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Theme:
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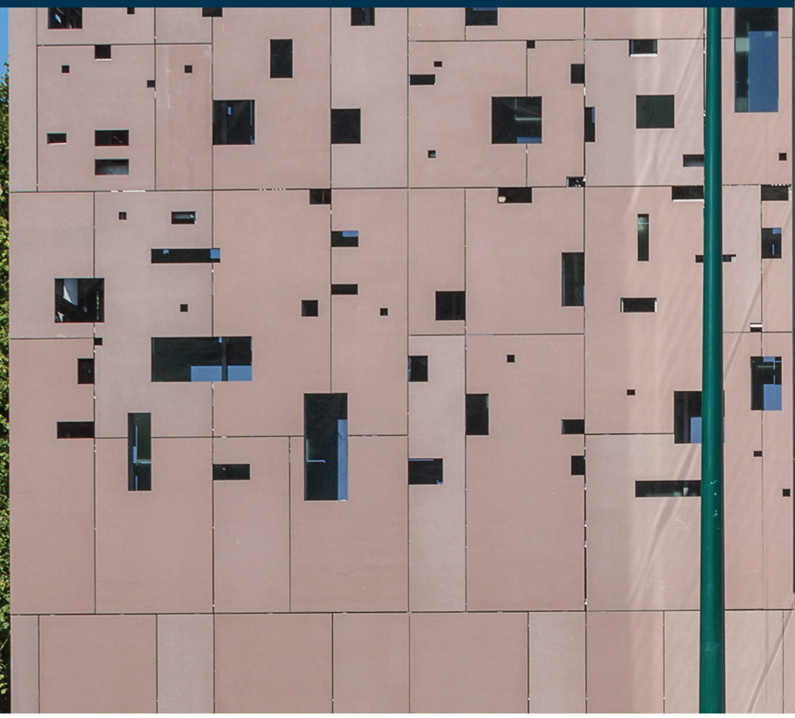

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CASTeL

MEI 9

Foreword

It is with great pleasure that we present the following proceedings from the Ninth Conference on Research in Mathematics Education in Ireland (MEI 9), which took place in Dublin City University (DCU) in October, 2023. The conference entailed two keynote presentations, two symposia, a panel discussion and a number of oral paper and poster presentations. The conference theme for MEI 9 was Conceptualising Success in Mathematics Education.

The discourse on success in mathematics education is populated with reference to student achievement and proficiency on standardised examinations – a distinguishing, tangible measurement taken to represent the success, or by association, failure, of a child, student, or adult, in their studies of mathematics. Scholars, however, contend that taken-for-granted ways of operating that privilege some individuals and exclude others should be uncovered and challenged to not just better understand, but to transform mathematics education. Therefore, MEI 9 served to provide a space to investigate what – and who – constitutes success in mathematics education, and how this is achieved.

As part of MEI 9, the keynote presentations, which were delivered by Professor Susanne Prediger and Dr Niamh O' Meara, and the panel discussion, involving Dr Thérèse Dooley, Dr Zita Lysaght, Dr Vasiliki Pitsia and Dr Gerry Shiel directly addressed the conference theme. These invited contributions ensured that MEI 9 afforded the opportunity for those working in the field of mathematics education to challenge existing conceptions and perceptions of success, and initiate new meanings, collaboratively developed, fitting with the needs of the informed, equitable, and socially-just citizen of the 21st century.

These proceedings contain 43 papers from 77 authors. Each paper was peer-reviewed by conference participants and subsequently edited and refined by authors, before being accepted for publication in this electronic conference proceedings. The papers document research from all levels of education, with contributions from scholars researching early childhood education, primary and secondary schooling, third-level education and adult learning. These proceedings capture the important research work in mathematics education that is being carried out across the continuum of education.

We are very pleased that MEI 9 was successfully hosted in DCU. We acknowledge the significant work done by all of the Organising Committee and Scientific Committee, and others within DCU, who assisted with the preparation for and facilitation of MEI 9. We were very fortunate to have a number of sponsors for the conference: the National Council for Curriculum and Assessment, CASTeL, Kingspan, the Irish Mathematical Society and Edco. We sincerely thank them for their generous support of the conference. We are very grateful to the two keynotes and panel members who accepted the invitation to contribute to MEI 9 and submitted papers for inclusion in the conference proceedings. Finally, we thank the authors who responded to the call, submitted and reviewed papers, and participated in the conference. We look forward to meeting with you all again for MEI 10 in 2025.

Aisling Twohill and Stephen Quirke

From task completion to learning progress: Shifting mathematics teachers' conceptualisations of success as a key challenge in professional growth

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Abstract

Heterogeneous conceptualisations of success in mathematics instruction influence what is strived for in mathematics classrooms, this is widely discussed for policy perspectives, student perspectives, and classroom observer perspectives. In each case, alignment to assessment has been identified as critical condition. This plenary paper adds to this discourse a fourth perspective, the teacher perspective, by summarizing research findings from professional development research projects that identify teachers' individual categories for successful mathematics teaching as critical for instructional practices and for their pathways of professional growth. In particular, the examples show that teachers' success category of task completion often dominates their practices. When they shift to success categories concerning learning progress, their pathways of professional growth can accelerate.

Keywords: Teacher professional development, conceptualisation of success, task completion

Introduction

The MEI conference theme "Conceptualising Success in Mathematics Education" is described on the website as follows: "The discourse relating to success in mathematics education is populated with reference to student achievement and proficiency on standardised examinations" and announces "a space to investigate what – and who – constitutes success in mathematics education, and how this is achieved". This plenary paper intends to contribute through teacher professional development (PD) research, by investigating and promoting teachers' conceptualisations of success for mathematics teaching and learning. Rather than presenting one classical research report, research findings from several PD research projects in our research group are synthesized. Section 1 gives with a brief overview on different perspectives on conceptualisations of success in mathematics classrooms, deriving why research on teacher perspectives could be essential. Section 2 briefly introduces the theoretical background, and Section 3 presents findings from several research projects on the role of a prevalent success category, task completion. Section 4 widens to further projects and Section 5 discusses the investigated teacher perspective in connection to other perspectives.

1. Different perspectives on conceptualisations of success

This section starts from conceptualisations of success of mathematics instruction, which can be defined as those prescriptions that guide different stakeholders' evaluations or actions for developing mathematics instruction. Section 2–4 will then narrow it down to teachers' implicit or explicit success categories that guide their instructional practices.

1.1 Policy perspectives and constructive alignment of state assessments

From a policy perspective, conceptualisations of success are prescriptions from state educational authorities as to what schools and teachers should aim at.

From a *policy curriculum perspective*, the most frequent approach to conceptualising success of mathematics instruction is to prescribe the learning goals that schools should strive to achieve. Since mandatory schooling had been established as a public responsibility some centuries ago, the learning goals were successively codified in national, state or district curricula (syllabi and textbooks, later also in standards). Since the 1960s, the scope of these official curricula were increasingly extended to more ambitious learning goals, from procedural skills in arithmetic to skills in algebra and geometry and then to also include conceptual understanding (Clements et al., 2013), further mathematical domains (e.g. set theory and probabilities), and since the 1980s increasingly also mathematical practices such as modelling, problem solving, arguing and others (Cockcroft Report, 1982; Silver & Lane, 1993). So, from the policy curriculum perspective, we might summarize that policy conceptualisations of the success of mathematics instruction have become more and more ambitious.

However, many observations of conditions for curriculum reforms to be implemented revealed that extensions in syllabi, standards, and curricula do not necessarily impact mathematics classroom practices as long as the state assessments do not cover the whole range of extended learning goals. This basic idea of *constructive alignment* was prominently articulated in the influential Cockcroft Report (1982) and later summarized as “What you test is what you get” (Burkhardt et al., 1990, see also Biggs, 1996). This observation led to efforts for enhancing the coverage of assessments to include all relevant learning goals, not only for high-stakes state assessments, but also for (summative and formative) assessment for learning (Silver & Lane, 1993; Burkhardt & Schoenfeld, 2018), resulting in an increasing discourse on conceptual understanding and mathematical practices as relevant learning goals in mathematics instruction. But already Burkhardt et al. (1990) emphasized that what is tested in state assessments can influence what is *taught*, but not necessarily what is *learned* by all students.

So, a second *policy perspective* concerns the increasing emphasis on *equitable access for all students,* so that not only a small minority gets access to ambitious learning goals (Clements et al., 2013). While in most OECD countries, the goal of formal institutional access to schooling is widely achieved (Callahan, 2005), the dominant *conceptualisation of success with regard to equitable access* also refers to all students’ opportunities for *achieving* in mathematics, not only their presence in classrooms (OECD, 2016). Most educational systems fail to provide such equitable access, as continuously documented by unequally distributed mathematics achievements in large-scale assessments for students from diverse socio-economic backgrounds, diverse language proficiencies, learning disabilities or other diverse abilities (OECD, 2016; Faragher, Hill & Clarke, 2016). Schools’ failure can often be traced back to opportunity gaps through low-quality teaching (Boaler, 2002; DIME, 2007).

The discourse about inequitable achievement has been problematized as a “gap-gazing fetish” (Gutiérrez, 2008) that is too restricted to too narrow educational goals and fuels deficit views on students from marginalized backgrounds. While I would totally agree that the persistent hint to social or immigration-related achievement gaps risks strengthening stereotypes and has its own contribution in reproducing inequitable learning opportunities (Boaler, 2002; Wilhelm et al., 2017), this deconstruction does not yet automatically lead to other constructive approaches to empower students from marginalized backgrounds. So, we need to continue to develop approaches for enhancing *all* students' mastery of ambitious learning goals, such as conceptual understanding of mathematics.

1.2 Student perspectives and the challenge of focusing assessments

With regard to *students'* learning, studies in learning sciences have repeatedly shown that attainment of ambitious learning goals (such as conceptual understanding or mathematical practices) requires students' deep cognitive engagement, whereas shallow cognitive processes can mainly serve more superficial learning goals (like facts or procedural knowledge) (Chi et al., 2018; Hiebert & Grouws, 2007).

But this might not necessarily be reflected in subjective conceptualisations of success from *students' perspective.* For students, a successful mathematics lesson might be one in which deep cognitive engagement was not necessary. Illusions of understanding for example can subjectively substitute deep cognitive engagement, as when students love certain YouTube videos more than their teachers' explanations even if they do not retain the presented knowledge aspects (Kulgemeyer, 2020).

Already Nicholls, Cobb, and colleagues (1990) investigated how students conceptualise success in mathematics classrooms and found that some students conceptualised success as being better than their peers, other students by gained understanding. The latter group believed also more in the worth of efforts in classrooms. This early finding resonates with Dweck's (1996) often-cited distinction into mastery orientation and performance orientation: students with mastery orientation strive for developing mastery in the content to be learned, whereas students with performance orientation strive for demonstrating their competence without necessarily improving mastery. Students with mastery orientation usually appreciate deep cognitive engagement more than students with performance orientation who might, e.g., misinterpret a productive struggle as a sign of bad performance.

For both, students with performance orientation and with mastery orientation, the alignment of assessments to the articulated learning goals is crucial: when the assessments suggest that shallow knowledge is sufficient, then students with mastery orientation might not even be aware of the depth that could be reached. Leber et al. (2018) showed that even for university students who voluntarily participated in a course with explicitly articulated conceptual learning goals, constructive alignment matters: for half of the students, a mis-aligned fact-oriented test was announced. The other half of the students who expected the aligned understanding-oriented test used significantly deeper elaboration strategies than in the group expecting a fact-oriented test.

To sum up student perspectives, students' individual conceptualisations of success and external success criteria set by assessments can impact students' learning processes substantially.

1.3 Classroom observer perspective

In several decades of classroom research, quality dimensions for effective mathematics instruction that lead to higher learning outcomes have been identified, in particular classroom management, high cognitive demands, conceptual focus, and instructional support (Brophy, 2000; Hiebert & Grouws, 2007; Pianta & Hamre, 2009). But research findings alone do not lead to increasing classroom quality: mathematics teachers in most countries continue to enact mathematics teaching with strongly varying qualities (e.g., TALIS study, OECD, 2020). Among many other factors such as some teachers' limited access to pedagogical content knowledge (PCK, Shulman, 1986) or challenging external conditions in under-resourced classrooms, it was shown that heterogenous *conceptualisations of successful teaching* (e.g., in the so-called visions of good teaching, Cobb & Jackson, 2021) can be an important factor explaining varying qualities of enacted teaching.

Heterogeneous conceptualisations of successful teaching have been documented in a study about observers' criteria for selecting effective teaching (Strong et al., 2011). Videoclips with teaching episodes were shown to school administrators, teacher leaders, mentors, and teacher educators to select the effective / non-effective teachers (by subjectively inferring who might induce high / low learning gains). The observers selected the effective teachers only with 31% – 47% accuracy (below probability of guessing), with individual success criteria that deviated substantially from the quality dimensions in the consolidated state of classroom quality research. Most often, observers applied surface criteria for successful teaching (e.g., small group work, use of visuals and manipulatives for hands-on activities), not the deeper structures of effective teaching (e.g., depths of minds-on activities).

These findings on *classroom observer perspectives* are remarkable because classroom observations by teacher educators and administrators can be considered as the typical assessment for teachers, and again, alignment with the regular goals seems critical but cannot be taken for granted. Indeed, alignment in the visions of good teaching between school administrators, headmasters, teacher educators, coaches, and teachers has been identified as a decisive condition for jointly developing teaching quality (Cobb & Jackson, 2021).

1.4 Research focus: Investigating and enhancing teachers' perspectives on success as critical factors for professional growth

The persistence of low-quality teaching, in particular in classrooms with many underprivileged students (Boaler, 2002; DIME; 2007), and the findings about the needed shared vision of successful teaching (Cobb & Jackson, 2021) raises the question about teachers' individual conceptualisations of successful teaching and the connection to their practices.

The deviations in conceptualisations that were sketched in the brief literature review from several perspectives, the policy curriculum perspective, the policy equity perspective, the student perspective, and the classroom observer perspective, suggest that also teachers' individual conceptualisations might be a) heterogeneous. In the following, we will argue that they might also be b) influential for teachers' practices, c) a hindrance for professional growth, and/or d) when shifted, then a potential motor for professional growth. Within the theoretical framework that will be outlined in the next section, the paper collects findings from several PD research projects of our research group with regard to three research questions:

- How are teachers' practices influenced by their conceptualisations of success?
- How are teachers' pathways of professional growth influenced by conceptualisations of success?
- How can shifts in teachers' conceptualisations of success be initiated, and how do they impact the professional growth of instructional practices?

2. Theoretical framework for describing and explaining the role of teachers' conceptualisations of success for practices and pathways of professional growth

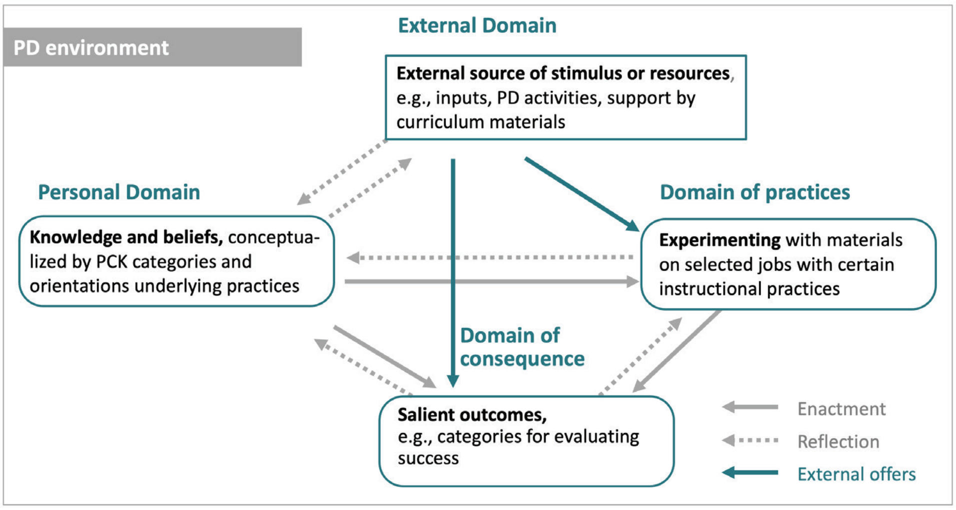
For describing and explaining the role of teachers' conceptualisations of success for their instructional practices, our research group draws upon the *Framework of Content-Related Teacher Expertise* (Prediger, 2019 adapted from Bromme, 1992 and Schoenfeld, 2010) that combines cognitive perspectives on teachers' knowledge and attitudes with situated perspectives on teaching practices (Depaepe et al., 2013). Five constructs are used to explain teachers' practices as recurrent answers for typical situational demands (jobs) by means of the underlying components:

- *Jobs*: typical and often complex situational demands that teachers have to master in classrooms (in each PD project, we focus on specific jobs of relevance for the PD content in view, for example fostering at-risk students' understanding).
- *Practices*: recurrent patterns of teachers' utterances and actions for managing the jobs – teachers' practices can be characterized by the underlying categories, pedagogical tools, and orientations upon which teachers implicitly or explicitly draw:
  - *Pedagogical tools*: tangible or visible tools applied to manage the jobs (e.g., facilitation moves, assessment tasks, manipulatives, or other instructional artifacts).
  - *Categories*: Categorical elements that filter and focus teachers' perceiving and thinking. These comprise, e.g., pedagogical content knowledge (PCK) categories that teachers explicitly or implicitly chose as their filters, but also teachers' individual evaluation categories for assessing the success of their teaching.
  - *Orientations*: Generic or content-related beliefs and pedagogical attitudes about mathematics and its teaching and learning that implicitly or explicitly guide the teacher's perception and prioritization of jobs (see Schoenfeld, 2010, p. 29).

For explaining and promoting teachers' pathways of professional growth, we draw upon the well-established *Interconnected Model of Professional Growth* by Clarke and Hollingsworth (2002), which has been widely used, not only for describing and explaining, but also for designing PD programs promoting professional growth in modes of action and reflection. The model refers to four analytic domains (adapted by Prediger, 2024, see Figure 1): the *external domain* (with external sources of information, stimulus, or supportive curriculum materials for teachers), the *personal domain* (teachers' knowledge or attitudes), the *domain of practice* (which refers to a broad meaning of practice, but in which experimentation with new teaching practices in the above narrow sense can take place) and the *domain of consequence* (with salient outcomes such as students' motivation or learning gains to be observed). The model identifies different mechanisms by which change in one domain can be associated with change in another. Rather than claiming simple mechanisms of transmission from the *external domain* via changing the *personal domain* to the *domain of practice* and then to the *domain of consequence*, Clarke & Hollingsworth emphasize an "interconnected, non-linear structure" between these domains and identify different "particular 'change sequences' and 'growth networks', giving recognition to the idiosyncratic and individual nature of teacher professional growth" (2002, p. 947). An example of such an individual change sequence is when teachers experiment in the domain of practice, monitoring students' thinking in the domain of outcomes, and thereby expand their knowledge about student thinking in the individual domain, which is then the result of the change sequence rather than its start. The slightly adapted model that draws upon the introduced constructs of jobs, practices, categories, orientations, is printed in Figure 1.

Figure 1

Interconnected Model of Professional Growth (adapted by Prediger, 2024, from Clarke & Hollingsworth, 2002)



3. Empirical insights into the relevance and persistence of the success category of task completion in the Mastering Math project

In this section, the framework is illustrated and substantiated by empirical insights into our long-term Mastering Math project that aims at fostering the understanding of underprivileged students who are at risk of failing due to limited learning opportunities in earlier school years (Prediger et al., 2019; 2023). Within the Mastering Math project, curriculum materials with formative assessment tasks and enhancement tasks were developed and offered to teachers that help to discover these kinds of missing conceptual understanding and turn teachers' attention to the needed understanding of basic concepts in which the multi-digit procedure is grounded (Hiebert & Wearne, 1996). In other contexts, our Mastering Math intervention program for fifth graders was shown to be effective for giving students safe access to the understanding of basic concepts, with significantly higher learning gains than in the control group (Prediger et al., 2019). However, the learning gains varied substantially and relied heavily on the effectiveness of the teachers, so further PD research was required for optimizing support for all teachers and teacher teams. Within the PD research, we found that teachers' practices were driven by two main pairs of orientations (Prediger et al., 2023):

- *Conceptual orientation* rather than *procedural orientation*: Whereas goal setting and monitoring practices in procedural orientation mainly focus on procedural knowledge elements and prioritize them as learning goals, goal setting and monitoring practices in *conceptual orientation* focus on conceptual understanding of procedures and concepts (Zohar et al.,2001). With respect to differentiation practices, teachers tend to choose procedural tasks for low-achieving students and more conceptual tasks for higher achieving students (Beswick, 2007). When evaluating the success of teaching, teachers with a more conceptual orientation usually apply the *success category of conceptual learning progress*, whereas teachers with a more procedural orientation can apply the *success category of procedural learning progress*.
- *Long-term rather than short-term orientation:* Whereas goal setting and monitoring practices in short-term orientation only focus on the superficial performance in actual learning content, a long-term orientation helps teachers to focus on deep and sustainable mastery of learning content (Dweck, 1996; Watson & de Geest, 2005). In the following, we will show how the short-term orientation can completely change teachers' evaluation practices when only the *success category of task completion* is applied instead of monitoring students' (conceptual or procedural) learning progress.

3.1 First vignette: Paul's differentiation practice with the success category of task completion minimizes learning opportunities

The vignette was first published and analyzed in Prediger (2024). In the very first phase of our PD design research project Mastering Math, researcher facilitators from our Dortmund research team visited active schools in which teacher teams collaboratively developed their teaching practices to increase access to mathematics for at-risk students. In the first meeting with the facilitator researcher, one teacher team reported proudly that during the last nine months, they had differentiated their teaching material to adapt to their fifth graders' diverse mathematical abilities, through task-based individualized settings. After the meeting, the teacher team joined the research-practice-partnership.

In the first meeting, (the mathematics teacher here called) Paul reported about Suleika, one of their students with learning difficulties, and showed two of her products on multi-digit subtraction (in Figure 2):

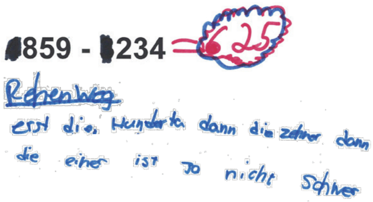
> Paul: Suleika can calculate the subtraction well, only the carries pose problems for her. But we can handle this successfully by differentiated tasks: I only give her subtractions without carries.

Described within the Model of Professional Growth (Figure 1), Paul, Maria, and their colleagues collaborated intensively, questioned and developed new practices of differentiation (*domain of practice* in Figure 1). However, Paul's utterance was a characteristic expression of this community's differentiating practices (repeatedly articulated in the conversation): students receive individualized tasks that are optimized on a level that they can master. Within this conceptualisation of success (*in the domain of consequence* in Figure 1), the collectively established differentiation practice proved to be successful, as Suleika was able to complete her tasks.

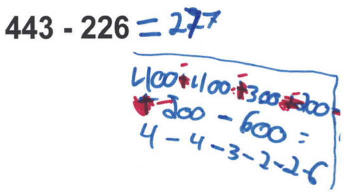
However, in spite of these teachers' intense work, they could not develop more productive practices for enhancing Suleika's learning. Although Suleika's second product reveals serious struggle with place value understanding (see Figure 2), this was not treated by the teachers. Although the teachers agreed on the success of their instructional changes, Maria, Paul's colleague, had spent three months to convince the group to participate in the Mastering Math PD program with the university, as she wanted students to continue learning.

Figure 2

Snapshot from a community: Monitoring Suleika's learning



Translation: *Strategy: first the hundreds, then the tens, then the ones, it's not that difficult*



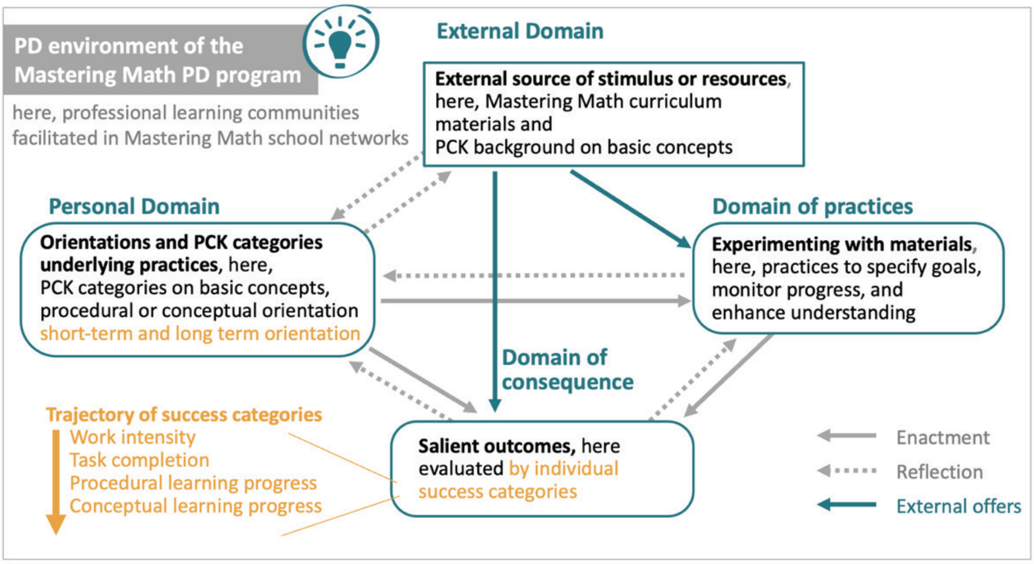
Maria's additional concerns that students "always forget" had also not yet entered the teachers' shared attention. For a deeper analysis of Suleika's assets and difficulties, the teacher team would have needed some PCK categories to unpack the error: Suleika mastered the basic skills of subtraction facts up to 10 and used them for multi-digit subtraction without carry. Subtraction with carry, however, is based on the conceptual understanding of decomposing numbers into digits. Suleika could not build on her mastery of multi-digit subtraction without carry due to limited fundamental place value understanding, visible in the highly indicative decomposition of 443 into 400-400-300 rather than 400+40+3.

As the teachers invested enormous efforts, it is important to understand why they were not reasoning about an accessible learning trajectory for Suleika from understanding basic concepts before acquiring procedures. This is not only a deficit of PCK categories, but also a consequence of their conceptualisations of success:

Within the Framework of Content-Related Teacher Expertise, the rationality of these teachers' differentiation practice can be traced back to a certain success category they applied to evaluate their teaching experiments: the teacher team's iterative pathway of action and reflection was driven by the shared idea that good inclusive classroom are adaptive to students' abilities, and they realized it using the pedagogical tools of differentiated tasks and activity settings of individualized learning. The teacher group chose their differentiation practices by the *shared success category of task completion*. Applying this category based upon a short-term orientation, the teachers evaluated the short-term success by observing if the student was able to complete the task with the given support and simplifications. Rather than focusing on how they can leverage their students' achievement to the zone of proximal development along the learning trajectory, these teachers optimize their differentiating practices in a way that all students can succeed to complete the task, even if the learning opportunities are taken away. Meanwhile, it was Maria's discontent with students not learning that led her to search new external input in initiating the research-practice-partnership within the Mastering Math PD project. In contrast to Paul, she adopted a long-term perspective in evaluating her success, yet needed three months to convince the others to search for more.

Figure 3

Substantiated Mastering Math Model of Professional Growth (Prediger, submitted)



The initial theorizing from this vignette later led to the substantiated Mastering Math Model of Professional Growth as depicted in Figure 3 (from Prediger, submitted). It is already printed here as an advance organizer and strengthened by further empirical insights in the next subsections.

From early experiences with teaching practices such as Paul's, we derived the need in the Mastering Math project to support teachers in monitoring students' conceptual understanding of basic concepts such as the place value understanding or the meaning of multiplication. So, we developed the Mastering math curriculum material (Prediger et al., 2019) with not only enhancement tasks following learning trajectories for all relevant basic arithmetic concepts (as identified by Gersten et al., 2009) in 45 modules, but also developed 45 conceptually focused formative assessment sheets, as suggested by Silver & Lane (1993).

In the Mastering Math PD program, teachers were invited to experiment with the curriculum materials, were offered some inputs for the PCK background of basic concepts, and in the Mastering Math school network meetings, the experiments were prepared, discussed and reflected (Figure 2). Once Maria could convince Paul and the rest of the team to shift their evaluation attention to students' learning, the team started to extend their differentiation practices.

3.2 Second vignette: Lia's compensation practice with the success category of task completion flattens deep conceptual learning opportunities

The second vignette with the case of Lia (taken from Prediger, submitted) illustrates that the formative assessment sheets can indeed support teachers in extending a procedural orientation into a conceptual orientation and start with enhancement practices for conceptual understanding.

However, the success category of task completion can still be persistent and a hindrance for students' deeper engagement. (The mathematics teacher here called) Lia was highly dedicated to supporting her at-risk students, so she participated in the Mastering Math PD program. She was satisfied about her teaching, but discontent with students' remembrance: "We invest a lot in training multiplications such as 23 x 6. But after three weeks, my low achievers make again 20 x 6 + 3. They simply forget too much and too quickly."

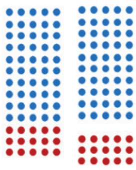
After a year in the Mastering Math PD program and experimenting with the Mastering Math curriculum materials, multiplication was one of the last contents we worked on. Lia reported proudly:

> "Only by these formative assessment tasks, I have realized that many of the kids don't know the meaning of multiplication. I invested a lot in making all of them draw full dot arrays, not only 3 dots in vertical and 5 dots in an L-form, because we must see all 15 dots. With the dot array, one student surprised me by arguing *why* 13 x 5 must be 10 x 5 + 3 x 5. Of, course, justification was too hard for the others."

The representations that Lia mentioned are printed in Figure 4, together with an advance organizer on the analysis of her pathways of professional growth.

Figure 4

Lia's changes in mathematical practices over three PD sessions and six months of experimenting with the Mastering Math curriculum materials

	Lia's practices in 1st PD session	Lia's practices after six months in the PD program
Self-report of practices	Train procedures 23 x 6 = 20 x 6 + 3 wrong 23 x 6 = 20 x 6 + 3 x 6 correct	Switch between representations: 3 x 5 can be represented by dot array  3 x 5 wrong 3 x 5 because it has 15 dots Welcome spontaneous use of dot array for justifying procedure, but "too hard for the others"  13 x 5 = 10 x 5 + 3 x 5 because both have 85 dots
Analysis		
Practices for three jobs	Set procedural learning goals Monitor correctness of procedures Foster simply by training	Set procedural and conceptual learning goals Monitor task completion Foster by supporting the task completion
Pedagogical tools	Procedural training tasks	Assessment tasks and enhancement tasks with multiple representations
Underlying categories of thinking and perceiving ex-/implicitly used	Categories for learning goals: • Procedure without errors Success category: • Task completion • Forgetting	Categories for learning goals (not yet further unpacked) • Dot arrays as the relevant representation • Procedure without errors and justified Success category: • Task completion • Still not conceptual learning progress
Underlying orientations	Procedural orientation Short-term orientation	Procedural and conceptual orientation Short-term orientation

According to the analysis presented in more detail in Prediger (submitted), Lia's pathway of professional growth can be described as follows (Figure 4): In the first PD session, Lia reported on training procedures and students forgetting them. From her report, we infer that she set procedural learning goals, and monitored only the correctness of students' procedures, so her main category of perceiving students' performance were procedures without error. Her main pedagogical tools were procedural training tasks, her fostering practices were restricted to training procedures without further learning opportunities. In total, her practices seemed to be consistently shaped by a procedural orientation. However, her short-term orientation was already slightly questioned by her discontent about students' forgetting, while the category of forgetting still assigns the responsibility of this not-lasting success to students, not to the teaching.

After three PD sessions and six months of experimenting with given curriculum material, Lia reported to have widened her practices, as she set also conceptual learning goals and monitored whether students could complete also conceptual tasks. In a certain way, this replicates findings about the role of formative assessments for installing conceptual learning goals (Burkhardt & Schoenfeld, 2018).

However, Lia continued to foster students by supporting them in completing tasks by correcting errors, even if these tasks now involve multiple representations. Her support for overcoming the misconception that 3 x 5 could be represented by an L-form (see Figure 4) was still restricted to the hint that 15 dots had to be visible, but not by articulating the underlying multiplicative structures by, e.g., three rows of fives (Götze & Baiker, 2021). She was not aware that by only alluding to the total number of dots, she could not yet enhance students' deep understanding because students continue simply switching representations. She widened her purely procedural orientation (often found in teachers of at-risk students Zohar et al., 2001; Beswick, 2007) by aspects of a conceptual orientation, but her surprise that a student justified the procedure with the dot array and her decision not to set the justification as a learning goal for all students indicates that she still fluctuated between procedural and conceptual orientation.

Lia illustrates a typical step on teachers' pathways of professional growth transcending from procedural to conceptual orientation, but yet stuck in a short-term orientation. Once having decided to set also conceptual learning goals, it can still happen that in the enactment of fostering practices, the conceptual learning opportunities are still limited by their in-the-moment focus on quick task completion rather than addresses mathematical structures in depth.

3.3 Prevalence of success category of task completion beyond the two cases

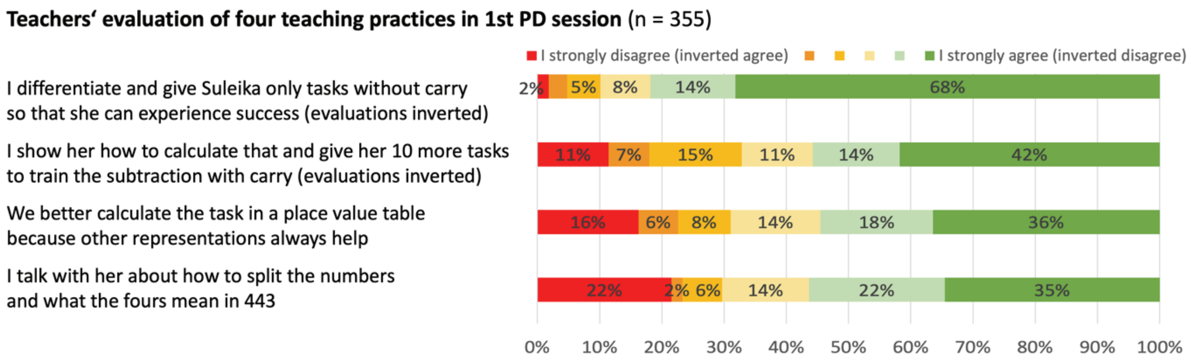
Teachers' individual *success category of task completion* instead of *learning progress* has been empirically identified in various case studies within the Mastering Math project and kindred projects (Prediger, 2024) and also by other researchers (Watson & Geest, 2005; Herbst, 2003). It is mostly connected to a short-term rather than long-term orientation (Prediger et al., 2016) and can co-occur with procedural or conceptual orientations (Prediger et al., 2023). In various contexts and for different jobs, we could show that in short-term orientations, other practices turn out to be more rational than in long-term orientations (Watson & Geest, 2005; Prediger et al., 2016). For example, many fostering practices reveal immediate support is optimized to make students complete the task (we call them compensation practices, Prediger et al., 2023) and substitute deep learning opportunities (necessary in real enhancement practices) by shallow support not adaptive to the real learning needs.

Although the case of Paul might be an extreme case, there is still a considerable percentage of teachers who share his ideas. This can be shown by the following quantitative data from n = 355 mathematics teachers in the beginning of the first PD session. Teachers were shown the student case of Suleika's erroneous subtraction (see Figure 2) and asked to evaluate four teachers' approaches in dealing with Suleika' error on a 6-point likert scale. The results depicted in Figure 5 reveal that only 68% of the teachers completely rejected Paul's differentiation practice whereas 32% disagreed to some degrees (only 2% disagreed completely).

But also the evaluation rates for the second teaching practice can be interpreted as an evidence of prevalent short-term orientations and procedural orientations. Only 42% of the teachers fully disagreed to the tell-and-train practice for subtraction procedures (second practice in Figure 5), while only 35% of the teachers fully agreed to the need to work on Suleika's place value understanding (fourth practice in Figure 5). This most productive practice intended to be established in the Mastering Math project was fully rejected by 22% of the teachers in their first PD session.

Figure 5

Quantitative evidence for prevalence of short-term orientations and task completion (from SchuMaS project led by the author)



These results give also quantitative indications that the unproductive short-term oriented practices are still widespread among teachers and need to be intensively discussed in the PD program.

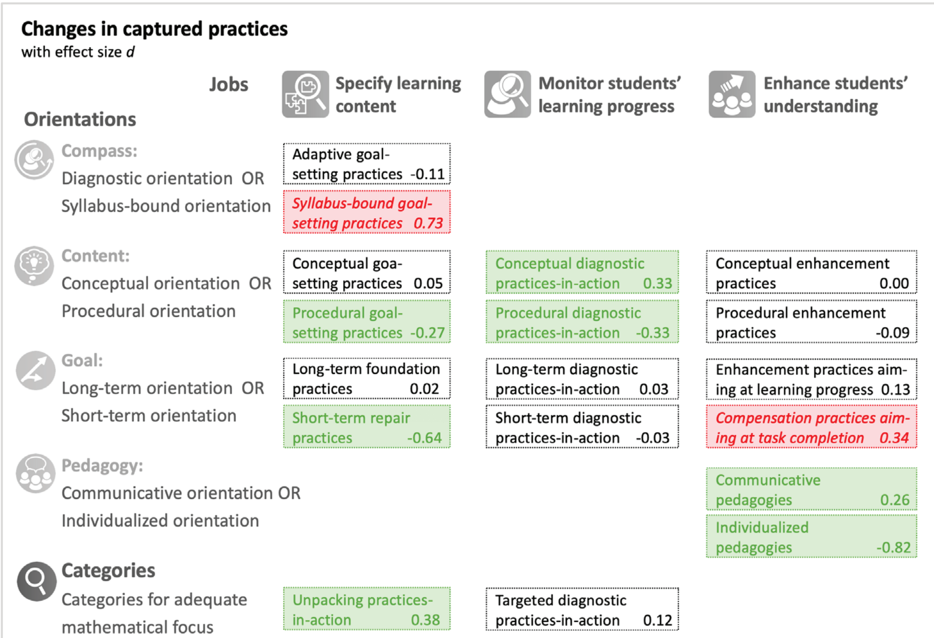
3.4 Partial shifts in success categories during the PD program

Although the *Mastering Math intervention program* proved effective on students' learning gains already in the first *field trial in 2015/16* (published in Prediger et al., 2019), the large differences between the teachers' degrees of effectiveness (Brophy, 2000) led us to continue to improve and investigate the Mastering PD program with more focused professional learning opportunities for teachers.

In order to unpack the PD learning content in more detail, we developed the already mentioned model of teacher expertise for fostering at-risk students' understanding of basic concepts (Prediger et al., 2023), by systematizing the early case studies and earlier suggestions of what teachers need to learn (e.g., by Brodie, 2013; Beswick, 2007; Gheyssens et al., 2020; Watson & Geest, 2005; and many others). The specified model of expertise contains several practices for the three jobs*: specifying learning content* (including unpacking mathematics learning content and setting learning goals for students, Morris et al., 2009), *monitoring students' learning progress* with respect to these unpacked learning goals, and *enhancing student's understanding* with respect to the unpacked learning goals (see Figure 6). For each job, we identified pairs of practices related to dual orientations (among them those found in the two vignettes in Section 3.1 and 3.2, procedural versus conceptual orientations, and short-term versus long-term orientations, but also others discussed in Prediger et al., 2023).

Figure 6

Results from the evaluation study of the Mastering Math PD program: effect sizes d from pre-PD to post-PD for each captured practice (light green color marks significant effects in intended direction, italics red marks in unintended direction)



For the *evaluation study on the PD level* conducted in 2019/20, we operationalized the practices in a standardized questionnaire of practices that teachers reported to adhere on a 5-point likert scale (from 0 to 5). We investigated the changes in (n = 95) teachers' self-reported practices from the pre-questionnaire administered in the first PD session to the post-questionnaire after 9 months of PD program (consisting of weekly experimentations with the Mastering Math curriculum materials, prepared and reflected in six PD sessions of 3 h each). The comparison of self-reported practices is summarized in Figure 6 by the effect sizes. Green colors marks significant changes in the intended direction and red color marks significant changes in the unintended direction (more details in Prediger et al., 2023).

For the practices in *procedural and conceptual orientations*, we identified significant changes for specifying practices and monitoring practices in intended directions (with effect sizes between $d = 0.27$ and 0.33), and a more focused unpacking of conceptual learning contents (as captured by vignette-based items, in the last row of Figure 6, with $d = 0.38$). For the enhancement practices, the changes in self-reports were not significant.

For the practices in *short-term or long-term orientation*, we found a significant and substantial reduction of goal-setting practices directed only to short-term repairs (with $d = 0.64$) with an intended push towards focusing basic concepts. However, for the enhancement practices, significantly more teachers ($d = 0.34$) than pre-PD reported compensation practices, even if this circumvents the enhancement of students' understanding (an example item of this scale is "When I realize that my low achievers lack basic concepts for solving a task, I scaffold them until they solve it nevertheless."). This means that after 9 months of participating in the PD program, teachers' short-term orientation had increased and the success category of task completion was even more dominantly articulated than before.

Although the Mastering Math PD program in 2019/20 showed many promising, intended effects with respect to a higher conceptual orientation and more long-term goal-setting practices, we had to realize that we still failed to shift teachers' individual success categories from task completion to learning progress (Prediger et al., 2023).

4. PD learning opportunity for shifting the success categories from task completion to learning progress

4.1 Task completion as success category in other DZLM projects

Within the research network of the German National Center for Mathematics Teacher Education (DZLM research network), task completion as a prevalent success category has also been identified in other contexts. I give only some examples with also external references to show the wide prevalence of this success category:

- In language-responsive mathematics teaching, some teachers only adopt approaches of simplifying language (Prediger, 2019). These compensation practices serve well to achieve task completion without language obstacles, yet they do not provide learning opportunities for amplifying students' language. Hence, "amplifying rather than simplifying" is the motto by which Walqui & Bunch (2019) try to shift teachers towards a stronger focus on students' language learning progress in a long-term orientation.
- Also in language-responsive mathematics teaching, many teachers tend to scaffold students' language production strongly (e.g., by sentence frames or prefabricated texts), yet without considering the fading out, so that students cannot sufficiently learn to express their ideas autonomously (Prediger, 2019). Although this kind of support can be efficient for guaranteeing task completion, van de Pol et al. (2010) emphasize that without planning and enacting the hand-over to independence, the term scaffolding should not even be used. In our PD program on language-responsive teaching, we have therefore

split scaffolding into two teacher jobs, supporting language and successively developing language (Prediger, 2019).

- In inclusive classrooms, many researchers have identified the distinction between what we call compensation practices (optimized for short-term task completion) and real enhancement practices (optimized for students' long-term learning progress), e.g., by distinguishing instructional adaptivity (differentiating the demands for allowing task completion) from curricular adaptivity (setting differentiated learning goals along students' learning trajectories, see Janney & Snell, 2006). In our MATILDA video study, we found that compensation practices (for language, mathematical prior knowledge, attention, etc.) occurred twice as often as enhancement practices (Prediger & Buró, 2021). So, we started to make this difference explicit in the PD programs and provided more professional learning opportunities for enhancement practices.
- Even in the PD program for enhancing mathematical potentials, when we worked with teachers who were willing to launch rich mathematics exploration tasks, we found that the success category of task completion still dominated the ways they monitored and intervened in students' problem-solving processes. Rather than strengthening typical germs of rich mathematical practices, they supported task completion in effective ways that yet reduced again the learning opportunities, this time for the general mathematical practices (Prediger, et al., 2016).

In each of these projects, teachers were highly engaged and wanted the best for their students' mathematical experiences. However, each time, the prevalent success category unnecessarily reduced the richness of students' mathematical learning opportunities. In each of these projects, we saw that for those teachers who shifted from task completion to learning progress as dominant success category, the pathways of professional growth profited substantially.

4.2 Consequences for enhanced PD opportunities

In the re-design of our PD programs, we have started to draw several consequences from the sketched findings about the prevalence of short-term orientations and the persistent success category of task completion. The following four consequences were most important:

- Explicit reflection opportunities on why task completion might be an insufficient success category because it might reduce rich learning opportunities to more shallow ones.
- Offers for formative assessment tools that support teachers in evaluating students' learning progress with respect to the PCK categories in view.
- Establishing long-term coherence of curricula as an important quality feature of quality mathematics instruction.
- Including more information about long-term learning progressions over several years to see the relevance of each content element in the long run.

Our current experiments with the enhanced PD opportunities in the SchuMaS PD program reveal first, humble indications that the combination of the four design consequences might enhance teachers' shifts from task completion to learning progress as major success category and might influence the ways they evaluate teaching practices. In the future, we need to investigate more systematically in how far we can succeed in promoting the growth of teachers' long-term visions of successful mathematics teaching.

5. Discussion

The paper started with a broad view on policy perspectives, student perspectives and classroom observer perspectives on how to conceptualise success of mathematics instruction. Whereas the research community and policies seem to agree that students' learning gains should be the ultimate goal (with extended visions of mathematical literacy including multiple ambitious learning goals, see Silver & Lane, 1993), teaching practices observable in classrooms do not seem to be always in line with the instructional practices that have proven most effective for reaching these ambitions learning goals (Hiebert & Grouws, 2007; Brophy, 2000; Chi et al., 2018). In order to understand this realization gap, this plenary paper has focused teacher perspectives and investigated one particular aspect of teachers' conceptualisation of success. The case studies and some selected quantitative results revealed that teachers' *highly prevalent success category of task completion* can heavily influence their practices. This was already mentioned by others (Herbst, 2003; Watson & Geest, 2005), but still requires further investigations. In this plenary paper, I have collected findings how the success category of task completion becomes visible in different jobs for fostering at-risk students' mathematics learning, in setting goals, unpacking learning goals into detailed content elements, monitoring students' progress and fostering students' learning (Prediger, et al., 2023; Prediger & Buró, 2021).

Whereas constructive alignment of curriculum and assessment has widely been discussed, teachers' weak adherence to (ambitious, conceptually focused and also formative) assessments is not only connected to too narrow assessments (Silver & Lane, 1993; Burkhardt & Schoenfeld, 2018), but also to (often unconscious) routines that optimize the teaching for immediate task completion rather than for productive struggle (Henningsen & Stein, 1997; Chi et al., 2018). Although this heavily contributes to shallow learning opportunities rather than deep cognitive engagement (Chi et al., 2018), it is still rational in a certain teacher perspective that was explored here. It is rational when evaluating the teaching by immediate and struggle-free task completion.

As a consequence, PD programs should put more effort not only in improving the tasks and the assessments, but also in shifting teachers' success categories from task completion to learning progress, also in the in-the-moment decisions of daily teaching practices. The transparent report about a partial failure in our PD program (Prediger et al., 2023) strengthens the argument that much more effort is needed to understand the background of persistence (e.g., by investigating the teachers' performance and mastery orientation in the classical sense of Dweck, 1996) and to develop and investigate PD learning opportunities to support the overcoming of the unfaithful success category.

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What constitutes success in mathematics education in Ireland and what obstacles stand in the way of this success: A decade in review

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Abstract

Success in mathematics means many different things to many different people. In Ireland, from a policy perspective, the notion of what it means for students to be successful in mathematics at post-primary level has changed in recent years. This change in perspective coincides with the implementation of mathematics curriculum reform and incentives to increase the uptake of study of advanced mathematics. This paper focuses on the evolution of the notion of success in mathematics education over the past decade and the role of different initiatives, which impacted, in different ways, teachers' and students' perceptions of success. Drawing on various research findings, the paper identifies the challenges – teacher knowledge, mathematics instructional time and students' attitudes and motivations – brought about by the initiatives which have limited access to 'success' for many students. Finally, the author will discuss that while the shift in what constitutes success was admirable it has had some many unforeseen consequences. It is these consequences that will need to be considered by all involved in mathematics education in future re-conceptualisations of success at post-primary level in Ireland.

Keywords: Project Maths, Bonus Points, teacher knowledge, time, student motivation

Introduction

Since 2012 the post-primary mathematics education landscape in Ireland has changed dramatically and with these changes came a change in what many perceive constitutes success in mathematics education. In this paper, I will discuss the initiatives that have brought about significant change to mathematics education in Ireland over the past decade, analyse their impact on stakeholders' perceptions of success in mathematics at post-primary level, and investigate the (unforeseen) challenges that have affected the uniform adoption of the perception of success espoused by Government agencies and policy makers. The identification of these challenges has been the kernel of much of my own research in this time period.

2010 and the Introduction of Project Maths

In 2010, a revised mathematics curriculum, which had been piloted across 24 schools between 2008 and 2010, was rolled out across all post-primary schools in Ireland. This reform movement was initiated for two reasons. Firstly, according to Byrne et al. (2021), at the turn of the new millennium there was mounting evidence to suggest that the mathematics curriculum at the time was not serving the needs of the Irish people and was not producing graduates from the post-primary system equipped with the knowledge and skills needed for a knowledge economy. Many believed that this deficiency in students' mathematical capabilities stemmed from an over-reliance on rote learning in Irish mathematics classroom (Lyons et al., 2003) and the declining attitudes towards mathematics among post-primary school students (NCCA, 2005). Secondly, there was a need for better alignment between the primary and post-primary curricula in Ireland. In 1999, the primary school mathematics curriculum was reformed while the previous full-scale reform at post-primary level was conducted in the 1970s[1]. As such, the alignment between the two curricula was tenuous at best, and a reform to the post-primary mathematics curriculum at both Junior and Senior Cycle was necessary to modernise a dated mathematics curriculum and to improve the alignment between the two curricula. These two shortcomings led to the introduction of the revised curriculum, known locally as *Project Maths*.

Project Maths was inspired by the Realistic Mathematics Education [RME] movement as this was seen as the *"most fashionable approach among mathematics educators"* at the time (NCCA, 2005, p.6). However, learning from the mistakes of previous curriculum reform efforts, those responsible for the design of this new curriculum were cautious not to overly depend on one single ideological standpoint, and so instead were inspired by appropriate aspects of the RME movement while at the same time ensuring that the new curriculum was aligned with international best practice and the needs of the Irish economy (O'Meara & Milinkovic, 2023). This approach led to the implementation of a curriculum which sought to "*teach mathematics in a way which promotes real understanding, where students can appreciate the relevance of what they are learning and its application to everyday life…"* (Project Maths Implementation Support Group [PMISG], 2010, p.12).

Project Maths advocated for a fundamental shift in the approach to mathematics teaching, learning and assessment (Cosgrove et al., 2012). As such, the notion of what it meant to be successful in mathematics changed. Success, as elicited by the reformed curriculum, was achieved through students developing and demonstrating conceptual understanding of a topic and being able to use mathematics to solve authentic problems (PMISG, 2010). Central to achieving this success was the role of students in the construction of their own knowledge, which was a shift from the procedural, top-down, approach to teaching favoured in the past (Lyons et al., 2003). Finally, successful mathematics teaching and learning resulted in students viewing the subject as an interconnected body of knowledge rather than a collection of isolated ideas. This revised understanding of success in mathematics was fundamentally different to what had been promoted previously and highlighted the lofty ambitions of this curriculum reform. However, the realisation of this perception of success encountered many challenges.

Challenges facing Implementation of Project Maths

Teacher Knowledge

The success of any reform movement depends on the teachers who are required to interpret and enact it (Spillane, 1999). With such significant changes to the teaching and learning of mathematics the demands placed on teachers changed, in particular in relation to

[1] In the 1970s the curriculum at both Junior and Senior Cycle was reformed, similar to the reform implemented most recently. In 1987 an updated curriculum was introduced at lower secondary level but this did not involve major reform nor was the focus on improving the alignment with the primary school curriculum (Oldham, 2007). Subsequently to ensure the mathematics curricula at upper and lower secondary school were aligned the curriculum at upper secondary school was then updated separately in 1992

the knowledge they needed in order to teach effectively. Project Maths aimed to secure a change in teaching approaches and practices and as Putnam et al. (1992) suggest such change also requires a change in a teacher's knowledge base. In order for students to develop conceptual understanding and problem-solving skills, as espoused by the new curriculum, teachers had to expand their own knowledge base to ensure they were in a position to help students achieve this success. While Shulman (1986) proposed three domains of knowledge for effective teaching, namely subject matter content knowledge, pedagogical content knowledge and curricular knowledge, many researchers, such as Ball et al. (2008), spoke of the need for models of teacher knowledge to be continuously refined and updated in line with changes in mathematics curricula and teaching approaches. As such, the changes brought about by Project Maths also brought about the need for different knowledge domains among teachers. It was these different knowledge domains that I identified in my own PhD study via a model labelled the *Ladder of Knowledge* (see Figure 1). This model, which built on the work of Shulman (1986) and Ball et al. (2008), outlined the three knowledge domains necessary to teach the revised curriculum effectively (subject matter knowledge, pedagogical knowledge and knowledge for effective teaching) and also provided sub-domains which allowed teachers to transition between these key knowledge domains (knowledge of real life applications, knowledge of connections, historical knowledge, knowledge of school maths and transforming knowledge) (O'Meara, 2011).

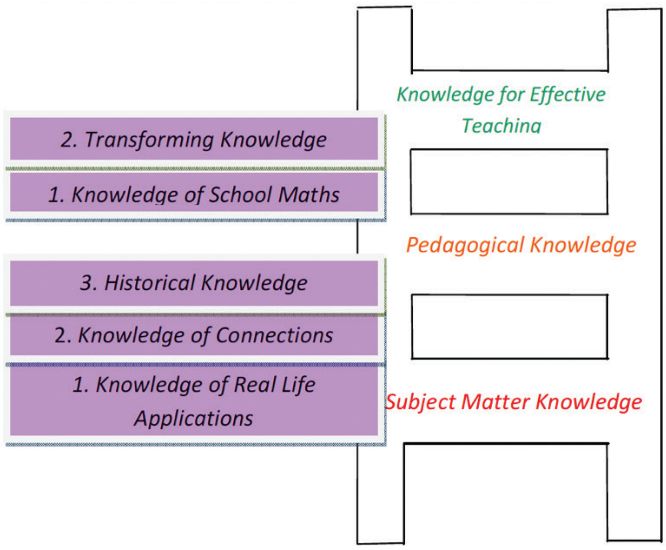

Figure 1. Ladder of Knowledge

While the provision of a model of teacher knowledge helped to identify the knowledge domains needed to facilitate student success and the effective teaching of mathematics, further research, indicated that work was still needed to help teachers to develop the different types of knowledge outlined in the model. The work of O'Meara (2011) showed that in-service teachers felt confident in only one domain, subject matter knowledge, and deficient in all others. This was also apparent when their knowledge was assessed, with issues identified across many of the domains, including knowledge of applications, knowledge of connections, historical knowledge and transforming knowledge (O'Meara, 2011). Subsequent studies have reported similar findings; for example, O'Meara et al., (2020a) found post-primary teachers did not possess sufficient knowledge of connections and knowledge of school mathematics, primarily due to unfamiliarity with the mathematical content and pedagogical approaches employed at the upper end of primary school. Consequently, post-primary teachers lacking this knowledge could not align their pedagogical approaches with those experienced by students in earlier years of schooling. Studies have also found that Irish pre-service teachers do not possess the required knowledge in many other domains included in the Ladder of Knowledge; for example, O'Meara and Fitzmaurice's (2022) study illustrated pre-service teachers', in the penultimate year of their teacher education programme, demonstrated superficial understanding of the applicability of mathematics. Less than 12% of the sample in this study were able to provide an accurate, uncontrived real-life scenario that would require one to perform subtraction or multiplication of rational numbers, while no pre-service teachers could provide such a scenario for the operation of division. These studies indicate that teachers, at both in-service and pre-service level, do not possess the requisite knowledge to facilitate students to succeed in the new curriculum. It must be acknowledged, however, that this is through no fault of their own, given that teachers have not been given sufficient opportunities to develop the array of knowledge types proposed in the Ladder of Knowledge. This shortfall in learning opportunities for teachers has been identified in a number of studies (O'Meara, Johnson & Leavy, 2020; Fitzmaurice, O'Meara & Johnson, 2021), and until such opportunities are provided to teachers, mathematical success for Irish students – as described by Project Maths – will be extremely difficult to achieve.

