tur integral tenglamasi [4] ekanligini e'tiborga olsak, (17) integral tenglama $[-T,0]$ oraliqda uzluksiz bo'lgan yagona yechimga ega [4]. Demak, $I_0$ masala ham yagona yechimga ega.

Shu bilan $I$ masala to'la hal bo'ldi.

Adabiyotlar


1. Nasriddinov, O., Maniyozov, O., & Bozorqulov, A. (2023). Xususiy hosilali differensial tenglamalarning umumiy yechimni topishning xarakteristikalar usuli. *Research and implementation*.
2. Maniyozov, O., Shokirov , A., & Islomov , M. (2023). Matritsalarni arxitektura va dizayn soxasida tatbiqi. *Research and Implementation*. извлечено от https://fer-teach.uz/index.php/rai/article/view/1007.
3. Jo'raeva, D. (2022). Buziladigan oddiy differentsial tenglama uchun birinchi chegaraviy masala. O'zbekistonda fanlararo innovatsiyalar va ilmiy tadqiqotlar jurnali, 2(13), 456-461.
4. To'xtasinov, D. F. (2018). Didactic Bases Of Development Of Logical Thinking In Schoolchildren. *Central Asian Journal of Education*, *2*(1), 68-74.
5. Ergashev, T. G., & Tulakova, Z. R. (2021). The Dirichlet problem for an elliptic equation with several singular coefficients in an infinite domain. *Russian Mathematics*, *65*(7), 71-80.
6. Tolipov, N., Isaxonov, X., & Zunnunov, M. (2023). Shar tashqarisidagi soha uchun garmonik davom ettirish masalasi. Research and implementation.
7. Madibragimova, I. M. (2023). Matematika darslarida muammoli ta'lim. *Principal issues of scientific research and modern education*, *2*(6).
8. Yusupov, Y. A. (2018). Algorithms for adaptive identification of parameters of stochastic control objects. *Chemical Technology, Control and Management*, *2018*(2), 74-77
9. Saidov, M., & Isroilov, S. (2023). To'rtinchi tartibli bir jinsli bo'lmagan tenglama uchun aralash masala. Research and implementation.

## ВНИМАНИЕ!

При использовании материалов журнала необходимо указывать источник
Авторы статей несут ответственность за содержание статей и за сам факт их публикации.
Редакция не всегда разделяет мнения авторов и не несет ответственности за недостоверность публикуемых данных.
Редакция журнала не несет никакой ответственности перед авторами и/или третьими лицами и организациями за возможный ущерб, вызванный публикацией статьи.

---

## ATTENTION!

When using journal materials, you must indicate the source.
The authors of the articles are responsible for the content of the articles and for the very fact of their publication.
The editors do not always share the opinions of the authors and are not responsible for the unreliability of the published data.
The editorial board of the journal does not bear any responsibility to the authors and / or third parties and organizations for possible damage caused by the publication of the article

---

## DIQQAT!

Jurnal materiallaridan foydalanganda manbani ko'rsatish kerak.
Maqola mualliflari maqolalar mazmuni va ularning nashr etilishi fakti uchun javobgardirlar.
Har doim ham mualliflarning fikrlari tahririyat nuqtai nazarini ifodalamaydi va nashr etilgan ma'lumotlarning haqiqiyligi uchun javobgar emas.
Jurnal tahririyati mualliflar va/yoki uchinchi shaxslar va tashkilotlarga maqolaning eʼlon qilinishi natijasida yetkazilishi mumkin boʻlgan zarar uchun javobgar emas.