ISSN: 2455 – 5428
International Journal of Current Research and Modern Education
Impact Factor 6.725, Special Issue, July - 2017

SUPER MAGIC LABELING OF CYCLE, WHEEL AND P_n^2

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Cite This Article: Dr. D. Ramprasad, "Super Magic Labeling of Cycle, Wheel and P_n^2", International Journal of Current Research and Modern Education, Special Issue, July, Page Number 75-78, 2017.

Abstract:

In this paper, we discussed P_n^2 , C_n and W_n. Further we introduce the new concept weak edge set of super magic labeling at the end.

Key Words: Super Magic Labeling, Super Magic & Weak Edge Set

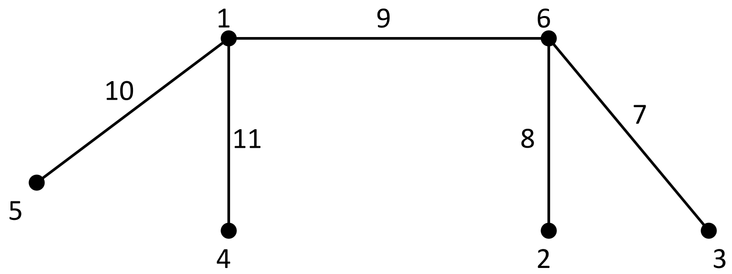
1. Introduction:

We consider finite undirected graphs without loops and multiple edges. We denote by V(G) and E(G) the set of vertices and the set of edges of a graph G, respectively. The set of vertices adjacent to x in G is denoted by $N_G(x)$ and $\deg_G(x) = |N_G(x)|$ is the degree of x in G. Let G be a graph with p vertices and q edges. A bijection f from $V(G)\cup E(G)$ to {1, 2, …., p+1} is called an edge – magic labeling of G if there exists a constant s (called the magic number of f) such that f(u) + f(v) + f(uv)=S for any edge uv of G. An edge magic labeling f is called super edge magic if f(U(G)) = {1, 2, ….p} and f(E(G)) = {p+1,…., p+q}. A graph G is called edge – magic if there exists an edge – magic labeling of G.

2. Preliminaries:

Definition 1: An edge magic labeling of a graph *G*(*V*, *E*) is called a super edge magic labeling of graph *G*, if $f(V) = \{1, 2, ..., p\}$ and $f(E) = \{p+1, p+2, ..., p+q\}$

Example:

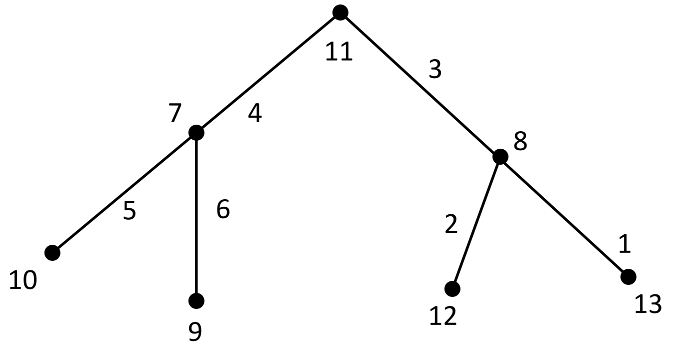


Definition 2: A graph is said to be super edge magic if it has a super edge magic labeling.

Definition 3: The super edge magic strength of a graph *G*, *sm*(*G*) is defined as the minimum of all *c(f)* where the minimum is taken over all super edge magic labeling *f* of *G* if there exists atleast one such super edge magic labeling. That is $sm(G) = \min\{ c(f) : f$ is a super edge magic labeling of $G\}$.

Definition 4: An edge magic labeling of a graph $G(V, E)$ is called super magic labeling of G if $f(E) = \{1, 2, ..., q\}$ and $f(V) = \{q+1, q+2, ..., p+q\}$.

Example:



Definition 5: The super magic strength of a graph *G*, *sms*(*G*) is defined as the minimum of all $c'(f)$ where the minimum is taken over all super magic labeling f of G if there exist atleast one such super magic labeling. That is $sms(G) = \min\{ c'(f) : f$ is a super magic labeling of G}.

3. Basic Results:

Theorem 1: $sms(C_n) = \frac{1}{2}(7n+3)$ where $n \ge 3$ odd and $n = 2m+1, \ m \ge 1$.

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Impact Factor 6.725, Special Issue, July - 2017

Proof: Let the vertex sequence of C_n be $v_0, v_1, v_2, ..., \; v_{n-1}, v_0$ and let edge sequences be $v_i \; v_{i+1}; \; i = 0,1,..., \; n-1$. Consider a super magic labeling $f : V(G) U E(G) \to \{1,2,..., 2n\}$ defined by

$$f(v_i) = \begin{cases} 4m - (i-1)/2 + 2, & 1 \le i \le 2m+1; \; i \text{ is odd} \\ 3m - i/2 + 2, & 0 \le i \le 2m; \quad i \text{ is even} \end{cases}$$

$$f(v_i, v_{i+1}) = i, \text{ for } \; 0 \le i \le 2m+1$$

Let us find the magic constant $c'(f)$

$$c'(f)(v_i v_{i+1}) = f(v_i) + f(v_{i+1}) + f(v_i v_{i+1})$$

$$= [4m - (i-1)/2 + 2] + [3m - (i+1)/2 + 2] + i + 1 = 7m + 5$$

Thus f is super magic labeling with magic constant $c'(f)$

$$= 7m + 5 = \frac{1}{2}(7n+3)$$

Thus C_n is super magic and $sms\,(C_n) \le \frac{1}{2}(7n+3)$.