Instructional Time

A second challenge which has the potential to limit the success experienced by *all* students in mathematics relates to the time available to implement the curriculum. Carroll (1989) described how academic success is dependent on variables representing the amount of time available to learn, the time needed to learn, and the time a student is willing to spend learning. More recently, a large body of literature has demonstrated strong, positive correlations between instruction time and student achievement (Benavot & Amadi, 2004). In the Irish context, a study by O'Meara and Prendergast (2017) found that 89% of Junior Cycle teachers and 92% of Senior Cycle teachers believed that Project Maths, and its new conceptualisation of success, had impacted on the time required to teach the curriculum. However, the majority of these teachers (88% at Junior Cycle and 79% at Senior Cycle) reported that the time allocated to mathematics had not changed with the introduction of the new curriculum. As a result, 62% of the teachers (n =316) strongly disagreed or disagreed that there was sufficient time available to achieve the goals of Project Maths at Junior Cycle, while the corresponding figure for Senior Cycle was 82%. Furthermore, this study uncovered significant variations in the time available to deliver the curriculum, with the allocated instruction time ranging from 120 to 300 minutes per week (pw) in first year; from 145 to 240 minutes pw in second year; from 145 to 249 minutes pw in third year; from 175 to 290 minutes pw in fifth year and from 180 to 290 minutes pw in sixth year. These variations coupled with the provision of voluntary, additional lessons outside of the school day by some, but not all, teachers meant that students, even in the same school, can receive varying amounts of instruction time. These students, regardless of the school they attend or the teacher they are assigned, are studying the same syllabi and preparing for the same examination with the same success criteria yet receiving various amounts of instruction time (Prendergast & O'Meara, 2017). Thus, until students are provided with equitable access to mathematics instruction, success for some will continue to be much more accessible than it is for others solely based on the school they attend or the teacher they are assigned.

The Bonus Points Initiative

In 2012 another significant change to mathematics education at post-primary level occurred with the introduction of the *Bonus Points Initiative* [BPI]. In Ireland, mathematics is not a compulsory subject at upper post-primary level; however, due in part to the matriculation requirements of Irish Higher Education Institutes, virtually all Irish students study mathematics at Senior Cycle. For example, in June 2022, 58,056 students sat the Leaving Certificate (the terminal Senior Cycle assessment) and 57,248 (98.61%) of these students sat a mathematics examination. While students do not generally have a choice about studying mathematics at upper post-primary school, they do have agency regarding the level they study. There are currently three levels of mathematics available to students: Higher Level; Ordinary Level; and Foundation Level. Higher Level is the most advanced form of mathematics that students can study. Ordinary Level covers many of the same concepts addressed at Higher Level, but not to the same depth. Foundation Level is a separate course of study centred on basic mathematical skills.

Researchers ascertain that the study of advanced mathematics (Higher Level in Ireland) facilitates the development of a variety of skills, such as decision making and critical thinking, that underpin a scientifically literate workforce, determine future academic success at third level and future income (Wolf, 2002; Chinnappan et al., 2008; Kennedy et al., 2014; Hine et al., 2015). Despite the importance of mathematics and the necessity for a mathematically literate workforce for economic growth and personal advancement, many countries worldwide, including Ireland, report low numbers of students studying advanced mathematics at upper post-primary level. Prior to 2012, the proportion of students studying Higher Level mathematics at upper post-primary level in Ireland was extremely low when compared to other school subjects. For example, for the 2011 Leaving Certificate, 15.8% of students opted for the Higher Level paper, with the corresponding figures for English and Irish being 63.7% and 32.3%, respectively. In 2012, policy makers in Ireland recognised the value of increasing this proportion due to the aforementioned benefits and so introduced the BPI. The BPI had two objectives: first, to increase the number of students opting for Higher Level mathematics; and second, to improve students' mathematical competency. In order to achieve these goals, the BPI incentivised students' 'successful' completion of the Higher Level mathematics course at upper post-primary level, and by consequence, assigned Higher Level mathematics a unique status.

In Ireland, students are accepted onto third-level courses based on a points system that equates students' performance in a subject-area for the Leaving Certificate with a point score. Undertaking a subject at Higher Level yields a greater number of points; for instance, the top grade in Higher Level, H1, is valued at 100 points, while the top grade for Ordinary Level, O1, merits 56 points. The student's total points from their six best subject is then used as the determining factor for their entry into third level, thus rendering the Leaving Certificate a gatekeeper to tertiary education. The BPI is situated in the context of this competitive ‘points race’ for third-level entry. The BPI awards students who achieve a passing grade (>40%) in Higher Level mathematics an additional 25 points. Consequently, a H4 grade (60%-69%) in mathematics is now valued at 91 points (66 + 25), while the same grade in any other subject yields 66 points. The BPI achieved one of its goals in that the proportion of students undertaking Higher Level mathematics has increased from 15.8% in 2011 to 37.1% in 2022; however, it has also had a significant impact on how students, parents and teachers now perceive success in mathematics. For many, success now means “enduring” Higher Level mathematics in the hope of “doing enough” to obtain 40% to secure the coveted additional 25 points, thus increasing their likelihood of securing a college place. This new perception of success has presented numerous challenges for students and teachers alike.

Challenges Associated with the BPI

Student Profile

The first challenge presented by the BPI relates to the new student profile in Higher Level mathematics classes. In a study conducted by Treacy et al. (2020), a large proportion of teachers (61.5%) voiced concerns regarding the BPI. For these teachers, the BPI has resulted in the need to provide greater support to “weaker” students and cater for “mixed ability” cohorts to a much greater extent than was previously the case, primarily due to “unsuitable” candidates now persisting with HL mathematics. According to Linchevski and Kutscher (1998), mathematics is one of the more difficult subjects for working with mixed ability groupings, while Hallam and Ireson (2003) suggested that such grouping is inappropriate for mathematics. The BPI was introduced without any apparent consideration for the impact that it may have on class profiles and as such, teachers received no training in dealing with the knock-on effects of the BPI, including guidance on how to develop teaching strategies to cater for more mixed ability classes.

(Extrinsic) Student Motivation

A second challenge that has affected the successful implementation of the BPI relates to students’ motivations for studying Higher Level mathematics. The findings from a study conducted by O’Meara et al. (2023) show that students are now predominantly extrinsically, as opposed to intrinsically, motivated to study Higher Level mathematics. For example, 46.2% of the 911 Higher Level students surveyed indicated that the BPI was the most influential reason behind their decision to pursue Higher Level mathematics, while a further 7.2% of students cited that the CAO points on offer for Higher Level mathematics was the determining factor. This is a cause for concern as research has shown that extrinsic motivators, such as the BPI, do not work over time (Adamma, Ekwutosim & Unamba, 2018). In fact, the presence of extrinsic motivational factors can lead to diminished intrinsic motivation among students (Biehler & Snowman, 1990). Therefore, the role that the BPI is currently playing in motivating students to study Higher Level mathematics may have negative effects on students’ affective reaction to the subject.

These challenges have meant that only one of the goals of the BPI has been successfully achieved. While the number of students studying Higher Level mathematics has increased, only 18.9% of teachers in the Treacy et al. (2020) study believe that the BPI has resulted in an overall improvement in students' mathematical ability. This assertion is supported when one compares students' results in the Leaving Certificate mathematics examination before and after the introduction of the BPI (Treacy et al., 2020). The challenges facing the BPI discussed here offer some insights into the potential reasons why the BPI has failed in its mission to improve the mathematical skills and understanding of post-primary students. The new notion of success associated with the BPI involves *"hanging on at higher level to gain bonus points"* (Treacy et al, 2020, p.1433), meaning that the BPI could be having an unintended negative impact on students' mathematical ability. Furthermore, fears now exist that if the bonus points are removed completely, as has been advocated by some teachers, Ireland may see a lower proportion of students opt for Higher Level mathematics than was the case when the Irish Government first set about rectifying this issue in 2012 (O'Meara et al., 2023).

Conclusion

The concept of what it means to be successful in mathematics has changed in Ireland as the curriculum has evolved and new initiatives have been introduced. Consequently, over the past decade, success in mathematics has been perceived in various, and in some cases, unintended ways by policymakers, teachers, parents and students in Ireland. The Project Maths curriculum espouses a view of success as students seeing and experiencing mathematics as an interconnected body of knowledge, and becoming adept problem solvers who recognise the prevalence of mathematics in the world around them. For students, mathematics has a unique status, offering additional, valuable points as part of the points race for third-level entry. In many cases, this has led to the need to *endure* in mathematics, with success equated to achieving a pass grade to secure the bonus points, often to the detriment of their disposition towards the subject. Many teachers believe that the success espoused by Project Maths is almost unattainable given the lack of support they have received for developing the requisite knowledge to teach in this way, the failure to recognise the greater time demands of teaching in such a manner, and the heightened demands of teaching larger cohorts with broader ranges of ability which has stemmed from the simultaneous introduction of the BPI. Thus, even if educational changes, such as Project Maths and the BPI, serve to promote a particular perspective of success, this may not be how it is received by others. Such initiatives may lead to unintended perceptions of success and potentially, inadvertently, adversely impact the very issues they set out to address. It follows then that any future initiative which may impact on how success in mathematics is perceived should be carefully configured, taking into account any potential, unintended consequences.

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What Counts as Success in the Assessment of Mathematics?

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Few topics generate such heated debate in education as assessment. Academics, policy makers, parents, employers, students and the general public all have views on assessment, be it low and high stakes, summative, formative, peer and or self assessment or whether it is authentic, dynamic, diagnostic, strengths based, ecological, inclusive, feeding back and forward or on teachers' assessment literacy (Black & William, 2018; Elder, Rood and Damiani, 2018; Douglas et al., 2012; Lauchlan & Jones Daly, 2023). Looney et al. (2018) talk of teacher assessment identity; there is also the power differential between the assessor and assessed (Imperio & Seitz, 2023), the backwash effect on pedagogy (Alderson and Wall, 1993), valuing what can be measured more than measuring what we value (Biesta, 2014), computer based assessment (Martin & Lazendic, 2018), test development (Downing & Haladyna, 2006), the use and abuse of standardised assessment (Penn, 2023), value added models (Marks, 2021), the influence of AI on assessment (Swiecki et al. 2022), international comparative assessment, opposition to what is called Baby PISA (OECD, 2015; Urban & Swadener, 2016) and of course the controversy in Ireland around teachers assessing their own students in State examinations (Molloy, 2022). This is before we get to validity and reliability and the purposes of assessment.

Instigating change in the area of assessment can be fraught with political and ideological aspects far removed from the evidence. Here we have an opportunity to listen and engage with four very different but complementary papers which assess the evidence as they pertain to assessment for success in mathematics.

Therese Dooley focuses on the key area of formative assessment and how it can be effectively used in the teaching of mathematics. She presents the findings from a recent review and outlines the complexity of measuring impact with different definitions and foci. The review points to initial very promising results with some more modest findings in recent research. She points to the potential of learning trajectories and how they can benefit formative assessment in combination with other pedagogical strategies. Dooley highlights the design and use of revelatory tasks as a window into student thinking and knowledge when combined with close observation, interviewing, examination of work samples, diagrams, concepts maps and reflections on learning as beneficial ways of utilising formative assessment in mathematics.

However, the review highlights self assessment as having the greatest impact on achievement followed by peer assessment, with both having greater effect than teacher directed assessment. Such assessment involves reflection on the quality of work and the extent to which goals were achieved and is linked to metacognition and self-regulated learning. Linking with Zita Lysaght's paper, the complexity of practice for the teacher is highlighted involving making decisions about feedback, understanding learning trajectories and misconceptions and how to respond to these effectively. It also entails a mindset change in developing a learning environment that is shared with students and a mitigation of the power differential in allowing students more autonomy in assessment as part of the learning process. A key question which arises here is how this is best done in the context of teacher professional learning?

Zita Lysaght focuses on the interaction between assessment for learning and the development of short term learning progressions in primary mathematics by pre-service teachers. She highlights the challenge for students of assessing where the learners are at in designing appropriate goals and how this links to their knowledge of learning trajectories in mathematics. Somewhat controversially she links student difficulties in executing this task to the criteria for entry to the B.Ed in Ireland in relation to attainment in post primary mathematics, arguing that the low expectations are a “hostage to fortune”.

There is no denying the levels of knowledge and skills involved in accurately assessing current student performance and using learning trajectory knowledge to craft next steps in learning and bridge the gap. A key question which arises here is in relation to what is reasonable to expect at initial teacher education level and what additional expectations should reside in the area of further teacher professional learning linking with Dooley? A related question is how best formative assessment is learned- what is the optimum balance between discrete, embedded and applied placement approaches?

Gerry Shiel explores new directions in summative assessment and highlights the affordances offered by technology in the area. Opportunities for interim assessment blocks and greater data based decision making are possible. He also highlights the use of adaptive testing which builds differentiation into the tasks. This is an element that does not feature yet in Irish assessment. He sees a greater merging of formative and summative assessment with greater use of summative assessment for diagnostic purposes. Politically, different systems have attached greater importance to the results of summative assessments and used the results in suspect ways for teacher and school evaluation. Other uses include their incorporation in value added models, measuring progress over time and controlling for socio-economic contexts. Systems also grapple with reporting mechanisms to parents and what approach best relays key contextualised information taking limitations into account. A key question here is how Ireland can strike a balance between using the benign features of summative assessment while avoiding the misuse of data for other purposes.

Vasiliki Pitsia traces Ireland’s involvement with international large scale assessment of mathematics achievement. From the late 1980s Ireland began to regularly participate in international assessment of mathematics including the OECD’s Programme for International Student Assessment (PISA) and the IEA’s Trends in International Mathematics and Science Study (TiMSS). PISA targets 15 year old students and TIMSS the equivalent of fourth class primary and second year post primary. She outlines the content areas, cognitive processes and application contexts for the assessments. Across both assessments Irish students have performed well with one exception. In addition lower-achieving students have also done relatively well but there are concerns about the lower than expected performance of higher achieving students. The one exception of PISA 2009 does present a salutary tale of the extent of the reach of influence of the results on policy in Ireland. While it subsequently has been shown to have been an over reaction with plausible explanations offered for the low performance in 2009, the impact on the teaching of mathematics has been significant from curricula reform, mandatory standardised testing in primary schools, a new numeracy strategy which impacted practice at primary level and the extension of teacher education programmes forming part of the strategy. A key question here is the extent to which such assessments should influence policy and practice and the process through which those decisions are made? Valisilki argues that current practice has much to learn from the assessments in moving away from state examination driven pedagogy to developing mathematical knowledge and skills more aligned with the intentions of PISA and TIMSS.

The papers present us with challenges for the future at classroom, initial teacher education, national and international levels across formative and summative assessment which impact hugely on success in mathematics.

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Formative Assessment in Mathematics: Students at the Centre of Learning

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In this paper the findings of a recent systematic review on formative assessment (Shiel & Dooley, 2022) that pertain to the teaching and learning of mathematics are outlined. Reference is made to the meaning of formative assessment, its effect on learning, the nature and source of feedback, learning trajectories, and revelatory tasks. It is argued that effective formative assessment encompasses more than a set of tools – it is dependent on a learning environment that is characterised by student autonomy in learning and assessment.

Keywords: formative assessment, feedback, self-assessment, learning trajectories, revelatory tasks

Introduction

A question that arises for teachers in Ireland (and elsewhere) is how formative assessment can be used effectively in their teaching of mathematics, particularly as it is an integral aspect of the Irish primary and post-primary mathematics curricula. While a thorough and detailed consideration of this question is outside the scope of this paper, a recent review conducted by Shiel and Dooley (2022) yields some pertinent findings. The purpose of the review was to examine how formative assessment might support the development of literacy and numeracy.[1] However, as no reviews of formative assessment in numeracy were found, the scope of the search was broadened to include the term "mathematics". An overview of relevant findings is provided below.[2]

Effect of Formative Assessment on Mathematics Learning

Despite over two decades of research on formative assessment there is no clear consensus on what it means. In 2009, Black and William proposed that assessment is formative "to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (p. 9). Adapted from their original interpretation (Black & Wiliam,1998), it is broad in scope. It includes, for example, different agents of assessment (teacher, self or peer), evidence of learning, decision making and next steps. Researchers in the field identify different aspects of formative assessment as appropriate for their context – some focus on formative assessment as a set of tools to be used to improve teaching (e.g., scoring rubrics, checklists, mastery tests), others consider it as part of the

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[1] The review was a component of a review of research in relation to literacy (including digital literacy) and numeracy commissioned by the Department of Education in Ireland (Kennedy et al., 2023).

[2] A bibliography of research cited in this paper can be found in Shiel and Dooley (2022).

teaching and learning process (e.g., effective use of teacher questioning, provision of feedback that supports students in setting new learning goals) and still others conceptualise it as neither one nor the other but some integration of the two.

Initial reviews showed the effect of formative assessment on achievement to be very promising; however, more modest, mixed effects are revealed in recent research. This is true for mathematics as well as for other subject areas. There are many reasons for this including the nature of the reviews, how achievement is defined and how formative assessment is conceptualised in specific research. Given the complexity of formative assessment, the focus of different reviews tends to be on features of formative assessment interventions. Findings are often ambiguous. Formative assessment with technology, for example, is found to have no more impact on achievement than formative assessment without technology. However, there is some evidence that online assessment is most effective when based on mathematics content that is aligned to a clearly defined hierarchy of skills. It is also concluded that, in such instances, feedback provided to students should be immediate and frequent. Other research points to the need for feedback to be provided to students within and between instructional units rather than within and between lessons (medium-term versus short-term). Much depends on the nature of the task being assessed. There is an indication in some studies that immediate feedback is more effective for lower-order learning than delayed feedback and vice versa. Coupled with this, and probably not of great surprise, is that feedback incorporating an explanation and focusing on strategies that helps students to bridge the gap between where they are and where they want to be is particularly beneficial for higher-order outcomes.

Learning trajectories focus on students' successively more complex way of thinking about a topic. Although nomenclature varies, they are increasingly a feature of curricula, e.g., they are included in the new Primary Mathematics Curriculum in Ireland[3] as "progression continua". Due to their relative novelty, research on effect of learning trajectories is limited. However, there is some evidence that they can benefit formative assessment by the teacher if they are used in combination with a range of other teaching strategies such as encouraging children to share ideas, take risks and learn from and with peers in mathematics lessons. Moreover, student-facing checklists[4] based on trajectories can support younger students to self-assess and set goals.

Key to effective formative assessment are revelatory tasks that provide teachers or learners or peers with a "window into students' thinking" (Wiliam, 2007, p.1069) so that decisions can be made about appropriate next steps. Interviews with students, observation of them engaging in cognitively challenging tasks, documentation of their talk and investigations of samples of work are some of the fruitful ways that formative assessment information can be gathered. In particular, student-generated diagrams, concepts maps and writing about learning show some promise as formative assessment tools in mathematics.

[3] The new Primary Mathematics Curriculum had not been published at the time of writing this paper; hence the information reported derived from a draft version of this curriculum (National Council for Curriculum and Assessment, 2022).

[4] Such checklists have been paraphrased so that they can be understood by students.

In the main, outcomes of reviews are nuanced – some researchers attribute this to the implementation of only certain principles of formative assessment, without consideration of the broader implications for classroom practice. However, self-assessment is highlighted as having a significant impact on achievement. It is a process whereby students reflect on the quality of their work, make decisions about the degree to which it meets particular goals and revise as appropriate (Andrade & Valtcheva, 2009). Many believe its powerful effect to be related to the role it plays in students' metacognition and self-regulated learning, both of which are significant factors in improving academic attainment (Muijs & Bokhove, 2020). Also emerging from the literature is the positive effect of peer-assessment on students' subsequent performance. Further studies are required to elucidate beneficial types of peer assessment (e.g., use of rubrics, written or dialogue component). However, there is little evidence that the provision of grades by peers – particularly in primary and post-primary schools – enhances learning. A significant finding about self- and peer-assessment is that they have much greater effect than either teacher-directed assessment or no assessment. While it remains difficult to pinpoint effective features of self- and peer-assessment, the overriding conclusion reached by many is that the most important factor underpinning effective formative assessment is a classroom environment where the student is at the centre of learning.

Final Remarks

Formative assessment extends beyond *tools*, important as these might be. It demands that teachers make decisions about the nature of feedback that they give to students and also its timing. It requires that they have an understanding of the misconceptions that students might have in particular areas of mathematics and the teaching and learning strategies that could help to address these misconceptions. It calls for them to be familiar with learning trajectories in various topics which they can use to design appropriate learning tasks and also to assess students' learning. But above all, it necessitates that teachers are empowered to develop a learning environment in mathematics where there is a focus on co-construction of knowledge by teacher and students, and where students are given autonomy in the assessment as well as in the learning process.

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Facing up to the challenges faced by pre-service teachers in developing learning progressions in Mathematics

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This paper aims to draw attention, via presentation and plenary discussion, to observed challenges faced by pre-service primary teachers when attempting to craft short-term learning progressions in Mathematics in preparation for School Placement (SP).

Keywords: Assessment for learning; teacher education; learning progressions

It is conjectured that, in Ireland, Department of Education (DE) (2023) minimum grade level entry requirements of 30% < 40% at Higher Level (HL) or 60% < 70% at Ordinary Level (OL) in Leaving Certificate Mathematics to access Bachelor of Education (BEd) and Professional Masters in Primary Education (PMEP) degree programmes gives hostages to fortune given the expectation that graduates will be competent to teach mathematics competently. Justification for this statement is based on experience of (a) teaching pre- and in-service modules on Assessment for Learning (AfL) to BEd and PMEP students and (b) evaluating their application of key AfL strategies in SP schemes and lesson plans. It is proposed that greater understanding and acknowledgement of the underlying content and pedagogical knowledge needed by novice teachers to develop appropriate learning progressions, even for relatively short periods of instruction, is needed urgently. Hence, the intended audience for this paper is teacher educators and curriculum developers with responsibility for developing pre-service teachers' capacity to translate curriculum aims and objectives in Mathematics into sequenced learning progressions.

While the term 'learning progressions' connotes different meanings depending on whose work is read (e.g., Shepard, 2018; Aulls, Harley, Getahun, & Lemay, 2020), defining features have been identified which are noteworthy. First, learning progressions detail the 'typical' sequence in which relevant knowledge, skills and competencies develop in a given domain. The expectation is that they provide a blueprint that can be adapted when planning for individual needs and capabilities. Second, the sequences are incremental in nature, i.e., they represent increasingly sophisticated learning as captured by the use of Bloom's (1956; Gogus, 2012) taxonomies of verbs, for example. Third, they focus on what is to be learned/mastered as distinct from how such learning is to be scaffolded (i.e., the conditions of performance to be used such as the resources and methodologies to be employed) (Black & Wiliam, 1998).

While it is axiomatic that learning involves progression, however small or insignificant this may appear (e.g., when working with pupils with moderate, severe and profound difficulties), enabling planned, incremental, vertical learning assumes knowledge of typical learning trajectories in defined domains. Linked with this, pedagogical expertise is required to determine the nature and extent of current performance relative to the overall learning outcomes as well as the capacity to 'close the gap' (Sadler, 1989). This is achieved by employing AfL strategies, notably reciprocal formative feedback between teacher and learner that drives assessment, learning and teaching decisions in real time.

In order to underscore how challenging this can be for pre-service teachers, particularly those pursuing a BEd or PMEP qualification with minimum performance in LC Mathematics, this paper focuses on a unique approach used with pre-service student teachers aimed at elucidating the steps involved in creating a short-term learning progression in a non-curricular domain. For the purposes of illustration, driving a car safely, something which the majority of pre-service have demonstrated knowledge and skill, is used. How this process may be applied in planning to teach Mathematic on SP such that informs both the sequencing of learning a priori, as well as the translation of incremental learning steps into inform daily learning intentions, conditions and criteria of performance, is explained. Attendees are invited to critique the approach adopted and consider its implications for curriculum development and teacher education.

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International Large-Scale Assessments of Mathematics Achievement in Ireland

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During the last few decades, international large-scale assessments of student achievement have attracted a lot of attention from educational stakeholders worldwide due to the quantity and quality of the information they provide, and Ireland is no exception. Results of both PISA and TIMSS across years have shown that Irish students perform consistently well on average in mathematics, however, some elements of their achievement have been a matter of concern. In this paper, areas of mathematics assessed by international assessments, strengths and weaknesses of Irish students' mathematics achievement, and potential policy directions to address some identified issues are discussed.

Keywords: international assessments; PISA; TIMSS; mathematical literacy

The predecessor of contemporary international large-scale assessments of student achievement and the first of its kind, conducted by the International Association for the Evaluation of Educational Achievement (IEA) in 12 countries between 1961 and 1965, is the First International Mathematics Study (FIMS). FIMS assessed 13-year-old students and its primary aim was to compare student outcomes across education systems (Mullis & Martin, 2006). Having mathematics as the assessment domain of the first international study was based on a number of reasons, including the universal value of the subject and its importance for progress in other subjects, the commonalities of mathematics curricula across countries, and the nature of the subject that allows for easier translation of the questions compared to other subjects where longer texts are used (Mullis & Martin, 2006). FIMS established the feasibility of international assessments across linguistically and culturally different contexts.

Ireland did not participate in FIMS. Ireland's first participation in an international assessment was in 1971 (in the civic education component of the Six Subjects Study), and despite that, it was not until the late 1980s to early 1990s that Ireland began to regularly participate in international assessments. Since then, mathematics achievement, alongside achievement in other subjects, has been assessed across different international assessments and student cohorts at both primary and post-primary levels. At the time of writing, mathematics achievement of Irish students has been assessed in all eight cycles of the Organisation for Economic Co-operation and Development's (OECD) Programme for International Student Assessment (PISA) (starting in 2000), and five cycles of the IEA's Trends in International Mathematics and Science Study (TIMSS) (starting in 1995).

PISA assesses the knowledge and skills of 15-year-old students worldwide, but it is not a curriculum-based assessment – it assesses student performance on "real-life" tasks that are considered relevant for effective participation in adult society and for life-long learning (OECD, 2021). In the most recent cycle of PISA, mathematical literacy is defined as

> an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens (OECD, 2018, p. 7)

The PISA mathematics domain comprises four content areas: (i) change and relationships, (ii) space and shape, (iii) quantity, and (iv) uncertainty and data, three cognitive processes: (i) formulate, (ii) employ, and (iii) interpret and evaluate, and four contexts: (i) personal, (ii) occupational, (iii) societal, (iv) scientific.

TIMSS is a curriculum-based assessment at grades 4 and 8 (corresponding to Fourth Class and Second Year in Ireland) and, in the most recent TIMSS cycle, its mathematics framework is organised around two dimensions: content, specifying the subject matter domains to be assessed (i.e., number, algebra, geometry and measurement, data and probability), and cognitive, specifying the thinking processes to be assessed (i.e., knowing, applying, reasoning) (Mullis et al., 2021).

Results of both PISA and TIMSS across years have shown that Irish students perform consistently well on average in mathematics (with the exception of PISA 2009, where a significant performance decline was noted) (McKeown et al., 2019; Perkins & Clerkin, 2020). However, while lower-achieving students have done relatively well in both assessments, the lower-than-expected performance and the relatively low proportions of higher-achieving students, especially at post-primary level, have been a matter of concern (Pitsia, 2021).

To help interpret what students' scores mean in substantive terms, the PISA and TIMSS performance scales are divided into proficiency levels or benchmarks that indicate the kinds of tasks that students whose scores are above a lower score limit are capable of completing successfully. These six proficiency levels and four benchmarks in PISA and TIMSS, respectively, are accompanied by descriptions of the skills and knowledge that students are able to demonstrate. Students performing at proficiency levels 5 and 6 in PISA and at the advanced international benchmark in TIMSS are usually considered to be the high-achieving students in a given subject. In PISA mathematics, students at proficiency level 5 "can develop and work with models for complex situations … [and] work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insight pertaining to these situations" (OECD, 2019, p. 105); while students at proficiency level 6 "can conceptualise, generalise and utilise information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts …. Students at this level can [also] reflect on their actions, and can formulate and precisely communicate their actions and reflections …" (OECD, 2019, p. 105).

In TIMSS mathematics, grade 4 students performing at the advanced international benchmark " … can solve a variety of multistep word problems involving whole numbers and show an understanding of fractions and decimals … apply knowledge of two- and three-dimensional shapes in a variety of situations … [and] interpret and represent data to solve multistep problems" (Mullis et al., 2020, p. 36), while grade 8 students at the advanced international benchmark "can apply and reason in a variety of problem situations" such as algebraic, geometric, statistical, and probabilistic problems (Mullis et al., 2020, p. 173).

The relatively low proportions of students (especially at post-primary level) demonstrating these kinds of advanced skills and knowledge in mathematics across all cycles of PISA and TIMSS (ranging mostly between 7% and 10%), along with the performance decline in PISA 2009 prompted the formulation and implementation of a number of policies and initiatives and new strands of research over the last fifteen years in Ireland. The development of curricula for Junior Cycle mathematics, including Project Maths that was implemented on a phased basis between 2008 and 2015 (e.g., Kirwan, 2015), the National Strategy to Improve Literacy and Numeracy among primary and post-primary students introduced in 2011 (Department of Education and Skills, 2011), constituting the first governmental policy document that established nation-wide targets concerning high achievement in mathematics at primary and post-primary levels and its interim report (Department of Education and Skills, 2017a), the bonus points scheme introduced in 2012 (Central Applications Office, 2012), the establishment of the STEM Education Review Group in 2013 (The STEM Education Review Group, 2016), and the Policy Statement on STEM education 2017-2026 (Department of Education and Skills, 2017b) all emerged to a great extent as a response to Ireland's results in international large-scale assessments, establishing these assessments as official indicators of Irish student performance and emphasising their key role in Irish educational policy-making. As noted by Harold Hislop, former Chief Inspector in the Department of Education, "the public and political interest aroused by PISA … deepened … interest in how well students are learning … and [led to] a commitment to tackling long-standing issues" (Hislop, 2011, p. 7).

The heightened focus of national policy and research on the results of international large-scale assessments, although crucial for driving change, is not sufficient on its own. Focusing on the impact of these results, and the associated policies, on teaching and learning is key. An important step going forward would be to strengthen the links of policy and research with practice in schools to be able to move from the contemporary practice, which, at post-primary level, is heavily focused on the teaching of the knowledge and skills students need to perform well on the state examinations, to the policy aspiration, which is raising the proportions of students with strong mathematical knowledge and skills, and improving the performance of existing high-achieving students.

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Summative Assessment of Mathematics: New Directions

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This paper outlines findings of a recent review of new directions in summative assessment (Shiel, 2022), prepared in the context of informing a new Literacy, Numeracy and Digital Literacy Strategy in Ireland. It looks at how summative assessment can be defined, the application of technology to summative assessment, and emerging trends in reporting. Links between formative and summative assessment are also considered.

Keywords: summative assessment, computer-based assessments, reporting

Introduction

> Summative assessment can be defined as 'the assessment of students that occurs at the end of a period of instruction. [It] provides a holistic measurement of an individual's knowledge, skills and dispositions. . . Summative assessment consists of a variety of different formats from multiple-choice exams to research papers to portfolios of student work.' (Nicholas, 2018, p. 1634).

In the Irish context, summative assessment relates to assessments such as the National Assessments of English Reading and Mathematics (NAREM) and the Programme for International Student Assessment (PISA), both of which are used to set national targets for student performance in literacy and numeracy and are essentially low-stakes assessments for most stakeholders. It also encompasses the standardised tests that are currently administered to pupils in the Second, Fourth and Sixth classes in primary schools, which are sometimes viewed as high-stakes, though certainly not to the same extent as census-based national or state-level assessments in Australia and the United States, where outcomes may also be used to evaluate the effectiveness of schools and teachers in promoting student achievement.

Summative assessment often enjoys a higher status than formative assessment, and this gives rise to the possibility that teaching and learning will focus more on preparing students for summative assessment, with less attention or recognition given to formative assessment. O'Leary et al. (2019) have argued that summative assessments should provide more formative information, especially for at-risk groups such as disadvantaged students, students for whom English is an additional language, and students with special educational needs. In the future, it is likely that formative and summative assessment will merge to a greater extent, particularly as technology facilitates the scoring of more complex assessment tasks. Looney (2019) has noted that electronic scoring systems can now score complex cognitive tasks such as problem solving and student collaboration on constructed response formats, enabling the use of more complex and real-life tasks in summative assessment.

Technology and Summative Assessment

In recent years, most international large-scale summative assessments, such as Trends in International Mathematics and Science (TIMSS) and the Programme for International Student Assessment (PISA) have partially or fully transitioned to computer-based assessment. Features such as adaptive testing (where test items are more likely to match the student's ability level) and computer-scoring of responses has led to greater efficiency in administration and scoring. Furthermore, test items have become more interactive or scenario-based, setting new challenges for students. In Ireland, schools and teachers have a choice as to whether they wish to administer computer- or paper-based standardized tests of mathematics at primary level, with separate norms available for both formats. However, features such as adaptive testing have yet to be incorporated.

A recent international development has been the use of interim summative assessments to generate formative assessment information. For example, interim assessment blocks (IABs) can be used throughout the school year to assess smaller bundles of content, (e.g., multiplication and division within 100; numbers and operations in base 10; time, volume and mass). In some cases, performance may be reported on the same scales as full standardised tests. Teachers may play a role in the selection of items for interim tests. Poortman and Schildkamp (2016) have provided evidence of a positive impact for Data-based Decision Making (DBDM), which involves the systematic analysis of existing data sources (such as interim assessments) within the school by teachers, to innovate teaching, curricula and school performance, and to evaluate progress.

Measures of Student Progress Over Time

In recent years, national and state-level assessment programmes in other countries have begun to report measures of student progress over time, alongside scores that reflect performance at a particular point in time. The Massachusetts Comprehensive Assessment System (MCAS) reports on performance with reference to student growth percentiles. These enable stakeholders to evaluate growth in performance relative to an intake measure such as performance on an earlier assessment. Although student scores derived from these and other valued added models are often used to evaluate teacher and school effectiveness (a practice not without problems), they may also be of value in monitoring the progress of individual students and groups.

Interpreting Performance with Reference to Socioeconomic Status

A number of national assessment programmes also support the interpretation of test scores by taking socio-economic context into account. The Australian National Assessment Programme – Literacy and Numeracy (NAPLAN), which is administered on an annual basis in Years 3, 5, 7 and 9, reports average performance at school level with reference to the performance of other students with similar socioeconomic backgrounds. This focus is intended to reduce competition between schools. In Scotland, post-primary schools can compare the performance of their school leavers on numeracy (based on national exams) against virtual comparators (groups of students of similar socioeconomic background). These approaches seem to be fair to both disadvantaged and non-disadvantaged schools.

Standards-Based Reporting Formats

The reporting of test scores (for example, STen scores) to parents and other stakeholders is not uncommon in assessment programmes in other countries. However, these are often accompanied by descriptors of performance. In England, parents and other stakeholders are informed if students have reached the expected performance level for their age group, albeit based on a single cut-off score. In Massachusetts, performance is also reported on with respect to levels such as 'partially meeting expectations', 'meeting expectations', and 'exceeding expectations'. These descriptors, if accompanied by information of what skills students have achieved, can provide stakeholders with more specific, standards- or curriculum-based based feedback on performance and shift the focus away from discrete numeric values.

Final Remarks

There are a number of ways in which summative assessments of mathematics, including standardised tests, can be made more relevant to the needs to stakeholders:

- Expansion of the sample-based National Assessment of Mathematics (2nd and 6th classes at primary level) to include at least some computer-based assessment tasks, as part of gradual transition to full computer-based assessment.
- Reporting of individual and group progress over time (for example, via use of student growth percentiles) and contextualising aggregated student performance by controlling for socioeconomic status, when performance is reported.
- The development of short, online interim assessments across a range of aspects of mathematics, beginning at primary level; these could be used by teachers on a needs basis throughout the school year, to support instructional decision making.
- The reporting of descriptors of achievement, alongside test scores in testing programmes with a view to providing stakeholders with more contextualised feedback.
- A consideration of the most appropriate time of the year in which to administer standardised tests (O'Leary et al., 2019).

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Conceptualising Success for Mathematics in Modernity: Augmented Reality, Data Science and Integrated STEM

We are living in a rapidly changing digital era where the emergence of innovative technologies, big data, artificial intelligence and the Internet of Things (IoT) make new demands on mathematics education and on what it means to be mathematically literate. These new forces are fast transforming the skillset that will be required for our young people to thrive in and contribute to modern society. This poses challenges for educators and requires us to rethink traditional approaches to mathematics education and necessitates that we replace traditional curricula and pedagogies with new and integrated approaches that better reflect the uses that mathematics is put to in this modern society. Children in our primary classrooms will require mathematical understandings that underpin and support their ability to engage with emerging technologies (for example, Virtual and Augmented Reality) and develop their data literacy and computational thinking skills. Furthermore, they will need to function in interdisciplinary STEM environments where mathematics, science, technology and engineering are highly integrated and interdependent. Today's learners will need to be able to communicate and collaborate in interdisciplinary teams to innovate and solve the problems of tomorrow's world.

In this symposium, we present three research projects being carried out at Mary Immaculate College that present new ways of conceptualising what it means to 'do' mathematics, and be successful in mathematics, in the 21st century. These projects attest to our belief that innovation in mathematics education needs to begin as soon as a child enters our education system. Consequently, we need to prepare our teachers to not only support the youngest members of society to become mathematically literate, but we need to help tomorrow's teachers reconsider and recognise what it means for children to be successful in mathematics.

The first presentation, from Aisling Leavy and Mairead Hourigan, reports on an integrated STEM project carried out in collaboration with 3 teacher educators, 28 student teachers and 60 6th class children and their teachers in Limerick city. The *Engaging with Beekeepers Using Data science* (BUDS) project situates statistics learning within a real-world STEM context and prepares children for the fast-emerging field of Data Science. The study provides compelling evidence of the capabilities of young children to grapple with big statistical ideas, such as statistical association and informal inference, when provided with big data about real world problems.

The second presentation, from Michelle Fitzpatrick, interrogates a series of integrated approaches to STEM education in initial teacher education across a 2-year period. It reports on an effort to support 27 pre-service primary teachers in unearthing productive mathematical learning opportunities within integrated STEM tasks and presents a framework for meaningful mathematics integration in STEM tasks. Preliminary findings suggest that the framework has potential to capitalise on mathematical opportunities across a STEM inquiry cycle. This study has implications for mathematics education in initial teacher education, where recent developments demand a balance of both disciplinary and interdisciplinary approaches.

The third presentation, from James O'Reilly, examines how Augmented Reality (AR) can be used fruitfully in the primary classroom. It reports on how a group of child participant researchers engaged with a prototype AR app over an 8-week period, developed by an Irish primary school teacher, and made suggestions about how the app could be improved. The suggestions were incorporated into the app. Children engaged in weekly iterative play-suggest-design-revise work with the app. Through analysis of multiple data sources, the paper describes how the AR app developed over the course of the research relates to the Irish Primary Mathematics curriculum, to the Irish primary classroom and to the children who helped to improve it.

The symposium will end with a critique and review of the studies lead by invited discussants:

Dr. Finbarr Sloane. Dr. Sloane is programme director at the National Science Foundation (NSF) in Reston, Virginia, USA. One of his current program responsibilities is in the area of *Harnessing the Data Revolution: Institutes for Data-Intensive Research in Science and Engineering.*

Prof. Eamonn Kelly. Prof Kelly is Associate Dean for Research and Professor of Educational Psychology at George Mason University.

Developing children's statistical understandings in the *Engaging with Beekeepers Using Data Science* (BUDS) project

Aisling Leavy and Mairéad Hourigan

Mary Immaculate College

The BUDS project situates statistics learning within a real-world STEM context and prepares children for the fast-emerging field of Data Science. Sensors placed within beehives capture and stream data about in-hive conditions (humidity, sound, temperature) to classrooms via web technologies. Using lesson study, teacher educators and 27 pre-service primary teachers designed, taught, revised and retaught a series of five extended data lessons to two 6th classes in a culturally and linguistically diverse city school. The series of connected lessons raised awareness of the importance and plight of the honeybee and developed statistical and data science skills through engagement with sensor data. Children explored distributions of data, identified data landmarks, and utilised measures of central tendency and variation to describe and compare data from two different beehives. The study provides compelling evidence of the capabilities of young children to grapple with big statistical ideas, such as statistical association and informal inference, when provided with big data about real world problems.

Keywords: Integrated STEM, Data Science, statistical reasoning, pollinators

Introduction

The function and purposes of statistics education are fast evolving and extend beyond school-level curricular goals. Many goals of statistics education, such as enabling students to critically 'read and write the world' (Freire, 1970, p. 4) and promote the development of critical citizenship (Skovmose, 2011) by deepening understandings of issues of global importance, are shared goals of STEM education. STEM education is the purposeful integration of the STEM disciplines to solve complex, real-world problems and "innovate to solve them" (Balka, 2011, p. 7). In this paper, we show how primary children can be supported to analyse meaningful real-world data and to utilise their evolving statistical and graphical understandings to reflect upon broader societal issues and start their journey to becoming active citizens and positive agents of change (Gutstein, 2016).

Background and study context

Many policy initiatives have been developed to protect pollinators. The UN Sustainable Development Goals[1] (SDGs) incorporate an emphasis on food security ('zero hunger') and biodiversity ('life on land'). The EU Pollinators Initiative[2] presents strategic objectives and a set of actions to be taken by the EU and its Member States to address the decline of pollinators in the EU and contribute to global conservation efforts. More locally, the All-Ireland Pollinator Plan (2021-2025) brings together pollinator initiatives across the island. Evident across global policies and initiatives is recognition of the role of local communities in protecting pollinators and in making their land more pollinator friendly.

The emphasis on the plight of pollinators, in this case the honeybee, situates the study of school mathematics within an integrated STEM context and highlights the critical role of

[1] https://www.un.org/sustainabledevelopment/sustainable-development-goals/

[2] https://ec.europa.eu/environment/nature/conservation/species/pollinators/index_en.htm

interdisciplinary contexts in supporting successful STEM learning. It presents a coherent integration of STEM disciplinary content and addresses the challenge of curricular overload by supporting learning across multiple domains. A focus on bees directly addresses the primary science (NCCA 2000) curriculum strands of 'Living Things' and 'Environmental Awareness and Care' through the emphasis on pollination, lifecycles, food chains, and habitats. The analysis of data collected through innovative technologies (see figure 1) and a web-based data science tool (Common Online Data Analysis Platform, CODAP[3]) and interrogated using online data analytic tools targets the 'data and chance' strand of the primary mathematics curriculum (NCCA 2000) and champions the key competency 'being a digital learner' identified in the Primary Curriculum Framework (NCCA 2023).

Figure 1

A beekeeper placing sensors in our two beehives

Methodology

Using Lesson Study, the researchers worked with 27 primary pre-service teachers (PST) to design, teach, revise and reteach a series of 5 extended lessons (for details of the research design see Hourigan and Leavy, 2019) focusing on the development of statistical reasoning within an integrated STEM context. The series of research lessons were taught across a three-week period in two 6th classes within a local school. The school was in an urban setting and represented high linguistic and cultural diversity with many children being EAL learners. Data collection methods included field notes, lesson plans, samples of children's work, researcher observations, photographs, audio recordings of children's group conversations, PST and researcher reflections, and PST group presentations. A systematic process of data analysis was undertaken where the raw data were initially organised into natural units using representative codes (Creswell 2009). Successive examinations of the data led to the identification of relationships between codes and subsequently the creation of overarching themes. Thus, data were analysed using a grounded theory approach, where the data steered the emerging theory. The study was granted ethics approval.

Five STEM extended lessons were taught: (1) Introduction to the *Honeybee*, pollinators and sustainability; (2) Using *central tendency* and *variability* to describe and summarise animal data; (3) Examining *distributions of data* – what are sound patterns within

[3] https://codap.concord.org

bee hives? (4) *Comparing distributions* – Using variations in temperature across time and between the bee hives to identify an ‘at-risk’ beehive (5) Exploring *relationships between variables* – is there a relationship between temperature and sound in the beehives?

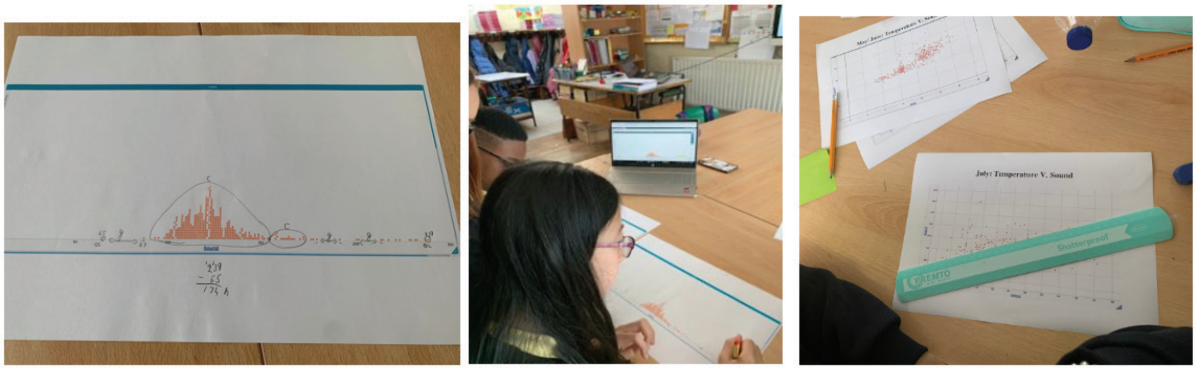
Findings and Conclusion

Preliminary analysis reveals insights into the statistical understandings of children (figure 2). Firstly, the *context of pollinators* facilitated deep and interconnected understandings of statistical measures and enabled sense-making and the development of meaningful inferences and decisions. For example, one child calculated ‘the average of the averages’ in an effort to make a reliable estimate, another described an outlier as “a loner, far away from home’, while others vehemently argued against the opening of the hive “until late in May otherwise the bees might die from the cold”. This use of real-world sensor data supported the development of sophisticated understandings of *variation* and the development of data-based (and contextual) inferences about factors that influenced fluctuations in data. Similar to findings of recent studies using statistical inquiry and technologies in primary contexts (Ben-Zvi et al., 2012; Hourigan & Leavy, 2021), children in this study demonstrated the ability to *reason about distributions* and coordinate understandings of central tendency and variability when making data comparisons and inferences. Challenges with *graphical literacy* emerged and involved conflating high frequency values with upper limits (confusing the mode as the maximum) and difficulties arising from moving between graphical representations. With regard to the latter, children tended to misinterpret what the variable (axes) represented, especially following engagement with time-series data as they misinterpreted higher range values on a line plot as indicating data collection points later in the day.

Our analyses revealed the *benefits accrued from the use of technologies*. Children were intrigued by how the sensors collected data and enjoyed the novelty of using laptops and software to analyse data. Of particular surprise was the motivational value arising from children’s interest in *making predictions* about statistical measures (central tendency, ranges, outliers, etc.) represented on the large A3 graph printouts and then checking these predictions using CODAP. Of equal importance were the *affective dimensions* and the high level of engagement and enthusiasm evident throughout the lessons. The use of emerging technologies to collect and analyse data promoted problem solving and critical thinking and provided these students with access to high-quality STEM learning. Our analyses also support what we know about the benefits of using simple *language-sensitive approaches*, such as visual representations and careful introduction of vocabulary, in supporting all learners in accessing mathematical learning. Finally, the study is a testament to the *power of child centred pedagogies in supporting children in accessing sophisticated statistical ideas* as was evidenced in their ability to explore the relationships between two variables, make informal inferences, and estimate lines of best fit between temperature and sound data.

Figure 2

Engaging in the analysis of beehive data collected from in-hive sensors



Alongside supporting the development of big statistical ideas, the BUDS project integrates content "into real world, rigorous and relevant learning experiences" (Vasquez, Sneider & Comer, 2013, p. 4), thus ensuring that students witness the potential of statistics to provide insights into both their future and the future of their communities.

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Mathematics *for* and *from* STEM: Conceptualising successful mathematics integration in STEM education

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The pace of STEM policy and curricular change in Ireland gives impetus to exploring more integrated approaches to STEM education. In turn, it challenges traditional, disciplinary approaches to teacher education, forcing us to reimagine how we prepare our future teachers for integrated STEM. This paper presents an effort to support 27 pre-service primary teachers in unearthing productive mathematical learning opportunities within integrated STEM tasks. It reports on pre-service teachers' experiences across two modules over two years. We outline the challenges experienced by both the pre-service teachers and teacher educators, before suggesting and trialling a framework for meaningful mathematics integration in STEM tasks. Preliminary findings suggest that the framework has potential to capitalise on mathematical opportunities across a STEM inquiry cycle. This study has implications for mathematics education in initial teacher education, where recent developments demand a balance of both disciplinary and interdisciplinary approaches.

Keywords: Mathematics education, integrated STEM, primary pre-service teachers

Introduction

There has been a recognised need for more integrated approaches to STEM (Honey et al., 2014). Although this has been met with enthusiasm from teachers (Hourigan et al., 2022), challenges abound which impede effective implementation (Margot & Kettler, 2019). One challenge is in maintaining a balance between the disciplines, with the underrepresentation of mathematics being particularly evident (English, 2016; Maass et al., 2019). Despite being generally accepted as the underpinning discipline in STEM, mathematics is often reduced to a service role in STEM tasks with little opportunity for genuine mathematical learning (Tytler et al., 2019; Forde et al., 2023). Meanwhile, teachers have reportedly struggled to identify opportunities for authentic mathematics teaching and learning within STEM tasks (Tytler et al., 2019). In this paper, we demonstrate how primary pre-service teachers (PSTs) can be supported in maintaining the spotlight on mathematics in integrated STEM tasks, by examining the role of mathematics teaching and learning for STEM and mathematics teaching and learning from STEM.

Background and study context

STEM education continues to attract global attention. The Irish policy landscape is no exception. Ambitious goals were set by the *STEM Education Policy Statement 2017-2026* (DES, 2017) mapping out a vision for STEM education engagement across the sectors. STEM policy is now aligning with curricular change at primary level. While mathematics and science are currently treated as isolated subjects, the new *Primary Curriculum Framework* (NCCA, 2023) sees the introduction of STEM education as one of five new broad curriculum areas. A STEM education development group has also recently been assembled, to prepare for the design of a national primary STEM curriculum.

Traditional teacher education programmes have focused on developing skills and knowledge in the individual disciplines. Pre-service teachers, therefore, are rarely provided with opportunities to develop pedagogical approaches to integrated STEM. Given the increased emphasis on interdisciplinary approaches, practices need to be reimagined, as teachers feel unprepared to teach in integrated ways (Shernoff et al., 2017).

Methodology

Participants were 27 primary pre-service teachers undertaking a mathematics specialism course as part of their Bachelor of Education programme. As part of a larger study investigating PSTs' evolving understanding of integrated STEM education, this paper follows the PSTs across two elective modules and reports on their experiences of integrating meaningful and authentic mathematics in STEM tasks. In the second semester of their third year (*Phase 1*), PSTs worked with three teacher educators across a 12-week integrated STEM intervention. As part of this module, PSTs designed and taught five STEM tasks to a fifth class in a partner school. PSTs and teacher educators then reflected on field practice, with a particular focus on the positioning of mathematics within each lesson. In the second semester of their fourth year (*Phase 2*), the same PSTs returned to the role of mathematics within STEM. Over five weeks, PSTs reconsidered the positioning of mathematics in STEM task design. They examined the role of mathematics during a STEM inquiry cycle, seeking out opportunities for rich mathematical teaching and learning at each stage of the process.

Data collection methods include pre-post intervention surveys, field notes, reflective journal entries, video recordings of lessons, post-teaching focus groups (*Phase 1*) and group-designed STEM tasks and interviews (*Phase 2*). Data from *Phase 1* were analysed using a grounded theory approach. Codes were generated and a constant comparison method was used to examine the data within and across each participant's data corpus. Data collected in *Phase 2* were deductively analysed based on the findings from *Phase 1*. Ethical approval was granted by the institution's research ethics committee and all considerations were adhered to.

Findings and conclusions

Phase 1

The pre-intervention data reflects high levels of confidence amongst the PSTs in their ability to teach mathematics. Participants attributed this confidence to success and enjoyment in their mathematics education modules, as well as in their professional school placement. Following the integrated STEM education intervention, however, there was concern amongst the PSTs relating to how mathematics could be meaningfully integrated in STEM. This was particularly evident following field practice. While video analysis and post-teaching focus group discussions suggest that the tasks were successful in promoting some STEM learning (notably science, engineering and 21st century skills), there was no evidence of planned, age-appropriate mathematical teaching and learning across the five lessons. Furthermore, the PSTs initially viewed the unambitious and incidental mathematical activities that were present (such as lower-order computation and measurement) as authentic mathematics integration and had significant difficulty in recognising opportunities to explore rich mathematical content and promote mathematical thinking.

We have detailed elsewhere (Fitzpatrick, et al., 2023) some possible reasons for the lack of mathematical learning in these STEM tasks. Firstly, by prioritising engineering practices (the initial source of concerns for PSTs), we, as teacher educators, underestimated the need to maintain the spotlight on mathematics in STEM and made assumptions about the ease with which PSTs would apply their experiences in mathematics education to more integrated approaches. Secondly, no curriculum-based learning outcomes were identified that would focus on the mathematical skills and understanding to be developed. Finally, the task parameters did not stimulate mathematical thinking or reasoning. Given the difficulties in realising productive mathematics within integrated STEM tasks, we suggest that conscious decisions must be made about *where* and *when* we position mathematics and STEM in our teaching. We argue that reflecting on mathematics learning opportunities before, during and after STEM tasks in the planning process, would support PSTs in recognising rich opportunities for mathematics in STEM inquiry cycles, allowing discrete disciplinary learning to inform meaningful integrated work and vice versa. In turn, we offer the terms **mathematics learning *for* STEM** and **mathematics learning *from* STEM** (Fitzpatrick, et al., 2023).

Mathematics learning *for* STEM. Specific disciplinary knowledge and skills, developed in preceding discrete mathematics lessons, to be utilised and developed in new ways during the STEM task.

Mathematics learning *from* STEM. New mathematical knowledge and skills developed during the STEM task. Furthermore, it may also refer to new mathematical concepts that the STEM task presents, whereby the STEM tasks act as both a rich context and a springboard for future mathematical inquiries.

Phase 2

Phase 2 of this study utilises the findings from Phase 1 as a framework for designing a mathematics-focused integrated STEM task. Attention was focused on the potential for spotlighting the ‘M’ in integrated STEM, by: ensuring tasks contain reference to relevant *explicit mathematics disciplinary content*; identifying *curriculum-based mathematics learning objectives*; setting *task criteria and parameters* that stimulate mathematical reasoning; and providing *mathematical materials* that promote mathematical thinking. PSTs were also guided to consider meaningful opportunities for discrete mathematical learning *for* and *from* STEM. Emerging findings suggest that the PSTs successfully used the framework to uncover potential for rich mathematical learning at different stages of the inquiry cycle. In their groups (5-6 PSTs), pre-service teachers were required to design integrated STEM tasks (based on *the honeybee*) that positioned mathematics as the central discipline. The groups each identified appropriate mathematical concepts and skills to be developed while remaining attentive to the characteristics of an integrated STEM lesson. Two lessons focused on the honeycomb structure, exploring geometry (shape and tessellation) and measures (capacity and volume). A third lesson also explored volume, this time presenting a Fermi problem to estimate the number of bees that would fit in a given area. The fourth lesson was centred around statistics, using data sets and maps to determine the best location for bee populations. The final lesson set children the task of designing a school ‘pollinator garden’, offering a series of dimension parameters to investigate measurement. While aspects of the tasks needed further development, there was a striking difference in terms of mathematical engagement, age-appropriate content, as well as an increase in mathematical cognitive demand, compared to earlier tasks designed. These preliminary findings, while limited, suggest that this framework has potential to support PSTs in foregrounding mathematics in integrated STEM tasks, and warrants further investigation. Data collection is ongoing.

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Teachers and learners teaching and learning: How does the use of participatory methods of research affect the design of an Augmented Reality app for the teaching of Mathematics

James O'Reilly

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Once confined to the realms of science fiction, Augmented Reality (AR) is an emerging technology that has applications to industry, entertainment and communication. As the technology becomes increasingly accessible, potential applications of AR to education are coming to the fore. This research seeks to investigate how the technology can be applied to the Irish Primary Curriculum in its infancy, with the aim of suggesting how AR can be applied to the primary classroom moving forward. As part of this research, child participant researchers used a prototype AR app developed by an Irish primary school teacher, and suggested how the app could be improved over 8 weeks. The suggestions made by the children were then added to the app before they were asked to make further suggestions. This paper describes how the AR app developed over the course of the research relates to the Irish Primary Mathematics curriculum, to the Irish primary classroom and to the children who helped to improve it through discussion of the analysis of data collected in a Researcher Structured Diary, some of the changes made to the app, the worksheet given to the children and transcripts from Focus Group sessions.

Keywords: Mathematics, Augmented Reality, Participatory Methods, STEM, Primary Education

Introduction

Despite being an emerging technology Augmented Reality (AR), it is predicted, has the potential to have an impact on teaching and learning in the future (Ibáñez & Delgado-Kloos, 2018). In recent years this technology has become increasingly accessible, particularly with the development of increasingly powerful handheld devices that can support a variety of AR applications. This research aims to consider AR for Irish primary education from its foundation, with the intention of suggesting how the technology may be effectively used in primary classrooms for the teaching of Mathematics and the development of 3D spatial awareness and pattern recognition. The AR app being used in this research, provisionally called PopCubes, was a piece of software which had been designed by an experienced Irish primary school teacher with consideration of the current Irish Primary Mathematics curriculum (NCCA, 1999). That the prototype software being investigated was designed solely by an Irish primary school teacher who does not have a background in software design is one novel aspect of this research. Adding to the novelty, this research adopted participatory methods. These participatory methods allowed child participant researchers to contribute to the design of PopCubes over a number of weeks, directly influencing and determining how the app changed and developed over time. This paper will consider how the use of PopCubes relates to Mathematics specifically, and the potential that the child participant researchers, and the teacher researcher, see for the use of AR in the teaching of Mathematics.

Software Used

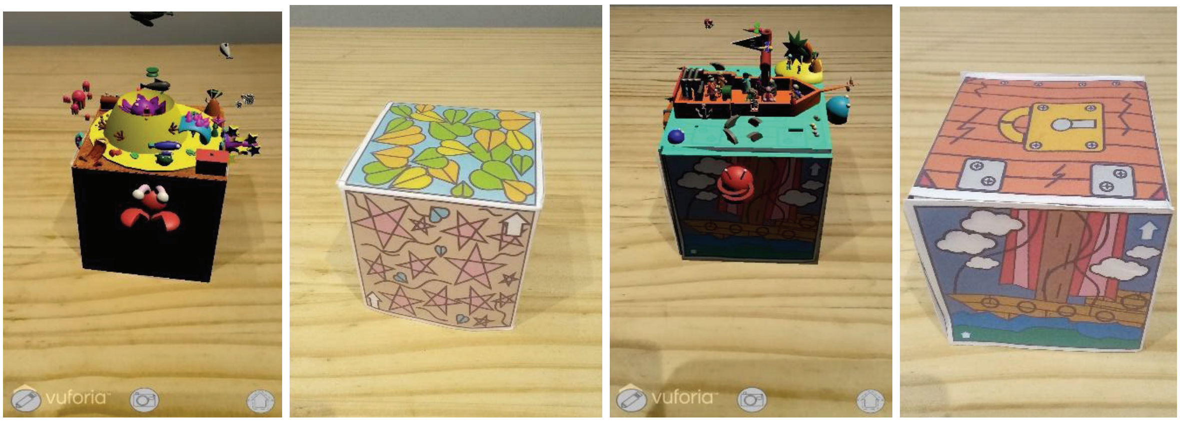
Before beginning an 8-week intervention with the children the decision was made to design and build an AR app, which resulted in the creation of PopCubes. This decision was made after an investigation into the AR apps available, none of which met the criteria of this research. The criteria being considered included age appropriateness, direct links to the Irish Mathematics curriculum and the exclusion of potential for engagement with individuals outside the school through the technology. Before beginning the development of PopCubes the researcher developed an App Design Tool which considered the Irish Primary Curriculum (NCCA, 1999), educational theory (Freire, 1972; Karl, 2012; Papert, 1980; Sadovnik, 1991), good pedagogical practice and Irish educational policy (DES, 2017; DES, 2015). PopCubes was developed, on and off, over the course of approximately a year. 2D assets for PopCubes were created using Microsoft Paint while 3D assets were built in Tinkercad, a free, web-based tool for 3D Computer Aided Design (CAD). PopCubes was developed using Unity, a free piece of software that can be used for the development of video games and apps.

One area of the Irish Primary Curriculum emphasised in PopCubes is the Strand of Shape and Space (NCCA 1999.) As they explored scenes in PopCubes the children were asked to complete a worksheet which asked them to explore aspects of Shape and Space and to use their understanding of the area to propose their own additions to the scenes.

When using PopCubes users point the camera of their device at a card cube, upon which a virtual 3D augmentation, referred to in this paper as a scene, appears. Figure 1 shows a photo of one of the card cubes alongside the cube as it appears in PopCubes. The user has access to three buttons, each of which serve a different purpose. The Enlarge button allows users to enlarge parts of the scene to take a closer look, the Pen button allows users to annotate what they can see on the screen and the Camera tool allows users to take a screenshot of what is on the screen at the time of the button being pressed.

Figure 1

Two cubes with and without their virtual augmentation



Methods

Before engaging in any research activities, ethical clearance was sought, and obtained, from the Mary Immaculate College Ethics Committee. Consent for the carrying out of the research was sought from the school principal and the class teacher before parental consent, after which assent was obtained from the children. Of the 22 children in the class, one child elected not to take part.

This research adopted qualitative methods with aspects of quantitative data collection. Much data was collected over the course of the intervention, some of which is beyond the scope of this paper. Each week children used the prototype software and gave feedback on how it could be improved by adding, changing or removing something. The children's suggestions were then implemented into the app before they used it again, after which they gave further feedback to be implemented. This piece will focus on data collected from the Focus Groups and Structured Researcher Diary, both of which were analysed through their transcription and coding (Braun & Clarke, 2008) after the intervention. Focus Groups were selected as they can allow for the collection of living, dynamic data (Hennink et al., 2012) and the Structured Researcher Diary was used to allow for the recording of reactions, hypotheses and difficulties encountered (Miles & Huberman, 1994). While many findings emerged over the course of the research, this paper will consider some findings that relate to the primary Mathematics curriculum (NCCA, 1999).

Findings

As the research began it became apparent that the children enjoyed using PopCubes. As the children used the app, they expressed positive affirmations of it saying "I like everything in it" (Pupil 2, Week 5) or that they "love" seeing their suggestions integrated into it (Pupil 21, Week 2). The children enjoyed exploring the scenes that appeared when they focused their camera on the cubes, telling others that they "like finding Robbie the Robot" (Pupil 5, Week 1), a robot hidden in each of the scenes, and even suggested that I make "finding stuff in the map a little harder" (Pupil 14, Week 3). Not all feedback was praise, however, with children saying "I don't really like it" (Pupil 21, Week 2) and that they thought the scenes did not look good "because it's all cubes" (Pupil 19, Week 8). Aspects of PopCubes had the potential to be distracting, as recorded in the Researcher Structured Diary (Week 1, Group 4; Week 6, Group 2).

The children did recognise some of the potential for learning through PopCubes. When asked if a child could learn from the app children responded "Yes. They could completely learn" (Pupil 13, Week 2). Children described engaging with the Primary Mathematics Curriculum describing "spheres" (Pupil 8, Week 2), discussing verticality (Pupil 12, Week 7) and saying that, through the app, users "could learn their shapes" (Pupil 16, Week 3). The researcher recorded that children were using "Maths language to ask one another questions" (Week 1, Group 2). The researcher recorded that children were using "positional" (Week 5, Group 4) and "directional" (Week 7, Group 4) language when describing aspects of the scenes. The children enjoyed a challenge (Pupil 5, Week 1) and children also made suggestions about the worksheet, suggesting questions such as "how many cubes can you find? How many spheres?" (Pupil 10, Week 1). From an early stage the researcher noted that engagement with Mathematics was strong when "working on sheets" (Week 2, Group 3). As the research progressed pupils suggested adding levels to the worksheet, adding a system of keys and locks which meant users could progress to the next level when they had completed a certain number of questions, a change which the researcher noted was exciting for the children (Week 3, Group 4).

Children seemed to enjoy taking part in the Mathematical aspects of PopCubes. On the worksheet children were asked to find 2D and 3D shapes hidden throughout the scene and children enjoyed exploring the scene and looking for shapes, saying “I love looking for shapes” (Pupil 8, Week 2) and “I love 3D shapes” (Pupil 16, Week 8).

This was reflected in the Researcher Diary, where it was recorded that children were “enthusiastic users of worksheets” (Week 4, Group 1). The 2D shapes were an addition to the scenes, a change made by the researcher after Week 2 because children were finding it difficult to identify faces of 3D shapes as 2D shapes, resulting in them being unable to complete the worksheet. This addition allowed for closer links to the 2D shape aspect of Shape and Space, as recorded in the Structured Diary (Week 3, Group 2). Towards the end of the intervention the children described finding the worksheet easy, in particular finding the 2D shapes. The finding of 3D shapes was described as “more difficult” but still “easy” (Pupil 6, Week 8).

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Mathematics Learning Support in Ireland: Where We Have Been, Where We Are Now, Where We Are Going

Mathematics learning support (MLS) has become a core part of the support systems offered by Higher Education Institutes (HEIs) in Ireland over the past fifteen years. The premise behind MLS is that it is free, additional support offered to students to help them to navigate the mathematical demands of their degree programme. Prior to the university campus closures that took place during the COVID-19 pandemic, this support was offered almost exclusively in person, with a mixture of open drop-in sessions and appointment-based support offered, depending on the HEI. Students generally opt to avail of this support of their own accord, although some might be advised to attend by a lecturer or tutor.

The Irish Mathematics Learning Support Network (IMLSN) was formally established in 2009, as a means to promote MLS, particularly through running workshops and conferences promoting best practice in MLS and providing networking opportunities for those who work in the field. As a network, the IMLSN has successfully received research funding to run a number of different projects over the years, and has produced two national reports on MLS in Ireland.

This symposium focuses on different aspects of mathematics support in Ireland that are most relevant today. One paper considers the need for systematic reviews in MLS research in order to ensure findings from individual studies can be combined and utilised. Another paper explores the impact of mathematics support in the wake of the campus closures that took place during the COVID-19 pandemic. A third looks at a project which aims to provide useful mathematical resources for MLS on the IMLSN website, which often include interactive exercises tailored for the Irish HEI environment. Through this variety of papers, we hope to stimulate some important awareness and discussion of issues in MLS among the mathematics education community

A Scoping Systematic Review of Mathematics and Statistics Support Evaluation Literature: Lessons Learnt

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Individual studies have demonstrated that mathematics and statistics support (MSS) provide academic and social benefits to students. However, results from individual studies can not necessarily be generalised. Systematic reviews are an important next step in the evolution of MSS research as they allow for findings from many studies to be combined and analysed. In this paper, the authors reflect on a recent scoping systematic review that they conducted and discuss what lessons can be learnt from it for planning and aiding future MSS research. For example, the authors propose the use of consistent terminology in MSS research, and a greater awareness of international MSS research.

Background

Lawson et al. (2003, p.9) consider mathematics and statistics support (MSS) as support ‘offered to students (not necessarily of mathematics) which is in addition to their regular programme of teaching through lectures, tutorials, seminars, problems, classes, personal tutorials, etc.’. This may be, for example, mathematics support centres, bridging courses, and workshops. Individual studies (Matthews et al., 2013) indicate that MSS assists students academically and socially. Building on Matthews et al. (2013) and the broader overview of MSS from Lawson et al. (2020), and in order to gain a deeper insight into how diverse types of MSS resources are being evaluated and whether they benefit students, the authors have conducted a scoping systematic literature review (Mullen et al., 2023).

Systematic reviews are an important next step in the evolution of MSS research as they allow for findings from many studies to be combined and analysed. Drawing on the hierarchy of evidence pyramid (NSW Government, 2020), systematic reviews and meta-analyses provide the strongest evidence and lowest bias for a research question as they are a formal process that draw on multiple studies as sources of evidence for their results. However, they can only be undertaken once a sufficient level of primary studies has been conducted. A scoping review, one type of systematic review, usually predates a full systematic review or meta-analysis. Scoping reviews ‘may examine the extent (that is, size), range (variety), and nature (characteristics) of the evidence on a topic or question; determine the value of undertaking a systematic review; summarise findings from a body of knowledge that is heterogeneous in methods or discipline; or identify gaps in the literature to aid the planning and commissioning of future research’ (Tricco et al., 2018, p.1).

When developing the protocol for the scoping review in question (Mullen et al., 2022), the authors discussed the approach to identifying MSS research, the eligibility criteria for including studies, and what general aspects of research would be useful for future researchers to know, e.g., funding sources. The scoping review has identified considerable variation in

MSS literature e.g., in the level of description of the MSS provided, in the reporting of the study design, and the evaluation approaches undertaken. This paper focuses on the description by Tricco et al. (2018) of scoping reviews as a way of aiding the planning and commissioning of future research. Therefore, this paper aims to present the authors' reflections on the identification of, and the methodological description provided by, MSS research.

Methodology

Studies were included in the scoping review if they met the following criteria during screening: a) published in English from 1st January 2000, b) the MSS provision was formally organised, optional for students and based within a higher education institution, c) studies included evaluation of the impact of MSS on students using either statistical methods or qualitative analysis. The scoping review included the following stages (Mullen et al., 2023):

1. Identification: Relevant studies were retrieved from pre-selected databases (ACM Digital Library, Australian Educational Database, Eric, PsychInfo, Scopus and Web of Science) using a pre-defined search string. The search string combined terms/synonyms for higher education and MSS (see Mullen et al., 2022), and returned studies where these words appeared in the study's title, abstract or keywords. The database search identified 3,009 records (including 1,314 duplicate records). Additional relevant pre-selected sources (MSS websites, reference lists of relevant studies, conference proceedings, MSOR Connections etc.) were searched and 189 further studies identified.
2. Round 1 Screening: Titles and abstracts of the database studies (n=1,695 with duplicate records removed) were screened to check if they met the eligibility criteria. In total, 169 met the criteria.
3. Round 2 Screening: Between the database search and additional sources, 349 relevant studies were found (full text) and screened (based on the full text). Subsequently, 149 studies were included in the scoping review.
4. Data Extraction: Pre-defined data items were extracted from each study.
5. Dissemination: Reporting of the scoping systematic review.

Descriptive statistics and summaries of the studies (for example, types of data collected, location of the institutions providing MSS, number of studies that received funding, and ethical approval status) are included in Mullen et al. (2023). This symposium paper reflects on the lessons learnt by the authors when undertaking the scoping review.

Results and Discussion

Identification of Mathematics and Statistics Support Research

In what ways is MSS beneficial for students? Systematics reviews can provide answers to questions such as this by drawing on multiple sources of evidence through a formal process, and therefore reducing bias in the answer. However, an essential part of this process is identifying the sources of evidence to draw on in the systematic review. The main approach is to create a search string, which is informed by the research question(s) and use the search string on different databases. A secondary search can also be conducted to draw on extra sources (which are generally identified using discipline knowledge).

Considering the main approach, studies need to be identifiable through matching terms in the search string to terms in their abstracts, keywords, and titles. This emphasises the importance of consistent and clear terminology in MSS research. The third author used their international contacts to identify different terms that are used for MSS e.g., learning assistance centre, calculus centre, and mathematics tutoring. It is likely that some terms used for MSS were not identified for the scoping review (Mullen et al., 2022). The authors propose that a series of keywords, which could be used internationally, may be established to make identification easier. In addition, a second consideration is that studies must be located on the pre-selected databases for them to be identifiable. For example, the Web of Science, one database that was used for the scoping review, proved not to contain all sources of MSS literature. The editors of Web of Science decide which journals, books and conference proceedings are included on it. While new sources can be submitted for consideration for inclusion, the editors decide based on a set of quality criteria what is included. For example, for a journal (and hence the journal's articles) to be included, the journal must have a registered ISSN, have a peer review policy, have a defined publication frequency, be transparent in their ethical requirements for authors, have stable citation activity etc. Some systematic reviews will only search databases as it ensures the literature meets these criteria.

The secondary search is an effective way of including additional sources of evidence and broadening the sources of evidence to other scholarly outputs (e.g., conference proceedings, websites, and reports). Using their discipline knowledge, and drawing on colleagues' knowledge, the authors identified additional sources of information (e.g., Sigma (UK) Network for Excellence in Mathematics and Statistics Support website, references from Matthews et al. (2013) and Lawson et al. (2020), and Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics (Delta) proceedings). The largest source of additional studies for screening came from MSOR Connections (n=101). It is likely that some international sources were missed; particularly from countries where English is not the first language. It would benefit future systematic reviews on MSS if sources of MSS research are known internationally and studies are open access.

Eligibility Criteria

Which studies are included in a systematic review will also be impacted by the eligibility criteria. For example, following extended discussions concerning the diverse types of MSS, the authors chose to include only studies where the MSS was formally organised and had voluntary engagement by students. In one case a study was excluded as the support, a workshop on maths anxiety, was mandatory for all incoming students. The eligibility criteria were kept quite broad for the scoping systematic review (Mullen et al., 2023) as the purpose of scoping reviews are to identify gaps in the literature and provide a broader overview of the research area (Tricco et al., 2018). Full-systematic reviews tend to have more stringent eligibility criteria as they are trying to answer specific research questions. Common eligibility criteria for systematic reviews may be a minimum sample size, a specific study type (quantitative or qualitative) or design (e.g., interviews, questionnaires, third-person reporting, pre- and post-analysis), specific demographics (based on age, gender, educational level etc.), location of study, or evaluation of specific measures (e.g., retention and confidence constructs). During Round 2 Screening of the scoping review, the authors discussed how some studies provided less methodological detail. Subsequently, the categories of different data items to be extracted from the studies were adapted. For example, as some studies did not explicitly state the study design, a new category of 'not specified' was created. An alternative approach, used by (scoping) systematic reviews, when methodological information is not stated explicitly, would be to exclude these studies. Similarly, it might be that the systematic review is examining a specific type of MSS support (e.g., drop-in maths support centre) and a study where the support is not described in sufficient detail (e.g., described as a maths support centre only) would be excluded even though the study is relevant for the review.

Conclusion

While systematic reviews are considered as a rigorous approach to synthesising the research on a topic, the authors hope this paper highlights the subjective nature of them and the importance of establishing a protocol for them which has been discussed in detail. To move forward with MSS research and evaluation, the authors propose that MSS research uses consistent terminology and explicitly states the methodology being used, and that there is increased awareness of MSS research internationally.

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The Impact of Mathematics Support upon Student Retention: The Student Voice

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A student survey undertaken by the Irish Mathematics Learning Support Network (IMLSN) in 2014 found that students reported a positive impact from mathematics support upon student retention. However, during the COVID-19 pandemic and subsequent university campus closures, engagement with mathematics support reduced greatly. To ascertain whether students still perceive that positive impact from mathematics support, an anonymous student survey was carried out with Dublin City University, with a total of 492 responses received. Of these, 164 students had used the services of the Maths Learning Centre. These students were asked about whether they had considered dropping out of their programme due to mathematical difficulties. Just under a third of students stated that they had thought about this, but two-third of these students credited the Maths Learning Centre with influencing their decision not to drop out. In this short paper, we give voice to their words in relation to these struggles, and consider the ongoing impact of mathematics support following the COVID-19 pandemic.

Introduction

In 2014, the Irish Mathematics Learning Support Network (IMLSN) undertook a national survey investigating student perception of mathematics support in higher education (O'Sullivan et al., 2014). The survey received responses from 1,633 first-year students across nine higher-education institutes (HEIs). One of the questions they considered related to the reported impact of mathematics support upon student retention (Ní Fhloinn et al., 2014). They found that, of the 573 responses, 125 (21.8%) students stated that they had considered dropping out of their course or college due to mathematical difficulties. Students were then asked if mathematics support had influenced their decision not to drop out, and of the 110 responses, 69 (62.7%) felt that it had.

During the university campus closures that took place due to the COVID-19 pandemic, the vast majority of mathematics support services continued to provide support in an online format. However, engagement fell dramatically, with 82% of Irish HEIs reporting lower numbers than usual (Hodds, 2020). Within DCU itself, in the four days leading up to the campus closures in March 2020, there were 256 visits to the Maths Learning Centre (MLC), where there were only 98 online visits between 19th March and 5th May 2020 in total (Howard & Ní Fhloinn, 2022). Since the return to in-person learning, engagement with mathematics support has increased, although there has been some (as of yet anecdotal) evidence that student attendance on campus is lower, resulting in fewer in-person visits. Therefore, the research question we aim to revisit in this short paper is "Did students report that mathematics support has an impact upon student retention?", in order to ascertain if students still report a positive impact between mathematics support and retention, as we have returned to in-person learning.

Methodology

An anonymous student survey was designed, based largely on the questions asked in the IMLSN survey (O'Sullivan et al., 2014). An MLC tutor visited first and second year mathematics modules during semester 2 2023 and asked students to complete the survey. It could be completed either online (via a Google Form) or paper-based, as student preference dictated. All students opted to complete it online. In addition, QR codes linking to the survey were posted on desks in the MLC. The quantitative data was analysed using Excel.

Results

There were a total of 492 responses to the survey, of which 164 students had used the MLC services (either the drop-in centre, or online Zoom sessions). 56.2% of respondents identified as male, 43.1% as female, and 0.6% as non-binary or other. Almost 4% self-classified as mature students (meaning they were over the age of 23 when they began their studies). Two-thirds of respondents were in first year, with a further 30.3% in second year, 1% in third year and 2.2% in fourth year. This was largely due to how the survey was distributed, and also because a high proportion of students in DCU do not study mathematics beyond the first or second year of their degree programme.

There were 158 responses (from those who had attended the MLC) to the question "Did you ever consider dropping out of your course/college because of mathematical difficulties?" with 31% stating that they did. Of these, 37% were undertaking a mathematics degree programme (either Actuarial Mathematics, or Common Entry into Actuarial and Financial Mathematics), with a further 6% studying a concurrent mathematics post-primary education degree. Students commented on the level of mathematics within their module ("*Felt like cannot cope with difficulty of material*"), their fear of failure ("*I feared that I would never pass my modules and be forced to leave*"), falling behind ("*Wish to drop out absolutely all the time. Cannot keep up with the maths*"), problems with lecturers ("*lecturers can lack empathy, hardly any female representation*"), and the knock-on effect on their other modules of struggling with mathematics ("*I struggle with my physics module also and I believe that this is because of my difficulty with maths*").

Of these, two-thirds (33 students) stated that the MLC influenced their decision not to drop out. Students mentioned the support they receive from tutors ("*They can help explain difficult stuff*"), how it encouraged them to study with their peers ("*It's huge to have somewhere to work with my peers with guidance from experienced tutors*"), and how it has increased their mathematical understanding ("*The MLC has helped with understanding*"). They also spoke of how it helped them to put their concerns into perspective ("*it made me realise that I just needed someone to help me understand the questions that I had*"). In addition, a further 10 students, who said they did not consider dropping out at any point, stated that this was because of the MLC in the first place ("*I know there's someone to go to*").

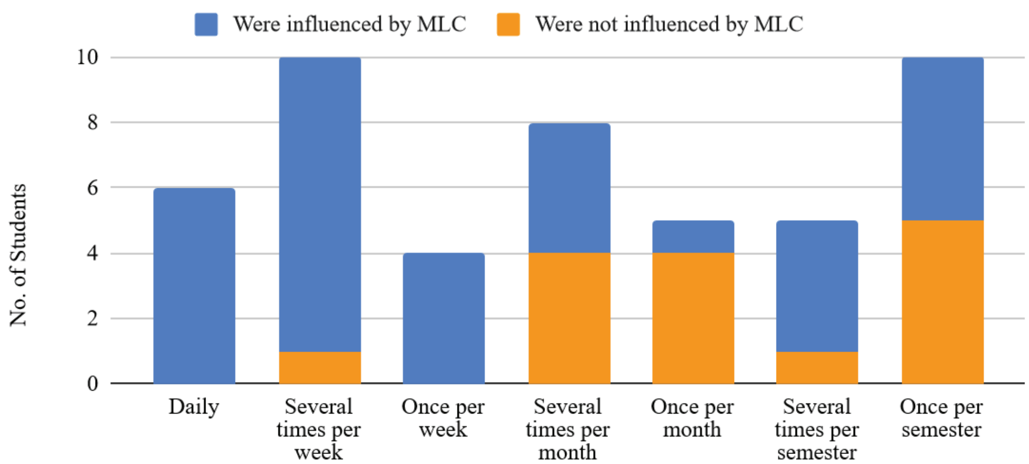
Students were asked what week they first used the MLC. Of the students who considered dropping out due to mathematical difficulties, 3 students did not fully provide information on when they first used the MLC, and only 6 students first used the MLC in semester 2. The following analysis therefore only considers students who considered dropping out who first used the MLC in semester 1. On average, students who stated that the MLC influenced their decision not to drop out first used the MLC two weeks earlier than students who stated that the MLC did not influence their decision. A two-tailed t-test was performed to ascertain if this data was statistically significant. The null hypothesis was rejected at the $\alpha=0.05$ level ($p=0.013$).

Students were asked how often they use the MLC, on a 7-point scale from "Daily" to "Once per semester". Of the students who reported wanting to drop out, almost all students who used the MLC once per week or more frequently stated that the MLC influenced their decision not to drop out, with only one outlying student (see Figure 1). A two-tailed Mann-Whitney U test was performed to ascertain if this was a statistically significant result. The null hypothesis was rejected at the $\alpha=0.05$ level ($p=0.008$). This significance is also evidenced in student comments: students who came more frequently were more likely to mention specific tutors ("*[Tutor] helped me infinitely*", "*MLC saved me, [Tutor] and [Tutor] especially*"), with students who came daily or once a week constituting the vast majority of these comments.

It is worth noting that there was no significant correlation between these statistics (when a student first used the MLC, how frequently a student used the MLC) and whether or not the student considered dropping out of their course programme.

Figure 1

How often students who considered dropping out used the MLC in Semester 1 and how much the MLC impacted their decision to remain



Note. Students who were not influenced by the MLC to not drop out (orange) mostly used the MLC infrequently.

Discussion and Conclusions

The results of this survey undertaken in DCU echo the previous results of the IMLSN survey (O'Sullivan et al., 2014), but in fact, are more stark than the earlier findings, in that a total of 31% of respondents had now considered dropping out as a result of mathematical difficulties, compared with 22% in the (albeit larger) earlier sample. Many of the themes that emerged from the national survey could also be found in the smaller sample of comments from DCU students, with a similar frequency distribution, although there was a noticeable absence of anyone mentioning the gap between mathematics in school and higher education. That said, it was striking that 43% of these DCU students were in programmes that they knew would contain all, or a high proportion of, mathematics, having chosen to be actuaries, work in financial mathematics, or become post-primary mathematics teachers.

There was no evidence of a reduction in the percentage of students who credited mathematics support with their retention in higher education, with 67.3% agreeing in our sample, compared with 62.7% in the national sample in 2014. The strong link between this positive influence and students' early and frequent engagement reinforces the validity of this result. This is a positive outcome in terms of providing evidence of the ongoing impact of mathematics support for students. In the words of one survey respondent who is undertaking a Physics degree,

> I have had a little bit of confidence for my maths abilities while doing the course, at times I felt like I would be more suited for a business course. But using the MLC, I can achieve greater things in the course I'm in now.

The challenge now remaining for mathematics support practitioners is to continue to reach out to those who do not engage but could benefit from mathematics support, particularly if lower student on-campus attendance becomes a part of our "new normal".

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The IMLSN Online Resources Website: Sharing Maths Resources

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As a reaction to the sudden obligation for Irish maths learning support centres to make suitable online resources quickly available for students due to pandemic restrictions, the IMLSN Shared Mathematics Resources Project was founded in 2020. The aim of this ongoing project is to provide easily searchable, reliable and targeted selections of notes, videos and interactive exercises in key topics in Mathematics and make them available on the IMLSN-website. In this paper we report on the status of this project: the criteria for choosing suitable resources; what topics are available so far; and feedback from users. The strength of this website is that resources are chosen with the Irish context in mind, are quick and easy to find, and that for most topics, interactive exercises with feedback are provided. The MEI symposium provides a welcome opportunity to discuss the progress of the project with the mathematics and statistics learning support community.

Background to the Project

One outcome from a survey on the online presence of maths support centres in Ireland and the UK (conducted in 2018) was that participating Maths Learning Support (MLS) managers “face a number of challenges including institutional control, staff time and skills and a myriad of technical issues.” The authors recommended considering how to be efficient when developing online resources: “... before an MLS practitioner decides to invest precious time and resources on developing their online presence, they should carefully consider why they are developing these resources. Are there similar resources out there that could be used? ...” (Mac An Bhaird et al., 2020, p. 206).

In 2020, as most maths support centres turned to online support due to Covid-19 restrictions, such challenges but also the desire to share resources became apparent. Participants at two Irish Maths Learning Support Network (IMLSN) workshops, held in June 2020 and September 2020 highlighted this issue as pressing[1]. The IMLSN committee scoped out a proposed response and surveyed the members of IMLSN about it. The response in the survey was clear: the vast majority said that a curated collation of maths learning resources from different institutions organised by topics would be beneficial for students and colleagues from their institutions. In particular, the challenge for students as well as for tutors to quickly find suitable resources from an overwhelming choice available online was emphasized: “I think it's important that the more basic principles of maths are easily accessed through resources like this.” As a response to this, the IMLSN Shared Mathematics Resources Project was founded. The aim of this project is, by using the existing IMLSN website, to curate and collate a choice of online resources to provide easily searchable, reliable and targeted selections of resources organised by themes. The resources are categorised by subject (mathematics, statistics, applied maths), topic (foundation maths, calculus, linear algebra, hypothesis testing, etc.), and format (videos, notes, interactive exercises).

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1 https://www.imlsn.ie/index.php/past-events/maths-support-in-covid-times-workshops/imlsn-workshop-12

With funding from the National Forum for the Enhancement of Teaching and Learning in Higher Education, two experienced tutors were employed to set up the first two chapters of the website. In the initial phase of the project, regular meetings took place to agree (i) on the layout and organisation of the website, (ii) on the organisation of the relevant mathematical topics, and (iii) on criteria for choosing suitable resources. This proved to be difficult as there is an overwhelming amount of excellent online resources available. This initial process was time consuming. However once the group had come to agreement, the development of the first two chapters proceeded smoothly and efficiently. To facilitate continuity of the project, the second author wrote guidelines on how to choose resources for the IMLSN website. Also, authors of chosen resources were contacted, and approval for using their resources on the website was provided. The first chapter on Foundation Mathematics was launched in June 2021. Since September 2021 a second chapter on Calculus is available to use for individuals and for further and higher education institutions in Ireland.

The project is ongoing: the current project team consists of four colleagues from four Irish HE institutions. We are currently gathering feedback from users. Based on this feedback we aim to develop the existing chapters further and to develop more chapters on a variety of mathematical topics such as Linear Algebra, Statistics & Probability, Applied Mathematics, and others. All chapters will be revisited regularly for updating and improvement, taking feedback from the IMLSN community into account. Data is being collected to provide insight in the usage of the resources, and we are planning to conduct further surveys with IMLSN members, teachers and students to gain insights into the usage as well as ideas and suggestions for improvement. Qualitative research will be conducted to find out how students use the different types of resources.

Preliminary Findings from our current Survey

Earlier this year we launched a survey to examine the experience of those who use online mathematics resources, with particular focus on the online mathematics resources on the IMLSN website. The plan is to use this feedback to improve and develop this website further. So far, 8 anonymous participants have filled out the survey. Of those who have used the IMLSN resources webpage, feedback has been very positive ("I think the resources on the IMLSN website are great and should be expanded"). One participant commented that IMLSN resources website could be "very beneficial" for institutions without an extensive pre-existing set of digital resources on their own website. Some maths support centres (MSCs) in Ireland are quite large, well-resourced and have created their own digital repository of resources for their students. However, these MSCs are in the minority. Responses to the survey so far indicate that having a well curated, easy to find, targeted set of good quality digital resources would be beneficial to mathematics tutors and lecturers in supporting their students.

Guidelines for Choosing Resources

Due to the abundance of high-quality online resources, it was necessary to develop amd apply some criteria to reduce the number of choices. We established guidelines for the collection of resources and how they should be presented to users. We will outline these guidelines below. Most of them were based on our own experiences and conversations with students and tutors. It is particularly difficult to choose between the wide range of videos which are available for free. To develop guidelines for choosing videos we were inspired by an article from Guo et al. (2014) which reports about a study of how the style of video production decisions affect student engagement. Their findings suggest that video length was by far the most significant indicator of engagement. (Guo et al., 2014, p. 44). The authors recommend to produce videos shorter than 6 minutes, to display the instructor's head at opportune times in the video, to film in an informal setting, to avoid recording classroom lectures if possible, to use motion and continuous visual flow (for example tablet drawing) in videos rather than presentation slides, to add support for rewatching and skimming, and for instructors to bring out their enthusiasm and to speak fairly fast. (Guo et al., 2014, p. 42).

We developed a directory structure in which we would list all the topics for inclusion along with their order of presentation. For each topic, we wanted a set of notes, one or more videos and interactive exercises for the user to engage with. These were chosen based on a particular set of criteria, which unfortunately were not always possible to completely satisfy.

Some characteristics we looked for in all types of resources included: (i) content that is in line with the "standard" Irish methods taught in LC and upwards; (ii) friendly and accessible presentation to boost the user's confidence; (iii) narrow scope – the resource should focus on the particular topic as much as possible, e.g. deal with adding and subtracting fractions, as opposed to all four arithmetic operations at once; (iv) a clear step-by-step approach from the author as opposed to concise remarks; (v) discussion of common mistakes; (vi) mobile-friendly resources given the modern trend of study via mobile devices.

Ideal criteria for videos included: (i) maximum duration of 10 minutes; (ii) good audio quality and subtitles if these are available), (iii) an informal and friendly presentation style as mentioned previously.

For written notes we looked for: (i) short documents with a soft upper limit of 5 pages; (ii) explanatory pictures if relevant; (iii) uncluttered pages; (iv) exercises, and preferably solutions, to complement the material.

With interactive exercises, we gave a brief overview and explained to the user what they would be asked to do. We also highlighted potential bugs or issues that the user may encounter. The key things to highlight were the facility for checking their method and answers and most importantly, if the exercise has randomised variants that the students can use for repeated practice. It was often difficult to find suitable, or any, exercises so we developed some original ones.

We also included brief descriptions for each resource which outlined what it covered, credited the author, and gave usage notes if relevant, e.g., technical aspects of interactive exercises for those less experienced with computers or the internet; particularly relevant pages in the PDF, or terminology differences between Irish mathematics education and that of other countries, typically the United States.

These guidelines were used by students from the University of Galway for a project where they gathered resources for the website. The students used these guidelines as a template for planning the structure of their contributions, and to inform their decisions on which resources to include based on the criteria outlined above.

Concluding Remarks

Several studies about findings from surveys as part of the SPIRIT Maths project[2] report that students viewed videos of worked solutions, an online bank of practice questions with feedback and a web portal with searchable topics as the most useful, effective, and favoured types of digital learning support (Lacey et al., 2022, pp. 21/22). Also, according to these studies students have a preference for resources provided or recommended by their own lecturers (Morari & O'Rourke, 2022, p. 18). The authors argue that this suggests that students don't make use of external online resources nearly as much as might have been suspected (Morari & O'Rourke, 2022, p. 19). This is in line with reports from our own students as well as from maths support tutors saying that the amount of available online resources can be overwhelming and finding the right resources can be very time consuming. The project aims to meet all students' preferences as described above. As it may not be possible for lecturers to provide or recommend resources on all material needed for their courses, in particular revision material, recommendations from familiar maths support centres may be a second-best alternative for students. This project aims to provide three types of resources, videos, notes and the preferred choice as mentioned above, namely interactive exercises with feedback. All are suitable for the Irish context and are well organised and easy to find. Lecturers and support centres are encouraged to provide a link to the resources website and recommend using it regularly to their students. We hope that this shared website will grow to be a useful tool for the community.

We value your feedback. Also, if you are interested in taking part in this initiative, please get in touch with one of the authors or the IMLSN. Everyone is very welcome!

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Aiming for success: Ireland's new Primary Mathematics Curriculum

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National Council for Curriculum and Assessment

A new curriculum provides an opportunity to propose a fresh new vision for children's learning and refine our understanding of what successful learning looks like. In Ireland, a new Primary Mathematics Curriculum has recently been published that sets forth a new such vision for children's mathematical learning. Tracing the development of the new curriculum, and explicating the rationale for curriculum change and the research base developed, provides an important context for the development of a new Primary Mathematics Curriculum in Ireland. This paper will extract and discuss the core components and features of the new Irish Primary Mathematics Curriculum, and the implications this will have on children's learning experiences and for teaching the curriculum in practice.

Keywords: Curriculum development, primary mathematics, curriculum enactment

Introduction

The National Council for Curriculum and Assessment (NCCA) is a statutory body of the Department of Education comprised twenty-five members appointed by the Minister. The remit of NCCA is to advise the Minister for Education on curriculum and assessment for early childhood education, primary and post-primary schools, as well as assessment procedures used in schools and examinations on subjects which are part of the curriculum.

To develop this advice, NCCA engages in robust discussions and deliberations with Council, its three supporting boards (Early childhood and primary, junior cycle, senior cycle) and the numerous development groups that are tasked with developing individual curriculum specifications and supports. The advice is also shaped by feedback from consultations with the public, schools and early childhood settings, education stakeholders and others.

A new Primary Mathematics Curriculum (DoE, 2023a) has recently been published. This new curriculum holds a new vision for children's learning in mathematics that represents a fresh and innovative conceptualisation of what it means to be 'successful' with mathematics. In this paper, this new vision for children's learning will be outlined in the context of the broader aims and goals of the *Primary Curriculum Framework* (DoE, 2023b). It will then discuss the rationale for curriculum change and describe the journey of curriculum development in the context of primary mathematics. Following this, the research base and core beliefs which underpin the new curriculum will be presented, in addition to the key stated aims of the curriculum. The paper will then detail the key components of the new curriculum specification before discussing key considerations for enacting the new curriculum in practice.

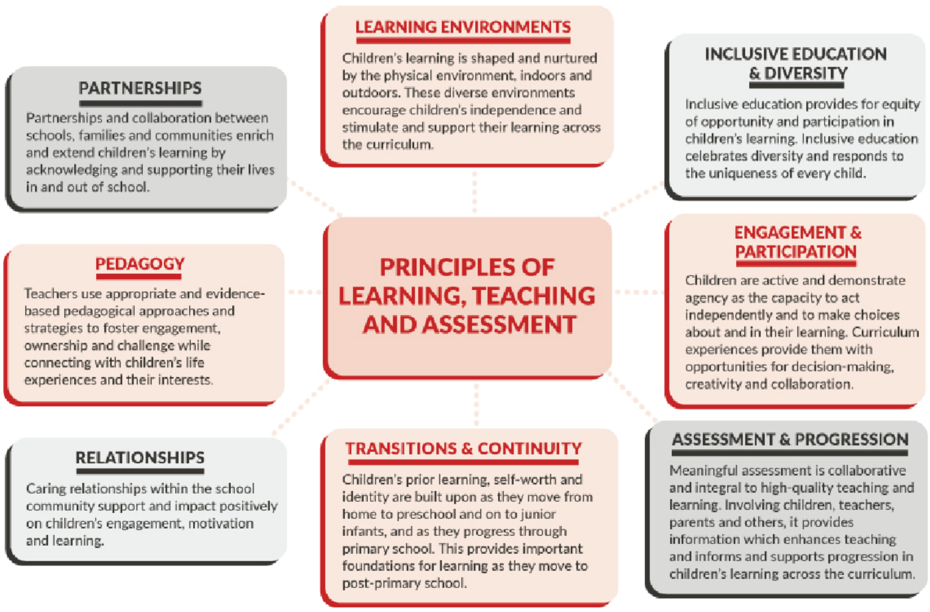
A new vision for children's learning in primary school

In Ireland, the *Primary Curriculum Framework* (DoE, 2023b) sets out a new vision for high-quality and inclusive learning, teaching, and assessment for all children attending primary and special schools. Developed by NCCA, the framework is informed by research, deliberation, and sustained work with school communities; it is the culmination of wide consultation and collaboration across the system. It aims to provide a strong foundation for every child to thrive and flourish, supporting them in realising their full potential as individuals and as members of communities and society. The curriculum views children as unique, competent and caring individuals, and it views teachers as committed, skilful and agentic professionals.

To achieve this vision, the *Primary Curriculum Framework* (DoE, 2023b) spotlights eight essential broad overarching principles.

Figure 1

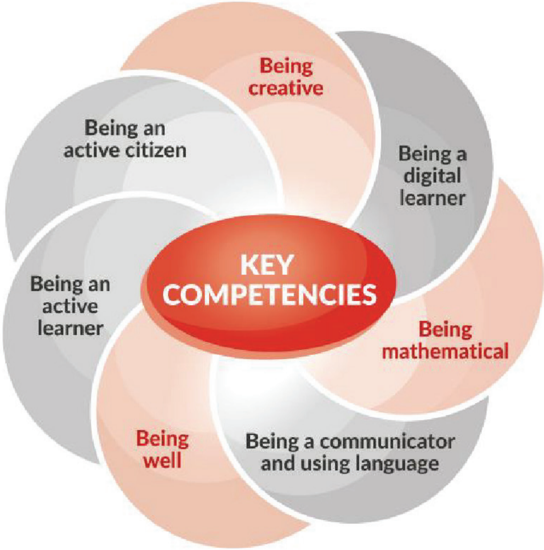
Principles of learning, teaching and assessment



As children progress through primary school, it is envisaged that they will develop competencies that allow them to interact and engage with the social world of their home, community and school, and navigate a wide variety of contexts and situations, not only in childhood but as they mature into adolescence and adulthood. The following seven inextricably linked key competencies, closely linked with the four themes of *Aistear: the Early Childhood Curriculum Framework* (NCCA, 2009a), and the eight Key Skills in the *Framework for Junior Cycle* (DES, 2015); encapsulate the essential knowledge, skills, concepts, dispositions, attitudes and values which enable children to adapt to and deal with a range of situations, challenges and contexts.

Figure 2

Seven key competencies



Curriculum change in primary mathematics

The current Primary School Mathematics Curriculum (DES, 1999) is based on constructivist principles and comprises five strands: Number, Algebra, Shape and Space, Measures, and Data, with Early Mathematical Activities an additional strand for junior infants only. Within these strands, the content is delineated by year and is articulated by a large number of learning objectives.

To develop a new Primary Mathematics Curriculum (PMC), a development group was assembled in late 2016 to begin the process of specification development. This group comprises fourteen members including a chair, representatives from a number of nominating organisations and a small number of co-options. The development of the new PMC sought to build on the success of the current 1999 curriculum and Aistear (NCCA, 2009a), and to address issues raised in a number of publications, including curriculum implementation reviews and evaluations (DES, 2005; NCCA, 2005, 2009b; Murchan et al., 2009) and international and national assessments (e.g. Eivers et al., 2010; Mullis et al., 2012; Sheil et al., 2014). The new curriculum also sought to respond to calls from teachers to reduce curriculum overload and allow for greater teacher autonomy and agency in managing teaching and learning in their classrooms (INTO, 2015).

Initially, the remit for the Early Childhood and Primary Mathematics Development Group was the development of a mathematics specification for children from junior infants to second class. This first draft specification (NCCA, 2017) was published for consultation, in late October 2017. Findings from the first consultation included calls to improve the accessibility of language used in the specification, further guidance on the use of Learning Outcomes as a basis for learning, teaching, assessment and preparation. To mitigate concerns about the removal of class-based learning objectives, teachers called for a greater level of specificity as to the core concepts which underpin each Learning Outcome (NCCA, 2018). This consultation gave rise to a number of new developments, including a revision of Chapter 6, 'Primary Mathematics in Practice', and the development of a suite of Mathematical Concepts and new Support Materials for teachers and parents.

Following this first consultation, the Minister for Education decided that the curriculum should be developed as a full specification from junior infants to sixth class. In 2022, a second consultation was conducted on a full specification of the PMC. Findings from this consultation revealed issues associated with teacher confidence and enacting the new curriculum. Stakeholders called for clearer explanations about the core changes arising in terms of children's learning and pedagogy in working towards this new vision for mathematics learning in primary schools. Additional themes such as alignment, supporting enactment and using Learning Outcomes also featured prominently (NCCA, 2023). Importantly, a key feature of the second consultation involved a greater emphasis on gathering the perspectives of children (Leavy et al., 2023). Amongst a wealth of insights shared by children in the consultation, they called for more collaboration in terms of their mathematics learning, as well as real-world, hands-on and playful learning experiences. Notably, in classes in which 'procedure-focused' and textbook-driven learning dominated, children generally reported negative emotions regarding their mathematical learning experiences. The findings from both the main consultation report (NCCA, 2023) and the report on the findings from children (Leavy et al., 2023) provided useful feedback to help refine the PMC specification and draft support materials; as well as shaping the design of new components of the Primary Mathematics Toolkit.

Research base to support developments

In addition to the consultation reports emerging from these two consultations, a rich research base underpins the development of the new PMC. In line with the two-phase development process described above, the initial research conducted to support developments includes a systematic review of the literature, concentrating on teaching and learning for children aged three to eight years. This includes an international audit of mathematics curriculum policy (Burke, 2014). It also features Research Report 17 which provides the theoretical underpinnings for the development of mathematics education in young people, and discusses current thinking and views on mathematics, specifically regarding definitions, theories, development and progression (Dunphy, et al., 2014). Additionally, Research Report 18 deals with current thinking on the teaching and learning of mathematics, investigating what constitutes good mathematics pedagogy and looks at appropriate structures for the development of mathematical knowledge for pre- and in-service teachers (Dooley et al., 2014). Finally, a background paper and brief for development of the draft PMC (NCCA, 2016) was subsequently produced.

In the second phase of developments, an addendum to Research Reports 17 and 18 (Dooley, 2019) and five short research papers which examined core mathematical concepts, skills and processes with which children engage across the five mathematical domains

(Delaney, 2020; Leavy, 2020; Nic Mhuirí, 2020a, 2020b; Twohill, 2020) were commissioned to complement the existing research base and offer insights on children's mathematical learning in the senior classes of primary school.

Core beliefs underpinning the new Primary Mathematics Curriculum

The new PMC makes explicit a set of core beliefs upon which the curriculum is developed. From the outset, the curriculum asserts that 'every child is mathematical'. As such, every child is considered to have an innate, intuitive and instinctive sense of mathematics which is deepened and developed over time as they engage in rich learning experiences. Accordingly, it acknowledges that a child's mathematical learning journey begins from birth, from which they learn mathematics through their interactions and experiences in their home environment, and later build on this learning through early childhood, primary and post-primary education.

The curriculum also sets out a solid stance on mathematics as a discipline for learning, and the implications this has for the kind of learning experiences that children ought to have in primary school. Mathematics is both a human and social phenomenon that is shaped by their social, cultural and educational experiences. Mathematics is a tool that helps us to make sense of our world so that we can think about, see and organise our everyday lives. Mathematics is also considered to be beautiful and worthy of pursuit and exploration in its own right. Finally, mathematics is everywhere and for everyone.

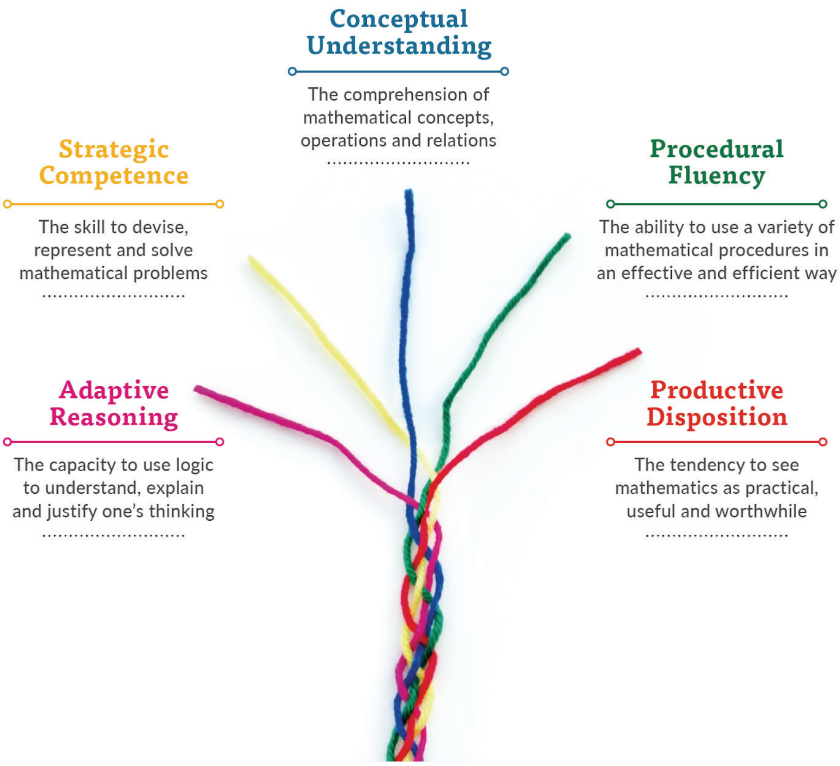
Given these beliefs, it holds that a child's primary mathematics education ought to tap into their innate ability to think and communicate mathematically. Moreover, it should provide opportunities to allow children to communicate and work collaboratively to co-construct knowledge and skills as they interact and collaborate to solve real-life problems. Rich learning experiences with mathematics should be characterised by playfulness, creativity, modelling, thinking aloud and maths talk. And importantly, children should be encouraged to have a positive disposition towards mathematics and develop their proficiency, confidence and resilience through their learning experiences in primary school.

Aim of curriculum: Mathematical proficiency - A new conceptualisation of success for children

The aim of the PMC represents a new conceptualisation of success for children's learning in mathematics, namely the development of mathematical proficiency. This term aims to capture the complex web of intricate knowledge, skills, abilities, and beliefs that children ought to develop to become more confident and competent mathematical learners (Kilpatrick et al., 2001). There are five aspects to mathematical proficiency, as presented in Figure 3.

Figure 3

Five aspects of mathematical proficiency



When considering mathematical proficiency, it would be incorrect to look at any of these individual strands independently, rather they comprise different parts of a complex whole that are interwoven and interdependent in the development of mathematical proficiency (Kilpatrick et al., 2001). Indeed, each of the five aspects can serve to complement and strengthen each other. For example, as a child develops their procedural fluency and becomes more automatic in applying procedures to solve problems, their strategic competence becomes more efficient and effective, enabling them to tackle increasingly complex and novel problems.

Mathematical proficiency is acquired and deepened over time through engagement with meaningful, sustained, engaging and playful learning experiences where children can problem-solve, practice procedures and skills, interrogate mathematical ideas, reason, argue and make connections with their learning. Such experiences provide the opportunity for all children to develop and achieve proficiency. Indeed, rather than being the gift of 'more able' children, the new PMC holds that mathematical proficiency is a necessary and appropriate aim for all children in primary and special schools (NCCA, 2016).

Structure of new curriculum

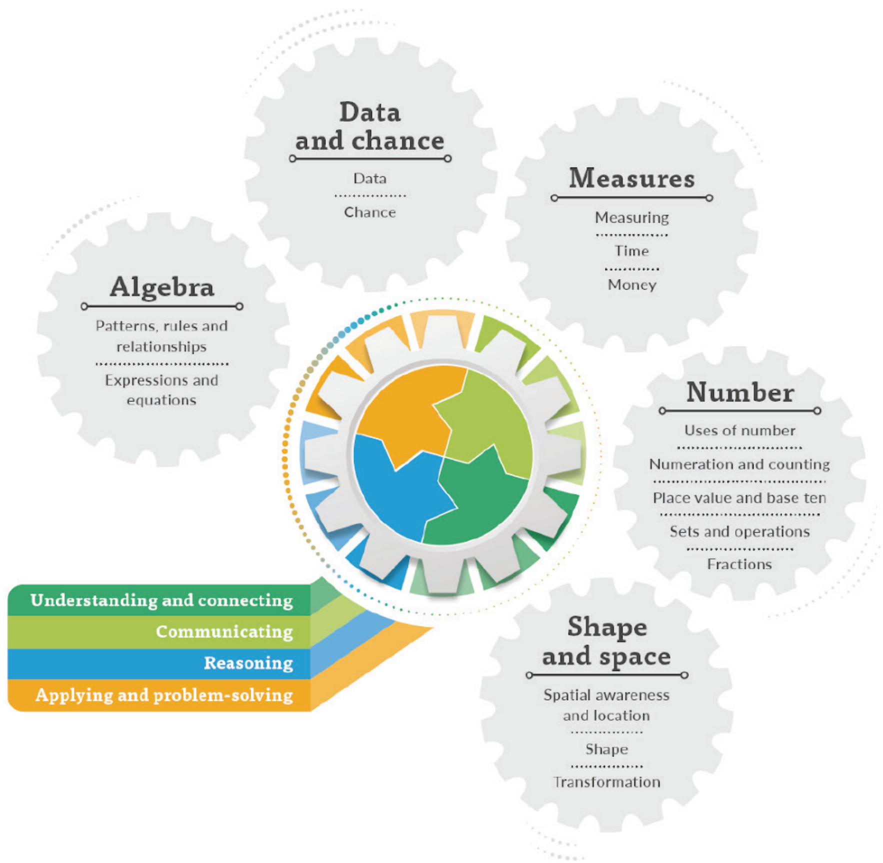
The new curriculum is structured according to five strands or categories of learning, namely, Algebra, Data and Chance, Measures, Number, and Shape and Space. Each strand has a set of strand units, numbering fourteen in total. Each strand unit has a set of Learning

Outcomes. Within the new PMC, Learning Outcomes are used to describe the expected mathematical learning and development for all learners at the end of a two-year stage, when due consideration is taken of individual abilities and varying circumstances. Learning Outcomes articulate big mathematical ideas, and encompass the knowledge, skills and dispositions that the PMC aspires for children to develop.

Importantly, the curriculum also puts forward Elements which describe the main categories of processes that children engage in through their mathematical learning. These mathematical processes are categorised into four Elements: Understanding and Connecting, Communicating, Reasoning and Applying and Problem-solving. These Elements are central to the development of children's mathematical proficiency, and involve children connecting, communicating, reasoning, justifying, representing, problem-solving, generalising and argumentation.

Figure 4

The strands and elements of the Primary Mathematics Curriculum



The new curriculum features a chapter 'Maths curriculum in practice' that describes the fundamental features of children's learning with the curriculum and the corresponding pedagogical practices that support and enhance learning. For 'Learning primary mathematics', this section outlines the kind of playful and engaging mathematical learning experiences that should be offered to children. These opportunities are categorised according to four curriculum Elements, as described above. In terms of 'Teaching primary mathematics', this section proposes five key practices that are essential to the provision of quality mathematical learning experiences. These practices are described in greater detail in the next section.

Finally, this chapter explores 'Assessing primary mathematics' as an integral part of mathematics learning and teaching. It views teachers as committed, skilled and agentic professionals who make key decisions every day about mathematical teaching and learning. These decisions are informed and shaped by the teacher's knowledge of the child and their prior learning, their knowledge of the curriculum and knowledge of pedagogy.

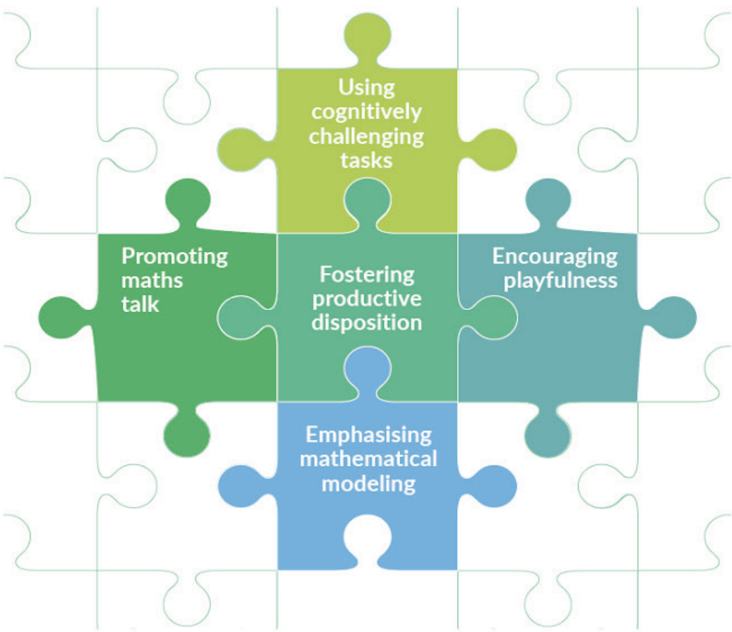
The PMC specification is accompanied by the Primary Mathematics Toolkit, an online space made up of four key components; mathematical concepts, progression continua, support materials and examples of children's learning. The Primary Mathematics Toolkit provides practical support for teachers and parents in building rich mathematical learning experiences for children.

Enacting the new curriculum

The curriculum puts forward five key pedagogical approaches, deemed as essential to the provision of quality mathematical learning experiences.

Figure 5

Five key pedagogical practices for the classroom



1. **Fostering productive disposition**: The first approach conveys the important role dispositions play in children's mathematical learning, highlighting how they can be nurtured or changed over time. It calls on teachers to emphasise the rich, useful and meaningful nature of mathematics in the classroom. The multiple ways in which children engage with mathematics, how they perceive mathematics and the rich

contexts in which mathematics is meaningfully presented to them, are what help form and shape their dispositions towards mathematics.