In the next part we prove that $sms\,(C_n) \ge \frac{1}{2}(7n+3)$ suppose there exists a super magic labeling f of C_n with $c'(f) = \frac{1}{2}(7n+3)$. Note that

$$nc'(f) = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e) \quad = 2 \sum_{i=n+1}^{2n} f(v_i) + \sum_{i=1}^{n} f(e_i)$$

$$= 2 \sum_{i=1}^{2n} f(v_i) - \sum_{i=1}^{n} f(e_i) = 2\frac{(2n(2n+1))}{2} - \frac{n(n+1)}{2}$$

$$nc'(f) = \frac{1}{2}\left[8n^2 + 4n - n^2 - n\right] = \frac{1}{2}\left[7n^2 + 3n\right]$$

$$nc'(f) = \frac{1}{2} n [7n+3]$$

$$c'(f) = \frac{\frac{1}{2} n [7n+3]}{n}$$

$$c'(f) = \frac{7n+3}{2}$$

Thus $sms\,(C_n) = \frac{(7n+3)}{2}$

Lemma 1: If a non trivial graph G is super magic then $q \le 2p - 3$.

Theorem 2: A wheel $W_n = C_n + K_1$ is not a super magic.

Proof: Lemma 1 is significant in the sense that it eliminates huge number of graphs from being super magic graphs. It is interesting to find families of super magic graph that satisfy $q \le 2p - 3$. Since W_n has $p = n+1$, $q = 2n$, $2p - 3 = 2(n+1) - 3 = 2n - 1$, does not satisfy the above in equality. Thus wheel is not a super magic.

Theorem 3: Disjoint union of 2 copies of c_3 is not super magic

Proof: Suppose that f be a super magic labeling of $2c_3$ with the magic constant $c'(f)$.

In $2c_3$ there are 6 vertices as well as 6 edges.

Adding all the constants, we get

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$$6c' = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e)$$
$$= (1+2+3+4+5+6) + 2(7+8+9+10+11+12)$$
$$= 21 + 2(57)$$
$$= 21 + 114$$
$$= 135$$

Since $6c'$ is an even number, it is not possible to obtain a super magic labeling for $2c_3$.

Theorem 4: $sms\ (P_n^2) = 6n - 6$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n^2.

Then $E(P_n^2) = \{v_i\ v_{i+1} : 1 \le i \le n-1;\ v_i v_{i+2} : 1 \le i \le n-2\}$.

Define super magic labeling f of P_n^2 as follows:

$f(v_i) = 3n - (2+i)$ for $1 \le i \le n$,

$f(v_i\, v_{i+1}) = 2i - 1$ for $1 \le i \le n-1$ and

$f(v_i\, v_{i+2}) = 2i$ for $1 \le i \le n-2$.

Let us find the magic constant $c'(f)$.

$$c'(f)(v_i v_{i+1}) = f(v_i) + f(v_{i+1}) + f(v_i v_{i+1}),\quad 1 \le i \le n-1$$
$$= 3n - (2+i) + 3n - (3+i) + 2i - 1$$
$$= 6n - 6,\ \text{for } 1 \le i \le n.$$

Now $c'(f)(v_i v_{i+2}) = f(v_i) + f(v_{i+2}) + f(v_i v_{i+2}),\ 1 \le i \le n-2$

$$= 3n - (2+i) + 3n - (4+i) + 2i$$
$$= 6n - 6,\ \text{for } 1 \le i \le n-1.$$

Therefore sms $(P_n^2) \le 6n - 6$. ...(1)

But since $q = 2n - 3$

$$qc'(f) = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e)$$
$$(2n-3)c'(f) = 2f(v_1) + 2f(v_n) + 3f(v_2) + 3f(v_{n-1}) + \sum_{i=3}^{n-2} 4f(v_i) + \sum_{e \in E} f(e)$$
$$= 2(3n-3) + 3(3n-4) + 3(2n-1) + 2(2n-2) + 4[2n + \ldots + (3n-5)] + [1+2+3+\ldots+(2n-3)]$$
$$= (6n-6)(2n-3).$$

Thus $c'(f) \ge 6n - 6$ and hence $sms(P_n^2) \ge 6n - 6$. ...(2)

From equations (1) and (2).

$$sms(P_n^2) = 6n - 6$$

Motivated by theorem 4 we introduce the concept weak edge set of super magic labeling.

Definition 6: Let G be any graph having atleast one super magic labeling set of non pendant edges whose removal decrease the super magic strength is called weak edge set.

Example: Consider the path $P_6 = (v_1 v_2 v_3 v_4 v_5 v_6)$ on 6 vertices. We know that $sms\ (P_6) = 20$

Deleting the set of edges $s = \{v_2 v_3, v_4 v_5\}$ from P_6, we get $3P_2$.

By theorem 4, $sms\ \ 3P_2 = 15$. Thus s is the weak edge set of P_6.

4. Conclusion:

In this paper, we proved that wheel W_n ad 2 copies of $C_3, 2C_3$ are not super magic graph. Further we introduce the new concept weak edge set of super magic labeling.

5. References:

1. D. G. Akka and Nanda S. Warad, Super magic strength of a graph. Indian J. Pure Appl. Math., 41(4) (2010): 557-568.
2. S. Avadayappan, R. Vasuki and P. Jeyanthi, Magic strength of a graph. J. Pure Appl. Math., 31(7) (2000): 873-883.
3. S. Avadayappan, P. Jeyanthi and R. Vasuki, Super magic strength of graph. J. Pure Appl. Math., 32(11) (2001): 1621-1630.
4. J.A. Bondy and U.S.R. Murthy, Graph theory with Applications, Macmillan, London (1976).
5. H. Enomoto and A.S. Llado', T. Nakamigawa and G. Ringel, Super edge magic grapsh. SUT J. Math., 34(2) (1998): 105-109.