2. **Encouraging playfulness with mathematics**: Highlighting the role of playful learning for all children across all stages of primary education, this approach provides teachers with an opportunity to engage with children in purposeful and sensitive ways. By infusing playfulness in children's learning experiences, this helps provide an important context for mathematical thinking and the development of mathematical language and concepts.
3. **Emphasising mathematical modelling**: Mathematical modelling involves children using mathematics to understand and describe a problem-context and determine meaningful solutions to problems. It foregrounds the importance of exploration, sense-making, conceptual understanding and flexibility in thinking. Children can be supported to form models through a process of testing, revising and expressing their interpretation of different mathematical ideas, experiences, problems and situations, typically posed to them as questions or challenges.
4. **Using cognitively challenging tasks**: Viewed as rich, higher-order learning opportunities, cognitively challenging tasks can be used to appropriately stretch and challenge children's conceptual understanding as they encounter significant mathematical ideas and situations. Such tasks can provide all children with the opportunity to access mathematics, while offering the potential for deeper engagement.
5. **Promoting maths talk**: Maths talk is a collaborative process where children's thinking, strategies and ideas are expressed, shared and/or exchanged. Providing a space for maths talk allows children to reflect on their own understanding; define, present and justify their ideas; make sense of and critique their own ideas and those of others; and develop their ability to express and articulate their thinking.

While presented separately, these practices are dynamic and naturally link with each other. They foster an inclusive learning environment and culture where children engage in rich and meaningful learning processes.

Practicalities and conclusion

A curriculum change can, for many, necessitate a shift in the role of the teacher and the mindset they hold towards mathematics. Viewing the role adopted by teachers as critical, Dooley and colleagues (2014) highlight how teachers are responsible for structuring the learning environment and developing the learning experiences for children, thus enabling mathematical learning to take place. The authors point out that for many teachers, this will involve embracing pedagogical approaches that will be markedly different from what they experienced as mathematics learners. In working with the new curriculum and embracing the new pedagogical approaches that are outlined above, teachers will be asked to reflect on their own views of mathematics. Teachers will also be challenged to consider the dispositions they themselves hold towards mathematics, their perceptions of 'playfulness' with mathematics and their beliefs about children's innate mathematical abilities. Confronting these considerations will be an important starting point for bringing responsive pedagogical approaches to life in mathematics classrooms and successfully enacting the new curriculum.

Following approval by the Minister of Education, the new PMC is expected to be published in Autumn, 2023. It holds a strong and ambitious vision for children in primary and special schools in Ireland to foster a love for mathematics and to develop as confident and proficient mathematical learners. Following its publication and introduction to schools, teachers, parents and children will have the opportunity to engage in robust conversations about the new curriculum, and to consider how this new vision and conceptualisation of success for children's learning in mathematics will be actualised.

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Testing Times: The Role of Standardised Tests in Irish Primary Mathematics Classrooms

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Drumlease NS

The relation of curriculum and pedagogy to social class and academic achievement has long been a central theme in the sociology of education. This study employs a mixed methods design approach (Teddlie & Tashakkori, 2009) across eight primary school classrooms at second and fifth class level in DEIS and non DEIS schools. A total of twenty-four mathematics lessons were observed, while eighteen mathematics lessons were video recorded. Focus group interviews with children, child questionnaires and drawings were also employed in the data collection. Eight teachers and eight school principals were interviewed. The focus of this paper is to examine how and in what ways primary school teachers react to statutory testing of mathematics. It is assumed that the content of the mathematics curriculum that is taught correlates with what appears in tests. This paper investigates what changes primary school teachers make in both the content of their mathematics lessons and their approach to teaching mathematics as a consequence of national standardised testing.

Keywords: primary mathematics, standardised testing, social class

Introduction

In the field of mathematics education, the issue of privilege and marginalisation has been examined in terms of social class (Atweh et al.,1998; Cooper & Dunne,2000; Straehler-Pohl & Gellert, 2013; Zevenbergen, 2001; Zevenbergen & Lerman, 2001). Students' learning opportunities are shaped by educational conditions and policies that originate beyond the classroom and often beyond the school. Student's lack of mathematical proficiency is, in the eyes of policy makers, to be blamed on teachers, within the context of their infrastructure and their institutions, while increased surveillance set within an audit culture, is the order of the day (Walshaw, 2010). With government education policy focused on standards, there is a danger that 'diversity' as constructed in national statistics is understood mainly in its relation to attainment. These somewhat 'uni-dimensional' constructions feed into policy, prompting interventions targeted at low attaining groups such as those in designated disadvantaged contexts. Such policies offer 'an impoverished understanding of difference' and risk perpetuating stereotypes and 'deficit thinking' among teachers (Ainscow et al., 2010; Tang & Ginsburg, 1999).

Students can be unintentionally marginalised in the classroom as a result of unstated norms and power structures, and the factors that shape participation are not limited to conditions inside the classroom. Research on the influence of culture on participation in mathematics classrooms indicates that students who are not enculturated into norms associated with mathematical discourse are at a disadvantage (Atweh et al, 1998; Boaler, 1997; Cooper & Dunne, 2000; Lyons et al., 2003). According to Cobb (2001)

> "students' home communities can involve differing norms of participation, language, and communication, some of which might actually be in conflict with those that the teacher seeks to establish in the classroom" (p.471).

Success in the mathematics classroom requires cultural knowledge that can serve to privilege the middle class or those who are the linguistic majority (Lerman & Zevenbergen, 2004; Zevenbergen, 2000). Communication norms associated with mathematics can be a source of 'cultural confusion' for students, either restricting participation or weakening the intended benefits of such participation (Rousseau & Tate, 2008). Students' participation in the mathematics classroom is influenced by organisational and social structures.

Standardised Tests

Research into standardised testing highlights the extent to which these tests influence what knowledge is taught, the form in which it is taught, and how it is taught (Au, 2010; Harlen, 2007, O' Leary et al.,2019). When student test results are used in accountability systems, teachers and schools will perceive the no-stakes, standardised tests to be high stakes. Teachers begin to view themselves and their effectiveness as teachers of mathematics on the basis of student outcomes on tests. Consequently when such tests are perceived to be high stakes for teachers and schools, the assessment system risks succumbing to teaching to the test, narrowing of the curriculum, teacher cheating and student exclusion (Anagnostopoulos, 2005; Lam & Bordignon, 2001; MacRuairc, 2009a, 2011; McNeil, 2000; Morris, 2011).

Bernstein's theory of educational transmission is useful in understanding the testing regime in Irish primary mathematics classrooms. For Bernstein (1971), school standardised testing regimes are based upon a visible pedagogy which is realised through strong classification and strong frames and it is this type of pedagogy which transmits symbolic property. As a result schools are directly comparable as to their successes and failures. If student access to visible pedagogy is delayed for too long, examination success may be considered to be in danger. The research of Cooper and Dunne (2000), Anthony and Walshaw (2007) and Lubienski (2002) shows solidarity with Gee (1996) and his argument that students from disadvantaged groups are less likely to adopt the preferred discourse of the classroom because it conflicts with their home or community-based discourse. The corollary of this is that students who are not able to adapt to the discourse of mathematics education, which includes assessment, are more likely to be marginalised from that community. The key role played by language in mathematics education and assessment is succinctly summarised by Durkin and Shire (1991):

> Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language (p.3).

The difference in performance in mathematics has the potential to influence a student's future opportunities. The negative impact of standardised testing is notable in schools succeeding in connecting curricula and teaching to the realities of students' cultures, backgrounds and economic conditions (MacRuairc, 2009a; McNeil, 2000). According to Au (2008) high stakes tests serve to reproduce dominant social relations in education. Bourdieu (1973) and the three key concepts – field, habitus and capital, and their complex interactions, help to illuminate issues of domination and reproduction in education. Capital plays an important role in the relationship between field and habitus. While Bourdieu (1986) describes two main forms of capital (economic and symbolic), this research study into mathematics classrooms will focus on cultural and linguistic capital (a form of symbolic capital). Grenfell (2008) describes cultural capital as a synonym for status or position, and resources that one brings to the field.

School success is predicated on such cultural capital so that middle class students who are familiar with the dominant culture will perform better academically while the mismatch between home and school cultures serves to disadvantage working-class young people. Bourdieu (1990) describes the middle classes as having a better 'feel for the game'. In the case of high stakes testing, the 'rules of the game' are set by the standardised nature of the assessment and, in many cases, by a standardised curriculum, which privileges certain kinds of knowledge. Middle class students can be expected to 'produce' the correct kind of knowledge within an examination. Social fields are often not level playing fields and those who begin the game with particularly valued forms of capital and "well-informed" habitus are always at an advantage. In schools and education, it is clear how and why those who have a privileged position and have learned how to play school well, have an investment in maintaining and reproducing the operations of the field.

Standardised testing, streaming/tracking systems in schools for mathematics and pronounced differentiated teaching practices in this subject, as well as other gate-keeping controls, ensure that a differentiated hierarchy of access is produced that emulates, assists, (re)produces, and is (re)produced by the hierarchy within capitalist relations of production. Mathematics high status in the "social division of labour of discourses" (Bernstein, 2000) within schools and society, makes it a high stakes game to play, and its "strong grammar" (Bernstein, 2000) provides it with significant cultural cache for those with the luck and privilege to have access to it as knowing subjects and citizens (Swanson, 2010 p.249). There is a notable absence of debate in Ireland in relation to the impact of a policy of mandatory testing on children from working class, marginalised communities (MacRuairc, 2009a). MacRuairc (2009b) argues that there is an inadequate consideration of the complexity and diversity of issues entrenched in the term 'educational disadvantage' when it comes to policy. Many of the initiatives and policies that underpin them are strongly positioned within a functionalist, meritocratic perspective (MacRuairc, 2009a). Targets included in many policy documents are 'hopelessly aspirational', preventing the setting of more measured, attainable targets (MacRuairc, 2009b). Standardised testing can lead to a loss of motivation among teaching staff and poor results, particularly in schools designated as disadvantaged.

Methodology

Going inside mathematics classrooms, observing, listening and speaking with research participants, offers invaluable insights into how mathematics is taught, learned and assessed. This study employs a mixed methods design approach (Teddlie & Tashakkori, 2009) across eight primary school classrooms at second (7 and 8 year olds) and fifth class level (10 and 11 year olds). To allow for comparative analysis of social contexts, it was necessary to enlist two DEIS schools and two non DEIS schools at both second and fifth class level. All eight schools were co-educational. Pseudonyms were used for the research schools and participants. Semi-structured interviews were employed when exploring teachers' experiences of and attitudes towards standardised testing. Class profiles were created based on the standardised test scores provided by the research schools.

Table 1

Overview of Research Schedule

Time period	Activity
September	Piloting of Children Questionnaire & Visiting the research schools
October	Observation visits to schools and focus group interviews
November	Teacher Interviews (6 female, 2 male)
February	School Leader Interviews (4 male, 4 female)

Table 2

Overview of Research Methodology

	Data Collection Instrument	N	Analytical Approach
Quantitative Instrument Design	Child Questionnaire	164	Quantitative Analysis in SPSS
Qualitative Instrument Design	Classroom observations	24	Thematic Analysis in Nvivo
	Interviews with Children	40	Thematic Analysis in Nvivo
	Interviews with Teachers	8	Thematic Analysis in Nvivo
	Interviews with School Leaders	8	Thematic Analysis inNvivo

Findings and Discussions

Across the four DEIS schools in this study standardised tests are identified as a key source of challenge and discomfort for teachers. According to these teachers pupils appear to be deficit in the adequate levels of literacy and understanding in order to function in the test:

> "I think the language of the maths standardised test is a problem for them. They have to read it, understand the words and then figure out what they have to do. It is asking an awful lot", (Ms Jones, 2nd Class, DEIS).

The social and linguistic differences among pupils is frequently cited among teachers:

> "when my own children went through school standardised tests were easy for them because it's their language...disadvantage is a totally different ball game and the standardised tests are not easy for them" (Ms Keane,5th Class DEIS).

This study found that teachers perceive school success to be predicated on cultural capital so that middle class students who were familiar with the dominant culture performed better academically while the mismatch between home and school cultures served to disadvantage students in DEIS schools:

> "I used to think it was unfair when I taught in a disadvantaged school...you worked your butt off and then you would do the standardised tests and ...you just wanted to stick your head in the oven" (Caitriona,Non DEIS).

Teachers in this study frequently made reference to pupils being 'upset' with their confidence 'knocked' and feeling negatively towards mathematics. According to one second class teacher she felt 'sorry' for the pupils who reported finding the test "too hard and I didn't understand it" (Ms Cooper, 2nd Class, DEIS). A negative construct of pupil ability is shaped by the test, not alone confined to second class but also shared among fifth class pupils:

> "you hear them asking each other what Sten they got and if one said a Sten of 2 and someone else said a Sten of 5 or a Sten of 6...you can see them ranking themselves ability wise based on how they compare with their peers..yeah it is hard for them" (Ms Keane, 5th Class, DEIS).

Assessment is a powerful tool in mathematics classrooms. For both second and fifth class teachers in this study, particularly in DEIS schools, teaching goals focused on improving test scores for their pupils. According to teachers in DEIS schools, the testing experience is one of struggle with the linguistic challenges that the test presents. In these schools classroom instruction focused on trying to address the literacy skills necessary for testing as well as the mathematical skills. Practice in test-taking skills was a routine classroom activity. Pressure associated with standardised tests was not felt equally by all schools. The influence of testing on the culture of the school was strongly found among DEIS schools rather than their middle class counterparts who displayed immunity to such pressure. For some teachers in DEIS schools pressure was something they put on themselves as well as what was imposed externally through the impact it was having on their pupils and how pupils in turn viewed themselves as learners of mathematics.

Conclusion

Assessment is an essential tool in both teaching and educational accountability. At the same time there is a strong need to place testing in its proper context and to be realistic of its strengths and weaknesses. Furthermore, there needs to be a greater understanding of the centrality to good teaching and to students' progress of assessment for learning which is intrinsic to pedagogy rather than detached from it (Alexander, 2010). There is a responsibility on those who develop national standardised tests, and upon those who advise and guide them, to attempt to ensure that the tests fully reflect not only the wide mathematical content that is in a national curriculum, but also that the nature and mode of assessment matches the philosophy of learning and teaching mathematics which occurs in classrooms.

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Evaluating the impact of student-centred computational activities on sensemaking and computational thinking in Financial Mathematics

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In this paper, we evaluate the computational lab component of a mandatory undergraduate Computational Finance course. We were interested to determine if and how the computational design of each lab contributed to students' sensemaking around the concept of computational thinking. We also examined the interplay between financial mathematics and computational thinking for students. Using thematic analysis, we analysed six students' responses to ten weekly surveys, one conducted at the end of each lab for ten weeks in 2020/2021. A set of 11 categories describing the main themes from the students' responses were generated with 'Improved Computational Thinking' being the most significant for student learning. Results also show that the learning outcomes of each lab were clear to almost all six students for all ten labs.

Keywords: Sensemaking, financial mathematics, computational thinking, inclusive computational practices.

Introduction

In recent years, scientific computing and computational thinking have become key drivers in many STEM areas. For future researchers, computing competences must be brought to the forefront of undergraduate education in universities, in particular for mathematical subjects (Lockwood et al., 2019). If university courses were designed to include computational thinking, students would almost certainly graduate with at least a foundational understanding of this important skill (Caballero & Hjorth-Jensen, 2018). In this setting, Financial Mathematics (FM) is a new and highly interdisciplinary mathematical subject, where computational thinking skills are fundamental for covering high-level quantitative finance roles as well as perform academic research with significant real-world impact.

At the same time, there are growing concerns that university students of computationally rich courses are not fully aware of the mathematical ramifications of their code design, its output and its real world meaning (Sand et al., 2022). As computer technology advances at an unrelenting pace, students are being exposed to more black box methods which have the potential to mask the principles of the underlying mathematics. This proposed research study aims to contribute to the FM curriculum, showing how tailored student-led computational practices designed for a Computational Finance module delivered at University College Dublin (UCD) provided opportunities to foster sensemaking in FM and improve computational thinking. We analysed students' responses to weekly surveys to address the following research questions:

RQ 1 To what extent do inclusive computational practices engage students in sensemaking in Financial Mathematics?

RQ 2 To what extent do inclusive computational practices engage students in developing computational thinking?

Despite the incredible growth of Financial Mathematics programs, the Computational Finance curriculum is under-researched; our study aims to contribute to this. A similar study is found in Barana et al. (2023), where the authors investigate the effectiveness of inclusive computational practices in providing opportunities for the *co-creation of knowledge* in Computational Finance within a Computer Supported Collaborative Learning environment.

Theoretical Framework

Designing engaging enquiry-based learning environments in science modules is fundamental to developing computational thinking. *Computational thinking* is a common phrase to which numerous definitions have been attached. Aho (2012, p.832) defines computational thinking as "the thought processes involved in formulating problems so their solutions can be represented as computational steps and algorithms", while Wing (2014) states that computational thinking is "the thought processes involved in formulating a problem and expressing its solution(s) in such a way that a computer - human or machine - can effectively carry out". This is the definition we adopt in this work. More precisely, we refer to the definition given by Lockwood et al. (2019, p.3), which aligns Wing's to a mathematical context, as "the practice of using tools to perform mathematical calculations or to develop or implement algorithms in order to accomplish a mathematical goal".

There are numerous advantages to incorporating computational thinking into undergraduate courses. As mentioned in Caballero and Hjorth-Jensen (2018), educators could enhance students' learning and understanding, leading to deeper insights being gained from the taught material. Educators have the potential to expose their students to a myriad of real-world examples and more complex problems at an earlier stage. These practices can undoubtedly lead to students having a more well-rounded and applicable education upon graduation. With this aim, Caballero and Hjorth-Jensen (2018) have developed an "inclusive computational" framework which we adopt in this study. *Inclusive computational practice* is a concept encompassing several computing-related activities, such as: (1) students working alone or in groups with algorithms to understand the underpinning ideas of a mathematical or financial model; (2) having students adapt lines of code to suit a similar, but different, problem; (3) having students inspect and comment critically on computational inputs and outputs. Inclusive computational practices have been used extensively to design physics undergraduates' modules at Michigan State University and Georgia Tech, USA (e.g., Caballero et al., 2012; Irving et al., 2017; Caballero & Hjorth-Jensen, 2018). They have included computation and computational thinking as a central element, and not simply as a tool in the design process. Since 2020, Author 2 has redesigned many computational practices including (1)-(3) above, for a Computational Finance module at UCD, with the aim to provide students with opportunities to foster computational thinking and sensemaking in FM. A comprehensive description of the module design can be found in Perrotta (2021), while a focus on the design of inclusive computational practices for FM is presented in Perrotta and Dolphin (2021).

Many researchers agree on an intuitive definition of *sensemaking*; however, the related literature is fragmented. The definition of sensemaking given by Odden and Russ (2018) has been adopted in Perrotta (2021) and in Perrotta and Dolphin (2021) as well as in this study. Odden and Russ describe sensemaking as "a dynamic process of building or revising an explanation in order to *figure something out* - to ascertain the mechanism underlying a phenomenon in order to resolve a gap or inconsistency in one's understanding" (2018, p.13). According to Odden and Russ, the sensemaking process begins when "something is puzzling or unexpected" or when "there is some gap in existing knowledge, individual facts or ideas conflict with one another, or some combination of these" (2018, p.6).

The Research Methodology: a pilot study

In this study, we attempt to shed light on how undergraduate students make sense of connections between computing and financial mathematics via their work in groups in the student-led labs activities described in Perrotta and Dolphin (2021). We analysed responses to three questions per student over a ten-week period during the Spring of 2020/2021 using a Google Survey. We performed a qualitative analysis, using the approach to thematic analysis developed by Thomas (2006). Thomas (2006) outlines a set of procedures to carry out qualitative analysis, aimed at new researchers or researchers from non-social science areas so that they need not be fluent in the underlying philosophical or technical terms often embedded in thematic analysis, and qualitative analysis more generally. Thomas developed an inductive approach which involves "detailed readings of the raw data in order to derive concepts, themes, or a model through interpretations made from the raw data by an evaluator or researcher" (2006, p.238), with the aim of an inductive approach being to "allow research findings to emerge from the frequent, dominant or significant themes inherent in the raw data, without the restraints imposed by structured methodologies" (2006, p.238). The output of this inductive approach is then "the development of categories into a model or framework that summarises the raw data and conveys key themes and processes" (2006, p.240). Given the non-social science context of our research study and the fact that we analyse students' short responses instead of recorded conversations, we decided to adopt Thomas' approach.

This research study was conducted in 2020/2021 in the Computational Finance module ACM30070, a core module for Stage 3 of the BSc in FM, at UCD. The module is also optional for Stage 3 of the BSc in Applied and Computational Mathematics (ACM). The class comprised 50 students, 35 FM and 15 ACM students, 40 were male and 10 females. The module was created in 2017 and then significantly redesigned by Author 2 in 2020 to include several *inclusive computational practices* in the module's labs. These inclusive computational *practices* were designed to enhance students' computational learning and sensemaking in FM within a collaborative computer supported learning environment (Perrotta, 2021; Perrotta & Dolphin, 2021). Weekly lab activities were student-led and involved the participation of the lecturer, a tutor and a teaching assistant as facilitators. The same lab structure was proposed each week with each applied to different computational practices. Each lab lasted two hours. During the first hour, students worked in groups on computational modelling, pseudo-coding, data analysis and other related computer-based activities as (1)-(3) above. In the second hour, each group chose a representative to present the group's results to the whole class. The tutor guided groups in presenting their results and encouraged them to agree on a shared conclusion. There were seven groups participating in the activity (six with seven students and one with eight), which remained constant for the term. Students were grouped according to: their GPA, (i.e. similar GPA, on average), their pathway (5 FM and 2 ACM), and gender balance (at least two females per group, otherwise no females). The impact of co-construction of knowledge during these lab activities is investigated in Barana et at. (2023). After each lab, students were invited to complete a Google Form survey to critically reflect on class activities. The weekly survey ran for ten weeks in Spring 2021; so, each student completed ten surveys. The survey contained qualitative and quantitative questions and the responses constitute the dataset for this study. The definition of sensemaking was included in the survey preamble so students were familiar with it. Ethical approval for this research study was granted in Spring 2020 (LS-20-05-Perrotta). In this paper, we analyse open-ended responses to the following questions, extracted from the Google Survey:

> Were the expected learning outcomes of the lab clear? Please explain.
>
> Was there any content area or computer code that was unclear prior to the lab that is now clear? Please explain.
>
> Can you describe in detail how the use of computation facilitated your sensemaking?

For this pilot study, we selected six out of 50 students, each of whom worked in a different group. This selection was made to represent three different proficiency levels: two students came from two high-performing groups, two from two medium groups and two from two low-performing groups. We analysed the six students' responses to the ten weekly surveys for Q1, Q2 and Q3 using Thomas' approach to thematic analysis. These six specific students were selected as their answers were well elaborated giving ample material for category generation in the qualitative analysis. It is noteworthy that lab attendance was not compulsory, so sometimes students missed a lab and consequently did not complete the survey. This justifies the frequencies shown in Table 1 below (see Labs 4, 5, 6, 10). Also, some questions' answers were left empty even when a student attended a lab. In conclusion, we analysed a total of 165 responses from the pilot study.

Guided by Authors 2 and 3, Author 1 coded the subset of six students' responses for all ten weeks to generate the initial set of 17 categories. Author 2 then independently repeated this process, generating a set of 21 categories. Author 1 and Author 2 merged their sets of categories, refined them, and resolved any discrepancies. They defined a set of 15 categories. At this stage, Author 3 independently coded survey responses for six students from four labs (labs 1, 2, 3, and 7). These labs were chosen because each of the 15 codes appeared at least once among them, but also because most codes were heavily highlighted. He started from the set of 15 categories applied to each response and further refined them, merging some categories into a unique one. At this time, authors also decided to perform qualitative analysis only on the responses to Q2 and Q3, while for Q1 instead we used a binary variable, with 1 indicating clear learning outcomes (LOs), and 0 indicating otherwise. The frequencies of the binary variable used in Q1 are shown in Table 1.

At the end of this process, the three authors agreed on the final list of 11 categories described below:

1. **Choosing Effective Computational Tools (CECT):** Student indicates that they have a better handle on choosing the most efficient computational tools with which to solve a FM problem.
2. **Coding as a Means to Apply a Method (CMAM):** Student discusses the use of coding to apply/implement a FM method about which they have learned.
3. **Coding for Real World Applications (CRWA):** Student acknowledges the importance of coding to solve real world applications.
4. **Coding to Sensemaking (C2S):** Student demonstrates sensemaking as a result of a coding activity.
5. **Debugging/Troubleshooting for Learning (DTL)**: Student indicates that they learned something through the debugging/troubleshooting stages of the lab.
6. **Financial Mathematics to Improve Computational Thinking (FM2CT)**: Student indicates that their FM knowledge helped to improve their CT skills.
7. **Group Discussion Aided Understanding (GDAU)**: Student acknowledges that the group discussions aided their understanding.
8. **Improved Computational Thinking (ICT):** Student improved their CT skills utilising the coding skills element of the lab.
9. **Unclear about Modelling (UM)**: Student indicates that they are unclear about the FM modelling concepts present in the lab.
10. **Understand the Theory (UT)**: Student shows that they have gained a better understanding of the FM theory through this lab activity.
11. **Use of Prior Learning to Build New Knowledge (UPL2BNK):** Student acknowledges their prior learning as a foundation on which new knowledge has been built.

Results

Table 1 shows the frequencies of the binary variable used in Q1. LOs were clear for at least 80% of the six students in all the labs, and for all of them in six out of ten labs. Moreover, LOs were clear to 100% of the students in the last four labs, when students would have been familiar with the proposed activities.

Table 1

Frequencies of 0/1 in Question 1

	Lab 1	Lab 2	Lab 3	Lab 4	Lab 5	Lab 6	Lab 7	Lab 8	Lab 9	Lab 10
Frequency of students who found the LOs clear	5/6	6/6	5/6	4/5	4/4	4/5	6/6	6/6	6/6	5/5

Table 2 below shows the observed frequencies of the 11 categories that we identified performing a qualitative analysis of Q2 and Q3 answers.

Table 2

Frequencies of Categories in Questions 2 and 3

Category	Count	%
ICT	46	27
FM2CT	21	12
UT	19	11
UM	15	9
GDAU	14	8
CECT	13	7
UPL2BNK	13	7
C2S	11	6
CMAM	8	5
DTL	7	4
C2RWA	6	4

These results show the positive effect of the lab design on the interplay of FM and CT. Indeed, each notion supports the other in terms of students' sensemaking and without the computing element of the module this would be highly unlikely to occur, based on Author 2's experience of teaching FM without such accompanying computational labs. The category "Improved Computational Thinking" occurred 46 times, indicating that students noticed their computational thinking skills being improved as a direct result of the implemented inclusive computational practices. The next most popular categories were "Financial Mathematics to Improve Computational Thinking" and "Understand the Theory", which occurred 21 and 19 times, respectively. An example of a segment in which both ICT and UT occurred is shown below:

> We used the aaBin function to price put and call options […]. The most educational part for me was to not[e] the vast difference in options prices when the time to maturity and the spot price is 10 euro above the strike price. […] This prompted much speculation and debate in our group as to the reason why […]. We also used Fincad to evaluate the put-call parity with a time to maturity of two years now […]. This computational aspect allowed me […] to solidify the truth of the put call parity.

Here, the thought process involved in formulating the problem, testing potential approaches to solutions, scaffolding the process in computational steps, and finally coming to a conclusion is very well described. Also, the student makes explicit the fact that this lab activity contributed to solidifying their FM knowledge. Conversely, an example of an extract in which the category FM2CT occurred follows:

> Before it was mostly just copying text and analysing the lines, which is of course vital, but this lab allowed me to try implement this in designing my own code

> from scratch. I found that if there was […] lines of code that I only half understood before I really had to figure out what they meant and why they worked for my code.

From this extract, we can see that the FM model behind the practice strongly contributed to fostering the student's computational thinking.

The category "Coding to Sensemaking" occurred 11 times. In the following extract, the student shows that he has completed the sensemaking process described by Odden and Russ (2019):

> I think that the labs that require us to design some code prior to the lab help me to understand the computational part of the course best. It's one thing to read the other code and implement it but the opportunity to write your own forces you to understand the deeper intricacies in the code and you become more aware of the parts that you do and do not understand, which you can then fix.

We highlight that detecting sensemaking from an asynchronous response to a Google Form is a hard task, since it heavily relies on the ability of each student to remember what happened in class and to then put the entire learning process into words. We expect a higher number of occurrences of the C2S category once the entire dataset of responses has been analysed.

Summary and future research

We performed a pilot study to investigate the effectiveness of tailored student-led computational lab activities in sensemaking in FM (RQ1) and in developing computing thinking (RQ2). We selected six students out of 50 undergraduate students attending the UCD Computational Finance module in 2020/2021 and analysed their responses to a weekly survey over 10 weeks. To frame our study, we referred to the Lockwood et al. (2019) definition of computational thinking in mathematics and to the Caballero et al. (2018) inclusive computational framework. We performed a thematic analysis (Thomas, 2006) to generate 11 categories from the students' responses. Results are shown in Tables 1 and 2 and are in line with the available literature in which inclusive computation is used to foster sensemaking in physics undergraduates. The category "Improved Computational Thinking" occurred 46 times, while "Financial Mathematics to Improve Computational Thinking" and "Understand the Theory" occurred 21 and 19 times, respectively. This result allows us to conclude that the proposed activities successfully contribute to the enrichment and knowledge of FM and students' computational thinking. The technology and computation used in this module played a key role, since they mediated and fostered the learning process. We are currently planning to extend the analysis to the entire dataset during which we will further refine the categories, seek links between them and define summarised themes. We will also investigate how computation combined with a collaborative learning environment fosters the learning process. To this aim, we will correlate our results with those of Barana et al. (2023).

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Investigation into Students' Understanding of Graphs Representing a Qualitative Scenario

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We have investigated students' understanding of graphs relating to qualitative scenarios. The students in question were first year undergraduate science students doing non-physics majors. The students (~300) were split into five groups; each group was given a different set of questions comprising a qualitative description of a physical scenario. In each case we asked students to draw a graph representing the motion involved, and to explain why they had chosen to draw the graph as they had. This study has allowed us to gain an insight into what students attend to as they draw these graphs. In this paper, we present empirical results of an analysis of the overall shape of the students' graphs and their accompanying explanations. Our findings show that students are much more likely to draw straight line graphs than curves (even when a curve is more appropriate) and that their reasoning is greatly dependent on context.

Keywords: physics education, graphs, qualitative graphs, graph construction

Introduction

Being able to draw and interpret graphs is undoubtedly a very important part of both mathematics and science. Deacon (1999) discusses the importance of being able to draw graphs and not being overly reliant on graphing software. In an Irish context, the importance of graphs is clear as they feature in the primary school curriculum and the Junior Cycle Maths and Science specifications (NCCA, 1999, 2015, 2018). Many studies have examined students' understanding of distance-time graphs in quantitative scenarios (Wemyss & van Kampen, 2013; Hale, 2000). In this paper we focus on students' conceptualisation of qualitative scenarios.

There is a large body of research in relation to students' abilities of both constructing and interpreting graphs. Overviews of existing research were carried out by both Leinhardt *et al* (1990) and Glazer (2011). However, few of these studies focus solely on construction tasks involving qualitative data. One such study where students are asked to construct graphs based on qualitative statements was carried out by Hattikudur et al (2012), however their study focuses solely on linear graphs and students' understanding of the *y*-intercept.

The research was carried out in an Irish university involving first year undergraduate science students enrolled in a Physics for General Science module. In this empirical paper we describe the methods used and the results obtained from this study so far.

Methods

The students involved in this study were first year undergraduate students, who were non-physics majors enrolled in the module Physics for General Science. 328 students were studied, and these were split into 5 groups. Each group was given a different set of paired questions in an end of semester exam. Figures 1 and 2 show an example of the paired questions involving a ball rolling down a track; Figures 3 and 4 a beaker being filled with water. The students' responses were coded according to graph shape and their accompanying explanations.

We are interested not only in the accuracy of the students' responses but also in their reasoning. We used open coding techniques (Otero and Harlow, 2019) to capture the students' explanations. In many cases students' responses were fragmented. For this reason we chose students' separate arguments as the unit of analysis, rather than their responses in their entirety. Initially two principal researchers independently coded the same two sets of ten responses (one relating to a track question, the other to a beaker question) and discussed their codes. Once the key features to code for had been established, we started to develop a codebook in an iterative way (Anfara et al., 2002). The codebook was refined by again independently coding two different sets of student responses, taking care that as much as possible of the students' responses could be captured in simple dichotomous codes. After both parties agreed on the usability of the codebook, a third principal investigator used their finalised codebook to code a sample of responses to test for validity and found it worked very well. While this kind of inductive coding is not necessarily tied to a theoretical framework, it aligns well with our constructivist views of education, in that we seek to identify what aspects of the given data and the qualitative graphs the students constructed they attended to, and what bits of knowledge they activate in their reasoning.

The Activity – Track Questions

Three of the five groups were given paired questions involving a marble rolling down a frictionless track. They were asked to complete two position versus time graphs. All three groups were given an experimental setup diagram with an accompanying paragraph of text which explicitly said that the track was frictionless, points a, b and c were evenly spaced, and that the stopwatch was started when the marble reached point a. They were asked to continue the accompanying graph (Figure 1) and to explain why they had completed that graph that way. The groups were then given one of three different tracks, and were again asked to complete a position-time graph representing the motion of the marbles and to explain what they drew. These tracks are shown in Figure 2.

Figure 1.

Experimental set-ups in the first of the paired questions. (i) Initial experimental setup for track questions, and (ii) accompanying position-time graph for track questions.

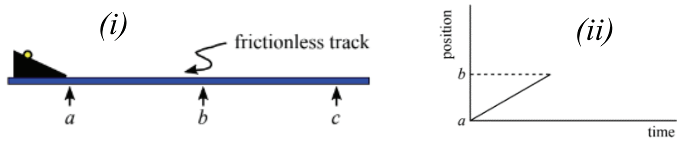
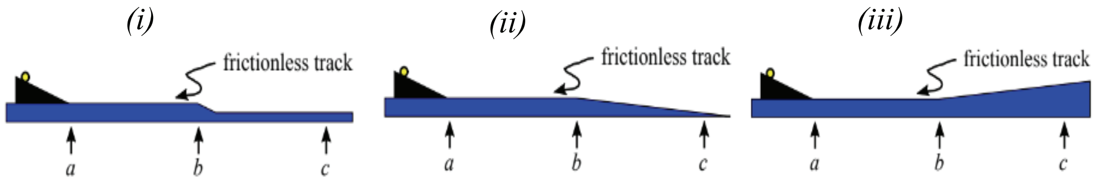


Figure 2.

Experimental set-ups in the second of the paired questions. Students were given one of these three settings. (i) step-down track, (ii) downward sloped track, (iii) upward sloped track.



The Activity – Beaker Questions

The other two groups were given paired questions involving water flowing into a beaker at a constant rate. Here, they were asked to complete two water level versus time graphs. Each group was given an experimental setup diagram with an accompanying piece of text stating that water flowed into the beaker as a constant stream, points a, b, and c were evenly spaced on the beaker, and that the stopwatch was started when the water reached level a. They were asked to continue the accompanying graph (Figure 3) and to explain why they had drawn it that way. Following this, each of the groups were given one of two different beakers and were again asked to complete a water level-time graph showing the change in height of the water. These beakers are shown in Figure 4.

Figure 3.

Experimental set-ups in the first of the paired questions. (i) Initial experimental setup for beaker questions, and (ii) accompanying water level-time graph for beaker questions.

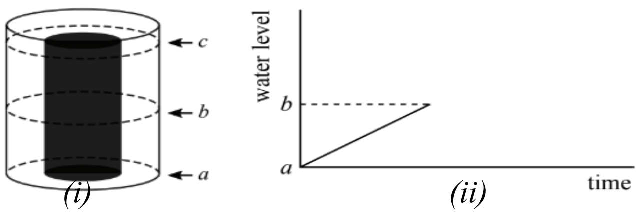
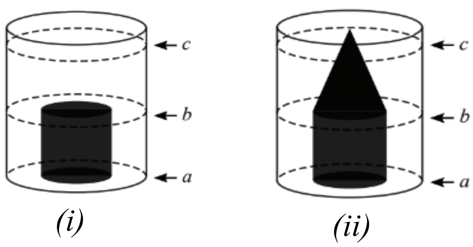


Figure 4.

Experimental set-ups in the second of the paired questions. Students were given one of these two settings. (i) half-cylinder in beaker, (ii) cone in beaker.



Data Analysis

Analysis of Drawings

In an initial examination of the graphs students drew, we noticed several trends that seemed to be applicable regardless of the context (track/beaker) or situation (shape of track/shape of object inside the beaker). We developed a set of independent and unambiguous categories of student drawings that was present in each data set. These categories were: *continues slope* (the student continued the line from a-b through to c); *steeper line* (the student drew a linear segment from b to c with a steeper slope than from a to b); *flatter line* (the student decided to draw a linear segment from b to c with a smaller slope than from a to b); *upward curve* (the student drew a curved line segment between b and c that increased in slope); and *downward curve* (the student drew a curved line segment with a decreasing slope). These categories were applicable to all data sets and, of the 312 total responses, these categories described all but 51 responses. In the step-down track setting, 12 students drew a line segment from b to c parallel in slope to that from a to b, but with a 'kink' or inflection at point b. We coded these responses as *parallel slope*. The remaining 39 responses we categorised as *other*.

Analysis of Explanations

On first examination of the responses, we noted that many students referred to the shape of the graph, the speed of the ball or the water level, and the end point. Sometimes students referred to graphical aspects ("I drew a curved graph because…"), sometimes to physical quantities ("the ball has constant speed"), sometimes to experimental features ("the cone is getting narrower"). Some students mentioned a reason or mechanism ("there is no friction, "gravity speeds the ball up", "the water flow is constant"). In our first attempt at systematic coding, we created three overarching categories: *shape*, *(initial) rate of change*, and *end point*. All three partial responses above would be categorised under *shape*, since they contain a reference to either the shape of the graph or to a physical or experimental feature that corresponds to it. The category *(initial) rate of change* refers to the speed of the ball or the rate of change of the water level at point b; the word "initial" only applies to curved graphs. In each of those overarching categories we checked whether students referred to graphical, physical, or experimental features. Finally, in a separate category we coded whether students mentioned a *mechanism*, making for a total of ten dichotomous categories.

Two of the investigators independently coded the same sample of ten responses to one of the paired questions involving a beaker, plus the same sample of ten responses to one of the paired questions involving a track. During individual coding and discussion of the codes it emerged that the categories captured the data quite well, but that some refinement of the categories would capture the responses better. The investigators also noted that they often asked themselves similar questions to determine whether a response should be coded as comprising a category. We established a codebook to provide a more robust set of criteria for coders to help them decide if responses referred to graphical, physical, or experimental features within the overarching categories of *overall trend*, *(initial) rate of change at b*, and *interval to be covered*, plus reference to a mechanism. For each of these ten categories the codebook, shown in Table 1, we provided a small number of questions coders could ask themselves.

The two principal investigators independently coded two new sets of 10 responses. They found that the codebook was easy to use, and that the ten zeros (not present) and ones (present) generated for each qualitative answer generally captured students' responses comprehensively and efficiently. The third investigator then applied the codebook to the same sample of responses to test the validity of the codebook. A Cohen's Kappa calculation on a subset of the data returned a value of 0.71 indicated excellent agreement.

Table 1.

Final version of the codebook used to analyse written explanations.

category		**criteria/questions**
overall trend	graphical	Do students mention the overall shape of the graph (straight or curved)? Do not code here for e.g. a kink at point b.
	physical	Do students say something about the speed being constant or that it speeds up or slows down throughout the process? Do not code if there is only a comparison to the speed on a-b.
	experimental	Do students say that after the step-down track, the track from b-c is flat/horizontal or sloped? Do students say that the cone gets narrower throughout b-c so that the volume is reducing as the water rises? Do students say that there is a new constant volume after the half cylinder?
(initial) rate of change at b	graphical	Do students compare their line to the initial line by using words such as: it got steeper / flatter / shallower, it curved up/down?
	physical	Do students describe the motion of the ball/rate of change of the water level by using words such as gets faster/slower, speeds up/slows down? Look for a distinction between a-b and b-c as opposed to the motion changing throughout b-c
	experimental	Do students comment on the change in the setup such as: there is a ramp / there is no cylinder in b-c / the track slopes?
interval to be covered	graphical	Do students discuss the length of the line? Do students discuss the interval between a-b and b-c in terms of the axes?
	physical	Do students discuss the overall time taken or distance travelled by using words such as "it took longer" / "more distance covered in less time"/ "it takes more time"?
	experimental	Do students discuss the total volume of water needed or use words such as "to fill the beaker"? Do students discuss the length of the track (which they may call "the distance" but not "the distance travelled") or height of the beaker?
mecha-nism		Do students refer to gravity / force / friction / constant stream of water?

To illustrate the coding process, we will discuss how we coded a student's written explanation to the graph they drew in response to the beaker question of Figure 4a: "*The line from b-c would still be straight as the stream is continuous. However, the volume to be filled by water in level b-c is now greater than level a-b because of the absence of the cylinder. It would take more time to fill b-c because the volume to be occupied has increased and so the slope would be less steep.".* In this case:

- "the line from b-c would still be straight" is coded as 1 under *overall trend graphical*
- "the stream is continuous" is coded as 1 under *mechanism*
- "the volume to be filled by water in level b-c is now greater than level a-b" is coded as 1 under *interval to be covered experimental*
- "the absence of the cylinder" is coded as 1 under *(initial) rate of change at b experimental*
- "It would take more time to fill b-c" is coded as 1 under *interval to be covered physical*
- "the volume to be occupied has increased" was already coded
- "the slope would be less steep" is coded as 1 under *(initial) rate of change at b graphical*

Table 2.

Sample coded response. G = Graphical, P = Physical, E = Experimental.

overall trend			**(initial) rate of change at b**			**interval to be covered**			**mechanism**
G	**P**	**E**	**G**	**P**	**E**	**G**	**P**	**E**	
1	0	0	1	0	1	0	1	1	1

Table 2 summarises how we coded this response. All 312 responses were coded in this way; a sample of responses were coded independently by multiple researchers to ensure the codebook is reliable and valid.

Results

In this paper we focus on the responses to the second question given to each group. We have analysed both the graphs the students drew and their written explanations. Firstly, the responses were categorised according to the shape the students drew. In Table 3, the correct shape for each group is in bold.

One of the key findings from categorising the data in this way is that a large fraction of students drew straight lines when a curve is required. This is seen most clearly in the downwards sloped track group, where over double the number of students (58% vs 25%) drew a steeper straight line as opposed to an upwards curve. We also saw a high percentage of straight lines instead of curves in the cone beaker group and the upwards sloped track group, though less frequently than with the downwards sloped track group. The students' written responses were coded according to the codebook shown in Table 1. The results are shown in Table 4.

Table 3.

Percentage of student responses categorised by shape.

	step-down track (***n*=59**)	**downward sloped track** (***n*=62**)	**upward sloped track** (***n*=69**)	**half cylinder in beaker** (***n*=74**)	**cone in beaker** (***n*=64**)
continues slope	14	14	9	9	6
parallel slope	12	0	0	0	0
steeper line	**58**	58	1	2	10
flatter line	6	2	33	**75**	43
upward curve	5	**25**	1	0	7
downward curve	0	0	**38**	0	**34**
other	5	2	16	14	2

Table 4.

Percentage of responses that contain each argument. G = graphical, P = physical, E = experimental.

	overall trend			**(initial) rate of change at b**			**interval to be covered**			**mechanism**
group	**G**	**P**	**E**	**G**	**P**	**E**	**G**	**P**	**E**	
step-down track	7	27	3	15	100	93	0	31	3	22
downward sloped track	10	10	0	18	87	87	0	39	0	23
upward sloped track	20	20	0	10	81	90	0	33	7	33
half-cylinder in beaker	9	14	3	20	27	72	4	69	77	24
cone in beaker	19	17	23	9	17	61	6	70	67	17

From Table 4 we can see that students in the track groups place a much higher importance on discussing the initial rate of change of the motion in comparison to the beaker students who favour writing about the interval to be covered. Our initial interpretation is that the main resources that are activated by the students are those relating to the change in the rate of change of the position at point b, or the total time the process takes from b to c. Thus, in situations where students should draw a curve, they tend to draw straight lines because they only consider what happens at b or at c, and not in between.

Conclusions

The two main findings of this study thus far are that a large proportion of the students drew straight line graphs rather than curves, and that the students' reasoning depends greatly on the context of the question. In the next stage of this study we will interview students so as to delve deeper into their reasoning and to further our understanding of why they chose to draw the graphs that they did. We will also provide a new set of different graphing questions that deal with resistance. We will vary the orientation to see if that makes a difference in terms of students' ability to plot position on the axis. One group will receive a selection of possible graphs and be asked to describe what caused them to select a certain graph over the other. This aims to see if when students are given the resources, are they better able to activate them than when they are asked to draw the graphs themselves.

Data Availability Statement

The data from this study is available from the authors upon reasonable request. The ethical approval number is DCUREC/2022/185.

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Investigation of the Degree to which the Study of the Old Applied Mathematics Course Affected Students' Performance in Other Leaving Certificate Subjects

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In light of the recent reform of the Applied Mathematics course – offered in the Senior Cycle of Irish post-primary education but taken by comparatively few students – questions of interest are whether taking the old Applied Mathematics course affected performance in other subjects and whether the revised course will do likewise. With regard to the old course, the issue was addressed in a study that used a mixed-methods approach: a small-scale qualitative element examining teachers' and students' experiences of the course, and a larger-scale quantitative analysis. The focus in this paper is on the quantitative aspect and considers effects on Mathematics. The paper reports on the difference in students' performance in Mathematics from the Junior Certificate to the Leaving Certificate, comparing those who took Applied Mathematics with those who did not. The results show that taking Applied Mathematics had a strongly positive relationship with a change in performance in Mathematics. The study also demonstrates that the students in the Applied Mathematics group can be further divided into subpopulations based on change in performance in Mathematics.

Keywords: Curriculum reform; applied mathematics; attainment; high-stakes examinations

Introduction

The subject *Mathematics* has a prominent place in the Irish post-primary curriculum. In the Senior Cycle, it is not compulsory but is offered by almost all students who sit the Leaving Certificate examination, taken in the final year of schooling. For a long time, the main focus – especially of the most advanced course, Leaving Certificate Higher level – was on pure mathematics. This has changed to some extent recently, with more emphasis on applications to real-life situations; however, topics such as mechanics that appear in some Mathematics courses (for example in England) still do not figure greatly. They do appear in another Senior Cycle course, *Applied Mathematics* (henceforth Applied Maths). Uptake is small, and not all schools offer it, but it plays an important role for some students.

Applied mathematics can be described as the study of the practical applications of mathematics to the real world and physical problems. The Irish Applied Maths course was dominated by just the one field, mechanics; its topics – such as projectiles, pulley systems and collisions – were tested in discrete examination questions, leading to a very predictable examination structure. Several attempts were made to revise the subject, without success. Eventually the National Council for Curriculum and Assessment (NCCA) produced a discussion paper designed to lead to substantial reform (NCCA, 2014). Controversially, it asserted that the course encouraged a procedural approach rather than problem solving; this clashed with at least some perceptions that there was a strong problem-solving element, perhaps even helping students to develop generalised skills that transferred to other subjects. A revised course has now been introduced (Department of Education, n.d.). While a strong mechanics component has been retained, new material – notably graph theory – has been introduced, and the course has been recast to focus explicitly on modelling. As with any curriculum change, questions arise as to gains and losses, and particularly in this case as to whether the criticisms of the old course were justified and the new one offers improvements.

The study on which this paper is based examined the old course, aiming to answer the following questions:

1. What were students' and teachers' experiences of Applied Maths?
2. Did studying Applied Maths develop problem-solving skills?
3. To what degree did the skills developed through studying Applied Maths transfer to different domains?

The work was the first author's final-year undergraduate project, which was supervised by the second author. A mixed-methods approach was used. The small-scale qualitative element chiefly addressed the first research question; the larger-scale quantitative analysis examined data from the State examinations – Junior Certificate and Leaving Certificate – and compared aspects of the performance in selected subjects of students who did and who did not take Applied Maths, with a view to answering the third question. This paper focuses on the quantitative component. However, in interpreting the quantitative findings, it draws heavily on the results of the qualitative component, which are presented in the literature review.

Literature Review

To address the research questions, three areas of research are relevant: problem solving, transfer of skills, and consideration of the old Applied Mathematics course. They are addressed in turn.

In order to consider *problem solving*, it is necessary first to clarify what is meant by a problem. Mathematics educators agree that a problem is a situation which

> … carries with it certain open questions that challenge somebody intellectually who is not in immediate possession of direct methods/procedures/algorithms etc. sufficient to answer the questions. (Blum & Niss, 1991, p. 37)

A consequence of this is that whether a question is a problem or only an exercise is dependent not only on the person tackling the question, but also at what point in time that person is doing so. What may be a routine exercise for a person today may have been a problem for them yesterday. In the extensive literature on problem solving, the work of Schoenfeld (1992) on the characteristics of *good problems* and Menary (2015) on *enculturation* is notable. According to this, a crucial aspect of mathematical development is acquiring the habits, beliefs, and attitudes of mathematicians as well as expanding one's knowledge base. If so, the development of problem-solving ability may be best encouraged by allowing students to experience problems in the way that an "expert" does. This suggests that students should be afforded the opportunity to act as mathematicians within the classroom.

Transfer consists of "[applying] previously learned knowledge with various degrees of adaptation or modification of that knowledge in completing a task or solving problems" (Hung, 2013, p. 27). This has frequently been described as a highly important goal of education, though according to Haskell (2000) it is very difficult to achieve in any significant way. Johnson (1995) differentiates between *near* and *far* transfer. However, it might be more accurate to think about transfer on a continuum; Thorndike's theory of transfer asserts that the amount of transfer from one situation to another is proportional to the degree of similarity between the two situations (Woodworth & Thorndike, 1901).

With regard to *the old Applied Maths* course, there is a paucity of research. The discussion paper produced by the NCCA (2014) as a background to reviewing the course provides some context on its history and low uptake, noting that fewer than 3% of students in the Leaving Certificate cohort presented the subject. As indicated above, the paper asserts that it is procedure-focused, reportedly being studied through intensive practice of past examination papers in particular; comments from third-level students are included in support of this view. A response by the Irish Mathematics Teachers' Association (2014) provides additional context on the history of Applied Maths, mentioning past unsuccessful attempts at reform. In contradiction to the NCCA view, and drawing on feedback from members who taught Applied Maths, it asserts that the subject promotes problem solving.

The study by Curran (2023) addresses the conflicting claims. Small convenience samples of teacher volunteers (four, all experienced teachers of Applied Maths) and third-level students (seven, all of whom had taken the subject) were interviewed about their general experience of the subject and were asked about the pedagogical and learning approaches they believed were encouraged by the curriculum and examinations. While the findings from such small convenience samples may not generalise, they offer potential insights. The teachers and students agreed that the examination papers were predictable, consisting of about 75% routine/familiar material and about 25% that was challenging/novel. However, the teachers emphasised understanding and problem solving as the keys to success, whereas the students focused more on practice and repetition. Curran (2023) points out that apparent discrepancies may be somewhat reconciled as suggested by the words of one teacher interviewee:

> You mustn't forget that in the month we did studying it, we started not knowing how to learn to do those questions. Especially if it's well taught, in other words, in, you know, not just, here's a nifty little way, but you have to understand what's going on and you understand why. (T2) (Curran, 2023, p. 56)

In other words, the problem-solving element may figure in (appropriate) initial teaching and learning, rather than when students are approaching and taking the examination. A possibly relevant finding here was that there was more reported diversity in the teachers' pedagogical approaches than in the students' learning strategies. With regard to transfer, the teachers and students agreed that Applied Maths helped Mathematics and Physics, but the students had a more conservative view of the benefits for other subjects, whereas the teachers had a greater appreciation for the role of Applied Maths in developing general problem-solving skills.

Methodology

This section describes the methodology for the main, quantitative element of Curran's (2023) project. Analysis of data from the third and fourth wave of the *Growing up in Ireland* study was undertaken. This study follows the progress of approximately 8,000 children, beginning in 2006 when they were eight years of age, and includes data on their Junior Certificate and Leaving Certificate results for English, Irish, and Mathematics – the three subjects taken (in both examinations) by the vast majority of students. Also included is whether students took Applied Maths in the Leaving Certificate. Research question 3 above was modified for this data set as follows:

3a. Did students who took the old Applied Maths course improve from the Junior Certificate to the Leaving Certificate in Mathematics, English and/or Irish, in relation to their peers?

Access to the relevant examination results was obtained from the Irish Social Science Data Archive, with help from Prof. Emer Smyth of the Economic and Social Research. The dataset in question is very detailed and had to be narrowed down significantly. While some information such as socioeconomic factors could be relevant in a larger study investigating who has access to and opts to take Applied Maths, the author did not have permission to use data other than students' results. Anyway, the issues that could have been addressed in such a study were outside the scope and timescale of this already substantial undergraduate project.

Data was cleaned by excluding participants with missing or incomprehensible data (for example, Leaving Certificate points for Mathematics greater than the maximum possible score of 125: *points* being the metric used to map Leaving Certificate examination grades onto a numerical scale to rank-order students applying for places on third-level courses). For each subject, Leaving Certificate results were ranked by the points; Junior Certificate results were ranked first by level – Higher / Ordinary / Foundation – and then by grade. The rankings were then transformed into percentile scores. Correlations between the scores were investigated; for example, correlations between students' Mathematics scores at Junior Certificate and at Leaving Certificate were computed.

For each student, the difference between the two percentile scores for a subject was calculated so as to measure improvement/regression of students *relative to their peers*. (It should be noted that students taking Applied Maths are usually high performers overall. The focus here is therefore on gains/losses over the period in which they studied Applied Maths, rather than just on Leaving Certificate scores.) A logistic regression was performed to investigate the relationship between this difference and whether or not a student took Applied Maths; the regression coefficient was tested for significance. Also, clustering techniques were used to investigate the possible existence of subpopulations within the data. The approach was authenticated by consultation with statistician Jason Wyse in Trinity College; technical details are outside the scope of this paper.

Results

Results presented here are limited to the findings for Mathematics. First, it should be noted that results in Mathematics at the two levels are highly correlated (r = 0.80). This suggests that skills tested in the Mathematics Junior Certificate and Mathematics Leaving Certificate examinations have very significant overlap.

Table 1

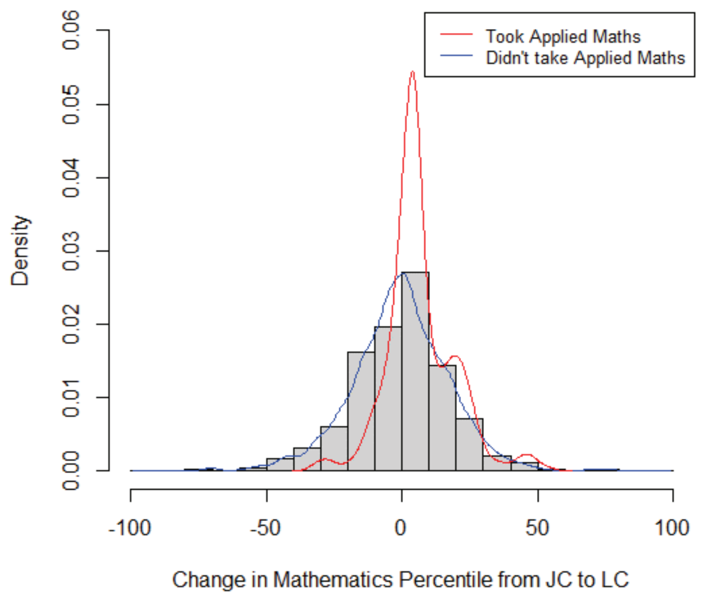
Average Mathematics Results (Applied Maths Students)

Applied Maths Students	Average Percentile
Junior Certificate Mathematics	81.76
Leaving Certificate Mathematics	88.52

The percentile rankings confirm that, on average, Applied Maths students performed above their peers in both Junior Certificate and Leaving Certificate (Table 1). Thus, the issue of interest is whether the Applied Maths students performed above their peers *to a greater extent* in the Leaving Certificate. Table 1 shows that there was a substantial improvement in percentile ranking from Junior Certificate to Leaving Certificate for those who studied Applied Maths, with a mean increase of 6.76 percentile points. A logistic regression determined this relationship to be highly significant with a p-value of 1.28×10^{-11}. However, while students who took Applied Maths improved in Mathematics by a considerable amount on average, relative to their peers, it is not immediately evident if this can be attributed directly to the skills developed by Applied Maths.

Figure 1

Distribution of Change in Mathematics Results: Applied Maths and Non-Applied Maths students



The distribution of the results paints a more complete picture. As can be seen from Figure 1, the distribution of changes in performance in Mathematics among Applied Maths students is entirely different from that of the rest of the population. The "bumps" in the distribution of Applied Maths students' scores suggests the existence of subpopulations:

- a small minority for whom Applied Maths has a significant negative impact (a decrease of 10-30 percentile points)
- the majority for whom Applied Maths has a positive impact (an increase of 0-10 percentile points
- a significant minority for whom Applied Maths has an extremely positive impact (an increase of 20-45 percentile points).

Clustering techniques confirmed the existence of the subpopulations within the data; details of this analysis are outside the scope of the paper.

Overall, therefore, it can be said that the students who take Applied Maths do, as a whole, improve in those skills tested in Mathematics, relative to their peers (research question 3a). To what degree this might be attributed to Applied Maths itself is discussed below.

Discussion

The results from the quantitative study give a positive answer to the modified research question 3a for Mathematics. Since Mathematics and Applied Mathematics are cognate subjects, perhaps this is not surprising in light of Thorndike's theory of transfer noted above (the amount of transfer from one situation to another being proportional to the degree of similarity between the two situations (Woodworth & Thorndike, 1901)). However, the original research question 3 is not fully addressed, as the findings do not indicate the nature of the skills that may have transferred to the study of Mathematics, except insofar as they are examined in the Junior and Leaving Certificate Mathematics examinations. This is now discussed in relation to the other literature reviewed, particularly with regard to the debate as to whether the old Applied Maths course promoted problem-solving in the sense that the term is used in mathematics education literature, distinguishing problems from routine exercises.

Confirmation by clustering techniques of subpopulations in the data only for the Applied Maths students indicates there are subgroups of students who are experiencing Applied Maths in some way differently. The question arises as to what these different experiences might be. The comment from a teacher in Curran's (2023) qualitative study, quoted above, may provide a clue; the teacher indicated that topics could be *introduced* in a way that developed understanding and problem solving, whereas later the focus would be on preparing for (predictable) examinations through intensive practice. If the old Applied Maths course were taught in the manner that this teacher describes, that is as a guided investigation, the first year's work would consist of students being introduced to most of the topics that they would cover. In that first year, students would experience mathematics in the way that mathematicians do: as an exploratory and investigative process, which can develop confidence in one's competency as a problem solver and the perseverance associated with that. Exploring the deep questions at the heart of topics offers all the characteristics of a good problem outlined by Schoenfeld (2016). If topics were treated in this way, the old Applied Maths course may well have had the potential to improve problem-solving skills. It is conjectured here that such an experience of the old Applied Maths course characterises the subpopulation of Applied Maths students who improved dramatically in Mathematics.

Unfortunately, it seems that most students did not have the experience of Applied Maths outlined above. In such instances, teachers may have put more emphasis on procedural fluency throughout the course than on constructing understanding and problem solving. One might expect students who had this experience of Applied Maths to improve somewhat in Mathematics purely due to the additional practice. It is conjectured that this characterises the subpopulation of Applied Maths students who improved in Mathematics, but by less than 10 percentile points. (No conjecture is offered here with regard to those who regressed.)

Overall, the evidence suggests that the old Applied Maths course had the potential, though often left unrealised, to facilitate significant near transfer (in Johnson's (1995) terminology) of problem-solving skills, if appropriately taught. The richness of the topics and the narrow scope of the course could promote this, as they allowed the opportunity for students to delve deeply into a small number of rich investigations.

Conclusion

The main aim of the study on which this paper is based was to investigate the effect that studying the old Applied Maths course had on other subjects. This paper reports findings with regard to Mathematics. The strength of the old Applied Maths course appears to have been in its potential – not always realised – for extremely significant near transfer to Mathematics, via the development of students' problem-solving skills and confidence in their capabilities as mathematicians. If the style of teaching that encouraged this could be more broadly implemented, it would represent a very powerful tool in mathematics education.

It is important to be aware of the limitations of this study. There are many factors that cannot be controlled for, so the overlap between studying Applied Maths and improved performance in Mathematics may not be causal. In particular, the mechanisms behind the presence of subpopulations among Applied Maths students, while informed by qualitative and quantitative research and the present literature on the subject, are merely conjectures, and are not backed by concrete evidence.

Additional research is required to investigate the effect that studying Applied Maths had on subjects other than Mathematics. (The work has already been done for English and Irish; however, the findings are outside the scope of this paper.) The first author hopes to be granted access to the more detailed files of the study, which will allow investigation of all the subjects that appear in both the Junior Cycle and the Leaving Certificate. Furthermore, the study provides a template for analysing the effect subject "x" has on performance in subject "y". There is little reason going forward to restrict "x" to be Applied Maths.

Further research within the next decade will be required to analyse whether the strengths of the old course have been kept in the new Applied Maths course, and whether the weaknesses of the old course have been improved upon. Initial research could perhaps utilise data from the infant cohort of the *Growing up in Ireland* study, the majority of whom will be sitting their Leaving Certificate examinations in 2024. The findings could then help to shape implementation and perhaps adjustment of the new course, and further data might be collected to monitor its role in the years to come.

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Online Synchronous Maths Support Attendance Post-Covid

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In the autumn semester of 2021, Dublin City University's (DCU) students returned to campus after just over a year of remote learning. This return had some caveats: COVID-19 restrictions limited how students could interact with lectures, tutorials and maths support. To continue catering to students during this period, the Maths Learning Centre (MLC) provided support both in-person (at a reduced capacity to accommodate COVID-19 restrictions) and synchronously online via bookable Zoom sessions — a continuation of the support provided during COVID-19 lockdowns. As Covid restrictions were lifted in September 2022, and the MLC drop-in service returned to its pre-Covid operation, the MLC continued to provide this online support. In this paper, we will examine the online MLC attendance over the last few years and discuss this in the context of in-person attendance.

Keywords: COVID-19, online support, mathematics learning support, student engagement

Introduction

The Maths Learning Centre (MLC) is a free additional support available to all DCU students studying any kind of mathematics (Jacob & Ní Fhloinn, 2018). This service is built to work alongside lectures and tutorials, so that students can address mathematical difficulties they may have. Outside of extraordinary circumstances, the MLC operates as a drop-in service in the main library on DCU's Glasnevin campus. The room can hold about forty students, and is staffed by two to three tutors, depending on how busy the room is at that point in the semester. Students register their attendance by scanning a QR code, located on each desk in the room. Students attending the MLC can avail of one-to-one and group help from the tutors, as well as having access to a small textbook library and a library of photocopied take-home revision sheets (mainly comprising revision sheets from mathcentre.ac.uk).

In March 2020, the MLC was forced to rapidly change tack in order to provide maths support in an online environment. For the first time, the MLC offered synchronous online maths support, via Zoom drop-in sessions. Howard and Ní Fhloinn (2022) report on the attendance of the online support provided during the 2020–2021 academic year. There was a considerable drop in student engagement when maths support moved to being solely online. The attendance patterns were similar to those generally observed in the drop-in centre, in that the peaks in attendance happened at the same usual points during the semester. They hypothesise that the demand for online maths support would likely continue alongside that for the more traditional in-person support.

In the 2021–2022 academic year, DCU made a tentative return to in-person learning, with the caveat of COVID-19 restrictions. In order to accommodate these restrictions, the MLC reopened its in-person service on a reduced timetable, and required that students book their attendance in advance in order to ensure that the number of students in the room stayed below 24 (an approved number which allowed adequate spacing between students in the room). Alongside this in-person support, the MLC continued to provide online support in the form of evening Zoom sessions, which remained well attended despite the in-person return.

In September 2022, DCU returned fully to in-person learning, dropping its COVID-19 restrictions. The MLC followed suit, returning the drop-in centre to its original operation pre-Covid (ie. pre-booking was no longer necessary, and students once again registered their attendance by scanning a QR code in the room). The online synchronous support, however, continued at designated times in the evening. The attendance for online support dropped significantly in Semester 1, which prompted a change in how this support was offered. Consequently, in Semester 2, the MLC offered online support by asking students to send an email directly to organise a session, which could then take place at a time that suited both the student and tutor. It appears from attendance data that student engagement has grown since, and so this paper serves to examine these changes in support with a view to providing best practice and engaging students.

Literature Review

Owing to the rapid shift online in early 2020, many publications sought to explore these changes in higher education in Ireland. Some outline the experiences of moving education online (Casey, 2020; Ní Fhloinn & Fitzmaurice, 2022) while others discuss student/teaching perspectives (Hyland & O'Shea, 2021; Mac an Bhaird et al., 2021; Meehan & Howard, 2020). O'Shea (2022) gives an overview of much of this research, broken into the broad themes of student/lecturer perspective, maths support, and assessment.

Hodds (2020) details some of the early major changes to maths and stats support that happened as a result of COVID-19 lockdowns in an international setting, with a focus on the UK. They found that most institutions provided some form of online support, but a huge number of institutions saw a major reduction in the number of students engaging with support once support had moved online. Many practitioners felt uneasy or underprepared for providing online support of a similar standard to traditional support methods. Ní Fhloinn and Fitzmaurice (2022) echo this in their discussion of lecturers' experiences moving online; lecturers felt more pressured to improve their performance in 2020-2021. The general consensus was that the future of maths and stats support would be hybrid, with more resources put into in-person support.

Mullen et al. (2022) go into further detail and discuss some of the pedagogical changes that happened in the provision of maths support as tutors moved online, via a series of interviews with tutors and students in University College Dublin (UCD), Ireland and Western Sydney University (WSU), Australia. Some of the tutors interviewed noted the difficulty that came with the "lack of body language or non-verbal communication" (p. 73); tutors found it harder to properly diagnose students' problems both due to this communication problem and since students often could not share their work for technical reasons. Gilbert et al. (2021) found similarly that communication issues constituted a huge drawback to online support — practitioners were not getting the usual visual feedback from students, and there was a "loss of the personal connection" (p. 4). Mullen et al. note that tutors found themselves talking more in online sessions; one tutor worried that they were "talking at, rather than to, students" (p. 74). Some UCD tutors noted that students tended to prepare more for online than for in-person. This was also acknowledged by tutors in DCU (Howard & Ní Fhloinn, 2022).

On January 13th 2023, the Irish Mathematics Learning Support Network (IMLSN) hosted a workshop to explore the experiences of maths support centres in semester 1 of the 2022-2023 academic year (IMLSN website, 2023). Of the eleven Irish Higher Education Institutions (HEIs) represented, nine provided a hybrid support of online and in-person, one institution provided solely online support, and one provided solely in-person support. Generally, online attendance was not very high. The rapid student return to in-person over online support came as a surprise to some of the workshop attendees. Indeed, this general preference for in-person support over online support appears to be present in DCU.

Methodology

From September 2020 to December 2022, the online support scheduling was handled by the in-built appointment scheduling feature of Loop, DCU's virtual learning environment (VLE). This feature allows the user to book a given slot, where that slot is attached to a given tutor. These data were exported from Loop to include student numbers and dates and times of appointments. Since these data come directly from student logins on Loop, there is no need to validate student names and numbers. For Semester 2 of 2022-2023, bookings were made via email and so the attendance data were collated manually by the first author, who staffed those appointments. Names and emails were cross referenced with Loop to validate them.

For the in-person data: in the 2021-2022 academic year, in-person attendance was booked in advance via a Google form, which asked students for their student number and degree programme, as well as time and date of the booking. These data were collated into a spreadsheet, and the data were cleaned and validated by cross referencing with Loop. Drop-in attendance data from the 2022-2023 academic year (where students scanned a QR code to register their visit) were tracked using Loop, in the same way that online sessions were tracked in previous years. As such, these data could again be directly exported from Loop without validating student name and numbers. Some students double or triple booked themselves at some points, and so these data were cleaned to remove the duplicate bookings.

Pivot tables were used in Excel to organise the data for use in tables and charts.

Results

Structure of Online Support

During the lecture terms of 2020-2021, four appointment slots were made available every weekday before 2pm, and a further four slots available four days per week in the evening. Slots were also available in the study week before Semester 1 exams, and in the weeks leading up to Semester 2 exams and the resit exams in August 2021.

In September 2021, the MLC continued its online support alongside the newly restricted in-person support. For the online support during Semester 1, two appointments were available four evenings per week. This was halved in Semester 2 to only two evenings. Again, appointments were made available during the various study periods before the Semester 1, Semester 2 and August resit examinations.

Coming into September 2022, the MLC again ran a hybrid service of online and in-person support, with the in-person support returning to its pre-Covid drop-in format. Two online appointments were available two evenings per week, starting in week 6 of the semester. The online support was only offered from week 6 onwards due to the expectation that online engagement would be low, owing to the return of unrestricted in-person support. This expectation seemed to come true, and only five of these appointments were booked. However, engagement increased in the two study weeks before the Semester 1 exams began, and more appointments were made available to accommodate this influx. Note that the in-person support was only available during the first study week, and not the second.

During Semester 2, online support was offered from week 2. No appointments were made available via DCU's VLE; instead, students were asked to email to request an appointment. Despite fewer students in Semester 2 taking maths modules (a consequence of how DCU's maths modules are timetabled), more bookings were made during this semester than in the previous one. In anticipation of the same influx of appointments as during the study weeks of Semester 1, appointments were once again made available through the VLE to accommodate students for the study week before the Semester 2 exams. However, the appointments were much less sought after than expected, and only six of the fourteen made available were booked. Table 1 gives a breakdown of the number of appointments made available over the three years in question.

Table 1

Numbers of online appointments made available by time period

	S1 Lectures	S1 Study	S2 Lectures	S2 Study	Resit Study	Total
2020-2021	368	60	424	160	196	1208
2021-2022	70	16	44	4	40	174
2022-2023	28	52	17*	14	—	94

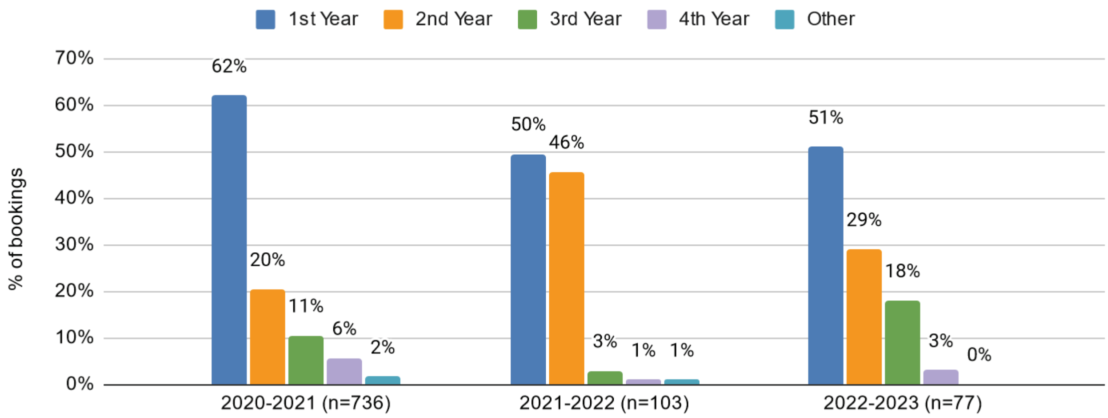
Note. S1 and S2 refer to Semester 1 and Semester 2 respectively. 'Lecture' refers to the lecture term, and 'Study' refers to the study period leading up to exams. This paper is written following the Semester 2 exams in 2023, and so no data are given for the 2023 resit study period.
*Since students emailed to request appointments in Semester 2 of 2022-2023, no explicit number of appointments was made available; the figure supplied is the number of bookings made.

Overall Engagement with Online Support

Over the course of the 2020-2021 academic year, 736 (61%) of the available 1208 appointments were booked. Most of the bookings were made by first-year students, and nearly 30% of the first-year bookings were made by Actuarial Mathematics (ACM) and Common Entry into Actuarial and Financial Mathematics (CAFM) students, whose course programmes contain only mathematics modules. See Figure 1 for a breakdown of the appointments booked by different year groups.

Figure 1

Breakdown of bookings for online support by year group



There are several significant contrasts between these statistics and those of the 2021-2022 academic year (Chi-square test, $p<0.001$). A similar proportion (59%) of the available appointments were booked over the year. The second-year students made up an unexpectedly large portion of the overall bookings (Figure 1), and nearly half (45%) of all second-year bookings were made by ACM or CAFM students. In contrast, only two of the first-year students who attended were ACM or CAFM students.

This contrasts again significantly with the 2022-2023 academic year (Chi-square test, $p=0.003$). An unexpectedly large portion of all bookings were made by third-year students (Figure 1). Only one of the fifty-eight total first- and second-year bookings was made by an ACM or CAFM student (one student in ACM1). Over one third (38%) of the bookings made by third- and fourth-year students were made by students in ACM or FIM (Financial Mathematics: the CAFM common entry cohort splits after two years into ACM and FIM).

A total of 240 distinct students engaged with online support in the 2020-2021 academic year. Due to Covid lockdowns, there was no in-person support against which to compare this figure. For the 2021-2022 and 2022-2023 academic years, the engagement of students with in-person and online supports is shown in Table 2. In both years, roughly one tenth the number of students engaging with in-person supports engaged with online supports.

Table 2

Breakdown of students who engaged with MLC supports in-person, online or both.

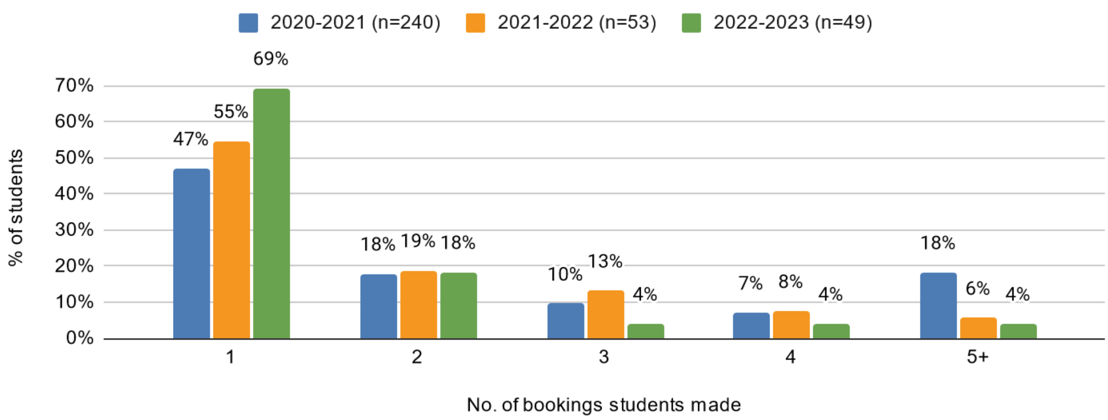
	In-person only	Online only	Both in-person & online	Total
2021-2022	518 (90.7%)	21 (3.7%)	32 (5.6%)	571
2022-2023	454 (90.2%)	34 (6.8%)	15 (3.0%)	503

Repeated Engagement with the MLC

Over half (53%) of the students who engaged with the MLC in 2020-2021 booked two or more sessions (Figure 2), which Howard and Ní Fhloinn (2022) note is in line with in-person numbers from previous years. This repeated online engagement drops significantly over the following two years (Chi-square test, p=0.025): most students in 2021-2022 only booked a single session, and no student booked more than six appointments. Again in 2022-2023, most students only booked a single session, one student booked six sessions, and no student booked more than six. This drop may be expected given the much smaller count of available appointments; that being said, in the 2020-2021 academic year there was a small cohort of students who booked upwards of twenty appointments over the course of the year. See Figure 2 for a visual representation of this shift towards fewer engagements.

Figure 2

Number of online appointment bookings made per student



On a broader timeline, some students engaged with online support over multiple years. In 2021-2022, of the thirty-one students who were not first-year students, just over half (52%) had engaged with the service in the previous academic year. There were also three repeating first-year students who had engaged the previous year. However, only three of the twenty-eight non-first-year students who engaged with online support in 2022-2023 had engaged previously, with an additional two repeating first-year students. There were three students who engaged with online support all three years in question.

Discussion

The most notable trend in the data appears to be the continued attendance of ACM and CAFM students, starting with first-year students in 2020-2021 through to third-year students in 2022-2023. In Figure 2 the impact of this can be seen, as it distorts the typical number of students from each year group usually seen in attendance at in-person support — more second-year students attended in 2021-2022 than would be expected, and more third-year students attended in 2022-2023 than would be expected. Where most course programmes that involve a mathematics module only do so in first-year, the ACM and CAFM students continue to have mathematics modules through all years of their course programmes.

The ACM and CAFM students in first-year in 2020-2021 appear to have become comfortable attending online support, doing so at high volume even when in-person support had returned. In contrast, first-year ACM and CAFM students in 2021-2022 seem to have had a strong desire to engage in-person and not online. This perhaps does not come as too great a surprise, given the now documented preference for in-person engagement over online.

One of the motivations for this paper was to analyse the effects of the changes made coming into Semester 2 of 2022-2023 as to how online support was offered. Despite typically higher engagement with support in Semester 1, nearly three times as many bookings for online support were made during the lecture term of Semester 2 when students emailed to book instead of booking through the VLE. Alongside this, the first author (who staffed the online support from the study period of Semester 1 onwards) found that students who had booked via the VLE were less likely to actually attend the session they had booked. Many students forgot they had made the booking despite email reminders, and some cancelled their session last-minute, meaning that other students would not have the opportunity to book that session. During the lecture term of Semester 2 when students emailed to book, only one student did not attend their appointment, and there were no cancellations.

Now that in-person support is back to its pre-Covid operation, it appears from these data that most students much prefer to attend in-person rather than online appointments. That being said, the repeated engagement of the ACM and CAFM students with online support highlights an addendum: it appears in their case that once the students had initially engaged online, they were comfortable continuing to engage online even when in-person supports had returned. The goal then would be to make online support accessible enough for a student to attend once; they may find the experience more worthwhile than they expect.

As we move further from the influence of COVID-19 lockdowns and restrictions, we may continue to see the dissipation of these lingering effects on the provision of maths support. The benefit of the new ubiquity of online support has yet to be fully explored, and so further research is warranted into what makes online support more alluring and accessible to new students. In addition, further investigation into the effects of online support on student engagement, performance and confidence — especially compared to the effects of more traditional in-person support — could help to illuminate how best to provide that support.

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Problems for Teachers? Views of Members of the Irish Mathematics Teachers' Association on Current Issues in Mathematics Education

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This paper examines two surveys carried out by the Irish Mathematics Teachers' Association (IMTA) in 2022 and 2023. They were designed to elicit members' views on issues identified by the IMTA Council as priorities, and hence to enable the Association to carry out its role of advocacy on behalf of its members. The issues and the survey findings are considered using the lens of recent Irish research on matters affecting curriculum implementation for post-primary Mathematics: time allocated to Mathematics; uptake of the highest level course; possible loss of procedural fluency; and teacher shortage. Teachers' stated concerns mirror these matters closely and reinforce the need for the underlying problems to be addressed.

Keywords: mathematics teacher voice; Irish Mathematics Teachers' Association

Introduction

Mathematics education depends crucially on the mathematics teachers who work with students inside and outside school classrooms. Thus, teachers' views are very important, for example both in identifying areas of mathematics education that are proving difficult or contentious and in contributing to system-level discussions on how problems can be addressed. To facilitate the latter process, bodies that represent teachers have a responsibility to learn their constituents' views and transmit them to appropriate audiences.

The Irish Mathematics Teachers' Association (IMTA) (https://imta.ie) is one such body. Founded in 1964 and with 1340 members in February 2023, its roles include supporting post-primary mathematics teachers via various forms of continuing professional development (CPD) and providing them with a voice in advocacy and negotiations about matters of relevance in mathematics education. This paper addresses the advocacy role. Its aim is *to report and discuss IMTA members' views as given in their responses to surveys carried out by the IMTA in 2022 and 2023, viewing them through the lens of current research.*

The following section of the paper offers a literature review. There are two aspects; one provides context and outlines recent curriculum changes, while the other focuses on Irish research that highlights problems affecting curriculum implementation. The rationales and methodology for the two surveys are then described, focusing chiefly on the more recent one. Findings are reported and discussed in the light of the literature, and conclusions are drawn.

Review of Literature

Curriculum Structure and Changes in the Last 20 Years

Post-primary education in Ireland is divided into two cycles, Junior and Senior, each with assessment leading to certification: respectively, the Junior Cycle Profile of Achievement (recently replacing the Junior Certificate) and the Leaving Certificate. Most school subjects are offered at two levels, Higher and Ordinary, but Mathematics has also been provided at Foundation level in both cycles. The Leaving Certificate acts as a gatekeeper to higher education. A student's grade in each subject is converted into "points" (initially in the range 0-100, but see below); the student's six best "points scores" are added, and the total is used to rank-order qualified applicants for over-subscribed third-level courses (Oldham, 2007; O'Meara et al., 2020). Within this general framework, two significant changes in the Mathematics curriculum were introduced in the last twenty years, involving major developments both for curricular intentions and for their implementation by teachers.

The first such change was the initiative known as "Project Maths". This aimed to increase the emphasis on problem solving and real-life applications; also, it set out to alter the dominant classroom culture – which focused overmuch on procedural fluency at the expense of conceptual understanding – by encouraging use of student-centred pedagogies. The "Project Maths" curricula for both Junior and Senior Cycle were phased in over several years from 2008. Adjustments to the style and format of the State examinations mirrored the aims of the initiative, one relevant feature being removal of choice from the examination papers (Berry et al., 2021; Byrne et al., 2021; Oldham, 2007, 2019; Shiel et al., 2020). It should be noted that changes in underlying philosophy and intended classroom practice can be very challenging for teachers, especially if their beliefs about mathematics and teaching clash with the new focus, or if there are practical difficulties. A common difficulty is shortage of time, for instance for content "coverage" using student-centred pedagogies (Johnson et al., 2019).

It was hoped that the changes would promote greater uptake of the Higher level Leaving Certificate course: historically low compared with the case for the other two subjects (English and Irish) taken by almost all Leaving Certificate candidates. To support the new curriculum, the higher education institutions agreed that, from 2012, 25 "bonus points" would be added to the Mathematics points scores of applicants who had achieved a satisfactory grade in Leaving Certificate Higher level Mathematics, irrespective of the subjects they intended to study at third level. Uptake indeed increased markedly, from around 16% in 2011 to over 30% by 2017. This is discussed by Shiel et al. (2020) in the context of a wider exploration of aspects of the "success" (however conceptualised) of Project Maths.

The second significant change was part of wider reform of the post-primary Junior Cycle curriculum. For all subjects, the "specifications" are now framed so as to address generic "key skills" such as being creative, working with others, and communicating; also, to reduce emphasis on a terminal examination, there are "classroom based assessments" (CBAs) which contribute to the Junior Cycle Profile of Achievement. For Mathematics, the revised specification was introduced in 2018, with the main alterations being reduction in the number of papers in the State examination, abolition of the Foundation level course, and introduction of the CBAs (Byrne et al., 2021). The CBAs involve investigational work: an approach which has been addressed for many years in other jurisdictions (for example in England and Wales), but which has been slow to take root in the Irish system (Oldham, 2019). It is intended that some mathematical concepts and skills would be learnt *through* doing the CBAs, rather than the material being taught and learnt first and then applied; altogether, in fact, the revised specification reflects a focus on problem-based learning. Again, this is challenging for teachers, especially if it is at variance with their beliefs and concerns about mathematics education (Johnson et al., 2019). The COVID pandemic led to changes in implementation of the revised curriculum, so research on the implementation is in its infancy.

Problems in Implementation

Problems caused initially by the radical nature of the Project Maths initiative are now a matter of history (Byrne et al., 2021). However, recent Irish publications identify ongoing issues, notably: the time allocated to Mathematics lessons, together with associated problems of curriculum implementation (with implications for further CPD); the effect of bonus points on students' uptake of the Higher level Leaving Certificate course; overall perceived decrease in procedural fluency; and the shortage of entrants to the teaching profession.

The question of *time* is prominently addressed. A study by Prendergast and O'Meara (2017) noted that, while the average time allocated to Mathematics as a proportion of overall teaching time was broadly in line with that in many OECD countries, the annual total was adversely affected by Ireland's short school year. It follows that, if content "coverage" is to be on a par with that offered elsewhere, teachers will have problems in moving towards the recommended – but time-consuming – student-centred pedagogies. Reports (for example: Berry et al., 2021; IMTA, 2012) consistently reveal that teachers are short of time.

It was noted above that students' *uptake of Higher level Mathematics* in the Leaving Certificate had grown markedly. The driving factor, according to the students themselves, is the award of the *25 bonus points* (O'Meara et al., 2023). However, an unintended outcome reported by teachers is that some students who attempt the Higher level are not suited to it (Berry et al., 2021; O'Meara et al., 2020). This places yet more pressure on teaching time.

A related problem – but one that applies more broadly than to the Higher Leaving Certificate – is that of poor *procedural fluency*, notably in algebra and perhaps reflecting conceptual misunderstanding for key aspects (O'Brien & Ní Ríordáin, 2021; Prendergast & Treacy, 2018). Whether this is a consequence of curriculum change is discussed by Shiel et al. (2020). They report that performance of Ireland's 15-year-old students in PISA (the Programme for International Student Assessment) has mainly stayed almost constant over recent years of testing. However, procedural fluency is not the main focus of PISA tests; a more appropriate measure may be the diagnostic test used by the University of Limerick for first-year students enrolled in science- or technology-based courses. Shiel at al. (2020) note that the decline in test scores over the period 1998-2013 antedates the impact of Project Maths and may have a variety of causes. Whatever the reasons, it is a matter for concern.

The final issue to be mentioned here – one that is fairly new in the Irish context – is the severe *shortage of entrants to the teaching profession* especially in some disciplines, including mathematics (O'Doherty & Harford, 2018). The issue receives media attention, notably when teacher unions hold their annual conferences; it has recently prompted a leading article in the prestigious *Irish Times* newspaper ("Teacher shortages", 2023, p. 11).

Methodology

The first of the two IMTA surveys described in this paper was carried out in April 2022, to collect feedback from teachers both on *the 2018 Junior Cycle curriculum* and on *their reactions to possible reforms for the Leaving Certificate programme* (due at some still unspecified future date). It also sought information on issues caused by the COVID pandemic, but that aspect is omitted here. The survey was drawn up by IMTA Council members and had two parts, one for the Junior Cycle and one for the Leaving Certificate. Both focused heavily on assessment issues, though the former also mentioned teaching time and student readiness for the Leaving Certificate. Most items were quantitative, chiefly of Likert type (with five responses ranging from *Strongly agree* to *Strongly disagree*); the Leaving Certificate part also asked about preferred options for the some of the issues (the number and timing of examination papers; choice on the papers; and coursework design and marking). For a qualitative element, comments were invited via open-ended items. The two survey parts were set up using Microsoft Forms and the links were emailed to all members (1050 at the time).

The second survey was undertaken in February 2023 to provide further data when the IMTA was invited (at short notice) *to make a submission to the Joint Oireachtas Committee on Education, Further and Higher Education, Research, Innovation and Science* on the future of science, technology, engineering and mathematics (STEM) in Irish education. Again, the survey was drawn up by members of the IMTA Council. It reflects Council members' perceptions of key priorities, and strongly echoes the issues in the literature reviewed above: course and assessment structures, teaching time, bonus points, students' procedural fluency, and teacher shortage. The style was similar to that of the first survey, with a simple format to encourage responses, and the link to the survey was emailed to all 1340 members.

Data were downloaded into spreadsheets. To relate responses to research on the various issues, coverage here is wide rather than deep. For quantitative items, frequencies (percentages) are reported; for the qualitative aspect, the authors independently studied the comments and then agreed on themes. The main focus is on the – broader – second survey.

Results

For the first survey, there were 362 responses to the first part and 387 to the second. The most striking results for Junior Cycle were the wish to retain Foundation level (only 5.3% agreed or strongly agreed with the statement "The removal of the foundation level exam is a good thing") and the negative opinions on CBAs (just 9.9% agreeing or strongly agreeing that "CBAs are beneficial to students"). Also, 84.8% disagreed or strongly disagreed that "One exam paper is sufficient for [Higher level] students." For Senior Cycle reforms, 77.3% of respondents "would need to see more [detailed proposals]." However, reduction to one examination paper (18.0% agreement) was unpopular, while question choice was heavily endorsed (92.4% agreement). Reactions to coursework at this level were nuanced, but CBA-style work was opposed (93.2%) and only 4.5% agreed with teachers grading their own students. Details are available at https://imta.ie/junior-cycle-reform-members-survey-2022/ and https://imta.ie/senior-cycle-reform-members-survey-2022/.

The second survey was completed by 444 members. An overview of responses to the IMTA Council's list of key priorities is provided in Table 1. With regard to Foundation level and CBAs in the Junior Cycle, and choice on examination papers, the findings reinforce those of the first survey. Among other issues, it can be seen that poor algebraic skills, and the need to shorten the Higher level Leaving Certificate course, attracted much agreement; the idea of a non-calculator element in examinations is less popular; and the items relating to improving teachers' pay and conditions received overall support but drew many neutral responses.

Table 1

Responses to issues identified by IMTA Council as key priorities – percentages (N = 444)

Item	SA	A	N	D	SD
Foundation level must be reintroduced at JC level	54.7	28.2	8.1	8.1	0.9
JC HL course is too long and hard to deliver with schools losing contact time for Maths	48.0	33.3	5.9	9.3	3.6
Algebra skills at JC HL have been eroded. This is influenced by new JC HL assessment	58.2	30.7	5.7	4.8	0.7
A second HL paper must be reintroduced at JC level	54.2	27.7	8.8	7.7	1.6
Numeracy skills are worrying – consideration should be given to having a non-calculator style JC exam	19.8	28.3	18.6	25.1	8.3
Validity of CBAs at JC – consider doing 1 at most	86.3	11.7	1.1	0.7	0.2
Revamp LC HL Bonus Points sliding scale only (H1=25, H2=20, H3=15 etc)	42.2	27.5	9.9	12.2	8.1
There must be an element of Choice on LC papers…	74.5	18.1	3.2	3.4	0.9
LC HL course is too long – some items need to be [cut]	66.7	19.4	5.4	6.5	2.0
Urgent measure to address serious shortage of qualified mathematics teachers – bonus pay, tax incentives, etc	53.3	23.4	15.4	7.0	0.9
Fees for PME/Education [i.e. teacher education] courses with maths element should be paid by State	25.9	27.0	26.8	15.7	4.5
Rental support should be available to STEM teachers … where accommodation costs are prohibitive	29.8	30.5	25.5	11.0	3.2

Note. SA – Strongly agree; A – Agree; N – Neutral; D – Disagree; SD – Strongly disagree.

JC – Junior Cycle; LC – Leaving Certificate; HL – Higher level.

Possible alternatives to the way in which bonus points might be allocated for Higher level Leaving Certificate, other than by the sliding scale specified above, were explored in a further question. Almost half of the respondents (49.3%) agreed with the sliding scale as given; however, there was support also for the current system (20.0%), for giving bonus points only for entry to relevant courses (15.1%), and for abolishing them altogether (15.5%).

With regard to teaching time for Mathematics, respondents were asked to indicate how many periods per week were assigned for their classes in each year of the two cycles. Table 2 presents the data for 40-minute and one-hour periods. The variation is striking, and the fact that some teachers meet their First Year classes only twice a week is a matter for concern. Also, asked whether they could teach the Higher level Leaving Certificate course in the time available, a disturbing 61.0% chose the response "To get the course done I have to do extra classes with my students" – that is, meeting them outside the "normal" school timetable.

Table 2

Number of classes per week for each year – percentages

School year	40-minute periods (N = 243)					School year	1-hour periods (N = 201)			
	3	4	5	6	7		2	3	4	5
First year	8.7	52.2	36.7	2.4	0	First year	6.7	80.3	12.1	0.8
Second year	3.2	43.3	50.2	2.5	0.7	Second year	7.1	70.8	21.2	0.9
Third year	1.8	28.4	62.9	5.5	1.5	Third year	0	73.0	24.8	2.2
Fifth year	1.2	2.1	36.6	55.6	4.5	Fifth year	0	12.9	75.4	11.6
Sixth year	0	2.7	25.2	60.8	11.3	Sixth year	0	6.9	72.5	20.6

Note. During Fourth Year (offered in some but not all schools), the major focus is not on examination curricula.

There were 115 responses to an item asking about issues omitted from the list in Table 1. From analysis of the responses, three main themes emerged: *shortage of time*; *assessment issues* (further ideas on the number and structure of papers and question choice; some comments on difficulty level and wordiness of questions; bonus points); and *curricular matters* (return of Foundation level Junior Cycle, the gap between Junior Cycle and Leaving Certificate courses, poor standards, individual topics that could be cut or reintroduced, and negative aspects of the CBAs). Further comments (398) were provided in the context of the "class time" / "periods per week" items. While there was some distinction between responses made with regard to the present courses and those that focused on possible course changes to fit the time allocations, overall they constituted a heartfelt plea for more time.

Discussion

Results are now considered through the lens provided by the Irish research literature described above. There is strong overlap between issues examined through academic research and those identified by practitioners: which is not surprising, as some researchers have specifically worked to obtain teachers' opinions. Additional aspects addressed in the IMTA surveys – teachers marking their own students' work for State certification, and the number and timing of State examination papers – refer to issues that apply more generally to (especially) Senior Cycle education, and are not unique to Mathematics.

Issues common to the research and survey findings – time allocated to Mathematics (with implications for the content and processes included in the curriculum), low uptake of

Higher level courses, and the apparent decline (for whatever reasons) in procedural fluency – highlight problems in implementing curriculum change successfully. Such problems are not new; they have affected previous phases of curriculum reform (Byrne et al., 2021; Oldham, 2007, 2019). Thus, claims about insufficient or variable teaching time go back at least to the Leaving Certificate courses of the 1980s. With regard to uptake of the Higher Leaving Certificate, this actually increased from below 12% in the early 1990s to over 18%: the outcome of a curriculum change in which uptake issues had to be balanced against perceived maintenance of standards (Oldham, 2007). Addressing that balance is an ongoing challenge. So is the frosty attitude to CBAs; it seems to reflect that teachers have not bought into the intended focus on problem-based learning, as well as indicating practical difficulties.

Conclusion

The surveys reported here were carried out by the IMTA as part of its advocacy role to ascertain teachers' views on key issues. The rich findings and their alignment with academic research have prompted wider circulation. They give voice to one part of the mathematics teaching community – members of the IMTA – and so probably reflect opinions of specialist teachers rather than those for whom mathematics is not their main subject. Within the IMTA, response rates (percentages in the thirties in each case) are reasonable for email surveys of this type, but outcomes may be skewed towards the views of those who chose to participate.

The fact that reported problems echo those from earlier periods of curriculum change points to their intransigent nature. They relate not just to Mathematics curricula but to the place of mathematics in Irish education, and call for urgent system-level consideration of relative priorities. It is tempting to refer to a four-dimensional challenge: length (of courses when appropriately taught and learnt), breadth (number of topics included), depth (per topic), and time (as a proportion of the overall curriculum) – all with implications for success in terms of pedagogies that can be used and meaningful standards that can be attained. We owe it to our teachers to address the issues.

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"*I'm not that bad in maths after all*": The Influence of Rich Tasks on Primary Students' Learning in Mathematics

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This study reports on the implementation of rich tasks designed to enhance student learning in mathematics at primary level. This study was conducted over a five-month period, involving 113 sixth-class students (aged 11-12 years) in a primary school in Ireland. Ten rich tasks were designed and implemented that were aligned with the primary mathematics curriculum and supported the development of students' productive dispositions in mathematics. The design principles used for developing these rich tasks in mathematics are discussed and exemplified. Semi-structured interviews were carried out with three teachers that collaborated with the researchers on facilitating the implementation of these tasks. This study discusses the teachers' experiences of implementing rich tasks in mathematics and their perceptions of the influence of these tasks on their students' learning. The teachers developed their understanding of the role of rich tasks in mathematics. The teachers identified that the use of rich tasks provided opportunities for students to develop their mathematical thinking, as well as fostering positive student attitudes and engagement in mathematics.

Keywords: Rich tasks, Primary mathematics, Mathematics learning, Productive disposition

Introduction

Research highlights a decline in students' motivation and interest for learning mathematics as they progress in levels of education (Deieso & Fraser, 2019; Widlund et al., 2018). As a result, they develop negative attitudes towards mathematics (Yao et al., 2018). These negative attitudes can strongly impede the development of students' mathematical identities and impact on their academic progression. A critical construct that significantly influences young learners' engagement in and attitudes towards mathematics is their productive disposition. Productive disposition is defined as 'the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics' (NRC, 2001, p. 131). In other words, productive disposition is a positive outlook towards mathematics that enables a person to put effort in solving a mathematical task with the belief that the problem can be solved. Teachers' instructional practices and classroom learning environment are key factors in shaping students' attitudes and dispositions for learning (Kaur et al., 2022).

Students with a productive disposition exhibit curiosity and a willingness to explore and extend their learning (NRC, 2001). However, the domain of productive dispositions has received little attention, as it is not as easily accessible and measurable as other domains of learning (Grady, 2016). Furthermore, common pedagogical practices employed in mathematics provide less opportunities to develop productive dispositions (Ally & Christiansen, 2013).

Rich tasks have been identified as a valuable approach for designing mathematics activities that address students' learning needs and interests (Bobis et al., 2021). Rich tasks refer to authentic learning opportunities that are accessible to a range of students, promote productive struggle and are characterised by real-life applications of learning (Piggott, 2018; Sheffield, 2003). Several characteristics and learner outcomes of rich tasks have been identified, e.g. rich tasks have a focus on inquiry, improve questioning, promote reasoning and problem solving, encourage collaboration and provide opportunities for critical thinking (Piggott, 2018). Such tasks can provide a useful scaffolding tool to enhance student learning in cognitive, behavioural and affective domains. Careful design and implementation of rich tasks in mathematics can enrich students' learning experiences and promote positive attitudes towards the subject (Schoenfeld, 2013). With an aim to support students' mathematics learning and foster their productive dispositions in mathematics, this research took place as a task-based intervention where rich tasks were designed and implemented with students aged 11-12 years. The study was conducted over a five-month period, involving 113 sixth-class students in a primary school in Ireland. Ten rich tasks were designed and implemented that were aligned with the primary mathematics curriculum (NCCA, 2022) and supported the development of students' productive dispositions in mathematics. Semi-structured interviews were carried out with three primary teachers that collaborated with the researchers on facilitating the implementation of these tasks with the students in their classes.

This study discusses the teachers' experiences of implementing rich tasks in mathematics and their perceptions of the influence of these tasks on their students' learning and addresses the research question: *What are teachers' experiences and perceptions of the influence of rich tasks on student learning in mathematics at primary level?*

Methodology

The task design principles for this study were developed through an iterative process using an Educational Design Research (EDR) approach. EDR is a genre of research in which the iterative development of solutions to practical and complex problems provides the context for both empirical investigation and theoretical understanding (McKenney & Reeves, 2013). The process of developing task design principles matured through an amalgamation of research and practice (McKenney & Reeves, 2013). This amalgamation took place in three iterative cycles. The initial set of these design principles (Iteration 1) was developed from a synthesis of research literature on primary-secondary transition in mathematics, published during the period 1990-2020 (Kaur et al., 2022) and a review of literature around rich tasks. These design principles were refined (Iteration 2) using the data generated from a teacher professional learning programme that aimed at supporting mathematics learning across transitions (designed as part of the broader study).

Over the course of the workshops, teachers participated in discussions about the richness of tasks, engaged in co-designing rich tasks and explored the opportunities provided by a given task to enhance student learning. The continuum of richness framework drawn from Johnston-Wilder and Mason (2004) provided the basis for critical appraisal of the tasks. This framework outlines eight distinct dimensions of rich tasks where each dimension can be considered as a spectrum, ranging from routine or closed to more open or rich tasks e.g., tangential to essential, passive to active and closed to open. A further refinement of the iteration 2 led to the iteration 3 of task design principles, which was carried out through a) an analysis of the tasks implemented by participating teachers with their students; b) teachers' reflections on their experiences of implementing these tasks and c) the findings from the influence of these tasks on students' learning. The resulting design principles are presented in figure 1.

Figure 1

Task Design Principles

Context	Well-pitched	Meaningful and relevant	Students' personal experiences	Multiple representations
Coherence	Connection between math domains	Connection to prior knowledge	Connection with informal experiences	Sequencing activities within task
Student agency	Equitable access	Autonomy	Equity of opportunities	Accountability
Reflection	Generating discussions	Critique and argumentation	Justification	Addressing misconceptions
Adaptability	Teacher agency	Modes of presentation	Extension activities	Differentiation

An Example of a Rich Task Implemented

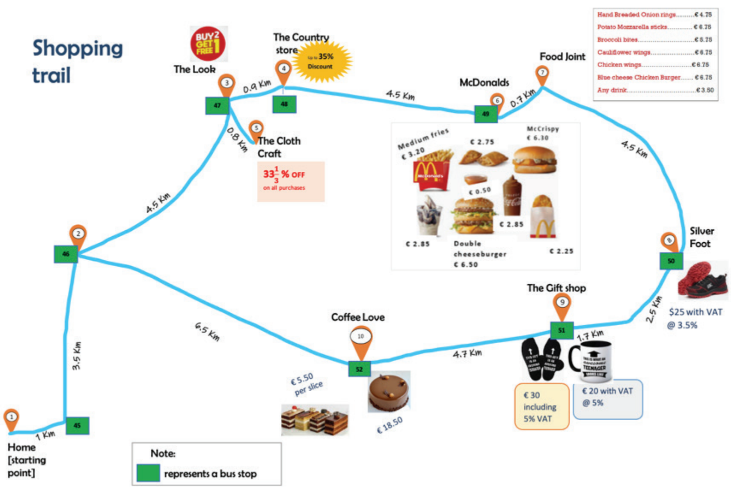
Students are given a route map for a shopping trail and a to-do list of several jobs (figure 2). For each job, students are required to decide the best shopping option and justify their decisions with reasoning. The task concludes with a whole class discussion on whether it is realistic to be home by the specified time, having completed the list of jobs.

This task is based on a real-life context, providing opportunities for connecting new knowledge with existing prior knowledge and informal experiences. Many students used their personal experience of travelling in a bus to estimate the time taken by the bus to cover a given distance. The element of reflection embedded within the task helped address student misconceptions. Many students changed their decisions after listening to others' ideas and reasoning. The discussion also allowed the teacher to identify the challenges that students had in completing the task and differentiate the task accordingly.

To evaluate the impact of the intervention, students' learning experiences were collected using a variety of methods such as class observations, written feedback, whole class discussions, task reflection forms, and student focus group interviews. The focus of this study is to investigate teachers' views on their experiences of implementing rich tasks and their observed influence on student learning. Semi-structured interviews with three teachers were conducted after the intervention. Data gathered from these interviews was analysed using a deductive thematic analysis.

Figure 2

Example of a Task from the Study



Findings

Influence of Rich Tasks in Fostering Productive Dispositions

Teachers appreciated student-led active learning and play-based techniques used in the tasks designed in this study. One of the observed influences of students' engagement in these tasks is a change in their outlook towards maths. In particular, for students who generally needed extra support in maths and had negative dispositions towards the subject, an increase in the level of confidence was reported.

> 'I think the most positive thing for me as a teacher that's come from all of this is the level of confidence that they now have in their own ability and their eagerness to actually participate'. (Teacher A)

> 'I found that it got the children thinking in a way that they wouldn't have done before, and they engaged in topics that they wouldn't like'. (Teacher B)

One teacher shared an example of two students in their class who received maths support and had a low self-esteem regarding their abilities in maths. Engaging in tasks that were open-ended and had multiple ways of engaging, helped them believe in their own abilities rather than comparing themselves to others in the class.

> 'It would literally be like you could see their insides churning and that doesn't exist at the moment because they have a little more of a positive outlook. They also realised, you know what, I might want to do the same as everyone else but in my own way, and I'm okay with that'. (Teacher A)

Another case of a student was highlighted by this teacher where the student had received maths learning support since junior infants and was so afraid of maths that she had just assumed that she was going to get it wrong. A significant increase in this students' confidence and self-esteem as a result of this intervention, was reported by the teacher.

> ‘With her up until this study, I would have to go down and work one to one with her for everything, even for the most simple task. And I do think that possibly maybe that's kind of a question for the level of intervention that young to do that to a child that no, it's probably not ideal. But she, I mean, she turned around and said ‘I'm not that bad in maths after all’. (Teacher A)

Long Lasting Effect of Students’ Engagement

Although the intervention was for block periods during the five-month implementation of the study, the same kind of engagement was observed by teachers in their maths classes that were not part of the intervention. For instance, one teacher shared that while teaching on the topic of circles, students were asked to find the circumference of the circle without giving them the solution directly, which would normally be the case.

> ‘So I gave them wool. I gave them, you know, measuring equipment, all that kind of stuff… they had lots of different strategies.. So they actually were talking through many different ways of figuring it out and knowing well that there was something that they were slightly missing, but they had multiple ways of getting really close to the Circumference, which was great’. (Teacher A)

The teacher shared that before participation in the study, their students would have just reverted to looking for a direct formula or solution, but now they were more open to discovering the circumference of the given circle by trying different strategies and discussing those strategies among themselves. The element of reflection embedded within the tasks through maths talk, critiquing and listening to others’ ideas, justifying, reasoning and convincing others with constructive arguments through respectful dialogue and conversations, facilitated many students to open up without the fear of getting wrong.

> ‘Just the way they could debate with each other that kind of stood out as well, like they were quite confident in challenging each other’. (Teacher C)

A similar effect on engagement in learning was observed in other subject areas. ‘*It's that lasting influence over that way of thinking is going across the board*’ (Teacher A). The teacher shared that those who would not usually be interested in writing or reading started to engage in conversations during their English classes.

> ‘It's very similar to the back and forth that they have [with] their partners in maths class. So it's kind of like they've been given that tool to hold, like, respectful dialogue about a topic and it's going into other areas as well, so I think it's a huge influence’. (Teacher A)

An interesting finding that emerged from teachers’ observation is that students who the teachers would describe as previously struggling with maths, enjoyed the tasks more than those who they would describe as the higher achievers. The fact that these tasks did not have one correct answer and that a problem can be approached in multiple ways, motivated them to participate without any reservations or fears. The higher achievers (as described by the teacher), on the other hand, found it hard to engage initially as they were used to giving one concrete answer and therefore struggled more to justify and explain their solution strategies.

> 'That was an eye opener to me, because two [students] in particular would be very strong at maths and would have all the questions, the competitions, any problems like that they would know how to do it. But that did, I suppose, stood out to me that they found it more difficult than the children of lower ability'. (Teacher B)

Positive Impact on Teachers' Professional Learning

The teachers involved in the study expressed that their involvement in this study has changed the way they look at maths and their teaching practices.

> 'I've questioned the way I have taught things in the past and how I could teach it differently and using the tasks and using your approach'. (Teacher B)

Teachers shared that the task-based intervention proved highly beneficial not only for their students but also for them as it offered professional learning for them in a practical context.

> 'I just think these tasks - the rich tasks are a different level altogether and they're very beneficial once you know we go through exactly what you're targeting with what you want to achieve with it'. (Teacher B)

> 'I'm always focusing on -It's not the end product like it's not the answer, it's the process, and I think these sort of tasks highlight that further'. (Teacher C)

The cross-curricular aspect of tasks highlighted to them the significance of application of such tasks to all subject areas and to connect student learning meaningfully. Going forward, they plan to incorporate similar tasks in their classroom teaching building on the learnings from this study:

> 'We're always trying to elicit prior knowledge through conversations and stuff, but that's one child normally speaking and all of us listening, whereas the way that [this intervention] lessons were structured was that everyone was actively engaged the whole time'. (Teacher A)

Teachers' Reflections on the Design Features of Tasks Implemented

When discussing the tasks that were implemented in their classroom, and tasks more generally, teachers appreciated the elements of group work, maths talk, reflection, connection with other maths domains, open-endedness and the cross-curricular nature of the tasks.

> 'I think it has the ability to target each different group in a classroom differently and they'll get different outcomes of it….the lower tiers will get the confidence that there's no right or wrong answer, But the higher achievers will get the frustration, but will actually start to think. well, you know, what's the different ways of, you know, applying this'. (Teacher B)

The teachers commented that with opportunities for student-led learning provided by the tasks, students took more responsibility for their work than they would usually do and were eager to explore the ideas they had. In particular, the feature of maths talk was something that stood out for all the teachers and students. This is something teachers had not used earlier in their maths teaching. They also appreciated the element of reflection embedded within the tasks and the opportunities created by tasks for critique and discussion through maths talk and debates. Overall, the teachers reflected positively on their experiences of implementing rich tasks in this study and the influence of the intervention on their students' learning. Teachers showed a willingness to continue with the use of those and similar tasks in their teaching in future.

Discussion and Conclusion

This study is an important contribution to the existing body of literature around rich tasks. Findings from teachers' interviews provide first-hand evidence of a positive impact of rich tasks on primary students' mathematics learning. Teachers reported that the use of rich tasks provided opportunities for students to develop their mathematical thinking, as well as fostering positive student attitudes and engagement in mathematics. Increased student confidence and self-esteem was also reported by the teachers. These findings are consistent with existing literature on rich tasks suggesting that rich tasks can serve as a valuable tool to enhance primary students' learning experiences in mathematics and promote productive dispositions in mathematics (Bobis et al., 2021; Schoenfeld, 2013).

This intervention also shows promise as a model of teacher professional learning, providing opportunities for teachers to reflect on their teaching practices and to inquire how and what they can do differently going forward. Teachers' experiences of implementing rich tasks based on the design principles presented in this paper, are extremely positive. The teachers developed their understanding of the role of rich tasks in mathematics and the opportunities they can provide for active student learning. A key concern identified by teachers is the time required to embed these tasks within the prescribed time-bound curriculum. Teachers commented on how the curriculum is dictated by text-book companies and the pressure to plan their teaching lessons based on the sequencing of topics according to the prescribed workbooks. These findings are consistent with how such challenges limit the time for incorporating student-led learning in their normal classroom lessons (Ingram et al., 2020). However, despite these concerns, the teachers in this study showed a willingness to incorporate rich tasks in their regular teaching practices to improve student learning in mathematics. The teachers also reflected on the importance of a learning environment in which a task is conducted. In fact, what makes a task 'rich' is the learning environment that offers opportunities for student-led learning and where students can construct their own knowledge (Piggott, 2018). Teachers shared that the tasks in this study offered a positive learning environment that helped promote students' productive dispositions towards mathematics learning.

To conclude, this study offers great potential for supporting student learning in mathematics. More specifically, the study demonstrates a shift in the belief system of students from a fixed mind-set to a growth mind-set (Boaler, 2015) and for teachers to see the value of student-led, active and inquiry-based learning through rich tasks.

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Challenges of ChatGPT for Mathematics Learning and Teaching

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We discuss ChatGPT (OpenAI, 2022), and its potential uses in mathematics education. ChatGPT can process and respond to natural language inputs with outputs that are human-like. It can output translations, text generation, conversations, and question answering, write code, perform statistical analysis as well as write exam items and answers, poems, and short stories (Topsakal & Topsakal, 2022; Zhai, 2023). Yet, it confidently "hallucinates" responses, often with made up "facts" and fabricated references. It also can repeat and mimic human falsehoods and other misinformation from its database (Lin et al., 2022). Critically, ChatGPT may generate text records that are offensive, sexist, racist or that teach immoral perspectives (Krügel et al., 2023). Its performance on certain standardized tests is impressive. For use in K-12 science, technology, engineering and mathematics (STEM) education, however, low accuracy for STEM content was found for GPT-3 over 57 tasks (e.g., mathematics, high school and undergraduate topics like computer science, high school biology, and other areas such as moral disputes, and econometrics) (Hendrycks et al., 2022). How should K-12 mathematics educators understand ChatGPT: a tool that produces – unpredictably and without assessments of quality– (a) dependable, (b) questionable, or (c) false and toxic output, or these in combination?

Keywords: ChatGPT, mathematics learning, teaching

Chatbots and ChatGPT

AI and other chatbots are based on large language models (LLMs), a technology that is trained on massive amounts of text from the internet. Chatbots have been used with teachers in language education (Ji et al., 2023), including chatbots for teacher education, yet rarely used in mathematics education (Belda-Medina et al., 2022). Indeed, in the very recent past, LLMs were available only to skilled researchers (e.g., Rae et al., 2022). However, on November 30, 2022, the company OpenAI released to the public an easy-to-use chatbot for GPT-3, called ChatGPT (OpenAI, 2022). For a comprehensive review of the development of ChatGPT from GPT-3, see Lovegrove (n.d.) and Kasneci et al. (2023).

The uptake of ChatGPT, globally, is phenomenal. By January 2023, 100 million downloads of ChatGPT had occurred. Since its release, ChatGPT has evolved to ChatGPTPlus, which had more features than the free version. In March, 15, 2023, OpenAI released the latest version of their LLM (GPT-4), which now is the model underlying ChatGPT-Plus (OpenAI, 2023a).

ChatGPT can process and respond to natural language inputs with outputs that are human-like. It can output translations, text generation, conversations, and question answering, write code, and perform statistical analysis as well as write exam items and answers, poems, and short stories (Topsakal & Topsakal, 2022; Zhai, 2023). Some predict that ChatGPT will soon "design experiments, write and complete manuscripts, conduct peer review and support editorial decisions to accept or reject manuscripts" (van Dis et al., 2023, p. 224).

ChatGPT: A Disruptive Chatbot

ChatGPT is a double-edged tool, however. The LLM on which ChatGPT is trained is proprietary, so its shortcomings and biases are difficult to document (Brundage et al., 2018; van Dis et al., 2023). For those wishing to understand what may be happening in the ChatGPT black box, see an analysis of the behavior of open LLM such as LLaMA (Touvron et al., n.d.).

Since the GPT models are trained on the internet, there are significant biases and other flaws in the LLM. Detoxifying LLM is a challenging task. First, humans have to review and reject deeply offensive output. The subsequent stage of "fine-tuning" the chatbot requires reinforcement learning with human feedback (RLHF). During RLHF, the machine "learns" what type of output is rewarded by human training. RLHF may itself introduce additional errors given the cognitive biases and limitations of the trainers (Azaria, n.d; Tabassi, 2023). For papers on RLHF and a range of topics from misinformation, AI value-alignment, bias, immorality, and cross-cultural dialogue, see Glaese et al. (2022). OpenAI (2023b) remains openly concerned about its chatbot's toxic nature and uses.

Most importantly, because the LLM has no "model of the world" (Hendrycks et al. 2022), it may display falsehoods in addition to toxic output. ChatGPT may draw on and generate untruthful, non-representative or biased information, write false reports, manufacture plausible sounding but fictitious concepts, and report citations that do not exist or are not germane to the argument in its reports (Cabanac et al., 2021; Lehnert, 2023; van Dis et al., 2023). A comprehensive review of ChatGPT in education may be found in Maynard (n.d,).

Teacher and Student Use of ChatGPT

In summary, ChatGPT is a tool that produces – unpredictably and without assessments of quality– (a) dependable, (b) questionable, or (c) false and toxic output, or these in combination. Despite all the concerns that have arisen about ChatGPT, across the US, the Walton Family Foundation (2023) reported use of ChatGPT by teachers and students.

When ChatGPT generates ostensibly dependable text output, the task for the student and teacher will to be to establish that the output actually is reliable and trustworthy. Since ChatGPT has no comprehension filter for grade-appropriate output, the student may be unable to process this text and understand what it is communicating, leading to confusion. Unlike for a vetted textbook, the additional cost of checking ChatGPT output may be significant for the student and the teacher. If the text is accurate, and comprehensible, the student and teacher may enter into productive conversations about it. However, students and teachers will still need to worry about document provenance (e.g., plagiarism, taking credit for machine-generated text). Students need to be able to cite sources and submit only their own work (see International Baccalaureate, 2023).

However, the LLM also has "holes": areas where it does not have adequate or authoritative text repositories on which to draw (it does not have access to data prior to 2021, and is unlikely to have had access to science articles behind paywalls). Indeed, the accuracy rates for GPT-3 on mathematics (Cobbe et al., 2021; Hendrycks, et al., 2022) and science content is low (Hendrycks et al., 2022; Wei et al., 2022). Critically, even when ChatGPT has "insufficient information," it confidently "hallucinates" a plausible response, often with made up "facts" and fabricated references. It also can repeat and mimic human falsehoods and other misinformation from its database (Lin et al., 2022).

When faced with misinformation, adults, even scientific reviewers (van Dis et al., 2023) and children can show exaggerated ideas about their own expertise, and may blindly trust the authority of sources due to the so-called Dunning-Kruger effect (Kruger & Dunning, 1999). A student working alone may not know that the output is scientifically suspect, including incorrect propositions, reasoning or "facts."

The task for the student and teacher in this case is to learn how to read, understand, and to reject this text output. Importantly, the teacher and the student must not pass this information along to others or store it in a digital store where its error may be compounded. For the teacher, there is the risk of grading the student's paper as acceptable, missing material that is "hallucinated." Here, the teacher and student must engage in critical thinking (e.g., Ennis, 2018).

Finally, ChatGPT may generate text records that are offensive, sexist, racist or that teach immoral perspectives (Krügel et al., 2023). Some researchers believe that chatbots can be designed to be more prosocial (Ganguli et al., 2023). However, OpenAI's moderation tools to curb offensive output for ChatGPT can be side-stepped (Hacker et al., 2023).

For toxic cases, the task for the student and teacher, and for school administration will be to establish and reinforce a STEM education milieu in which scientific integrity is upheld and societal core values such as fairness, diversity and inclusion are sustained.

Students are more likely to cheat when the risk of detection is assumed to be low. While cheating on high stakes tests is not new (Lancaster, 2020), tools like ChatGPT provide easy access, and low-cost means of violating academic standards. For complex assignments, which portion(s) are due to ChatGPT is difficult to ascertain. The problem of academic fraud not only impacts K-12 education, but continues throughout one's career (The Economist, 2023).

Developmental Issues for Children

Children move through different developmental stages. Research is needed on how tools like ChatGPT will interact with human cognitive capabilities (Markauskaite et al., 2022). For example, what are the implications for the development of executive control, grit and other cognitive and affective states when reasoning is handed off to a tool like ChatGPT (e.g., Diamond & Ling, 2016)? New models of how children learn, preK-12, must be developed to account for the greater integration of AI into education (Poquet et al., 2021).

Some researchers have begun to examine how children self-regulate their learning with AI (Järvelä et al., 2020; Molenaar, 2022). Will an over-reliance on ChatGPT for reports and homework lead to passive and shallow learning (e.g., failing to learn arithmetic in lieu of using pocket calculators)? Or will the ease of ChatGPT-generated text output deprive a child of the need to develop persistence and grit, especially for rigorous courses that need to be taken over a number of years (e.g., Ashford et al., 2016)?

In short, the implications for K-12 mathematics education are being explored, like the AI program for teachers and students, Khanmigo (Kahn Academy, n.d.). For general users, the use of in-context prompting to get the LLM to give more useful responses has value – an approach known as "prompt engineering" (Aman's AI Journal, n.d.). The goal of this paper is to foster critical dialogue on this new force in education.

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Changing Teaching Practice to Support Spatial Skills Development in Primary School Children in Ireland

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Spatial skills are strongly correlated with achievement in Science, Technology, Engineering and Mathematics (STEM) and are also intrinsic in other curricular areas. Emerging research suggests that interventions aimed at building students' spatial skills may yield significant impacts on learning in educational settings (Gagnier & Fisher, 2020). Yet, despite this the intentional practice of teaching spatial thinking skills is largely absent in school curriculums. The aim of this research was to explore changing teaching practice to support the spatial skills development of primary school children in Ireland. It also aimed to examine how to leverage aspects of the Knowledge Transfer Framework (Gagnier & Fisher, 2020) to help infuse spatial skills into lessons, changed and informed practice. Key spatial enhancements were integrated into three lessons across the curriculum. An action research method focusing on a group of six primary school students (age 7-8 years) within the whole class setting was used. The findings of this research conclude that children benefit and respond positively to the spatially-enhanced lessons and it is possible to infuse spatial enhancements into lessons whilst meeting the objectives of the Irish Primary School Curriculum.

Keywords: Spatial skills, enhancements, integration, cross-curricular

Introduction

Spatial skills help individuals read maps, interpret charts, assemble furniture, visualise things we cannot see and aid us to represent these things in the mind, through drawing, construction, gesture and language. These foundational skills that start developing in early life have been found to contribute to children's learning and success in STEM and other areas such as the arts and physical education. Yet, despite this the intentional practice of teaching spatial thinking skills is largely absent in school curricula. The lack of focus in teaching and curricula in fostering spatial skills means we risk losing a significant cohort of potentially talented individuals (OECD, 2017). Spatial skills are associated with STEM careers and explicit attention to assessing and developing spatial skills may support teachers in identifying children with high spatial skills who might not be identified on other traditional achievement measures. Supporting spatial skills development in schools may also help under-represented groups gain access to STEM careers.

Literature

Existing literature, including meta-analyses of longitudinal studies over the last 50 years evidences the role and importance of spatial skills development in STEM and lifelong career achievement (Lowrie & Jorgensen, 2017). While early studies in this area sometimes focused on gender or 'natural ability' it is generally accepted that spatial skills are malleable through appropriate intentional interventions and experience (Cheng & Mix, 2014).

While some differences in achievement are noted across gender it appears likely these arise due to social and educational factors and will require a system-wide support structure from educational curriculum and practitioners, parents and other individual factor levels to combat stereotyping and cultural factors to promote and foster spatial skills and STEM interest in girls (Andrews et al., 2016; Clerkin & Perkins, 2019). Literature indicates that individuals from a lower socioeconomic background tend to fare less well on spatial ability skills and academic achievement but with appropriate interventions to support their development, these skills may partially or fully improve (Lowrie & Jorgensen, 2017).

The difficulties of incorporating spatial skills development in an already crowded curriculum and the ensuing call from scientists to spatialise the curriculum have been noted in a range of international and national literature (Irish Primary Principals Network, 2017; National Research Council Of The National Academies, 2005; NCCA, 2010; Ministry of Education Ontario, 2014; OECD, 2017). The Knowledge Transfer Framework (Gagnier & Fisher, 2020) is an example of how to translate scientific knowledge into educational curriculum and practice. While there is little explicit reference to spatial skills in Irish policy documents, proposed changes to curriculum align with the recommendations arising from international literature. Ireland has already taken steps to increase the focus on spatial skills development in children through the revision of the Draft Primary Mathematics Curriculum (NCCA, 2022) and the augmentation of the Shape and Space Strand and the integrated approach to teaching and learning in the proposed Draft Primary Curriculum (NCCA, 2022).

Methodology

The aim of this study was to explore how spatial enhancements might be incorporated into the Irish primary school curriculum subjects of Social, Environmental and Science Education (SESE), Mathematics and Art lessons and to investigate children's responses to spatially-enhanced lessons using an integrated cross-curricular approach. The research entailed observation of a focus group of six students from 2nd class aged 7-8 years, of mixed ability and gender. It leveraged aspects of the Knowledge Transfer Framework (KTF) developed by Gagnier and Fisher (2020) to teach lessons using five key spatial enhancement skills of Visualization Instruction, Sketching, Spatial Comparison, Spatial Language and Gesture.

The KTF draws upon research from cognitive, developmental, educational and implementation sciences to guide the infusion of spatial-thinking into a curriculum intervention. The framework offers practical advice on how to purposefully plan and infuse these spatial thinking skills into curriculum areas for both teacher and students to apply in teaching and learning. Leveraging and adapting the KTF with the Irish Primary School Curriculum learning objectives in mind, the lessons were purposefully designed using a cross-curricular integrated approach on the theme of Explorers to include spatial enhancements scaffolded by the teacher and subsequent spatially-enhanced student activities for children to respond and actively engage in the lessons. Figure 1 below provides an overview of these lessons.

Figure 1

Overview of Lessons

Lessons	Key Activities	Cross Curricular Integration
Lesson 1	Christopher Columbus Story: Students explore and develop the language of spatial properties and relations. Students draw map of how mid-1400s Europeans would have imagined the world to look like based on spatial properties as described by teacher. Students compare to modern-day map using a Venn Diagram to capture spatial (and other) similarities and differences. Students identify where Spain is on the map and in groups trace the route creating an old-world map.	**History**: Story, Change and Continuity **Strand Units:** Stories, Continuity and Change in the Local environment **Geography:** Natural Environments **Strand Units:** Planet Earth in space **Mathematics:** Data **Strand Units:** Representing and interpreting data **Art:** Paint and colour, Drawing **Strand Units:** Painting, Making Drawings
Lesson 2	Students explore and develop spatial awareness of cardinal directions and map work skills through pair/group work activities. Students introduced to the compass, map skills following and giving direction instructions and drawing a simple classroom map using keys/legends.	**Geography**: Natural Environments, Human Environments **Strand Units:** Planet Earth in space, Living in the Local Community **Mathematics**: Shape **Strand Units:** Spatial Awareness **History**: **Strand**: Story, Change and Continuity **Strand Units:** Stories, Continuity and change in the local environment
Lesson 3	Students complete a step-by-step activity that leads students through the process of building a boat. Collaboratively work and brainstorm the design/sketch/build of the boat, within a limited budget, Students use rich spatial language and demonstrate and explore their spatial skills in building the boat.	**Science**: **Strand** - Energy and forces, Materials **Strand Units:** Forces, Properties and characteristics of materials **Maths**: **Strand** - Measures **Strand Units:** Money, Capacity, Weight **Art:** **Strand** - Constructions **Strand Units:** Making constructions, Looking and responding

An action research method was used. This involved using participant observation to explore the children's response to spatially-enhanced lessons. McNiff (2013) states action research is always about improving learning, and improving is always to do with education and personal and professional growth. As such many people regard action research as a very powerful form of educational research. Data that was collected for this study included observation field notes, audio recordings of children's discussions, teacher lesson plans, the teacher's reflective journal, samples and photos of the students' work, and students' reflective learning logs. The data including transcriptions and pupil's sample work were coded using a hybrid of deductive and inductive thematic analysis codes (Fereday and Muir-Cochrane, 2006). A hybrid process of deductive and inductive thematic analysis used to interpret themes from the raw data.

Findings and Analysis

Whilst these lessons were focused on the curriculum learning outcomes of History, Geography, Science, Mathematics and Visual Arts, it is clear that the intentional teaching of spatial skills using the eight spatial enhancements proposed by the KTF (Gagnier & Fisher 2020), facilitated children's engagement in activities to meet these learning outcomes. Of paramount importance to all of the lessons is the use of spatial language as it underpins the learning throughout whether it is used on its own or in combination with another spatial activity such as gesture, sketching, comparing or visualisation. The findings of this study were amalgamated into the following themes: Responses to the Spatial Enhancements, Importance of Spatial Language, Collaborative Learning and Student Motivation. However, for the purposes of this paper, the theme of Responses to the Spatial Enhancements will be examined.

Responses to the Spatial Enhancements

As outlined, the five enhancements identified by Gagnier and Fisher (2020) include: Visualisation Instruction, Sketching, Spatial Comparison, Spatial Language and Gesture. Student activities were created that combined these various spatial enhancements and were incorporated into the lessons in fun and engaging ways. In this section, I summarise the findings of the responses categorising six of the most frequently used common student activities (*Say and Display It!*, *Diagram and Describe It!, Observe, Draw and Describe It!, Predict, Draw and Describe It!, Compare and Describe It!, Graph It!*) according to the five spatial enhancements.

Visualisation Instruction. Visualisation Instruction involves helping children to read and understand existing external representations as well as creating external representations that conveys spatial information such as maps, graphs and diagrams. Having read the story of Columbus, in lesson two the participants were tasked with creating a map depicting the route Columbus and his explorers would have taken to cross from Spain to the Caribbean. In this student activity the participants used the *Diagram and Describe It!* student activity to trace the route identifying cut outs of Europe and America, positioning them on the map and then drawing Columbus's boats showing the directionality of the route. Here to understand the process of navigation the children had to execute skill in self-location placing the three ships of the Nina, the Pinta and the Santa Maria in the middle of the Atlantic Ocean and drawing the bow of the ships in the westward direction of the Caribbean. This process effectively required the participants to initially create the map in their head of that journey and then represent it in drawing using the spatial skills of self-location and place recognition for navigational map reading (Lobben, 2007). The evidence shows this posed little difficulty for the participants although the order of the ships in the representational drawing needed to be corrected with the Santa Maria boat leading the ships to arrive first at the Caribbean.

Sketching. Research has shown sketching can facilitate reasoning and learning about spatial properties (Ainsworth et al., 2011, Nic Mhuirí, 2020). There were common student activities that included this spatial support in combination with other spatial skill-enhancements. *Diagram and Describe It!* discussed above*, Observe, Draw and Describe It!* and *Predict, Draw and Describe It!*

The student activity *Observe, Draw and Describe It!* included the sketching enhancement in combination with the use of spatial language. In lesson 2 the participants were tasked with helping Columbus discover the new land and were given a map of an island that contained symbols or keys. The participants had to follow the written instructions to locate and draw each of the symbols relative to the location of the campsite drawn on the map. It was evident from the data all participants were able to draw and describe the location of the keys on the map with ease. However, the extension activity to Map Your Own Classroom proved challenging for some participants. Data analysis showed that some children had difficulties with the concept of using keys as symbols and understanding scale and perspective. These difficulties were noted in previous research of Uttal and Sheehan (2014). A recommendation was made to provide the keys/symbols to use for the classroom map initially so that the participants could focus on the relational spatial skill as to where to draw the key on the classroom map first and then scaffold the learning until their level of understanding of the spatial relations of scales, overhead view or perspective and symbolic use of keys is developed. In lesson 3 sketching was also used to draw a picture of their boat and label it appropriately. Here children demonstrated a good sense of space and geometry at sketching the finished boat and was comparable to the initial sketched design.

The student activity *Predict, Draw and Describe It!* includes the spatial- enhancement of sketching in combination with language. In lesson 1 the teacher described how Europeans in the mid-1400s would have imagined the world to look like using spatial language. The participants were then tasked to draw a map the way the mid-1400s Europeans would have viewed the world based on the descriptive spatial language as described by the teacher. It necessitated the participants visualising in the mind's eye according to the description and then representing it through drawing. For some participants this was a challenging task as their only concept of what the people thought the world looked like was that of the modern map displayed on the SmartScreen. Through drawing rather than writing, the children were given an opportunity to build and manipulate mental representations from a different perspective and develop their spatial thinking and reasoning skills further. Thus, they demonstrated understanding aligning with the visuo-spatial demands of science learning (Ainsworth et al., 2011; NicMhuirí, 2020).

Spatial Comparison. Comparison is particularly useful to allow learners to notice critical spatial differences between two entities (Gagnier & Fisher, 2020; Newcombe, 2010). Analogy facilitates learning through the process of comparison identifying similarities and differences between two entities. The 'Compare and Describe It!' student activity incorporates this comparison enhancement in combination with use of spatial language. As referred to earlier the student activity Graph It! was preceded by a scaffolded discussion by the teacher and participants to identify spatial differences between the old-world map and the modern map in lesson 1 thus sharpening the children's focus on comparing spatial properties. Most participants were able to identify key similarities and differences although it was noted that this spatial skill needed to be practised more with the children. This spatial enhancement was also used in lesson 2 as part of the Map Your Own Classroom student activity as described earlier in the *Observe, Draw and Describe It!* The participants swapped their maps and had to follow the direction of the keys drawn on his/her partner's map and compare it by moving in the direction the map suggested. This was challenging for half of the participants as the success of this task was dependent on the children's spatial ability to draw the map of their classroom accurately as discussed earlier. In addition, comparing the map with the physical classroom revealed that map reading requires individuals to be able relate to where you are in the real world and compare to where you are on the map when reading a map. It also involves map rotation either mentally rotating a map as you make successive turns or physically turning the map to assess direction (Lobben, 2007). Some children were observed to turn their map to go in the direction they needed to go while others displayed the ability to follow their map by being able to self-locate.

Spatial Language. Like all language learning, spatial language can influence children's spatial cognition and if developed from an early age can equip children to represent and reason about the world. This role of talk is essential in mathematical thinking and problem solving (Nic Mhuirí, 2020). Newcombe (2010) states that children that understand spatial words understand spatial relation better when it is given a name and consequently perform better on spatial tasks. As evidenced throughout this study the use of spatial language has been a fundamental cognitive tool in completing all student activities in combination with other spatial enhancements. The one exception was in lesson 3 when having completed the building of their boat and testing their boat out in the water, the participants were tasked in step 5 of the boat building activity to reflect on the success of their boat *(Describe It!).* Prior to documenting and summarising the group's reflections in writing all participants engaged in reasoning why and why not the boat that they built was successful. Although the participants did use some spatial language using this spatial enhancement, the children's personal construction of meaning and understanding of the scientific reasons why the boat sank became most evident. However, it was noted that this assumes children have sufficient language to reason and have a good level of the English language, in particular those with English as an Additional Language (EAL) needs. In the latter case this evaluation was only made possible by the active involvement of the children in the practical and creative design and making process.

Gesture. The sole spatial enhancement that was consistently used across all lessons was the Say and Display It! spatial enhancement. This enhancement as previously discussed combines the use of language and gesture for children to describe and show their observations, experience and prediction of an activity used in the curriculum content. The evidence collected shows this is consistent with the findings of Newcombe (2010) who states children learn better when they use gesture with speech as they explain a problem. This is in contrast to just using speech alone. For example, in lesson 2, the participants had the opportunity to give evidence of their thinking by linking spatial language and representation through gesture when learning about cardinal directions. Gesture combined with spatial language provides deeper understanding and opportunity for children to develop spatial thinking skills in a meaningful way.

Significantly, the data showed that use of this spatial enhancement is useful for children in the early years who are still developing literacy skills and in particular children with EAL needs who may not have acquired sufficient language yet. This predicates with Goldin-Meadow and Alibali (2013) who argue that the act of gesturing plays a causal role in language learning by allowing children the opportunity to practise and communicate ideas they are not yet able to express in speech. Ng and Sinclair (2013) highlight the interallied role that gesture and language plays in developing mathematical thinking skills in children. One participant with EAL needs, lacked the language skills but was able to participate in the collaborative activity in lesson 3 nonetheless dually building his spatial and language skills.

Concluding Remarks

A fundamental aim of this study was to advance the research into how to spatialise the curriculum by attempting to do so with the Irish Primary School Curriculum. This was inspired by the OECD's call to governments and system leaders to recognise that spatial skills enrich the traditional educational focus on developing literacy and numeracy skills and to include this cognitive domain which is relevant to achievement in STEM and other curricular areas such as SESE and the Arts.

The findings identified opportunities for further skills development for participants particularly in the area of spatial enhancement of comparative analysis and description. From a teacher perspective this could be supported by facilitating additional and specifically focused practice lessons on this spatial skill. Factors found to challenge implementation of these lessons included supporting children with EAL needs and class size. Continued practice of collaborative learning was identified as an opportunity for further development as is promoted by the Irish Primary School Curriculum. As the teacher assumed the role as facilitator, the participants were motivated to manage their own learning through active open-ended discovery learning.

It is noted there have been significant changes to incorporate more spatial skills development in the Draft Primary Mathematics Curriculum (NCCA, 2022). The strand of Shape and Space has been expanded with additional learning outcomes and most notably now includes the exploration and visualisation of the effects of transformation on shapes. This new emphasis in mathematics is welcome, however, it is the author's opinion that there is an opportunity to develop spatial skills in a more holistic and integrated way. This study has shown how spatial skills can be infused in the non-STEM subject areas using the eight student spatially-enhanced activities in fun and engaging ways in lessons. These spatially-enhanced student activities offer an alternative method to achieve the subject area learning outcome whilst at the same time serve to develop children's spatial skills. From experience of this study, I would argue these eight student spatially-enhanced activities can easily be put into practice and used as an additional suite of activities to design and enact teaching that will meet that objective using appropriate engaging pedagogy. This is in line with the aims of the Primary School Curriculum which supports an integrated approach to teaching and learning with links and integration across subject areas to optimise curriculum integration whilst supporting progression in learning (NCCA, 2020). Considering this and in addition, in my experience, designing these lessons presented little difficulty in accomplishing this, as the Irish primary school curriculum already encourages and promotes active open-ended discovery learning.

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Currachs and Ethnomathematics

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Currachs are traditional boats, made of thin strips of wood, covered in cloth, but their use has declined in Ireland since the 19th century. When traditional practices need reviving or maintaining, ethnomathematics is often promoted as a way to contribute to this in mathematics classrooms. In this paper, we use the cultural symmetry model to examine opportunities and issues that could arise when ethnomathematics is used to explore the building of currachs. We investigate this as a research project, with potential links to what could occur in school. This model enabled us to discuss how the currachs used locally available materials to fulfil specific purposes and to consider how describing them as mathematics could add to understanding their construction. The model also provided opportunities to discuss how the design of currachs and why they are no longer being built in many places can be connected to the valuing of different knowledge over time.

Keywords: ethnomathematics, cultural practices, currach

Introduction

> "Lightweight and constructed of readily available, inexpensive materials, these uniquely Irish craft used scarce wood conservatively, relying instead upon the hides of locally raised livestock, or later upon tarred canvas" (Sikes & Meide, 2006, p. 5).

Currachs (see Figure 1) are traditional Irish boats whose use goes back to pre-history (Tully, 2008) and are present in much of the history of Ireland (Tully, 2008), including a brief possible mention in Tamsin's family history of the Kerin family from Ballyvaughan, in the north of County Claire.

Figure 1

Currach from the Arran Isles with a lob sail (Hornell, 1938)

Note: from Hornell (1938, Plate II).

Yet, since the time that the Kerin family left Ballvaughan in 1866, the building of currachs has almost completely been reduced to being built for races, with their original function as fishing boats, or to ferry goods around the coast, no longer much in evidence. In 2008, in a survey of traditional Irish boats in County Clare, no currachs were registered at Ballyvaughan (Tully, 2008). In County Clare as a whole, there were 70 currachs, of which 38 were of the

West Clare style. Not all of these were in use (Tully, 2008). The Heritage Council (2006) in a policy document noted that modern fishing regulations and safety concerns have led to the decline, which with an ageing population of those who knew how to build them was likely to result in their disappearance in the future.

Education is often advocated as a way to maintain and resurrect cultural practices (Trinick & Meaney, 2020). Yet, the revival of cultural artefacts and knowledge in schools comes with multiple considerations, particularly in countries with histories of colonisation. In mathematics education, ethnomathematics is often used as a basis for integrating a cultural practice or tradition into mathematics lessons in schools.

> Ethnomathematics is a research program incorporating history, anthropology, pedagogy, linguistics, and philosophy of mathematics with pedagogical implications that focus on the techniques of explaining, understanding and coping with different sociocultural environments. (Rosa & Gavarrete, 2017, p. 5)

Ethnomathematics is advocated as a way to engage cultural and/or underachieving groups in school mathematics by including aspects of Indigenous or cultural mathematics that are usually not considered valuable enough to discuss in schools (Meaney et al., 2022; Trinick & Meaney, 2020). For example, Gerdes (1985) identified situations in which mathematical elements existed in the daily life of Indigenous groups during the colonial occupation of Mozambique, but which were not recognised as such because of the colonisers' belief in the superiority of Western mathematics. However, as summarised in Meaney et al. (2022), several concerns have been raised about the inclusion of cultural practices into mathematics education. These concerns focus on how the use of ethnomathematics can result only in the valuing of the cultural practice or artefact as a way to learn Western mathematics. Similarly, discussing the artefact or practice can restrict it to being valued only as a connection to the past, romanticising the artefact or practice, without discussing its value within modern society. This suggests that before introducing currachs into school mathematics, there is a need to describe what an ethnomathematical gaze on currachs could contribute and to consider how the integrity of the cultural artefact could be maintained if it were to be included in school mathematics. Thus, our research question is: What opportunities and issues are raised when considering the ethnomathematics of currachs in educational situations?

To answer this question, we frame our investigation by using the cultural symmetry model Meaney et al. (2022). This model identifies the aspects—social, linguistic, cultural (knowledge and values), and mathematical—which affect the choice to use cultural artefacts or practices in the classroom. We begin by describing the curricular opportunities that could be connected to ethnomathematics, before briefly describing the cultural symmetry model.

Curricular Opportunities

The Irish post-primary (approx. age 12-18) education system operates on a centralised education model and contains very prescriptive syllabi, with a state examination after year three and a terminal state examination upon completion of post-primary education. These relate to the Junior Cycle (Years 1- 3) and Senior Cycle (Years 5 and 6), respectively. The introduction of a new mathematics curriculum in 2010 aimed to address issues relating to students' understanding of mathematical concepts, their ability to apply and problem solve, and to move away from over reliance on rote-learning procedures (Shiel & Kelleher, 2017). There was an emphasis on setting mathematics in context and making it relevant for learners. The reform involved a staggered rollout of the new specification over the course of five years with changes to the content of the syllabi, how mathematics is taught and how it is assessed, with some further revision in 2018. In the current Junior Cycle mathematics specification (National Council for Curriculum and Assessment/Department of Education and Skills [NCCA/DES], 2017, p. 5), mathematical proficiency is conceptualised not as a one-dimensional trait but as having five interconnected and interwoven components:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
- adaptive reasoning—capacity for logical thought, reflection, explanation, justification and communication
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one's own efficacy.

There is limited research which has examined the impact of the 2010 curriculum. Shiel and Kelleher (2017) concluded that it has had a small positive impact on performance, teaching approaches and students' attitudes. The Junior Cycle specification acknowledges that mathematics and mathematical ideas have evolved across societies and cultures over many years. While the move to utilising contexts and real-life problems to teach mathematical concepts is worthwhile, we suggest that ethnomathematics may provide another such possibility and could be incorporated into current curricular offerings to embed more strongly the role of culture and society. In particular, the current use of contexts by Irish mathematics teachers generally examines the mathematical concept. However, adopting an ethnomathematical approach could build on this and explore ideas such as social values and cultural roots and their connection to mathematics (Rosa & Gavarrete, 2017), while enabling the five interconnected components outlined above. Transition Year (TY, optional year between the Junior and Senior Cycles) could provide such an opportunity. TY is designed to be cross-curricular in nature and affords teachers the scope to plan learning experiences outside of the prescribed curricula. Adopting an ethnomathematical approach in TY may provide students with an opportunity to revisit their mathematical understandings, while developing an appreciation of diversity and cultures. It could also support a way to examine traditional artefacts and practices as contexts for learning mathematics.

The Cultural Symmetry Model

The cultural symmetry model was developed as part of a long-term project about the revival and maintenance of Māori language and culture through mathematics in

Aotearoa/New Zealand. It consists of three steps to cover different aspects that need consideration when integrating a cultural artefact or practice into mathematics education (Meaney et al., 2022). The cultural symmetry model was designed to find a balance between Indigenous cultural knowledge, including language, and mathematical cultural knowledge.

The first step is to identify why a group values the practices and artefacts, as a contribution to a local or societal cultural tradition. In regard to Indigenous cultural traditions, this is usually best done in collaboration with elders, who have expertise in regard to the practices and artefacts. This focus enables the revitalisation and maintenance of the cultural knowledge to occur through understandings about the original purposes of the practice and artefact. Ideally, the traditional language should be used to discuss this cultural knowledge. However, in some situations of colonisation, Indigenous languages have been lost or are in the process of being revived. In these cases, the choice of language needs consideration.

The second step involves examining the cultural artefact or practice from a range of perspectives, of which mathematics would be one. Different knowledge bases can deepen understandings about the practices and artefacts. Mathematics provides one way to describe the cultural artefact or practice. An outcome of this step is to illustrate how invisible socio-cultural aspects connected to a practice, such as cultural values, relationships, problem solving processes and knowledge can be ignored, when an artefact or practice is just viewed as a quaint, historical relic. By discussing the practice or artefact from a range of perspectives, students' language can be developed. This is particularly important when the students are second language learners of the language being used in the classroom.

The third step involves discussing how and why different kinds of knowledge are valued by different groups of people at specific periods of time, sometimes to the detriment of other groups. This provides an opportunity to raise how mathematical discussions about a cultural artefact or practices should not detract from cultural understandings.

Currachs and the Cultural Symmetry Model

In this section, we discuss each step of the cultural symmetry model and identify opportunities and issues that need consideration in using the design and building of a currach as a possible focus for mathematics education in schools. We use information about the currachs from County Clare, predominantly, but also draw on information about currachs from other areas. Information about currachs comes from several written sources, in particular Hornell's (1938) definitive article on the "The curraghs of Ireland", as well as films about currachs, available on YouTube, some of which provide audio-commentary, with many showing the building of currachs at different points in time. Our lack of experience of being in a currach means that we are restricted to being observers. Thus, rather than focus on their functions at sea, we investigate the ethnomathematics of designing and building currachs.

Step 1: The Cultural Tradition

Currachs were a product of their environment in that local resources were used in their design and building so that they could fulfil particular purposes such as fishing or ferrying items around the coast. Although currachs made of a tied frame of willow or hazel and covered with animal hide were used from prehistoric times in inland waterways of Ireland, changes in design and function occurred in the 1820-1830s (O'Sullivan & Downey, 2015). These changes produced the fishing currachs of the west coast of Ireland which were built using sawn timber in the hull, a double gunwale and tar covered cloth. In County Clare as elsewhere along the west coast, trees were scarce and only the frames were made of wood:

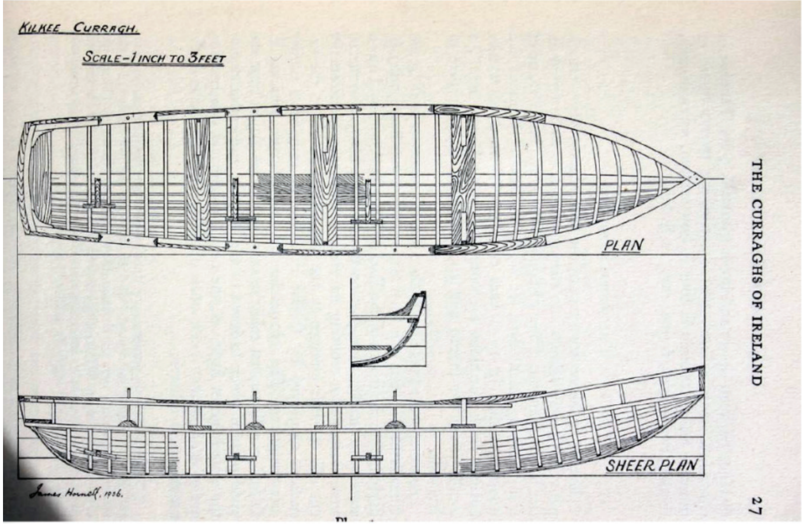
> Deal was used for the oars and gunwale (upper edge of the side of the currach). Oak was used for the frame, while elm and sallywood were also used in currach construction. Although a finished currach could reach up to 18 feet in length, be up to 4 feet wide and 3 feet deep, these vessels are remarkably light and manoeuvrable and could be easily beached. (The Clare Champion, 2002, p. 1).

Thus, the function of the boats was to be able to tackle the seas around the coast and to beach on the shoreline, not always an easy task. This could be achieved through using lightweight materials. However, the design of a currach differed according to locality. These differences seemed to be due to whether the currach was to be paddled or rowed; the sea conditions; and what the currachs were to transport (O'Sullivan & Downey, 2015; Tully, 2008). There were also differences depending upon the number of rowers. In more recent times, the use of currachs for racing have affected their design (Tully, 2008).

Nevertheless, a commonality between currachs which is different to most other boats is that they are built from the top. Figure 2 provides a conventional plan of a currach which shows the boat as though it was sitting on the water.

Figure 2

Kilkee currach



Note: from Hornell (1938, p. 27).

Yet, currachs are built from the double gunwale upwards, with the ribs being bent, usually after being steamed, and placed directly into the gunwale. This makes visualising the design "upside down" difficult, as noted by Shane Holland in Broderick's (2018) film on the *Currach Na Sceiri* (Skerries Currach), by those familiar with building other kinds of boats.

Valuing of the design and building of currachs needs to be done first from the perspective of its role for a cultural group. This situates the currach, not as a vehicle for developing mathematical understandings, but as having value within a cultural group for the function that it fulfils. Nevertheless, the discussion of the design and building of currachs provides some opportunities to highlight mathematical ideas, which are described in the next sections. There is a need to consider which language should be used to discuss currachs, depending on general fluency of teachers and students as well as specific fluency with boat building terms.

In the mathematics lessons, a tension could arise with expectations about fulfilling a highly prescriptive curriculum when so much time needs to be spent on discussing the cultural aspects of the artefact, before discussing any mathematics. However, if everyday situations are to be used to support students to see mathematics as useful, then time needs to be spent discussing those everyday activities as being meaningful in their own right.

Step 2: Currachs from Multiple Perspectives

The design and building of currachs can be discussed in many ways, related to the resources available, the traditions for building at a particular moment in time and the purpose that the currachs were supposed to fulfil. The changes can be discussed in relationship to the historical context. For example, the change to tarred cloth became an alternative to the use of animal hides, with the coming of the industrial revolution and the production of cotton that allowed the import of cheap canvas. Similarly, the design of currachs could be discussed in relation to the sea conditions and possibilities for landing a boat on the shoreline of the Atlantic Coast. Such discussions provide possibilities to make visible socio-cultural aspects connected to currachs, such as cultural values, relationships, problem solving processes and knowledge. They also allow for vocabulary and grammar enrichment for students who have not had previous experiences with boats or discussing boats in a particular language.

In these discussions, there are possibilities to make connections to strategic competency, adaptive reasoning and productive dispositions, from the Junior Cycle mathematics specification (NCCA/DES, 2017). For example, the measurements in the designs, including the scales in the plans (see Figure 2), provide opportunities for currachs to be discussed mathematically. Measurements provide opportunities to discuss differences between currach designs and reflect on how those differences might be affected by the environment and the resources available. For example, the shape of the bow provides opportunities to discuss different angles and mathematical functions (for example the sheer plan in Figure 2) but these discussions need to add value by extending the discussions about how the bow shape is affected by the sea conditions faced by those rowing the currachs.

As well, there are opportunities to discuss visual-spatial skills linked to being able to have a mental map of the currach to ensure that all pieces are cut and placed appropriately to ensure the efficient building of the boat. Mathematics contributes to discussions about the design and building of currachs, rather than the design and building of currachs being the vehicle for discussing mathematics. Mathematics needs to add value to these discussions, rather than detract from them, otherwise mathematics will not be seen as sensible, useful, and worthwhile, as designated by the Junior Cycle specification (NCCA/DES, 2017).

However, issues can arise in classrooms if the students do not have sufficient knowledge about the artefact or practice, or the valued knowledge about that cultural practice or artefact. Without this, a focus of mathematical understanding can be seen as disrespectful in that it questions cultural knowledge. This was the case in the study of traditional wayfaring in the Pacific in a study by Trinick and Meaney (2020). Consequently, there is a need to discuss who can decide which knowledge is valuable and in what circumstances. This provides a critical perspective on the valuing of some knowledge, such as mathematics, and how this might affect the valuing of other knowledge.

Step 3: Adding Value with Mathematics and Discussing the Politics of Valuing Knowledge

By discussing currachs from a mathematical perspective it is also possible to discuss how different knowledge systems come to be valued. For example, who decides whether a design is “upside-down”? Such a determination indicates that there is a general expectation about what is the “normal” way of building boats. Rarely in mathematics lessons is space given to discussions about who decides what kind of knowledge is valued in classrooms and in examinations and how valuing one kind of knowledge may affect how other knowledge is valued. This lack is in contrast to statements in curricula, such as the Junior Cycle specification, which acknowledge that mathematics and mathematical ideas have evolved across societies and cultures over many years. Yet, for example, discussions about visualising the design of currachs which deepen understanding about shapes and how they are described using algebraic notation, bring together geometric as well as algebraic understandings, two different kinds of knowledge, not easily developed otherwise. Therefore, although taking time in a mathematics lesson, discussions about valuing different kinds of knowledge can highlight how mathematics can be developed in the future, through connections to everyday situations.

Conclusion

Our aim was to consider the opportunities and issues that arise if ethnomathematics is to be used in bringing a cultural artefact or practice into mathematics classrooms. In particular, we looked at how the design and building of currachs could promote discussions about different kinds of knowledge, including mathematical knowledge, and how this could contribute to mathematics being seen as sensible, useful, and worthwhile as required by the Junior Cycle mathematics specification (NCCA/DES, 2017). Thus, ethnomathematics, through discussions about currachs, provides opportunities to deepen students’ understandings about mathematics, but also about how mathematics comes to overtake the importance of other cultural knowledge unless precautions are implemented and discussed.

However, in a prescriptive curriculum, mathematics teachers may struggle to find the time to raise discussions about how different aspects of knowledge come to be valued, especially if such discussions are not recognised in examinations. There is also a need to consider whether the currach is best revived as a meaningful cultural artefact, through discussions within a mathematics classroom. It may instead need better opportunities for cross-subject integration of projects that involve language, craft and mathematics. Such decisions cannot be made at the teacher level, but require the education system more generally to decide how the aims of the curriculum regarding the use of every day and cultural experiences can be integrated more appropriately in schools.

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An Investigation Into How First Class Children Model Multi-digit Addition and Subtraction Problems

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This paper reports on an investigation into how first class children model multi-digit addition and subtraction problems. This case study was carried out within a Delivering Equality of Opportunity In Schools Band One school. The children were presented with problems and thematic lessons were designed using the addition and subtraction problem-structures outlined by Van de Walle et al. (2020). The data was analysed according to the Ongoing Assessment Project Additive Framework (Hulbert & Ebby, 2017). This study demonstrates that first class children can use models such as manipulative materials, prepared images and the empty number line to determine successful solutions to word problems.

Keywords: mathematical modeling, addition, subtraction

Introduction

This paper focuses on how first class children model multi-digit addition and subtraction problems. This research was carried out in the author's classroom within a Delivery Equality of Opportunity in Schools (DEIS) Band One school. In this paper, the strategies and models the children used are explored and analysed.

Literature Review

Over the last number of decades, mathematical modeling has become more prominent in mathematics education literature (Blum, 2015) and has also come to the forefront of mathematical curriculum development in Ireland (National Council for Curriculum and Assessment (NCCA), 2022). Mathematical modeling is seen to bridge informal understanding with more abstract formal mathematical ideas (Kaiser et al., 2011). Therefore, it would seem fitting that the development of mathematical modeling would be one of the more significant goals of mathematics education (English & Sriraman, 2010). Using a variety of models offers multiple levels of learning (Suh & Seshaiyer, 2016). Consequently, the inclusion of mathematical modeling could enable all children to develop mathematical concepts at their own level.

Models

Engaging with models can provide children with the opportunity to model mathematical ideas and leads to unstructured problem solving (Suh & Seshaiyer, 2016). Manipulative materials and the Empty Number Line (ENL) are discussed below.

Manipulative Materials. Research emphasises that the use of manipulative materials aid children to make sense of mathematics at their own level. Their incorporation is not a new idea in mathematics education and holds many benefits, including helping students to focus on mathematical ideas and stimulating higher order thinking (Clements & McMillen, 1996). However, "busy hands don't necessarily mean busy minds" (Waite-Stupiansky & Stupiansky, 1998, p. 85). Therefore, it is imperative for students' mathematical development that when using manipulative materials, there is a conversation between the student and teacher which links concrete materials and abstract ideas (Maclellan, 1997).

ENL. The ENL is a model that can support "children in using a variety of sequential calculation strategies" (Murphy, 2011, p. 148). It is an internationally acknowledged model for developing mental mathematic strategies (Bramald, 2000) and was developed as many children were experiencing problems with two-digit arithmetic (Bobis & Bobis, 2005). It was originally created by the Dutch and was encompassed in their Realistic Mathematics Education (Murphy, 2011). Much of their improved performance in international standardised testing has been directly associated with its introduction (Bramald, 2000).

Methodology

Van de Walle et al. (2020) outlined three ways in which word problems can be structured; change (join and separate), part-part-whole and compare. Within each of these categories there are three subcategories, each naming a different way the question can be structured or framed (Van de Walle et al., 2020). These are outlined in detail in Table 1. When designing the research tasks, each category and subcategory was used to ensure that the children were exposed to every type of word problem. Each lesson focused on a different problem type, apart from lesson three, which revised the children's learning from lessons one and two. This research explores and documents first class children's use of models to record their strategies when solving multi-digit addition and subtraction problems. A cohort of fifteen children participated in six mathematics lessons. Their solutions were discussed with their peers and were recorded using the tools provided. These included cubes, lollipop sticks, prepared images, pencils and paper. Discussions of strategies took place after they had reached their own solutions and the teacher researcher then recorded their strategy using an ENL. As the lessons progressed, the children also had the opportunity to share their strategy with their peers, however, the use of the ENL was optional for them. Samples and photographs of the children's work, teacher observations and audio recordings of one group of children were used to collect the data during lessons.

Table 1

Problem type and structure (Van de Walle et al., 2020, p. 187)

Type	Result Unknown	Change Unknown	Start Unknown
Change-Join	Sandra had 8 pennies. George gave her 4 more. How many pennies does Sandra have altogether? 8+4=□	Sandra had 8 pennies. George gave her some more. Now Sandra has 12 pennies. How many did George give her? 8+□=12	Sandra had some pennies. George gave her 4 more. Now Sandra has 12 pennies. How many pennies did Sandra have to begin with? □+4=12

Change-Separate	Sandra had 12 pennies. She gave 4 more pennies to George. How many pennies does Sandra have now? 12-4=□	Sandra had 12 pennies. She gave some to George. Now she has 8 pennies. How many did she give to George? 12-□=8	Sandra had some pennies. She gave 4 to George. Now Sandra has 8 pennies left. How many pennies did Sandra have to begin with? □-4=8
	Whole Unknown	**One Part Unknown**	**Both Parts Unknown**
Part-part-whole	George has 4 pennies and 8 nickels. How many coins does he have? 4+8=□	George has 12 coins. 8 of his coins are pennies and the rest are nickels. How many nickels does George have? 12=4+□ or 12-4=□	George has 12 coins. Some are pennies and some are nickels. How many of each coin could he have? 12=□+□
	Difference Unknown	**Larger Quantity Unknown**	**Smaller Quantity Unknown**
Compare (situations of more)	George has 12 pennies, and Sandra has 8 pennies. How many more pennies does George have than Sandra? 8+□=12	George has 4 more pennies than Sandra. Sandra has 8 pennies. How many pennies does George have? 8+4=□	George has 4 more pennies than Sandra. George has 12 pennies. How many pennies does Sandra have? □+4=12
Compare (situations of fewer)	George has 12 pennies. Sandra has 8 pennies. How many fewer pennies does Sandra have than George? 12-8=□	Sandra has 4 fewer pennies than George. Sandra has 8 pennies. How many pennies does George have? □-4=8	Sandra has 4 fewer pennies than George. George has 12 pennies. How many pennies does Sandra have? 12-4=□

Findings and discussion

The findings offer an insight into the models used to solve addition and subtraction problems. The On Going Assessment Project (OGAP) Additive Framework (Hulbert & Ebby, 2017) was used to analyse the strategies employed. Strategies were sorted into three categories: counting (ones), transitional (tens) and additive. The progression begins with counting (ones), which is the most basic level and extends to the additive category, which is the most complex and efficient level. This section analyses the strategies used.

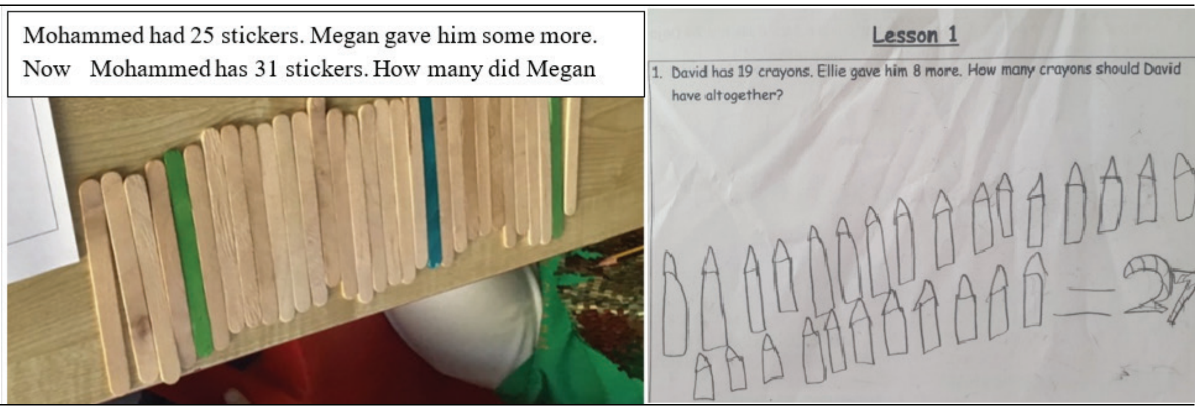
Counting (Ones) Strategies

Direct Counting. Direct counting was utilised in various ways, from using the resources to drawing images. Concrete materials were used regularly to model addition and subtraction problems in lessons one and two. It accounted for 67% of the strategies employed by two of the three groups in the first two lessons. This result could suggest that these children initially needed concrete materials to build meaning of the new mathematical structures (Clements, 1999). The concrete materials assisted them towards an answer as they "moved, grouped and re-arranged" to illustrate a problem (Putri et al., 2020, p. 83).

Figure 1.

Examples of direct counting strategies employed by first class children.

(i) (ii)



The above figure 1(i) highlights how the problem was modeled using concrete materials. Initially looking at Figure 1(i), the method by which the children solved this problem is not evident. It was noted in the field notes that in order to reach this solution, the children initially counted the full set of thirty-one lollipop sticks and then split them into two sub-sets, the stickers that Mohammed had and the ones Megan gave him. These actions replicated the problem structure of change unknown and suggests a good understanding. It could also suggest that the concrete materials were enabling the children to understand the part-part-whole relationship of numbers. Figure 1(ii) highlights another way in which direct counting was applied. Despite this solution looking unlike the previously presented solution, they are quite similar. The child drew images to replicate the problem presented, and although they were not using physical concrete materials, they created their own visual equivalent. Rellensmann et al. (2016) suggests that the drawing of images is a good starting point for conceptual understanding. This research seems to echo this, as all word problems were correctly solved by the children drawing images and they applied more complex additive and transitional strategies as the research progressed.

Counting With Prepared Images. The children utilised prepared images to count sets and solve the given problems. Giving the children images could be viewed as one way to aid children towards an adequate visual representation of the word problem, which many children have difficulties with (Boonen et al., 2011). 60% of the pairs of children used these images at least once over the course of the six lessons to solve a given word problem and for the most part were successful in finding its solution. Although this strategy falls into the most basic level of early counting strategies within the counting (ones) level of the OGAP Additive Framework (Hulbert & Ebby, 2017), its use was not without reason. As suggested in the framework, “ as students learn new concepts or interact with new problem situations and problems structures, they may move up and down along this progression” (Hulbert & Ebby, 2017, p. 2). Given that five of the six lessons introduced new problem types and structures, it is not surprising that this strategy was the most prominent used by two table groups.

Counting in Ones With ENL. Using the OGAP Additive Framework (Hulbert & Ebby, 2017), this strategy is seen as one that exists within the second tier of the more basic counting strategies, counting (ones). However, this framework fails to note the transition from using concrete materials to an abstract tool such as the ENL. This transition reveals that the children understand the problem and can represent this in an abstract form (Putri et al., 2020).

Counting On. The counting on strategy was achieved for the most part through the use of concrete materials. Similar to the direct counting strategy, some children drew images and counted on from them. This is more complex than direct counting and more efficient, however it still falls into the counting (ones) category (Hulbert & Ebby, 2017). Hulbert and Ebby (2017) do note that it falls into the top tier of counting (ones) category, suggesting that it is a stepping stone to early transitional strategies. Mc Intosh (2006) considers this counting on or back in ones problematic, as it is inefficient but this fails to realise that this was a development for some children, away from direct counting. However, there is some merit in Mc Intosh’s (2006) recommendations stating that teachers should encourage children who are secure in this method to move onto mental mathematics strategies. Counting on with the standard number line was an unforeseen strategy and was heavily influenced by the classroom environment. It was not until discussing strategies that it was realised its use to problem solve. Some children jumped back or forward in ones on the standard number line, which would classify as counting on with a visual model, counting (ones) strategy (Hulbert & Ebby, 2017). This strategy was particularly favoured by one group in lesson six. Before utilising this strategy, the children in this group relied solely on direct counting strategies using images or concrete materials. This is the lower level of the counting (ones) strategy (Hulbert & Ebby, 2017). However, this framework does not account for the fact that using the standard number line prompts direct counting (Clements, 1999). This is the case as children merely located a number on the number line and counted on the number being added. In order to utilise mental counting strategies, children would need to keep track of the number they are counting and how many they are adding (Clements, 1999).

Transitional (Tens) Strategies

Inefficient Use of ENL. The ENL was used inefficiently when solving three of the eighteen problems. Despite this, this strategy did not necessarily portray a lack of understanding of mathematical concepts. The data shows that some of the children did in fact understand the problems, but tried to create a unique solution to share it with their peers.

Efficient Use of the ENL. The efficient use of the ENL is "jumps by multiples of ten on a number line" and "jumps by a ten and efficient groups of ones" (Hulbert & Ebby, 2017, p.2). This strategy was applied by the one group in every lesson, with multiple solutions on the ENL becoming more prominent in later lessons. This research highlighted that the ENL can be used as a flexible tool by first class children in which they can represent their thinking. Van den Heuvel-Panhuizen (2008) found that for some children the ENL could be seen as constraining, however, it depended on the way in which it was taught. Within this research, this model was not taught explicitly and the children were introduced to it as a means of modeling and sharing ideas, which was not the case in Van den Heuvel-Panhuizen's (2008) research. Confining the children to an ENL could limit children's thinking as they could focus on using this abstract tool and not fully engage in understanding the mathematical problem.

Figure 2.

Examples of transitional (tens) strategies utilised by first class.

(i) (ii)

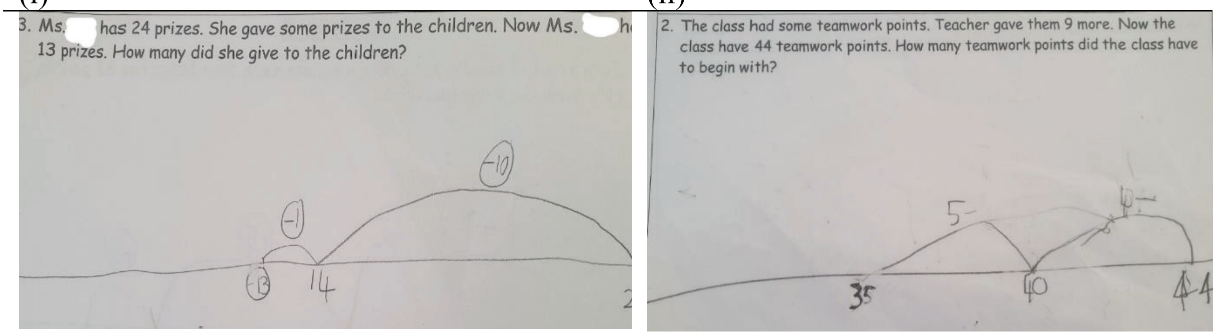


Figure 2(i) above demonstrates an efficient use of a model by jumping in "10s and efficient groups of ones" (Hulbert & Ebby, 2017, p.2). This was only one of the solutions produced on the ENL, which promoted flexibility and use of mental strategies in the research lessons.

Additive Strategies

Using 10s. Bridging ten is when "part of the second number is used to make the first number up to the next multiple of 10, and then the remainder of the second number is added" (Chesney, 2013, p.38). This strategy is one of the more complex on the OGAP Additive Framework, falling into the additive category. In this research using 10s usually went alongside the use of the ENL as a model. This is possibly a result of the children's strategies being modeled in the share sessions on the ENL. This was a favourite strategy employed by one group of children, equating to 20% of their strategies used. Figure 2(ii) outlines how one child modeled his 10s strategy using the ENL. This problem prompted the children to subtract, and this is visible from the subtracting sign on the first jump.

Fact Recall and Derived Facts. Fact recall is when a child remembers a number fact without having to work it out. In this study, doubles (i.e. adding the same number to itself) were recalled when solving two different problems. Recalling number facts from memory leads to a very efficient problem-solving strategy (Leutzinger,1999). When utilizing the strategy of derived number facts, known number facts are utilised to derive another. This is one of the more complex strategies utilised in these lessons and some of the children worked systematically, using previous number facts to work out another. Using this additive strategy portrays a deep level of understanding. It is interesting how this strategy was first employed by the children in lesson three, as this lesson was a revision lesson and contained problem structures already seen by the children. Using the strategy of derived facts recognises that the children are "developing more sophisticated ideas about number, recognizing ways to think about basic fact combinations" (Van de Walle et al., 2020, p. 184).

Flexible Compensation. Interestingly, this strategy was only seen in the first problem of the first lesson. After the introduction of the ENL, this strategy never made a reappearance. Perhaps, this demonstrates one of the weaknesses of the ENL, that flexible compensation is a strategy which is very difficult to record on the ENL. This was highlighted in the field notes; it was difficult to demonstrate the compensation strategy used in problem one on the ENL. It was noted that some of the children found it difficult to follow this strategy on the ENL until they questioned the children that used it. Possibly, this reasoning alone deterred some from using it throughout the lesson series, as they found it challenging to record it on the ENL.

Conclusion

This research has highlighted the potential of mathematics modeling in junior class within a DEIS context. This is particularly valid within the Irish primary education context, with the new Primary Mathematics Curriculum incorporating a high degree of mathematical modeling (NCCA, 2022). During this research, first class children used a range of models such as manipulative materials, prepared images and the ENL to determine successful answers to word problems. It confirmed that first class children can model multi-digit addition and subtraction problems.

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Investigating the Effect of Creative Mathematical Reasoning Tasks on Student Achievement: A Causal Inference Machine Learning Approach.

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In this study we investigate the impact of regularly assigning creative mathematical reasoning tasks on student achievement. Using a causal inference machine learning approach applied to Irish eighth grade data from TIMSS 2019, we find that assigning challenging questions requiring students to go beyond the instruction has a clear positive effect on mathematics achievement. Asking students to decide their own problem-solving strategies is also found to have a positive effect. In contrast, frequently asking students to practise procedures on their own is not associated with a positive increase in achievement. These results were consistent across all three cognitive domains of "knowing", "reasoning", and "applying". We therefore recommend the incorporation of creative mathematical reasoning tasks into most classes by teachers as an effective way to improve student achievement.

Keywords: creative mathematical reasoning, causal inference, TIMSS

Introduction

A growing body of research demonstrates the importance of students working on problems which require them to think creatively and develop their own problem-solving strategies. However much of the work to date has been based on small studies. In this paper, we will examine the effectiveness of regular exposure to creative reasoning tasks using data from an international large-scale assessment, namely the Trends in International Mathematics and Science Study (TIMSS 2019). Our study employs a causal inference machine learning approach based on Bayesian Causal Forests (BCF, Hahn et al., 2020). We apply our model to Irish eighth grade data from TIMSS 2019, and attempt to answer the following research questions: 1) What is the effect on achievement of regularly assigning creative mathematical reasoning tasks to students, and 2) How does this effect change across the three TIMSS cognitive domains of "knowing", "reasoning", and "applying"?

Literature Review

Lithner's (2008) framework describes two ways in which students can approach problems in mathematics. The first, called imitative reasoning can be further divided into two distinct types: memorised reasoning and algorithmic reasoning. Memorised reasoning refers to situations where a student can recall a complete solution and need only transcribe their answer without any further thought. Algorithmic reasoning is applied in situations where students instead recall a full procedure or algorithm from similar problems they have seen previously, and may follow the necessary steps to arrive at a solution, often requiring only trivial calculations to proceed from step to step. Lithner argues that the reliance of students on imitative reasoning can be a hindrance to the development of deeper levels of mathematical understanding, as the recollection of solutions and completion of procedures can be performed with very little “conceptual insight into what they [are] doing” (Lithner, 2008, p. 259).

When no complete solution or algorithm can be recalled, students must employ creative mathematical reasoning (CMR). CMR requires students to respond to novel situations by creating their own problem-solving strategies, with “plausible” mathematical arguments used to justify or support their choices (Lithner, 2008). Research has shown that students presented with tasks requiring CMR tend to perform better on tests (Lithner, 2017). Studies by Jonsson et al. (2014) and Wirebring et al. (2015) which involved assigning students to groups with either an algorithmic reasoning or creative mathematical reasoning focus to learning both found that students in the CMR group outperformed their peers in a test after the study.

Despite the many benefits of CMR, research has shown that much of the emphasis in textbooks is focused primarily on algorithmic reasoning, and that students are rarely given a chance to engage with problems requiring CMR. A review by Jäder (2020) of mathematics textbooks in twelve countries for example, showed that up to 90% of tasks could be solved by students reusing existing templates, or making only minor modifications to example solutions. Work by Mac an Bhaird et al. (2017) shows that this reliance on algorithmic reasoning tasks is also common in undergraduate service mathematics courses in Ireland.

Methodology

Data and Sample

The Trends in International Mathematics and Science Study (TIMSS 2019) is an international large-scale assessment which has taken place in many countries across the world every four years since 1995. As part of the study, students in the fourth and the eighth grade (fourth class of primary school and second year of secondary school in Ireland) are given a short assessment in mathematics and science to estimate their achievement level in these subjects. Participating eighth grade students also complete a short questionnaire on aspects such as confidence in mathematics, how many books and educational resources they have at home, the highest level of education received by their parents, and many others. The eighth-grade subset of the 2019 data provides a representative sample of 4118 second year students, with an average age of 14.4 years. Additionally, the 565 mathematics and science teachers, and school principals of these students complete a short questionnaire on teaching practices, years’ experience, and other school related factors. The combined dataset therefore provides a rich source of information on student, teacher, and school characteristics.

Creative Mathematical Reasoning Questions

To investigate the effect of regularly assigning tasks requiring CMR, we used the data arising from the following questions answered by teachers: “How often do you do the following in teaching this class?” 1) “Ask students to complete challenging exercises that require them to go beyond the instruction”, 2) “Ask students to decide their own problem solving procedures”, 3) “Ask students to practice procedures on their own”. The possible responses to these questions were: 1) “Every or almost every lesson”, 2) “About half the lessons”, 3) “Some lessons”, or 4) “Never”.

These questions tie in closely with the features of creative mathematical reasoning from Lithner’s (2008) framework. Asking students to decide on their own problem-solving procedure requires them to reason creatively about which strategies are best suited to a particular problem. Also, asking students to complete tasks which require them to “go beyond” the instruction means they cannot rely on template solutions or examples studied in class previously, requiring a creative solution to tackle an unfamiliar problem. Practising procedures, however, is more closely linked to preparing students for tasks requiring imitative reasoning. Of course, there will be a degree of subjectivity regarding what constitutes deciding a problem-solving strategy, or what going “beyond” the instruction means, but we believe the teacher' responses to these questions provide valuable insights into how regularly their students have opportunities to practise creative mathematical reasoning.

Cognitive Domains of Achievement

In addition to assessing overall student achievement in mathematics, TIMSS includes a specific evaluation of student performance in the three cognitive domains of “knowing”, “reasoning”, and “applying” (Mullis et al., 2017). Achievement in the “knowing” cognitive domain reflects a student’s ability to recall and recognise facts as necessary when working on a mathematics problem. High achievement in the “reasoning” domain requires students to demonstrate they can analyse, evaluate, and draw conclusions from mathematical situations, providing mathematical arguments to justify their decisions. Finally, students who perform well in the “applying” domain can successfully apply mathematical facts, rules, and procedures to both real life and purely mathematical problems.

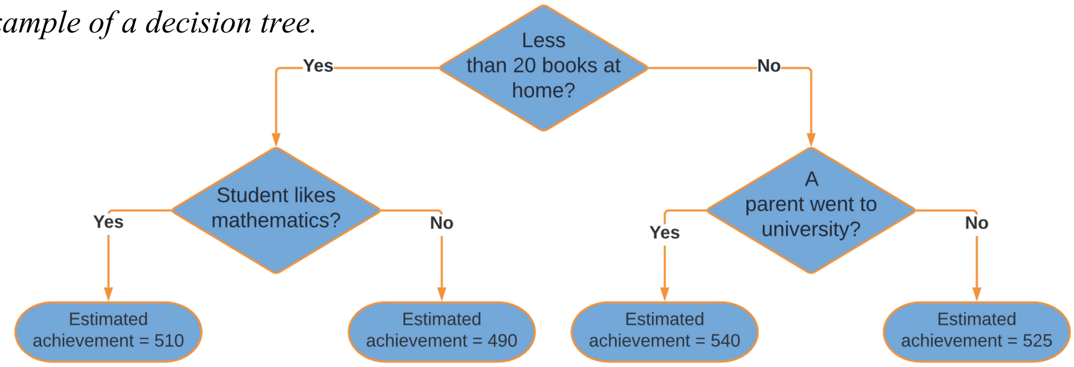
Given the focus of CMR on presenting students with novel tasks in unfamiliar situations, requiring them to creatively develop their own problem-solving strategy, we expect the “reasoning” domain to be more strongly influenced by the frequency at which teachers assign CMR tasks to their students. Similarly, as the “knowing” and “applying” domains are focused primarily on recalling and applying mathematical facts to familiar situations, we expect these domains may be less subject to influence from engagement with CMR tasks.

Bayesian Causal Forests

Our analytical approach in this study is based on an advanced causal inference machine learning algorithm called Bayesian Causal Forests (BCF, Hahn et al., 2020). BCF is a tree-based model, meaning it learns from the data by creating a set of decision rules which provide a pathway from the root of a tree to its terminal nodes. At the terminal nodes a prediction for each data point is provided. Figure 1 gives a visual example of a single decision tree from a BCF model. While this example appears quite simple, it is important to note that the size of the trees in a BCF model can increase or decrease in size depending on the complexity of the relationships among variables in the dataset. Moreover, the predictions made by a BCF model are typically based on the contributions of many decision trees which work together to each explain a small part of the variability in the outcome variable. In this way, BCF can capture very complicated relationships and interactions among variables in the data.

Figure 1

An example of a decision tree.



Unlike most machine learning models, BCF is capable of disentangling the causal effect of a specific factor from the impact of other confounding variables that are not of primary interest. The standard implementation of BCF is only applicable to a single outcome variable at a time, and only to situations where the factor of interest takes just two levels. Therefore, in order to apply BCF to the multiple cognitive domains of “knowing”, “reasoning”, and “applying”, we will use the multivariate BCF extension developed by McJames et al. (2023) and make an extra modification allowing for the inclusion of multiple treatment levels. Our model in this study can be summarised as follows:

$$y_{i,j} = \mu_j(x_i) + \tau_{j,1}(x_i)Z_{i,\ half\ of\ lessons} + \tau_{j,2}(x_i)Z_{i,\ every\ lesson} + \epsilon_{i,j}$$

In this formula, the Z_i values are dummy variables which indicate if a student belongs to the “half of lessons” or “every lesson” categories, and x_i represents the known characteristics associated with student i such as age, gender, socioeconomic background, the experience of their teacher, and many others. In total, each x_i comprises more than forty student, teacher, and school characteristics which we control for. The response $y_{i,j}$ is the achievement of student i in cognitive domain j, and μ_j provides the prognostic contribution of the variables we are not primarily interested in. Each τ_j represents the effect on cognitive domain j of receiving creative mathematical reasoning tasks with the specified frequency provided by Z_i, and $\epsilon_{i,j}$ is a random error term. The predictions from the μ_j and τ_j parts of the model are provided by decision trees, such as the example in Figure 1.

Results

A breakdown of the teachers’ self-reported responses can be found in Table 1 below. Of the 565 teachers in the sample, 46 teachers did not provide a valid response due to not being administered the questionnaire or failing to answer the relevant questions, so 519 valid responses were obtained. Interestingly, asking students to practise procedures on their own, which is most closely related to the category of imitative reasoning, is much more common than the two CMR task types.

Table 1

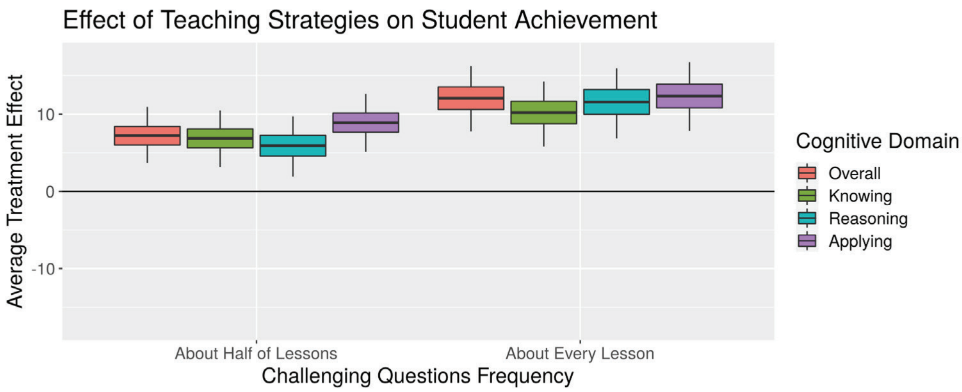
Creative mathematical reasoning frequency. 565 teachers. 519 valid responses.

How often do you do the following in teaching this class?	Never	Some Lessons	Half the Lessons	Every lesson
Ask students to complete challenging exercises that require them to go beyond the instruction.	3.2%	33.1%	38.5%	25.2%
Ask students to decide their own problem-solving procedures.	5.4%	40.3%	35.3%	19.0%
Ask students to practise procedures on their own.	0.3%	7.4%	20.2%	72.1%

Figures 2, 3 and 4 provide a visual representation of our results. The boxplots depict the credible intervals for the average treatment effects of using different question types at varying frequencies. We have constructed these boxplots such that the central body corresponds to a 50% credible interval, and the full extent of the whiskers covers a 95% credible interval for the estimated effect sizes. Overall mathematics achievement in Ireland follows an approximately normal distribution with an average of 524 and a standard deviation of 73. Therefore, an effect size of 7.3 would correspond to a 0.1 standard deviation increase in achievement. Effect sizes of this magnitude are common in education research and can be thought of as being medium in size (Kraft, 2020).

Figure 2

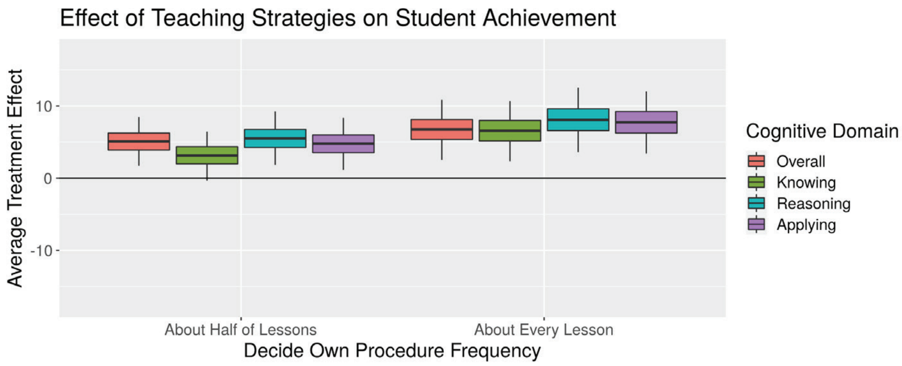
Effect of regularly asking students to go beyond the instruction



Looking at Figure 2, we see that relative to never, or only sometimes assigning challenging questions which require students to go "beyond" the instruction, there is a clear positive effect of assigning questions of this type in about half of the lessons. This positive effect is increased when the frequency of assigning the challenging questions becomes every or almost every lesson. A similar pattern can be seen in Figure 3 which shows the estimated effects of regularly asking students to decide their own problem-solving procedures. In contrast, Figure 4 shows that asking students to practise procedures on their own in "about half of the classes" or "about every class" is likely to have a negative effect on student achievement. While the effect sizes across the cognitive domains are very similar in all three cases, note that the positive effect of asking students to complete challenging questions is consistently higher than the positive effects of asking students to decide their own problem-solving strategy, suggesting this may be even more beneficial for improving student achievement.

Figure 3

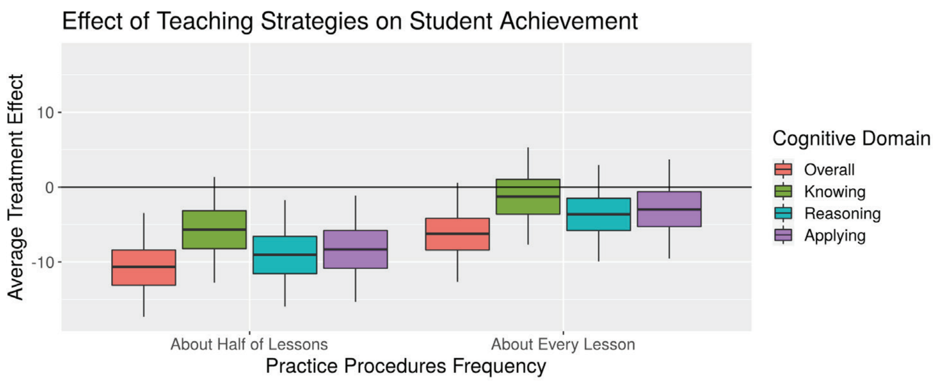
Effect of asking students to decide their own problem-solving strategies



The similarity of the effect sizes across the three cognitive domains and the overall achievement scale is surprising. We might have expected that engaging in creative mathematical reasoning would have less of an effect on the "knowing" domain, which is focused on memorising rules, procedures, and facts. If we look very closely at Figures 2 and 3, a very small difference in the effect sizes is visible, with the magnitude of the effect on the knowing domain slightly smaller than the other three domains, but this difference is too small to be significant. With more data and an increased sample size, greater differences across the cognitive domains may begin to be discernible with greater precision, however.

Figure 4

Effect of asking students to practise procedures on their own



Discussion

Our results which identify the positive effect of regularly assigning tasks requiring creative mathematical reasoning are in line with previous studies by Lithner (2017), Jonsson et al. (2014), and Wirebring et al. (2015). Our finding that frequently assigning tasks focused on algorithmic reasoning is likely to decrease achievement also supports these studies by highlighting the negative effect of relying too heavily on tasks of this type. Of the factors we investigated, the most beneficial impact was associated with assigning challenging questions which require students to go "beyond" the instruction. We emphasise, however, that tasks do not need to be difficult to encourage creative mathematical reasoning (Lithner, 2008). There only needs to be an aspect to the question, which is novel, requiring more thought than simply reusing previously studied solutions or algorithms.

Our short study makes two important contributions. Firstly, the studies mentioned previously included small, non-representative samples of students, and therefore the results from our study which uses a large representative sample of students from Ireland are more generalisable. Second, unlike the post-study tests used in these existing studies, our results are based on a standardised international mathematics assessment, and therefore provide an added amount of interpretability. For example, an effect size of nine on the TIMSS overall achievement scale corresponds to the difference between the mean mathematics achievement levels of students in Ireland (524) and the United States or England (both 515).

Based on these results, we encourage teachers to place a higher emphasis on assigning CMR tasks to their students. Only 19.0% of teachers in the sample reported asking students to decide their own problem-solving strategies every or almost every lesson, and only 25.2% reported assigning questions that go "beyond" the instruction at the same frequency. Therefore, given the positive effect that this can have on student achievement, we encourage all teachers to incorporate such questions in most classes where possible. This is especially important given previous findings which show that students are unlikely to be presented with CMR tasks in standard mathematics textbooks (Jäder, 2020). An opportunity therefore exists for teachers to help fill this gap.

Some limitations of this study include that our results only apply to eighth grade student data from Ireland. Additional studies based on data from other countries or grade levels would therefore be necessary to improve the generalisability of our findings. It is also important to note that the BCF model we have used makes a number of important assumptions. Crucially, the model assumes we have accounted for all possible sources of confounding. However, while we have strived to control for as many confounding variables as we can, it is certainly possible that some important confounding variables might not have been collected as part of the TIMSS study. Finally, as we noted earlier, the question given to teachers on how often they ask students to "go beyond" the instruction is somewhat subjective. This should be borne in mind when interpreting our results, as teacher specific reporting differences may exist.

Potential avenues for future research include the application of a similar method to investigate the effect of other teaching practices, such as how often teachers use mixed or same ability grouping during class time, or how often teachers use various strategies such as relating new concepts to a student's prior knowledge. Given TIMSS includes data on student achievement in both mathematics and science, a similar investigation focused on science achievement would also be of interest. Alternatively, using data from PISA may provide valuable insights.

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Conceptualising Success in Mathematics Education: Three Immigrant Families Discussing Saturday Schooling of Their Children in Norway

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In Norway, Saturday schools, which are usually established to provide possibilities for children from immigrant families to learn about heritage languages and culture, can also include mathematics learning. However, the reasons why parents want their children to learn mathematics in their home languages has not previously been investigated. Consequently, we present case studies of three immigrant parents. Each parent was interviewed individually. In each case, the main reason for enrolling their children was to gain more opportunities to develop their language skills. However, they had individual reasons for sending their children to learn mathematics which were connected to what the parents considered would increase opportunities for their children's future education. For each parent, mathematics was important for their children to understand and use in order to gain access to future educational opportunities.

Keywords: parents, Saturday schools, educational futures

Introduction

The focus of this research is to explore why three immigrant parents to Norway chose to enrol their children in Saturday schools to learn mathematics. Saturday schools are common throughout the world, including in Ireland (O'Brian & Long, 2012). Some, although not all, provide opportunities to learn mathematics. With increasing numbers of children who themselves or whose parents are immigrants in many European countries, such as Ireland and Norway (Heckman, 2008), there is likely to be an increase in enrolment in such schools. This is because Saturday schools, set up by parents, sometimes in collaboration with the home country education system, provide one way of maintaining language and cultural knowledge (Okumura & Obara, 2017). Knowledge about Saturday schools can provide a way for curriculum content to act as a bridge between an immigrant community's cultural knowledge, mathematical practices and social issues (Quintos & Civil, 2007). Yet, in their study of a Polish weekend school in Ireland, O'Brien and Long (2012) found that integration of the Irish and the Polish ways of teaching and learning mathematics was done at the student level. Other possibilities which do not put the weight of cultural integration around mathematics on students, are needed. One way to do this is to understand why parents enrol their children in Saturday schools.

Generally, students are enrolled in the local schools from Monday through Friday and attend schools in their home languages, usually on Saturday, hence their name as Saturday schools. However, in some places, they are given other names, such as weekend schools (O'Brien & Long, 2012). Aoki et al. (2023) suggested that the teaching of mathematics at Japanese supplementary schools, based on the Japanese national curriculum, would contribute to children being able to seamlessly continue their education when they returned to Japan. Similarly, O'Brien and Long (2012) noted that changing economic situations often meant that

Polish immigrants returned to Poland after the Great Financial Crisis of 2008, indicating that the need for children to integrate back into Polish schools was likely to be one consideration for parents in enrolling their children into the Saturday school. Taking a different perspective, Okumura and Obara (2017) stated that the intention for teaching mathematics in Japanese supplementary schools was that "the mathematical knowledge acquired in the supplementary schools can be applied to learning mathematics in local schools" (p. 25). The mathematics learnt in Saturday schools is considered not to be in opposition to what is taught in local schools but to be in alignment with it. However, as noted earlier O'Brien and Long's (2012) research showed that often the responsibility for integrating the two systems fell on the students. Although these two perspectives are mentioned as possible reasons for students to learn mathematics at Saturday schools, there remains a lack of research on the reasons why parents value their children attending these schools to learn mathematics (Ferstad, 2020). Therefore, our research question is: how do parents who send their children to learn mathematics in Saturday schools describe their educational aspirations for their children?

Previous relevant research

Research which has been done on the mathematics taught in Saturday schools (such as Okurmira & Obara, 2017; Aoki et al., 2023), has focused on how one or more languages are used to discuss mathematics. For example, Farsani (2015) investigated how gestures together with English and/or Farsi, the Persian language, are used as a resource to teach mathematics in a Persian Saturday school in England. He found that the gestures connected to a specific language created a different conceptual understanding of mathematical concepts. Similarly, Parvanehnezhad and Clarkson (2008) studied when children used Farsi, to solve mathematics tasks at the Persian Saturday school in Australia. When the difficulty of solving the problems increased, the children used Farsi more frequently. These studies provide insights into how the home languages were used in these Saturday schools when learning or using mathematics, but do not provide insights into how they contribute to the replication and maintenance of cultural knowledge, through the teaching of mathematics. In contrast, O'Brien and Long's (2012) study investigated different actors' views about attending Saturday schools. However, the focus was on students' views and parents' educational aspirations for their children were not discussed.

Yet, studies that investigated immigrant or minority parents' views about mathematics education have highlighted differences to what is taught and how in the schools that their children attend during the week. Civil et al. (2005) compared how minority parents in Barcelona (Spain) and Tucson (USA) perceived their children's mathematics education. In Barcelona, the parents largely accepted that their children had learnt new calculation methods, which resulted in the old methods being replaced. In Tucson, some parents considered that the teaching their children received in the local school was not good enough and the parents' calculation methods were more efficient. Similarly, Takeuchi (2018) referred to previous research that supported parents wanting to preserve mathematics from their home country. This suggests that some parents may not see their local schools as being respectful of the efficient methods that children bring with them from home and previous schooling experiences.

However, sometimes parents accepted that the children needed to learn new ways of learning and doing mathematics and so did not resist their own methods being replaced by these new ones. In Takeuchi's (2018) empirical research focusing on Filipino immigrants in Japan, when supporting their children to learn multiplication, preferred the Japanese memorization methods rather than using a calculation method with their fingers which they brought with them from their homeland. This seemed to be related to how they were positioned in society and their lived histories which influenced what they considered to be the valuable mathematics for their children to know. In Civil et al. (2005), a mother described how she wanted her child to focus on the mathematics that they taught in the USA, rather than continue with using mathematics from their home country. This was because the child would grow up in the USA and they did not consider that their family would return to Barcelona. The need to replace known methods with those taught in the local school seemed to be related to immigrant parents seeing their children's futures as being in the new country and to do well in that country required them to abandon what they themselves were familiar with.

Although not referring to mathematics specifically, Hall et al. (2002) referred to an example from a Tamil Saturday school, where the children attended because they wanted to improve their results in the Norwegian school. Thomas (2019) found that Tamil parents in Norway focused on ensuring that their children gained the most beneficial educational opportunities, "Whether we can afford it or not, Tamil parents do everything in their power to accommodate the educational needs of their children. They can work odd hours and sacrifice" (Thomas, 2019, p. 14). Therefore, this research goes some way into understanding how parents connect enrolling their children to learn mathematics in Saturday schools to the educational aspirations that they have for those children.

Methodology

Following the research of Takeuchi (2018) and Civil et al. (2005), qualitative interviews were used to elicit parents' opinions about why they sent their children to Saturday school. According to Johannessen, et al. (2016), the purpose of qualitative interviews is to obtain full descriptions, so that an issue can be illuminated from several aspects.

The interviews were semi-structured and conducted with three mothers (R1, R2, R3) in 2019 (Ferstad, 2020). At home, the families used Russian and Japanese (R1), Ukrainian and Russian (R2) and Tamil (R3). At the time of the interviews, R1 had been in Norway, just less than nine years; R2 for five years and R3, just over twenty years. R1 had two daughters, 10 and 7 years old. R2 had two sons, 6 years old and 1 years old. The oldest son had attended the Russian Saturday school since he was 5. R3 had three daughters, 25, 21, and 14, who had all attended the Tamil Saturday schools but the youngest one was the only one still at school. All of the children had their weekly schooling, in Norway, and had continuously attended the Saturday schools. The interviews were recorded and transcribed. Parents chose the language of the interview, R1 chose English, the other two Norwegian. The interviews lasted for 30 minutes (R3), 45 minutes (R2) and 90 minutes (R1).

In the interviews, parents were asked about the experiences of their children, about their motivations for enrolling their children, their children's participation both in their Norwegian education as well as their children's sense of belonging to the cultural groups represented in the Saturday school. Following Hill's (2020) research with teachers of high-performing, immigrant students in Germany, a thematic analysis was undertaken of the interview data. For this paper, we focus on the parents' views on: enrolment in the Saturday schools; learning the specialised mathematics terms in the home language; gaining their children an advantage in learning and using mathematical skills and knowledge. Transcript extracts for R2 and R3 were translated into English.

Findings

Although the parents sent their children to different Saturday schools, there were often similarities in the points that they raised. For example, the main reason for the parents to enrol their children in Saturday schools was to maintain and develop the children's skills in the home language.

R1: So, why we started, I mean basically everybody starts because of the language. You know, what you realise as a.. somebody who trains to maintain, your own mother tongue in your children, is that the, your regular family language exposure is very limited.

R2: We come from Ukraine and for us it is very important that our children, we have two, have knowledge of Russian and Ukrainian languages. That was the reason why we sent the children to Saturday school.

R3: I think it is very important that children learn their mother tongue.

As well, the parents highlighted the role that the Saturday schools played in developing their children's awareness of the importance of culture, heritage, and friendships with other children who spoke their home languages. This was often because there was a wish to maintain links with the home country. R3 stated, "we have been to my home country again and then my mother-in-law got a little angry because my daughter could not speak Tamil at first." The valuing of Saturday schools as supporting the development and maintenance of the home language and culture is not surprising as most Saturday schools were established with this intention (Okumura & Obara, 2017).

Each mother also described enrolling their children in mathematics at the Saturday school because it would support the development of the children's home languages and cultural knowledge. This can be seen in the following quote from R1:

R1: And, math is added, by all schools, math is added.. mostly on demand, because children are differently gifted mathematically. And for majority actually there's this need to go through things again. And then, but also I feel, for my daughter because she didn't need the mathematics, we still left her in the mathematics class because, ... The way you talk about mathematics in a language, right, it gives you a certain vocabulary. You use the phrases. You kind of need to know, what's multiplication, what's addition, what's subtracting…

Aoki et al. (2023) found that the Japanese teacher in the supplementary school in Sweden was able to emphasise through the language specific aspects of fraction representations. This was likely to extend the children's understanding of fractions, which were not easily recognisable in the Swedish language. Although these parents did not express it in the same way, they also considered learning mathematics in the children's "home tongue" to contribute to the children's mathematics learning in their Norwegian schools. For R2, Saturday school provided an advantage, as their child started in Saturday school a year earlier than starting in the Norwegian school.

R2: My son started at Saturday school last year, and the regular Norwegian school this year. So, when he started Norwegian school he understood the calculation method and which way to write 1, 2, 3, it was quite easy for him. And we asked our teachers if he can get extra tasks at regular school, and they said, "yes, he can". But as we understand that we are lucky because not all our friends have the same situation at school, some teachers say that, "he can just wait until next year", so we are very grateful to our teachers.

This advantage from attending Saturday schools, noted by the Ukrainian mother, was in alignment with what some Tamil parents in Norway have said about doing what they could to support their children's educational possibilities (Thomas, 2019). However, as noted by R2, not all the parents felt that the Norwegian schools were willing to provide extra opportunities for children who had earlier learnt mathematical knowledge and skills at Saturday schools. R1 repeatedly stated that the Norwegian school did not challenge her older daughter. In one story, she said that she begged her daughter to take her Japanese textbook to her mathematics class at school, but "she refused point blank because, "everybody would ask me, everybody would talk about it"". Her daughter did not want to stand out. This frustrated her mother who felt that her child was not receiving the mathematics education she needed. R1 found out about an extension programme available at the Norwegian school, but was unable to work out how to have her oldest child participate, because she did not have the knowledge of the schooling system:

R1: If you're a resourceful parent and you fight for your child, you get something and if you are not, I was starting as a foreigner and I didn't know anything about the Norwegian school, I.. I didn't know what to do ..

We found out too late that there was one child that was doing it. I would have jumped on and they would have gone together if the school had encouraged them to do it.

In O'Brien and Long's (2012) study, the alignment of two educational systems was on the shoulders of the students. However, the parents in our study are suggesting that they wanted to engage with the schools and find ways to make bridges between the two systems but did not always find the appropriate ways to do that. This suggests that the local schools could support parents to provide more input about their children which could be utilised in what was being offered to the children.

For R1, the Japanese Saturday school provided her two children with mathematics education which met their needs, which they did not receive at their Norwegian school. Thus, the experiences from the Saturday school reinforced this mother's concerns about their children's educational futures. Her concerns were in alignment with those raised by some of the parents in Civil et al.'s (2005) research. Both R1 and R2 considered, that because of possible changes in the parents' work, their children may have to move countries, there was a need for the children to have achieved an appropriate level of mathematics, in the new country, an issue raised also by O'Brien and Long (2012) and Okumura and Obara (2017).

R2: In Ukraine, we have very strong mathematics subjects. Like for example, if we decide to go back to Ukraine then it is good if my son understands mathematics at the same level as in Ukraine. Or if we decide to live in England for example. It is the same situation as mathematics being a very strong subject in England.

For the Tamil mother, the situation was different because she saw her children's futures being in Norway and so it was important that her children gained good results to have the best possible educational futures in Norway. Saturday schools provided support for her children to achieve better results in the Norwegian school system. This parent did not consider that the family would leave Norway and so the educational advantage for her children came from improving their grades in the Norwegian school, something that she did not think was possible without this extra support.

R3: Have sent the children to extra maths lessons because it is easier to do homework and …, if one is to explain, for example in the same mother tongue then it is easier to understand. But the children are good at Norwegian too, they understand. I have heard he is a very good maths teacher, and he is in different meetings and explains problems, and the children can easily understand, solve the problems. There are so many students who have had teaching with him before, and then they got a very good grade. At first, they only got 2, but eventually when they got extra tuition, they had 5 and 6.

In the Tamil Saturday school, the mother reported that the parents appreciated the teacher because he spoke Tamil and Norwegian. This meant that parents could get information from him about how best to help their children with their Norwegian school homework. As well, the teacher knew the Norwegian school system, so he could provide appropriate extra tuition to help the students to get better grades. This was important for the children's future educational opportunities in Norway. Parents and students frequently sought out extra mathematics support, particularly when it was time for Norwegian examinations. This role for Saturday schools had not previously arisen in regard to teaching mathematics, but clearly some immigrant parents have little opportunities to return to their home countries or to move for work easily.

Conclusion

In the interviews, the parents made clear that providing support for the children's knowledge of the home language was the main reason for enrolling their children in Saturday schools. Although learning the specialised language of mathematics in the home language was part of developing their children's fluency, the parents also indicated that learning mathematics at Saturday schools provided opportunities for children to achieve the educational aspirations that the parents had for them. This could include, the children gaining the knowledge and skills needed to be successful in other countries, where the parents considered a higher level of mathematics was required than was expected in Norway, as well as to gain higher grades in the Norwegian education system, necessary for opportunities for further education. In raising these points, it seemed that the parents had not necessarily accepted Norwegian school expectations about mathematics education and the possible educational achievement that their immigrant children could gain at the local schools.

Although only a small study, it is likely that there would be other immigrant parents who had similar educational aspirations for their children. In some cases, those parents would not have the possibility to gain extra mathematics education from the Saturday schools because of a lack of availability in the areas in which they lived. However, in countries such as Norway where schools are expected both to engage with parents about their children's education, there seems to be a need for more schools to find ways to do this more successfully with immigrant parents.

Saturday schools have remained mainly unknown in mathematics education research. If they have been researched the focus has generally been on issues to do with language in mathematics. However, our findings indicate the need for more general research about Saturday schools. For example, R2's experiences suggest that research is needed to identify the circumstances in which local schools are successful in making use of the knowledge and skills that children bring from Saturday schools. This could contribute to students not having to carry the responsibility to bridge the two educational experiences. Findings from this research could then be included in professional development and initial teacher education.

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Undergraduate mathematics students' views on assessed group work

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Previous research has examined the implementation of collaborative learning and the conditions under which it may be optimally effective. One means of implementing collaborative learning in the university setting is through assessed group work, whereby students are asked to work together to submit a piece of coursework for assessment. In the context of undergraduate mathematics, assessed group work is a relatively novel means of assessment, and previous research has documented undergraduate mathematics students' preference for the retention of the status quo of assessment through closed-book examinations. This paper builds on previous research, related to both collaborative learning and assessment, by examining undergraduate mathematics students' views on assessed group work. To this end, semi-structured, one-to-one, interviews were conducted with ten recent graduates from undergraduate degrees in the mathematical sciences. Data analysis was conducted using reflexive thematic analysis. Findings indicate an uncertainty among students as to the purpose of implementing assessed group work in undergraduate mathematics degrees. Additionally, participants demonstrated concern regarding the assessment and allocation of groups in any proposed implementation of assessed group work. We discuss potential implications for practice, highlighting the importance of considering students' views when evaluating and implementing alternative assessment methods.

Keywords: Assessed group work, Collaborative learning, Assessment, Undergraduate mathematics education

Introduction

While there is no well-established definition of collaborative learning, we will follow the broad conceptualisation of Johnson et al. (2014) and define it to be a set of methods in which students "work together to maximize their own and each other's learning" (p. 87). Much of the research which has been conducted on collaborative learning to date has focused on its implementation and evaluating the conditions under which it might be optimally effective (e.g., Johnson et al., 2007; Springer et al., 1999). One means of implementing collaborative learning in a university context is through assessed group work, whereby students are asked to work together to submit a piece of coursework for assessment.

Undergraduate mathematics degrees in the UK and Ireland, the setting of this study, are predominantly assessed through closed-book examination (Iannone & Simpson, 2011, 2022; National Forum for the Enhancement of Teaching and Learning in Higher Education [National Forum], 2016). This means that assessed group work is a rather novel concept for an undergraduate mathematics student. Previous research, examining undergraduate mathematics students' perceptions of oral performance assessment (Iannone & Simpson, 2015b), has advocated for the representation of students' views when evaluating and implementing alternative assessment methods. We hope to add to research on undergraduate mathematics students' assessment preferences, and collaborative learning in the context of mathematics, by addressing our research question: 'What are undergraduate mathematics students' views on assessed group work?'.

Literature Review

Slavin (1996) remarked that it could be considered a point of irony that researchers who have examined aspects of collaborative learning have "often operated in isolation from one another, almost on parallel tracks" (p. 44). There is no singular, well-established definition of collaborative learning and different researchers have focused on various aspects in their analyses of the issue (Slavin,1996). However, we will follow the broad conceptualisation of Johnson et al. (2014), defining collaborative learning as encompassing a set of methods in which students "work together to maximize their own and each other's learning" (p. 87). We additionally distinguish assessed group work as lecturer-mandated group work where students work together to submit a piece of coursework for assessment.

Previous research conducted on collaborative learning has generally focused on the implementation of collaborative instructional methods by teachers and lecturers, evaluating their effects and the conditions under which they may be optimally effective (e.g., Johnson et al., 2007; Springer et al., 1999). Slavin (1996) outlines four broad types of theoretical perspectives from which researchers can come to the issue of collaborative learning: motivational, social cohesion, developmental, and cognitive elaboration. Each perspective places emphasis on different aspects of collaboration between students, and this is reflected in researchers' recommendations for the implementation of collaborative learning. The focus of motivational perspectives is primarily on the "reward or goal structures under which students operate" (Slavin, 1996, p. 44), and places emphasis on all group members feeling a sense of individual accountability to the group. Social cohesion perspectives are concerned with the importance of positive interpersonal relationships between peers and contend that when students feel a connection with one another, learning and peer-teaching occurs as a natural consequence. The emphasis of developmental perspectives is on the learning which results from the resolution of cognitive conflicts and, as a result, promotes opportunities for students to share and hear opposing viewpoints. Adopters of a cognitive elaboration perspective believe that an effective means for students to learn is through "explaining the material to someone else" (Slavin, 1996, p. 50) and emphasise peer-teaching as a result.

Much of the research referenced above relates to a general educational context. Research on students' assessment preferences has demonstrated the importance of not automatically accepting findings from the general education literature as valid in a mathematical context (Iannone & Simpson, 2015a). Closed-book examinations are the primary means of assessment in undergraduate mathematics degrees in the UK and Ireland (Iannone & Simpson, 2011, 2022; National Forum, 2016). In spite of a general tendency, documented in the general education literature, for students to prefer less traditional forms of assessment, undergraduate mathematics students appear to favour the retention of the status quo of assessment, primarily, through closed-book examinations (Iannone & Simpson, 2015a). Students were found to prefer, and deem most fair, assessment methods they perceived as being best able to distinguish between *individual* students' mathematical proficiencies. To that end, it was found that "any assessment that can be completed in groups…may be deemed to be unfair" (Iannone & Simpson, 2015a, p. 1058). Additionally, some mathematics students have exhibited concern regarding the possibility of other students' performance impacting their grade in the context of assessed group work (Iannone & Simpson, 2017).

While little is known about undergraduate mathematics students' engagement in, and perceptions of, collaboration in the context of mathematics, mathematicians' views on the issue have been documented to a greater extent. The collaborative nature of mathematics emerged as a prominent theme in the work of Burton (1999), who conducted an interview-based study with 70 mathematicians from the UK and Ireland. In contrast to the often-purported image of the solitary mathematician, only four of the 70 interviewed mathematicians reported working on mathematics entirely alone and many spoke of a cultural shift towards a collaborative norm.

Methodology

As the aim of this study is to understand undergraduate mathematics students' views on, and experiences of, collaborating with their peers, a descriptive phenomenological design was chosen. One-on-one, semi-structured interviews were conducted in spring 2022 with ten recent graduates (graduating in 2019, 2020 and 2021) from degrees in the mathematical sciences from a large, Irish university. Initially, graduates known to the authors were invited to participate. From there a form of snowball sampling was used, with recruited participants asked to recommend other graduates who may be interested in participating. We sought participants' views on both assessed group work and informal, voluntary collaboration with peers undertaken outside of their lectures and tutorials. The interviews, which were conducted by the first author and lasted on average 30 minutes, were audio-recorded and transcribed.

We undertook reflexive thematic analysis (Braun & Clarke, 2022) based on descriptive phenomenology in a manner which was guided by Sundler et al. (2019). We were particularly guided by their discussion of paying attention to the importance of researchers recognising and questioning any "personal beliefs, theories or other assumptions that can restrict the researcher's openness", rather than "attempting to set aside one's experiences and assumptions" (Sundler et al., 2019, p. 735). At the time of data collection, the first author was a final-year undergraduate mathematics student and, having had personal experience of working with peers on group assessments, would be considered an insider researcher (Braun & Clarke, 2022). The second author is a mathematics lecturer and mathematics education researcher. The method's emphasis on researcher reflexivity allowed for these two disparate perspectives to be capitalised on and used as a "resource for knowledge production [...] rather than a must-be-contained threat to credibility" (Braun & Clarke, 2021, pp. 334-335).

Braun and Clarke's (2022) six-stage process was followed. In line with Braun and Clarke (2022), the second author took on the role of an "experienced qualitative researcher [and] supervisor" (p. 273), both participating in and supporting the first author in the analysis process. Both authors familiarised themselves with the data corpus through repeated reading of the interview transcripts and re-listening to the audio-recordings. Throughout this process, notes were made on any initial ideas, and both authors wrote reflections on their position in relation to the data and topic. For the purposes of this paper, our research question is concerned with participants' perceptions of assessed group work. Therefore, the data set which was then coded became "all instances in the corpus where that topic is referred" (Braun & Clarke, 2006, p. 79). This data set was coded by the first author using the software package NVivo. An inductive approach was taken, and no pre-existing theoretical framework was used. However, the first author was aware that her coding was likely influenced by prior reading of literature on assessment and collaboration and the issues highlighted therein. A mixture of semantic and more latent codes was used. For instance, while the code 'Individual marks don't really work' captures an explicit, overt meaning in the data, the code 'Different students have different goals' centred on the more implicit narrative that students should be free to work towards their own individual goals.

The first author then reviewed her codes and associated data extracts and wrote a short analytic summary of each. She then began to sort the code labels into candidate themes, which were explored in relation to one another and the entire data set, using visual thematic mappings. Through this process, focusing on whether there was something different about group assessment in mathematics as opposed to other subjects, she decided that a candidate theme was under-developed as it was not addressed fully enough in the data. The remaining three themes were then named and defined, and a final visual thematic mapping produced.

Findings

Existing research has identified closed-book examinations as the prevalent assessment method in undergraduate mathematics degrees in the UK and Ireland (Iannone & Simpson, 2011, 2022; National Forum, 2016). This is reflected in the fact that the mathematics graduates interviewed as part of this study reported limited experience of group assessment. Their experiences of assessed group work were largely confined to a small number of group projects, mainly statistics and programming related, completed in various modules, and the option, in a few instances, to submit problem sheets as a group as opposed to individually. In order to address our research question, participants were asked for their views on assessed group work based on their own limited experience and were also asked to respond to the hypothetical proposal of assessed group work being made a larger component of the 'assessment diet' in undergraduate mathematics. This paper reports three themes in relation to our research question: 'Is the purpose, necessarily, to learn more mathematics?', 'Not when my grade depends on it', and 'Navigating freedom and fairness in relation to group composition'.

A scepticism among participants as to whether group work has direct benefits to students' mathematical learning became the focus of the theme 'Is the purpose, necessarily, to learn more mathematics?'. This was particularly evident in discussions regarding the prospect of 'divvying up' work among group members in mathematics. For instance, some participants mentioned that when asked to complete a problem sheet as a group, perhaps the most obvious approach would be to divide up the relevant questions among group members. However, some participants did not deem this suitable in the context of mathematics. They felt it important that they understand all questions on a problem sheet, rather than just a subset: "If four of us do one question each, then I don't feel like I know how to do the other three questions" (Graduate 9). Other participants spoke of issues arising when each of the constituent group member's questions were checked prior to submission:

> It was mainly me checking their work to be honest. So I checked [another student's] work and then unfortunately there were a bunch of errors and issues with it that we then kind of needed to fix quickly. (Graduate 3)

Graduate 3's account implicitly raises the issue of heterogeneous student abilities in group assessments. Some participants deemed the purpose, and perhaps the value, of group work as allowing for the "delegat[ion of] some of the teaching" (Graduate 1) and effectively to:

> Get some stronger students and weaker students to work together and maybe the stronger students can learn by having to teach other people and the weaker students can learn from both the lecturer and the other students. (Graduate 3)

There was no concrete consensus as to whether assessed group work definitively benefits all students' learning and a number of participants hypothesised about whether the decision to implement assessed group work might stem from a place of pragmatism, in relation to grading, on the part of lecturers. Some participants did speak to the value they saw in group work in providing an opportunity for students to learn to work with others: "I'm not sure you necessarily learn more maths by submitting as a group, but I think you probably learn more skills, life skills as it were" (Graduate 4).

The value they saw in peer-teaching and in learning life skills meant that some graduates were not entirely opposed to the concept of assessed group work. However, many approached the prospect with a degree of apprehension. Their concerns, in relation to the assessment of group work, became the focus of the theme 'Not when my grade depends on it'. A recurring sentiment was that individual student's ambitions, in relation to assessment, ought to be respected. However, as Graduate 8 illustrates, tension can arise when individual student's goals are in conflict with one another: "some people [...] are happy enough to get a certain grade and that's completely fine, but it's not fair throwing them in with someone who wants to do better". The dilemma of how to reconcile heterogeneous student goals and ambitions appeared to lie at the heart of some students' issues with assessed group work: "There's two outcomes, either it's one or two people in the group working really hard that like, get everybody an A+ or it just brings everybody closer to an average" (Graduate 8).

Also relevant to the above theme were participants' discussions of how inequity in student workload can lead to conflict and feelings of resentment. It was felt by some participants that, *if* assessed group work were to be implemented, there should be some means of ensuring, or incentivising, individual members to be accountable to the group. Some suggestions included implementing a system of individual marks, or individual interviews between students and the lecturer in order to gauge understanding. As Graduate 1 phrased it, it was felt there ought to be "some reward for actually trying". However, some participants also expressed doubts as to the feasibility of implementing such mechanisms. While some felt that group work has the potential to be "super, super helpful" (Graduate 6), the graduates appeared dubious as to whether it would be possible to assuage their concerns in relation to assessment. Graduate 1 summarises this sentiment, saying "if there is a way of kind of preventing that, then I think it is really good. But I don't know what that would be".

It is possible to argue that many of the aforementioned student concerns regarding assessed group work seem to stem from assigned, heterogeneous student groupings. The issue of group composition forms the cornerstone of the theme ‘Navigating freedom and fairness in relation to group composition’. While the informal collaboration that students voluntarily engage in is beyond the scope of this paper, some graduates did contrast their experiences of assessed group work with a certain sense of freedom experienced in their informal collaborations. Graduate 8 discusses the flexibility, in the informal case, of being able to work with others if you wish but “if you don’t then it’s not like affecting anyone else either. It’s just your grade”. Others discussed a sense of ease in working with peers who you know well:

> [Assessed group work] was more kind of stilted because you didn’t know the people. When it was informal, you knew the people, everything flowed pretty easily. There was kind of an unwritten routine. (Graduate 7)

In spite of this, only two graduates reported being in favour of letting students self-select their own groups in the context of assessed group work. A number of participants deemed the prospect unfair, their reasons falling broadly into two categories. Some graduates felt that students were likely to pick their “smartest friends” (Graduate 2) and believed that it did not “seem like a very fair approach to [...] group people together by levels and tell them to work together on something they are going to be graded on” (Graduate 9). Others worried about potential social implications, and how “people in the module who didn’t know anyone else” (Graduate 7) may be unfairly disadvantaged. It was also felt that assigned, heterogeneous groups were reflective of the nature of collaboration students may be expected to engage in later in life. These beliefs are seemingly at odds with difficulties encountered when working in assigned groups, and the freedom experienced when working with peers in an informal setting. This apparent tension is captured by Graduate 8 who ponders:

> It’s a funny thing because [...] if you don’t get to pick your groups then you’re put with people who want to work differently. But if you just pick like, in essence I guess we kind of just picked our own group that was quite strong and then did well because of that so [...] yeah, I'm not sure. (Graduate 8)

Discussion

For those seeking an answer to the question of whether assessed group work has a role to play in undergraduate mathematics education, this paper offers no clear answers. However, our findings do suggest that any implementation of assessed group work requires careful thought, consideration, and an evaluation as to the purpose it is to serve. We, along with Iannone and Simpson (2015b), advocate that any such evaluation “should take into account the views of students” (p. 983), on which this paper does shed some light.

As previously outlined, the interviewed graduates questioned whether there are direct benefits of assessed group work to students’ mathematical learning. Adopters of a developmental perspective assert that “interaction among students on learning tasks will lead *in itself* to improved student achievement” (Slavin, 1996, p. 49). They emphasise learning through the resolution of cognitive conflicts which arise in students’ discussion of content. This diverges from our participants’ depiction of the process of dividing problem sheet questions among group members and the role of checking each member's contribution, in some cases, falling to one student. Research on students' assessment preferences has highlighted the importance of not automatically accepting findings from the general education literature as valid in a mathematical context. While not explicitly addressed in this paper we, along with Slavin (1996), highlight the need for "research at the intersection of cooperative learning and curriculum" (p. 62) in order to examine whether there is something different about assessed group work in the context of mathematics, as opposed to in other subjects.

Some graduates suggested that a potential purpose of implementing assessed group work could be encouraging students to engage in peer-teaching. This aligns with a cognitive elaboration perspective (Slavin, 1996) on collaborative learning. Slavin asserts the importance of rewarding a group based on the learning of its individual members in order to encourage students to engage with the group and to help one another. He argues against rewarding groups on the basis of a product which "could theoretically have been done by one group member" (Slavin, 1996, p. 54), as is the case with students working together to submit a problem sheet, and stresses that each group member should see their own personal success as inseparably linked to the success of the group. Relevant here is the graduates' discussion of the perceived inequity of a student feeling compelled to do additional work in order to compensate for differences in individual member's goals in relation to assessment. This raises interesting points to be considered by lecturers with a view to implementing assessed group work. Do they view the purpose of assessed group work as, in the words of one participant, allowing for the delegation of some of the teaching and, if so, how do they perceive the applicability of Slavin's thesis to an undergraduate mathematical context?

This paper also contributes to our understanding of students' preferences in relation to the assessment of undergraduate mathematics. Complementing the work of Iannone and Simpson (2015a, 2015b, 2017), it demonstrates that students' views in relation to assessment are thoughtful and nuanced, and ought to be considered in the evaluation and implementation of alternative assessment methods. As reported by Iannone and Simpson (2015a, 2017), participants in our study also indicated a preference for the assessment of their *individual* mathematical ability. They additionally exhibited concern regarding the allocation of groups and the assessment of any potential implementation of assessed group work. We therefore advocate for these two areas of concern to be addressed explicitly with students in any implementation of assessed group work in undergraduate mathematics degrees.

Finally, although beyond the scope of this paper we note some participants' favourable discussions of their engagement in informal collaboration with peers, outside the context of assessed group work. Of particular interest is the apparent alleviation of concerns regarding assessment and unfamiliar, heterogenous groups. Noting Burton's (1999) mathematicians' depiction of mathematics as a collaborative discipline, we hope to examine mathematics students' engagement in informal collaboration further in our wider study.

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Supporting teachers to engage with Structured Problem Solving in their Junior Cycle classrooms –developing Educative Curriculum Materials for use with Lesson Study

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Successive curriculum reforms in Ireland have led to an increased emphasis on problem-solving approaches to teaching mathematics. However, there is little research to suggest that classroom practices have changed significantly. In this paper we outline the design of a set of Educative Curriculum Materials (ECMs) as part of a professional development intervention, which aims to support teachers to incorporate Structured Problem Solving into their classroom practice. These materials are designed to be used in Lesson Study as part of a professional development intervention. We begin by describing Structured Problem Solving and the challenge it poses for teachers, before outlining the role professional development, and specifically ECMs, can play in supporting teachers with this approach. Finally we highlight some key features of the ECMs currently being developed.

Keywords: Structured Problem Solving, Educative Curriculum Materials, professional development, curriculum reform

Introduction

Over the past 15 years curriculum reforms in Ireland have encouraged teachers to adopt a problem-solving approach to teaching mathematics in post-primary classrooms. However, there is little evidence that classroom practices have changed as envisaged by the reforms (Byrne & Prendergast, 2020; Jeffes et al., 2013). Recent research has highlighted the high levels of concern teachers hold about these reforms and their discomfort implementing problem-solving approaches (Berry et al., 2021; Byrne & Prendergast, 2020; Neururer & Ni Shuilleabhain, 2022). Teachers are unsure how to incorporate problem-solving into their classrooms and lack well-designed resources to support them with this in their planning and teaching (Neururer & Ni Shuilleabhain, 2022). Several studies have examined how professional development interventions can support teachers to incorporate problem-solving approaches in their classrooms (Hourigan & Leavy, 2022; Sullivan et al., 2015). However, teachers' beliefs about what constitutes effective teaching and about their role in the classroom can impact their intention to implement changes to their classroom practice (Maass, 2011). Those initiating curriculum reform often fail to consider the teacher's seminal role (Spillane, 1999) and this failure at a structural, systems level may be contributing to the poor enactment of problem-solving practices in post-primary classrooms.

This paper outlines the design of Educative Curriculum Materials (ECMs) as part of a professional development intervention incorporating Lesson Study, which aims to support teachers to incorporate problem-solving into their daily practice. Building on the work of Fujii (2018), teachers will be invited to discuss, plan, conduct and reflect on SPS in the classroom within Lesson Study. The supporting material within the ECMs focus on the key role of the teacher in orchestrating discussions within SPS (further detailed below) and utilises the five practices highlighted by Stein et al. (2008) as a way to promote approaches to problem-solving. Building on research outlining the potential for curriculum materials to support teachers to adopt new practices (Lewis & Perry, 2017; Rezat et al., 2021), the intervention incorporates materials specifically designed for the Junior Cycle classroom. For the purposes of this study the materials align with the teaching of algebra at 2nd year (13/14 years), one topic where students' poor performance has been identified (Prendergast & Treacy, 2018).

Structured Problem Solving

The emphasis on problem-solving in successive curriculum reforms at post-primary level, first in 2000 and later in 2017, marks a significant departure from conventional mathematics lessons to a new way of considering teaching and learning in the mathematics classroom (Ni Shuilleabhain & Seery, 2018). In the former approach, the teacher demonstrated procedures for students to practice and there was little emphasis on mathematical reasoning and discussion (Prendergast & O'Donoghue, 2014). By contrast, the recent reforms view mathematics as "an interconnected body of ideas and reasoning process that students negotiate collaboratively with their teachers and their peers and as independent learners" (NCCA, 2017, p. 4). This approach requires an adjustment in the role of the teacher away from transmitting knowledge and demonstrating procedures, towards one of facilitating mathematical discussions and supporting students to engage with challenging mathematical tasks (Takahashi, 2021).

Stigler and Hiebert (1999) describe the outline of a problem-solving approach to teaching mathematics in Japan as *Structured Problem Solving (SPS)*. This pedagogical approach involves students learning new mathematical ideas, while grappling with carefully chosen mathematical problems. This contrasts with the "one-off", strategy-focused teaching of problem-solving which often occurs outside of the curriculum content (Takahashi, 2021). Structured Problem Solving broadly involves a four-phase lesson structure whereby a problem is posed, students work to explore the problem, different approaches and solution methods are discussed and, finally, the teacher summarises the lesson (Fujii, 2018). Hino (2015) adds an additional stage of "reviewing the previous lesson" in his outline of SPS in Japanese classrooms, while others (Stein et al., 2008; Sullivan et al., 2015) consider a 3-phase lesson structure in which the discussion and teacher summary are consolidated into a single phase. However, regardless of the lesson structure, students engaging with a problem and a carefully orchestrated whole class discussion about potential approaches to that problem are key elements of a SPS lesson. Indeed, the whole-class discussion has been described as the "heart" of the lesson (Takahashi, 2021).

Teaching through SPS is challenging. Teachers need to organise and manage the classroom discussion in a way that moves the learning forward. Stein et al. (2008) argue that without careful planning, including consideration and anticipation of which particular students' methods will be appropriate to discuss, in what order methods might be presented, and how disparate ideas and approaches might be connected to contribute to the mathematical goals of the lesson, the classroom discussion risks becoming "show and tell" with limited mathematical learning occurring. While in Japan the textbooks (which are approved by the Ministry of Education) support teachers in this approach (Watanabe, 2019), there are few resources or textbooks available to support teachers with SPS in other education systems (Takahashi, 2021). For many students and teachers, SPS is an unfamiliar pedagogy which necessitates a significant shift in their role in the classroom.

Rather than dispensing knowledge, demonstrating procedures, and sanctioning correct approaches, SPS requires teachers to facilitate students' engagement in problem-solving by orchestrating learning environments in which students can grapple with challenging problems and participate in productive mathematical discussions (Stein et al., 2008). This shift in role is challenging for teachers and, without expectations for what might happen or guidance on what a teacher could do, many teachers are left feeling they should avoid telling students anything, leading to a reduction in their self-efficacy (Stein et al., 2008).

Professional Development for SPS

Research suggests that effective professional development is grounded in teachers' practice, engages teachers as active learners, is collaborative and is ongoing (Darling-Hammond et al., 2017). Many studies have demonstrated that Lesson Study is an effective way to enable teachers to reconsider their role in the classroom and attempt new pedagogical practices (Lewis & Perry, 2017; Ni Shuilleabhain & Seery, 2018). However, recent research analysing lesson plans from post-primary teachers engaged in Lesson Study with an aim of incorporating SPS found the initiative not wholly successful (Ni Shuilleabhain et al., in press). Fujii (2018) describes Lesson Study and SPS as "two wheels of the same cart" in the Japanese context, where this model of professional development supports teachers in incorporating SPS in their practice. The cultural contexts of educational initiatives are important to acknowledge however (Stigler & Hibert, 2016) and there is therefore a need to consider other forms of structured professional development interventions which will allow teachers the opportunity to consider the required classroom reforms of SPS.

Educative Curriculum Materials in Professional Development

All teachers use curriculum materials of some form in their planning and teaching of lessons. Consequently, curriculum materials are well placed to influence and support teachers' practice. For those with a focus on teacher learning, a challenge arises in ensuring such materials are used effectively. This challenge is even greater when the materials in question aim to instigate change (Stein & Kim, 2009). However, curriculum materials have the potential to contribute to professional practice and teacher learning if they are designed with this purpose in mind (Ball & Cohen, 1996; Davis & Krajcik, 2005). Such curriculum materials which aim to support teacher learning as well as student learning are called Educative Curriculum Materials (ECMs) (Davis & Krajcik, 2005).

A key feature of ECMs is their transparency. Rather than merely providing instructions and strategies for teachers, they also provide the rationale for the various implicit pedagogical decisions (Davis & Krajcik, 2005). By making the pedagogical rationale for different suggested strategies explicit, ECMs help teachers make good decisions about adapting curriculum resources. By utilising ECMs in curriculum reform, teachers can be supported in changing their practices with reasoning and understanding, making adaptations to their practice within the spirit of the reform (Davis & Krajcik, 2005).

Although ECMs have the potential to contribute to teachers' professional learning, there is agreement that they are best used alongside other forms of professional development (Davis & Krajcik, 2005; Fuentes & Ma, 2018). Indeed, Fuentes & Ma (2018) highlight the need to investigate how teachers' use of, and learning from, ECMs can be enriched through other supports. Our research aims to investigate how engaging in Lesson Study can support the use of ECMs as a form of professional learning.

Design of Educative Curriculum Materials

The ECMs in this research have been designed to support teachers to engage with SPS in their classrooms. As discussed above facilitating mathematical discussions is a central component of SPS, however it is challenging for teachers and may demand a change in their role in the classroom. Thus, the materials aim to help teachers with the shift in role required of them, by incorporating specific guidance on how they might orchestrate these whole-class mathematical discussions. In particular, the five practices outlined by Stein et al. (2008) are employed. These five practices consist of *anticipating* students' mathematical responses to the task, *monitoring* students' approaches, purposefully *selecting* specific student responses for presentation, purposefully *sequencing* the order of student presentations, and *connecting* the student responses in a meaningful way. Each of these practices draws on, and benefits from, what was learned from the previous one. In the ECMs designed for this research, these five practices are highlighted throughout the materials with concrete examples provided and "callout boxes" encouraging teachers to engage and reflect on their use of these practices (see Figure 1). By reviewing the materials and tasks within them, teachers can develop a deeper understanding of the pedagogical practices required for SPS. They can choose to use the tasks in their research lessons or consider others, using the ECMs to support them with incorporating the five practices and with orchestrating productive discussions as part of SPS in their classrooms.

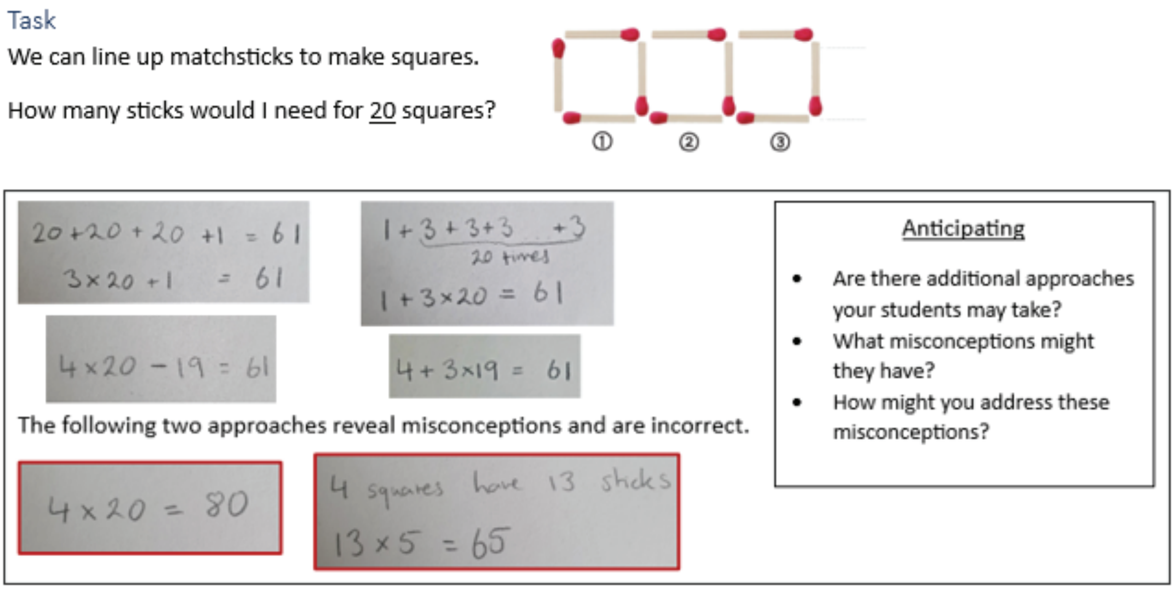
In designing ECMs, it is important to note that teachers frequently adapt and modify resources to suit their classroom contexts and learning goals (Calleja et al., 2023). Specifically in the context of SPS, Calleja et al. (2023) highlight how teachers often introduce scaffolds for their students in problem-solving and may deviate from the 4-phase lesson structure of SPS in their classrooms. While scaffolding students' learning can be an important element of building students' skills, it is important that such scaffolds offered by the teacher do not undermine the problem-solving element for learners (Foster, 2019). Rather than simplifying the task, or narrowly helping students to solve the problem at hand, teacher guidance should therefore aim to focus students' attention on key features of the task to enable them to engage productively with it (Calleja et al., 2023). In this context, the use of enablers and extenders can help teachers adjust the level of support provided to students without eliminating the problem-solving aspect of the task (Charalambous et al., 2022; Sullivan et al., 2015). The

ECMs designed for this research will suggest possible enabler and extender approaches for teachers to engage with. For example, an enabler might make a particular heuristic clear to a student or identify key features of the task so that a student struggling to start can be supported to make an attempt at the problem. Throughout the materials we endeavour to highlight and identify key features of possible scaffolded adaptations, so that teachers are supported in making informed decisions about how to incorporate, and possibly adapt, SPS in their classrooms in a way that the key features of SPS are not diminished (Calleja et al., 2023).

Building on the work of Davis and Krajcik (2005), and using the framework of Fuentes and Ma (2018), the materials are designed ensuring pedagogical decisions are transparent and with reasoning given for the inclusion of different elements. As outlined above, a key aspect of SPS is the discussion of student approaches in such a way that moves student learning forward. The materials therefore aim to support teachers with orchestrating such productive mathematical discussions, by incorporating specific guidance focused on the five practices of Stein et al (2008).

Figure 1

Sample of the features of the purposefully designed ECMs: Callout box drawing attention to the practice of anticipating; examples of possible student approaches and misconceptions



Future Phase of the Research

A range of ECMs will be designed and shared with teachers in two case study schools, who will trial them through school-based Lesson Study with their colleagues. Potential extenders and enablers will be included for various problems identifying how various tasks can be adjusted according to the students' prior knowledge and experience with problem-solving. By aiding teachers to anticipate students' responses in their collective planning in Lesson Study, it is hoped that the ECMs will prepare teachers to effectively facilitate whole-class discussions and, consequently, help them incorporate SPS into their classrooms (Fujii, 2018). It is also hoped that they will support teachers adopt the change in role demanded of them in incorporating problem-solving approaches in their practice, by providing concrete guidance for their in-the-moment decisions and pedagogical ideas.

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The bilingual advantage: Where we are and where to next.

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Evidence is now persuasive towards a bilingual advantage and adaptations in human cognition once a higher degree of bilingualism is established. However, contradictory evidence exists in supporting an advantage for bilingual students in mathematical attainment and there are no clear explanations for this etiology. Examining the executive function (EF) aspect may provide some valuable insights into the bilingual advantage, particularly with young children. While broader research has supported an EF advantage for bilingual children, there is a dearth in the cognitive literature within the discipline of mathematics education and younger learners. Concerns have also been raised in relation to the limitations of the research designs of many of the studies that currently exist. This paper examines some current research and where we are currently, while suggesting some key considerations for future research in this area. Some recommendations include the need to examine the underlying basis for positive outcomes relating to bilingualism and impact on mathematics attainment, with a focus on ambitious longitudinal studies to establish reliability of claims being made.

Keywords: Executive Function, Mathematics, Bilingualism, Young Children

Introduction

Approximately half of the world's children are raised in bilingual or multilingual homes. The Irish education system naturally caters for bilingual learning through the promotion of heritage language (Irish) medium education (immersion education) alongside mainstream English-medium education. Of particular interest in this context is the network of "Gaelscoil" primary schools across the country, who operate in the medium of Irish day-to-day but are in English-speaking communities. Currently, 8% of the primary school population in the Republic of Ireland learn through the medium of Irish and there has been a 28% increase in enrolments in Gaelscoileanna since 2010. With increasing migration and displacement, our mathematics classrooms are also becoming more diverse in terms of languages and cultures.

Evidence is now persuasive towards a bilingual advantage and adaptations in human cognition once a higher degree of bilingualism is established (Bialystok, 2018). However, contradictory evidence exists in supporting an advantage for bilingual students in mathematical attainment and there are no clear explanations for this etiology. Therefore, pedagogically, challenges exist in terms of catering for bi-/multi-lingual learners, and moreover, helping them reach their full mathematical potential. Although little is known about the connection between bilingualism and mathematical achievement in children (just that there is one), what has been established is the positive impact of bilingualism on executive function (EF) (Bialystok, 2018), a multifaceted set of mental skills inclusive of working memory, flexible thinking, and self-control (Miyake et al., 2000). I suggest that we need to revisit the conclusion made by Peal and Lambert (1962), the first researchers to provide reliable evidence of a cognitive bilingual advantage:

> The results of this study indicate the value of shifting emphasis from looking for favorable or unfavorable effects of bilingualism on intelligence to an inquiry into the basic nature of these effects. Perhaps further research may profit from this different emphasis. (p. 21)

This recommendation has yet to be realised to its full potential in general bilingual educational research and more specifically, in mathematics education research. Examining the EF aspect may provide some valuable insights into the bilingual advantage, particularly with young children, where significant development of EF skills is evident.

Executive Function, Mathematics and Bilingualism

Executive function relates to a set of cognitive processes that guide behaviours on tasks essential to learning, through a contribution to self-monitoring and regulation processes (Baggetta & Alexander, 2016). The most rapid development of these EF processes occurs in the early childhood years (Clements et al., 2016). Numerous studies with children have consistently demonstrated that bilingual children outperform monolingual children on well-established and validated battery tests associated with EF skills. These include inhibitory control (Simon Task, see e.g., Antoniou et al., 2016; Morales et al., 2013; flanker tests, see e.g., Costa et al., 2009; Attention Network Test, see e.g., Yang & Yang, 2016); mental set-switching (Dimensional Change Card Sort, see e.g., Carlson & Meltzoff, 2008; colour-shape task, see e.g., Barac & Bialystok, 2012); and working memory (complex span tasks, see e.g., Sorge et al., 2016). Similarly, studies focused on brain-imaging of infants and children demonstrate that acquiring two languages facilitates the cortical and subcortical brain regions operations that are associated with EF (see e.g., Arredondo et al., 2017; Krizman et al., 2015; Ramírez et al., 2016). From a bilingual education perspective, Nicolay and Ponclet (2015) found greater advantages in EF for 5-year-old children participating in a 3-year language immersion programme, than for the monolingual control group. Therefore, we can propose that acquiring two languages in early childhood can contribute to differences in EF between bilingual and monolingual children (Moreno et al., 2015).

Similarly, research has demonstrated a relationship between EF and mathematical skills. In order to solve mathematics problems, key cognitive resources are required. One essential resource is EF (Clements et al., 2016; Cragg et al., 2017). For example, working memory is required for remembering answers to different parts, when working out the solution to a complex problem (Cragg & Gilmore, 2014). Similarly, the EF processes of shifting and inhibiting have been found to be essential for mathematical achievement (Bull & Lee, 2014). Other essential resources for solving mathematics problems are domain-specific such as mathematical proficiencies (Cragg et al., 2017). Accordingly, several key researchers have offered a multi-component model to explain the skills underpinning mathematics inclusive of both cognitive processes and domain-specific mathematical knowledge (e.g., Geary, 2011). These frameworks include reference to mathematical procedures and concepts, and how they draw upon linguistic and spatial skills and in particular EF processes to support mathematics attainment. Therefore, these models suggest that EF is negotiated through domain-specific mathematical skills (Cragg et al., 2017) and that many elements contribute to mathematics ability (Geary, 2011).

Interestingly, longitudinal studies indicate that the connection between EF and mathematical attainment is one-directional, that is, EF contributes to the development of mathematical abilities, but mathematical abilities do not contribute to enhancing EF (Clark et al., 2010). Addendum to this, causal evidence of interventions designed to develop EF and accordingly increase mathematical achievement are weak (Jacob & Parkinson, 2015). Clements et al. (2016) conclude that developing both EF processes and mathematical proficiencies is essential for young children. They theorize several pedagogical mathematical activities that may support the development of EF, that is creating a bidirectional relationship, but conclude that much more research and developmental work is needed in creating high-quality mathematics education that would have a twofold benefit for mathematics attainment and EF processes. Given the well-established link between mathematics achievement and EF, I posit that the advantages in EF evident in bilingual children could impart benefits for mathematical cognition and ability for all students.

Most recently, work by Hartanto et al. (2018), when controlling for key covariates (e.g., demographic, socioeconomic status), found a positive relationship between bilingualism and mathematical competence for 4–7-year-olds. Similarly, Marian et al. (2013) examined the impact of a two-way immersion programme that combined English (majority language) with Spanish (minority language) on third, fourth and fifth grade students' performance on a State Standards Achievement Test. Findings demonstrate that bilingual students outperform monolingual students. However, when controlling for students from lower socio-economic backgrounds, this correlation was weakened. An fMRI study undertaken by Stocco and Pract (2014) found that bilingual students were significantly faster than matched monolingual students on tasks that required cognitive flexibility to combine arithmetic operations. Kempert et al.'s (2011) study illustrates the importance of bilingual students' (age 8) language proficiency for performance on more demanding mathematical word problems. When controlling for socio-economic backgrounds and cognitive and arithmetic ability, interestingly monolinguals outperformed bilingual students on simple word problems. However, this was not the case for more challenging word problems with distractors. It is possible to conjecture that the nature of the mathematical experiences of young bilingual children may impact on EF development, but a significant research gap remains in relation to understanding the EF advantages associated with bilingual children and connected benefits for mathematics learning.

Some key considerations

Both positive and null results in terms of the correlation between bilingualism and cognitive advantage continue to be reported (Gunnerud et al., 2020) suggesting some further factor(s) may be involved in the variable outcomes. Of note, previous studies have generally been restricted by limitations in the research design and methods, for example, sample size, age of participants, not controlling for key variables such as socio-economic status and language use, a focus on clinical settings, and single measurements of mathematics ability (Hartanto et al., 2018). In view of children's rapid cognitive development in pre-school/early primary, Hartanto et al. (2018) identify that "methodologically rigorous large-scale studies are vital" (p. 218) and should be longitudinal in design to establish reliability (Castillo et al., 2020).

Bialystok (2018) argues that the contradictory evidence in relation to the relationship between bilingualism and EF can be attributed to an "over-simplification" (p. 285) of the two concepts. The typical methodological approach employed to measure bilingualism tends to be language-type assessments. However, this portrays language and bilingualism as a categorical variable, that is limited and having a fixed number of possible values and assigning participants to a bilingual or monolingual group. Bialystok (2018) contends that this is "problematic for bilingualism: there is no agreed cutoff point determining when knowledge of another language is sufficient to designate "bilingualism", thereby introducing large variation into each group." (p. 285). She suggests that bilingual experience needs to be examined and taken into consideration when assigning participants to groups within a study in terms of degree of bilingualism. Accordingly, language proficiency tests, background information and language use/experiences should be examined in order to measure bilingualism and to group participants (Comishen & Bialystok, 2021). Taking such an approach to defining and measuring bilingualism may contribute to understanding the longitudinal relationship and to unpacking the causal relationship between bilingualism and mathematical cognition.

Similarly, EF is a complex construct that is difficult to examine, with many models currently existing (Bialystok, 2018). The most utilised model in bilingual research is that of Miyake et al. (2000), yet it has not proved to be effective at capturing differences between bilingual and monolingual learners (Bialystok, 2018). Bialystok (2018) argues for a focus on executive 'attention' which can be viewed as a continuous construct and would allow for examining how experience (e.g., classroom) impacts on the executive system and its functioning. This results in a focus on adaptation rather than transfer and I suggest that by combining both qualitative and quantitative measures as related to language experiences, EF will contribute to exposing the etiology of the relationship between bilingualism and mathematical cognition and lead to significant educational research breakthroughs.

Education itself is a multidisciplinary endeavour and multidisciplinary approaches are utilised in undertaking educational research. Challenges also persist with implementing novel methodologies in classroom settings and implementing such approaches with young children (Bhavnani et al., 2021). One such approach may be real time continuous monitoring of physiological cognitive responses (e.g. eye tracking, EEG) merged with observations of mathematics classroom experiences. Generally, cognitive neuroscience research is conducted in controlled laboratory settings. Such settings bear little resemblance to classroom and educational settings where key learning and development takes place. The use of traditional cognitive neuroscience methods in real-world social settings is starting to emerge due to the development of portable devices. However, these devices have a significant cost attached to them, as well as a specialised skillset in using them (Diiker et al., 2017). Nevertheless, Bhavnani et al. (2021) stresses the need for moving outside highly controlled settings and using multi-disciplinary research groups to undertake such cognitive neuroscience research in social settings to reap the benefits of such frontier research and its implementation at scale.

Where to next

There has been a focus more on the social aspect of learning rather than the cognitive impact of bilingualism within mathematics education research (Ní Ríordáin & Flanagan, 2022). If we are to examine in more detail the cognitive advantages of bilingualism and ultimately exploiting that advantage for learners, then we need to consider novel research designs and methodological approaches that facilitate such an exploration *in classrooms* and overcome key methodological limitations identified in previous studies (Bhavnani et al., 2021; Hartanto et al., 2018). This will involve connecting objective cognitive measures with phenomenological classroom data (Delahunty et al., 2018) and a design that is longitudinal in implementation. Such an approach would require a multidisciplinary approach and methodology that has never been utilised in the intended manner to examine bilingual mathematics learners; that is utilising and combining clinical approaches (such as EEG, heartrate and electrodermal measurement, eye tracking) with phenomenological data in classroom settings (rather than clinical/lab settings). Such educational settings and longitudinal studies carry high-risk and challenges (Diiker et al., 2017). If a conceptual link can be demonstrated and established, it would provide evidence of the cognitive etiology of the bilingual advantage as a 'lived' manifestation in young children, within the classroom environment. In particular, it could lead to a new understanding of the ways in which language(s) and EF interact and support mathematics attainment and to create advantages for bilingual learners. This new understanding can then influence pedagogic interventions that aim to capitalise on this relationship (Cragg et al., 2017) and to improve the mathematical outcomes for bilingual learners. Interventions in the early years provide the optimal opportunity for child development (Bhavnani et al., 2021). Moreover, it may provide some inwards in identifying "the basic nature of these effects" (Peal & Lambert, 1962, p.21) and conducting frontier research to provide a causal explanation for bilingualism and its impact on mathematics attainment.

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A Case Study of Changes in Assessment at University Level

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The COVID-19 pandemic necessitated major changes in assessment in university mathematics courses. In this paper, I will use Lithner's Reasoning Framework to analyse tasks from three instances of the same advanced calculus module from the years 2019, 2020, and 2021. The tasks from the years in question have been classified into those that require creative mathematical reasoning and those that can be solved using imitative reasoning. I investigate whether the opportunities for creative reasoning offered to students in this module changed significantly during the pandemic, and consider the implications of the move to e-assessment.

Keywords: advanced calculus, mathematical reasoning, tasks, e-assessment

Introduction

The COVID-19 pandemic had obvious short-term implications for university level mathematics education in Ireland (O'Shea, 2022). Traditional in-person final examinations were an early casualty in 2020; and research was carried out on lecturers' and students' views of the move to online assessment (Ní Fhloinn & Fitzmaurice, 2021a; Meehan & Howard 2020). Since then most final examinations have returned to their usual format; however, in the case of continuous assessment it seems as if the use of online quizzes and pdf upload assignments remains very common. The move to e-assessment is not new, but the pandemic certainly accelerated the change. Kinnear et al. (2022) recognise the need for research in this area and have developed a research agenda for e-assessment in undergraduate mathematics courses. One area of research identified is that of the capabilities of e-assessment, and in particular on the types of reasoning and understanding that can be fostered and measured using online assessment. Kinnear et al. (2022) note that researchers have analysed tasks used in traditional assessments but that very little analysis of this sort has been done in the area of e-assessment.

Studies have shown (Boesen et al., 2010; Jonsson et al., 2014) that the types of tasks assigned to students can affect their learning; and the use of tasks with lower levels of cognitive demand leads to rote-learning by students and a consequent inability to solve unfamiliar problems or to transfer knowledge to other areas competently and appropriately. De Guzman et al. (1998) advised university instructors to design problems that target the type of thinking they themselves value, and to resist assigning purely technical exercises, in order to help students appreciate the type of mathematical thinking required at university. It makes sense then, to investigate what kind of questions are used in undergraduate mathematics assessments. Some studies have been carried out on the types of tasks used in calculus courses at university level. In Sweden, Bergqvist (2007) analysed examination questions from 16 first-year calculus courses at four universities. She found that the majority of the examination questions (70%) could be solved using imitative reasoning alone, and that all but one of the examinations could be passed in this way. In Ireland, Mac an Bhaird et al. (2017) found that nearly 90% of graded tasks in non-specialist calculus courses could be solved with imitative reasoning, while in a specialist course the proportion was 36%. Note that both of these studies use Lithner's Reasoning Framework (Lithner, 2008) to classify the tasks, and neither focus on e-assessment.

In this case study, I will use Lithner's framework to classify tasks from three instances of an advanced calculus module taught in the Spring of the years 2019, 2020, and 2021. In that period, changes necessitated by the pandemic lockdowns meant that the continuous assessment for this module moved from hand-written assignments graded by tutors to online automatically graded quizzes and pdf uploads. The research questions for this study are:

RQ1: Are there differences in the proportions of creative reasoning tasks used in the three instances of this module?

RQ2: If we consider graded tasks only, is there a difference in the proportion of creative reasoning tasks between the three years?

RQ3: Is there a difference in the proportion of creative reasoning tasks between automatically graded quizzes and tutor-graded homework?

Theoretical Framework

The analysis of tasks in this paper is based on Lithner's Reasoning Framework (Lithner 2008). Lithner defines reasoning as 'the line of thought adopted to produce assertions and reach conclusions in task-solving' (Lithner 2008, p 257). This definition could apply to a wide range of student work from routine calculations to sophisticated proofs. He distinguishes between *imitative* and *creative reasoning*. Imitative reasoning (IR) has two main types: memorised and algorithmic. Memorised reasoning is characterised as

> The strategy choice is founded on recalling a complete answer. The strategy implementation consists only of writing it down. (Lithner 2008, p258)

While algorithmic reasoning is characterised as

> The strategy choice is to recall a solution algorithm. The remaining reasoning parts of the strategy implementation are trivial for the reasoner, only a careless mistake can prevent an answer from being reached. (Lithner 2008, p259)

In contrast, Lithner defines creative reasoning (CR) as having the following three properties:

> Novelty - A new (to the reasoner) reasoning sequence is created, or a forgotten one is re-created. Plausibility - There are arguments supporting the strategy choice and/or strategy implementation motivating why the conclusions are true or plausible. Mathematical foundation - The arguments are anchored in intrinsic mathematical properties of the components involved in the reasoning. (Lithner 2008, p 266).

The notes and other teaching resources available to students in the module in question informed the classification of the assessment tasks into the IR and CR categories.

Methodology

This paper is a case study which explores the changes in assessment over a three year period in a single module. We use Stakes's (2000) definition of different categories of case study, namely intrinsic, instrumental, and collective. In an intrinsic case study, the analysis focusses on a particular single case and no attempt is made to generalise about larger phenomena. Thus, this paper is an intrinsic case study, and the case is the continuous assessment in the MT202A module in Maynooth University.

The Module

I analysed the tasks associated with the course MT202A over the three years 2019, 2020, 2021. This module was delivered in the Spring semester of these years. The module is the fourth calculus module taken by students in the Mathematical Studies programme, and covers the topics of infinite series, multiple integrals and line integrals. I did not analyse tasks which appeared on midterm or final examinations.

In 2019, the module was taught in person with 23 lectures and five tutorials. There were five hand-in assignments (consisting of five or six tasks each) which were graded by a tutor, as well as practice problems for the students to work on at home or in tutorials. The assignments were worth 15% of the module mark, and tutorials took place after the hand-in dates.

In 2020, the module was scheduled to run as in 2019. It did so for the first six weeks of the semester but the COVID-19 pandemic meant that all teaching went online in March 2020. The final six weeks of the module took place during lockdown. Lectures were replaced with a suite of short videos and online notes. The hand-in assignments were replaced by pdf uploads which consisted of a single task each time. In addition, there were two multiple choice quizzes on the VLE (Moodle). Tutorials were replaced with online sessions on Teams. The assignments were worth 15% of the module mark and the quizzes were worth 10%.

In 2021, teaching remained online. The lectures were replaced by a suite of short videos supplemented by a class meeting once every second week on Teams. In these meetings, the lecturer summarized the main ideas and did some worked examples. The tutorials took place on Teams before the assignment deadlines; tutors and students worked through a set of tutorial problems. The assignments were replaced with four Moodle quizzes (designed using STACK) and one pdf upload assignment (which was graded by a tutor). There were also four practice quizzes, five sets of practice problems, and five sets of tutorial problems. The graded quizzes and the pdf upload together were worth 15% of the final grade.

In each of the three years, the lecturer assigned a small number of optional problems. These were aimed at high achieving students and usually required them to construct proofs.

The Analysis

To begin with, I gathered all tasks used in each iteration of the module. I created an SPSS file which listed each task, along with the year, the grading status (graded, optional, practice), the type of assessment (quiz, homework sheet, tutorial problems, practice sheet, practice quiz) and the medium of assessment (online quiz, on paper, pdf upload). For each task, I used the definitions of IR and CR above along with the resources available to the students to decide on the classification. I recorded both the reasoning classification and the reason for this in the data file.

Results

Number of Tasks

Let us first look at the number of tasks in the module in each of the three years. It is clear from Table 1 that the number of tasks increased in 2021, in fact there were more than twice the number of tasks associated with the 2021 instance of the module than were offered in the two previous years. This difference is statistically significant (chi-Square Test, n=469, dof=2, χ =82.712, p<0.001).

This difference is not a surprise, since, as we outlined above, each assignment in 2021 had an associated practice quiz and practice sheet plus a tutorial sheet. In fact, the numbers of graded questions in each of the three years were very similar (43 in 2019, 39 in 2020, and 41 in 2021) but the number of practice questions increased dramatically (67 in 2019, 61 in 2020, and 201 in 2021). Grading status is not independent of year with a much higher proportion of practice questions in 2021 than in the other two years (Fisher exact test, p<0.001).

Table 1

Classification of graded, practice and optional questions

Year	Grading Status	Imitative Reasoning	Creative Reasoning	Total
2019	Graded	29 (67.4%)	14 (32.6%)	43
	Practice	50 (74.6%)	17 (25.4%)	67
	Optional	0 (0%)	5 (100%)	5
	2019 Total	79 (68.7%)	36 (31.3%)	115
2020	Graded	33 (84.6%)	6 (15.4%)	39
	Practice	47 (77%)	14 (23%)	61
	Optional	0 (0%)	5 (100%)	5
	2020 Total	80 (76.2%)	25 (23.8%)	105
2021	Graded	32 (78%)	9 (22%)	41
	Practice	167 (83.1%)	34 (16.9%)	201
	Optional	3 (42.9%)	4 (57.1%)	7
	2021 Total	202 (81.1%)	47 (18.9%)	249
Total		361 (77%)	108 (23%)	469

Proportions of Imitative and Creative Reasoning Tasks

We can see from Table 1 that the proportion of CR tasks has decreased each time the module was run (from 31.3% in 2019 to 23.8% in 2020 and 18.9% in 2021). This difference is statistically significant as the classification of a task is not independent from the year (chi-square test, n=469, dof=2, χ= 6.903, p=0.032). We see that there is a statistically significant higher proportion of IR tasks in 2021 compared to the other two years. We will endeavour to investigate the reason for this difference. It would be tempting to say that the move away from written homework assignments to online quizzes could be the reason.

Proportions of Imitative and Creative Reasoning Tasks amongst Graded Tasks

If we look at graded tasks only (Table 2), we do not see a statistically significant difference in the proportion of CR tasks between the three years (chi-square test, n=123, dof=2, χ=3.438, p=0.173). We see that the biggest proportion of CR tasks in the graded assignments occurred in 2019 with the smallest in 2020.

Recall that in 2019, the graded tasks were all submitted homework tasks while in 2021 the majority came from graded quizzes. We investigated whether there is a difference in the proportion of IR tasks between tutor-graded homeworks and automatically graded quizzes (Table 2). The analysis shows that there is not a statistically significant difference between graded homework and graded quizzes. This is the case for all years combined (Fisher exact test, p=0.216), as well as each of the years 2020 (Fisher exact test, p=0.296) and 2021 (Fisher exact test, p=0.299). However note that in 2021, 40% of the pdf upload homework was classified as CR as opposed to only 19.4% of the Moodle quiz tasks.

Table 2

Classification of graded tasks by assessment type

Year	Assessment Type	Imitative Reasoning	Creative Reasoning	Total
2019	On Paper	29 (67.4%)	14 (32.6%)	43
	PDF Upload	-	-	-
	Moodle Quiz	-	-	-
	2019 Total	29 (67.4%)	14 (32.6%)	43
2020	On Paper	20 (80%)	5 (20%)	25
	PDF Upload	3 (75%)	1 (25%)	4
	Moodle Quiz	10 (100%)	0 (0%)	10
	2020 Total	33 (84.6%)	6 (15.4%)	39
2021	On paper	-	-	-
	PDF Upload	3 (60%)	2 (40%)	5
	Moodle Quiz	29 (80.6%)	7 (19.4%)	36
	2021 Total	32 (78%)	9 (22%)	41
Total		94 (76.4%)	29 (23.6%)	123

If we combine paper and pdf upload then overall the classification is independent of the mode of assessment (chi-square test, n=123, χ=2.8501. p=0.091). Therefore, we have not found significant differences in the classification of the tasks on graded written assignments (whether they are submitted in hard copy or via pdf upload) and those on graded quizzes.

Comparison of Practice and Graded Tasks

If we look at all three years together, then grading status is not independent of the reasoning classification (Fisher exact test, n=469, p<0.001) and this is also true if we look at each year separately. We see (Table 1) that the optional questions have a very high proportion of CR questions while the practice questions have a high proportion of IR questions. However, if we do not consider the optional questions at all and only include practice and graded tasks, then the reasoning classification is independent of the type of task (that is graded or practice). This is true in each year separately and overall.

Recall that the graded questions comprised written assignments and Moodle quizzes, and that the practice questions are taken from practice sheets, practice quizzes, tutorial questions and practice questions on the written assignment sheets. Looking at 2021 only (Table 3), we see that the practice sheets have the highest proportion of IR questions amongst the assessment types, but the differences are not statistically significant (Fisher exact test, p=0.179). However, if we combine all of the tasks which are not on practice sheets and compare them with the practice sheet tasks, then the proportion of practice sheet tasks that were classified as IR was significantly higher (chi-square test, n=249, χ= 4.6057, p=0.0319).

Table 3

Classification of the 2021 tasks

Assessment Type	Imitative Reasoning	Creative Reasoning	Total
Homework Sheet	17 (70.8%)	7 (29.2%)	24
Graded Quiz	29 (80.6%)	7 (19.4%)	36
Tutorial Sheet	36 (78.3%)	10 (21.7%)	46
Practice Sheet	95 (87.2%)	14 (12.8%)	109
Practice Quiz	25 (80%)	5 (20%)	25
Total	25 (73.5%)	9 (26.5%)	34

Discussion

In answer to the first research question, the task analysis has revealed significant differences between the proportions of IR/CR tasks in the three instances of the module. The proportion of CR tasks was not very large in any of the three years, although higher than corresponding modules in the Mac an Bhaird et al. (2017) study. We have seen that the proportion of CR tasks in MT202A has decreased during the pandemic. It would be tempting to conclude that the decrease was related to the move to e-assessment but when considering graded questions alone the difference between the years was not significant (RQ2). In particular, the quiz questions do not seem to have caused the decrease in the proportion of CR questions. There was not sufficient evidence for a significant difference in the proportion of CR tasks between automatically graded quizzes and tutor-graded assignments (RQ3).

We did see that there was a significant increase in numbers of tasks in 2021, largely as a result of structural changes which meant more practice questions, practice quizzes and tutorial problems. The analysis showed that the main source of IR tasks in 2021 were the practice sheets. These were designed in an effort to provide resources and support to students when they had no face-to-face teaching in lectures, tutorials or the Mathematics Support Centre. However, on reflection, these sheets may have been counterproductive given their lack of creative reasoning tasks (Jonsson et al., 2014), and it is possible that the large number of questions was overwhelming for students.

The extra work for the lecturer in designing these tasks and providing solutions should also be taken into consideration. Recall that the number of tasks in the module doubled in 2021. This, along with the work involved in preparing 1081 minutes of online instruction (766 minutes of video and 315 minutes of recap classes on Teams) for this module as opposed to the usual 1150 (i.e. 23x50) minutes of face-to-face lectures, represents a considerable increase in workload for the lecturer. It may explain why lecturers reported that teaching online during the pandemic was very time-consuming and stressful (Ní Fhloinn & Fitzmaurice, 2021b).

This current study clearly has limitations: it concerns just one module and one task-designer. It would be useful to extend the study to a range of modules to investigate the effect that the changes made in response to the pandemic had on reasoning opportunities in undergraduate mathematics assessments. It would also be prudent to investigate further how the move to e-assessment is likely to affect the provision of these reasoning opportunities in the future.

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Pre-service Teachers' Confidence in solving Numeracy Tasks

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In Ireland, teachers are required to teach numeracy across all subject disciplines. Therefore, Initial Teacher Education [ITE] standards require all teachers to possess an adequate level of numeracy themselves and profess positive dispositions towards teaching numeracy across the curriculum. This paper reports on a study that investigated 204 pre-service post-primary teachers' confidence in completing numeracy tasks. Analysis of questionnaire data from participants in three universities showed that pre-service teachers in the STEM disciplines are more confident in their ability to complete the numeracy tasks correctly than pre-service teachers in other disciplines. This is an issue of concern. If pre-service teachers of all disciplines are to teach for numeracy learning within their lessons, regardless of their subject discipline, then pre-service teachers of all disciplines will need to have an adequate level of numeracy themselves, along with confidence to support numeracy development within their specific subject discipline.

Keywords: Numeracy in Initial Teacher Education; Confidence in numeracy tasks; Numeracy for disciplinary learning

Introduction

It is clear that governments internationally have agreed that all citizens should have gained the necessary literacy and numeracy competencies to live and work in today's world (ACARA, 1997; Department of Education and Skills [DES], 2011). While there has been significant discourse both internationally and nationally regarding the development of numeracy within the school education systems, very often teachers are not confident in teaching numeracy across the curriculum. The Irish government has stressed the importance of all teachers teaching for numeracy learning across all subject disciplines (DES 2011, 2015). This paper reports on a study that explored the confidence of Irish pre-service post-primary school teachers in completing numeracy tasks correctly.

Background and Context

In 2011, the Teaching Council of Ireland, the regulatory body for the teaching profession in Ireland, published a policy on the continuum of Teacher Education. This document refers to equipping newly qualified teachers with competencies such as numeracy in order to address national priorities, such as a numerate society (The Teaching Council of Ireland, 2011). More recently, the Teaching Council of Ireland has set out Core Elements that need to be included in all Initial Teacher Education [ITE] programmes (The Teaching Council of Ireland, 2020). It states that pre-service teachers who are enrolled in an ITE programme should have the chance to improve their own numeracy capabilities and furthermore, should be supported in demonstrating their numeracy competencies within their subject discipline. This is a change from the Teaching Council's position in 2011. The most recent policy put forward acknowledges that pre-service teachers need to be supported in their mission to teach for numeracy learning, including their own personal numeracy capabilities and ITE programmes are required to support them in doing so (Teaching Council of Ireland, 2020). However, this can be a challenging task. For example, in the case of teaching for numeracy learning across the school curriculum, some teachers who are not teachers of mathematics or STEM disciplines may feel they do not have the expertise or adequate knowledge of numeracy to incorporate it into their lessons. Often teachers do not feel confident in their teaching capabilities to teach "mathematics", as it is referred to in the Literacy and Numeracy strategy, which in turn leads to higher levels of stress. Furthermore Callingham, Beswick and Ferme (2015) argue that if a teacher is not confident in their own mathematical capabilities, and teaches a subject such as Art or History, they will feel less equipped to develop mathematical ideas and less disposed towards focussing on numeracy. Therefore a challenge is evident whereby teachers' awareness of numeracy in their subject area needs to be addressed. Research conducted in Australia by Forgasz and Leder (2016) with teachers of primary and post-primary schools assessed their confidence in answering numeracy tasks. They found that 70% of teachers who studied mathematics at third level were confident in their answer being correct in comparison to only 40% of teachers who had not studied mathematics at third level feeling confident in the answer they provided. If teachers are expected to teach for numeracy learning across all school subject disciplines, then they must first have confidence in their own numeracy capabilities.

In some countries educators have been encouraging teaching numeracy across all subject disciplines and different contexts outside of the mathematics classroom, and they consider this approach essential in the development of students' numeracy capabilities (Bolstad 2019; DES 2015; Goos et al. 2019). As such, if students are to develop numeracy competencies outside of the mathematics classroom, then the teachers supporting these initiatives also need to have the confidence to support students in developing the necessary numeracy capabilities. As discussed previously, the literacy and numeracy strategy 2011 – 2020 is an effort by the Irish government and the DES to ensure that the young people of Ireland will be well equipped to use their numeracy skills in the workplace, which in turn will benefit the economic growth of the country (DES, 2011). Research has shown that in order for teachers to improve the numeracy capabilities of their students, teachers must first equip themselves with the necessary skills to develop their own understanding of how mathematical concepts and numeracy affect their own lives and their subject discipline (Goos et al, 2013; Forgasz and Hall, 2019). To gain insight into pre-service teachers' confidence in their own numeracy capabilities, responses to 7 numeracy tasks were analysed to address the following research question: How confident are pre-service post-primary teachers in their own numeracy competencies?

Research Design and Methodology

Pre-service teachers enrolled in the Professional Masters of Education (PME) programme in three different universities were invited to take part in this research study. Pre-service teachers were asked to complete a questionnaire at one of their general education lectures at the beginning of their second year on the PME. There were 204 pre-service teachers who completed this questionnaire.

Section C of the questionnaire consisted of 7 numeracy tasks in total; however, the numeracy tasks were presented in 6 questions. The first two numeracy tasks were part of question 1. Table 1 presents a short explanation of each numeracy task in the questionnaire. Pre-service teachers were asked to display their workings for each task in a text box provided.

Asking pre-service teachers to provide mathematical workings in the space provided enabled the authors to identify if the answer was correct or incorrect and furthermore allowed the authors to understand the pre-service teachers' mathematical thinking. Three of the numeracy tasks were published by the OECD as PISA test questions: Earthquake task (PISA, 2003), Car task (PISA, 2003) and the Salad dressing task (PISA, 2012). The other three numeracy tasks were developed specifically for this study.

Table 1

Explanation of each Numeracy task in the questionnaire

Numeracy tasks	Explanation
Time task	Calculate the difference between two Olympic swimmers' finishing race times for a 100 metre butterfly race. The results were presented in a table and the pre-service teachers had to subtract one decimal number from another decimal number (51.14 – 50.39)
Distance task	Joseph Schooling had a result of 50.39 in the 100 metre race and if the race was 30 metres longer, given that he was travelling at the same average speed as he did in the first race, calculate the new time he would finish the 130 metre race.
Earthquake task	A documentary about earthquakes and how often they occur is broadcast. A geologist stated "In the next twenty years, the chance that an earthquake will occur in Zed City is two out of three". Pre-service teachers were asked to use mathematical knowledge and understanding of statistics to predict an event occurring in this specific context. Pre-service teachers were provided with 4 different scenarios and asked to choose which one best reflected the meaning of the geologists statement.
Pie-chart task	Given a pie-chart, calculate the proportion of the pie chart (as a percentage) that represented the participants who chose biology as a subject for the Leaving Certificate.
Best Car task	Calculate the score of the "Best Car" given an equation. The "Best Car" is evaluated based on scores for safety features (S), fuel efficiency (F), external appearance (E) and internal fittings (T) and these were the variables in the given equation (Ca) = (3 x S) + F + E + T. Pre-service teachers had to substitute values into the equation and work out the final answer for the Best Car.
Salad dressing task	A recipe for 100mls of salad dressing has three ingredients which are Salad Oil (60mls), Vinegar (30mls) and Soy sauce (10mls). Pre-service teachers were asked to calculate how much salad oil is required to make 175mls of salad dressing.
Mobile Phone task	David uses 500 minutes per month and 15GB of data. Recommend the best mobile phone plan for David, given price tariffs for 3 mobile phone companies.

In addition to completing the numeracy tasks, pre-service teachers were asked to indicate their level of confidence (Confident, Not Confident or Unsure) in each answer being correct. Pre-service teachers' responses could therefore be analysed to compare correctness and confidence for each numeracy task.

Findings

Pre-service teachers were asked to indicate whether they felt "confident", "not confident" or "unsure" that their answer was correct for each numeracy task they completed. Some participants chose to leave the confidence questions blank. Table 2 displays the confidence levels which pre-service teachers chose when answering each numeracy task.

Table 2

Frequency of self-reported confidence levels for each numeracy task

Numeracy tasks	Confident N(%)	Not Confident N(%)	Unsure N (%)	Blank N(%)
Time task	161 (78.9%)	23 (11.3%)	19 (9.3%)	1 (0.5%)
Distance task	75 (36.8%)	63 (30.9%)	38 (18.6%)	28 (13.7%)
Earthquakes task	141 (69.1%)	31 (15.2%)	27 (13.2%)	5 (2.5%)
Pie-chart task	115 (56.4%)	47 (23%)	22 (10.8%)	20 (9.8%)
Best Car task	134 (65.7%)	31 (15.2%)	20 (9.8%)	19 (9.3%)
Salad dressing task	101 (49.5%)	45 (22.1%)	30 (14.7%)	28 (13.7%)
Mobile Phone task	28 (13.7%)	56 (27.5%)	37 (18.1%)	83 (40.7%)

Overall the pre-service teachers displayed a high level of confidence in most of the answers they had provided. The distance task, which was part of question 1, saw a lower level in pre-service teachers' confidence with just over one third of participants stating they were confident in the answer they had provided. This may be because pre-service teachers had to calculate a new time for finishing the race if it was 30 metres longer. Some pre-service teachers may not have carried out this type of numeracy task since they were in school themselves. The final question, which was the mobile phone task, involved much more mathematical computation than any other numeracy task and this question was the least frequently answered question. In relation to confidence in the answers pre-service teachers provided for the mobile phone task, a total of 121 (59.3%) answered the confidence question. This was the only question for which there were more pre-service teachers not confident in the answer they had provided than any other question they had answered previously. One reason pre-service teachers may have had lower confidence in their answer could be that there was too much mathematical computation and thinking needed to answer this question and those who answered may have just written down an answer without providing any evidence of their mathematical procedures to support their answer. The following section presents the analysis of comparisons between pre-service teachers' confidence and their ability to complete the tasks correctly.

Comparison between pre-service teachers' confidence and correctness

In order to compare pre-service teachers' confidence and their ability to answer the numeracy tasks correctly, four confidence categories were created:

- Category 1: Confidence Aligned (Confident and correct or not confident and incorrect)
- Category 2: Under Confident (Correct and not confident)
- Category 3: Over Confident (Incorrect and confident)
- Category 4: Unable to assess confidence (Sum of Unsure categories)

The results of this analysis are presented in Table 3.

Table 3

Comparison between confidence categories and numeracy tasks

	Confidence Aligned	**Under Confident**	**Over Confident**	**Unable to assess confidence**
Time Task	104 (51.7%)	9 (4.5%)	70 (34.8%)	18 (9%)
Distance Task	95 (58.7%)	19 (11.7%)	13 (8%)	35 (21.6%)
Earthquake Task	137 (68.8%)	20 (10.1%)	15 (7.5%)	27 (13.6%)
Pie-chart Task	130 (72.2%)	9 (5%)	21 (11.7%)	20 (11.1%)
Car Task	138 (76.2%)	19 (10.5%)	7 (3.9%)	17 (9.4%)
Salad Dressing Task	109 (63.7%)	17 (10%)	18 (10.5%)	27 (15.8%)
Mobile Phone Task	45 (38.8%)	17 (14.7%)	18 (15.5%)	36 (31%)

The Time task saw just over 50% of pre-service teachers align their confidence accurately, indicating that they were confident and correct, or not confident and incorrect. Furthermore, the Time task was the task where a considerable number of pre-service teachers (N=70, 34.8%) displayed over-confidence, in that they indicated they were confident their answer was correct, when in fact it was incorrect. It was interesting to observe the prevalence of over-confidence amongst pre-service teachers who obtained an incorrect answer. Confidence is often implicated in definitions of numeracy (e.g., DES, 2011; Goos et al., 2019), but this usually manifests as lack of confidence amongst those with poor numeracy capabilities. These findings present some worrying trends where some pre-service teachers exhibited confidence when the answer they provided was incorrect. This could have a negative effect on students' learning if these pre-service teachers continue to possess a sense of over confidence when it comes to teaching for numeracy learning within their classroom. The Mobile Phone task was the only task whereby less than 40% (N=45) of the pre-service teachers who answered the confidence question managed to align their confidence and correctness of the task and this was also the task with the highest number of pre-service teachers who were unable to assess whether they were confident or not in their answer. The next section presents the analysis on pre-service teachers' confidence in relation to subject specialism.

Comparisons between pre-service teachers' confidence in correctness and subject specialism

It may be reasonable to assume that pre-service teachers in the STEM disciplines would be more confident in their ability to complete the numeracy tasks correctly than pre-service teachers in other disciplines. O'Sullivan and Goos (2022) revealed that pre-service teachers in the STEM disciplines believed they were better equipped and had enough mathematical knowledge to teach numeracy than pre-service teachers of non-STEM disciplines. Pearson Chi-square tests and Fisher exact tests were conducted to check if there was an association between pre-service teachers' confidence in completing the tasks correctly and Subject specialism of the pre-service teacher.

Analysis found that there was a statistically significant association between pre-service teachers' confidence in completing six of the numeracy questions correctly and their subject disciplines. Further analysis identified pre-service teachers in the STEM discipline possessing more confidence in completing the numeracy tasks correctly than any other discipline. However, it was interesting to see the results for the Mobile Phone task, as this was the only numeracy task that did not show a statistically significant difference between pre-service teachers' confidence and their subject discipline. This means the Mobile Phone task was the only numeracy task that pre-service teachers in the STEM discipline were not more confident in completing correctly. Table 4 presents the number of pre-service teachers in each discipline who were confident their answer to each numeracy task was correct. The most striking observation was that pre-service teachers from the STEM disciplines possessed a very high confidence in their answers with over 80% of them confident that their answer was correct for six of the numeracy tasks, however this fell to less than 40% for the mobile phone task.

Table 4

Cross tabulation of pre-service teachers' subject discipline against their confidence in completing each numeracy task correctly

Confidence in correctness	STEM (% within discipline)	Sociology (% within discipline)	Practical (% within discipline)	Languages (% within discipline)
Time Task	34 (97.1%)	37 (77.1%)	14 (58.3%)	76 (80%)
Distance Task	28 (84.8%)	16 (41%)	7 (35%)	24 (28.9%)
Earthquake Task	30 (88.2%)	35 (74.5%)	12 (50%)	64 (68.8%)
Pie Chart Task	30 (90.9%)	25 (55.6%)	9 (45%)	51(60%)
Best Car Task	33 (97.1%)	32 (76.2%)	12 (57.1%)	57 (65.5%)

Salad Dressing Task	30 (93.8%)	23 (54.8%)	11 (52.4%)	37 (46.3%)
Mobile Phone Task	10 (38.5%)	5 (19.2%)	2 (15.4%)	11 (19.6%)

Conclusion

Forgasz and Hall (2019) argue that pre-service teachers' confidence includes not only their own abilities and understandings, but also their willingness to teach for numeracy learning and their confidence in doing so. Researchers argue that demonstrating positive dispositions towards numeracy learning is essential in teaching for numeracy learning (Goos et al. 2019). If pre-service teachers are not confident in their own numeracy capabilities and not confident in their ability to teach for numeracy learning, then they must work towards building and developing their own mathematical knowledge and confidence in mathematical capabilities prior to teaching numeracy for disciplinary learning. Forgasz and Hall's (2019) research in the field of teacher education and numeracy learning, shows that students need to also be supported in this endeavour and this support can be offered in their ITE programmes.

Analysis from this research study demonstrated the differences between STEM and non-STEM pre-service teachers and found that teachers qualifying in a STEM discipline were significantly better at completing the numeracy tasks correctly and more confident in their answers to the numeracy tasks. This finding is an issue of concern, because if pre-service teachers of all disciplines are to teach numeracy for disciplinary learning within their lessons, regardless of their subject discipline, then pre-service teachers of all disciplines will need to have an adequate level of numeracy themselves in order to support numeracy development within their specific subject discipline. Callingham et al. (2015) advocate that all teachers, irrespective of their subject discipline, need to be given the opportunity to learn about the nature of numeracy within their subject. Furthermore, the Teaching Council of Ireland (2020) have agreed that teachers need to develop their personal numeracy knowledge and also specified that all universities involved in preparing pre-service teachers must provide appropriate support. Researchers argue that a person needs to be comfortable and confident to use mathematical knowledge to solve real world problems (Goos et al., 2019; Venkat & Winter, 2015). If pre-service teachers are lacking confidence in their own numeracy capabilities, this may in turn have an effect on how they embed numeracy learning in their lessons. Therefore, it is crucial that pre-service teachers are afforded the opportunities to develop positive dispositions towards numeracy learning.

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Is fearr DEIS chun chainnte: An initiative to support teaching for robust understanding in post-primary mathematics classrooms.

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This paper outlines a research project which aims to support the pedagogical practices of Mathematics teachers in socioeconomically underprivileged post-primary schools. In Ireland, the socioeconomic background of a student continues to determine how likely they are to experience high-quality post-primary education and make the transition to further or higher education. This is particularly relevant in mathematics, which remains a gateway subject in accessing third-level education. This research aims to address such inequity by supporting teachers to incorporate student-centred practices in their mathematics pedagogy. The project is undertaken in two phases: First, case studies of high-quality mathematics teaching and learning will be conducted in four schools. The findings from the case studies will inform the design of an intervention, which will involve 10 pilot schools across Ireland in the 2023-24 academic year. The research team (and research Mathematicians) will collaborate with the schools and support the teachers to reflect on and reform their teaching by engaging in school-based Lesson Study. This paper provides an outline of the project and initial findings which will contribute to research on the teaching and learning of mathematics in Ireland.

Keywords: DEIS, post-primary mathematics, Lesson Study

Introduction

The socioeconomic status (SES) of young people impacts their educational achievement and is a predictor of their achievement in mathematics (OECD, 2016). A young person's mathematical achievement is, in turn, a predictor of earnings and employability (Rose & Betts, 2004) and so a cycle of inequality continues. While there has been a marked increase in the numbers completing and continuing on from post-primary education in Ireland recently, this has not impacted students equally. A student's SES continues to shape how likely they are to experience high-quality post-primary education and make the transition to further or higher education (Byrne & McCoy, 2017). This is particularly evident in the case of mathematics, which has become a gateway subject in accessing third-level education (McCoy et al., 2019), further impacting the diversity of those who enter the Science, Technology, Engineering, and Mathematics (STEM) fields and STEM teaching, again perpetuating the cycle of educational disadvantage. In this paper, we provide an outline of a two-year study intended to positively impact the teaching and learning of mathematics in schools designated as socioeconomically disadvantaged and, thereby, attempt to address some of the inequities of achievement and experience for learners in post-primary schools.

Mathematics in the Irish post-primary classroom

Mathematics classrooms in Ireland have tended to follow a traditional exposition approach, where a teacher demonstrates a method and students repeat that same method for a number of exercises (Byrne & Prendergast, 2020). While there is merit in mastering mathematical procedures, teaching mathematics exclusively in this way restricts students' development of conceptual understanding (Rittle-Johnson et al., 2001). Furthermore, experiencing mathematics in this way communicates the subject as a rigid, predictable subject that, often, students do not enjoy (Marchis, 2011). Learning mathematics in a classroom which emphasises one 'correct' procedure and which focuses on achieving the 'right' answer (as opposed to discussing how an answer was achieved) is also an issue that exacerbates the mathematical anxiety of students, particularly young women (Maloney et al., 2013). Such experiences are often lamented by mathematicians, who feel that students miss out on engaging and interesting mathematical insights that could benefit their understanding of the world around them, including the numeracy skills necessary for today's society. These 'traditional' experiences contrast with reform-oriented teaching, where students are encouraged to communicate their mathematical thinking as problem solvers and where the teacher acts as a deliberate and careful facilitator of students' learning (Takahashi, 2021). The most recent mathematics post-primary curriculum reform in Ireland has attempted to encourage the incorporation of such classroom practices, but research suggests that little change has occurred in how students experience the subject (Byrne & Prendergast, 2020). Such findings are particularly relevant in schools which are designated as socioeconomically disadvantaged, where students are more likely than their counterparts in other schools to experience mathematics in a procedural way, which emphasises rote-learning at the expense of understanding (Perkins & Shiel, 2016).

International studies have demonstrated that students with lower SES tend to experience lower self-efficacy and higher mathematical anxiety than their less-disadvantaged counterparts (OECD, 2016). In an attempt to prevent social and economic disadvantage from depriving students' access to and achievement in education, the Irish government launched the Delivering Equality of Opportunity in our Schools (DEIS) programme in 2005. The programme allocates additional resources to schools that cater for students with low SES. Research suggests, however, that not only are these resources inadequate, but that elements of the funding process perpetuates educational disadvantage (Fleming & Harford, 2021). Students in DEIS schools continue to, on average, have lower quality and less engaging classroom experiences and, in tandem, experience less success in high-stakes and other assessments (Perkins & Shiel, 2016). These students also possess higher anxiety and lower self-efficacy and self-concept in their mathematics learning (Perkins & Shiel, 2016). This has a detrimental effect on these young people's educational potential and Smyth et al. (2015) found that students in DEIS schools are 40% less likely than students in non-DEIS schools to study Mathematics at a higher level during Junior Cycle. Further, the incentivisation scheme of 25 additional 'bonus points' given to all students who pass Mathematics at higher level in the final post-primary high stakes Leaving Certificate exam, this has resulted in further educational inequity between students in DEIS and non-DEIS schools (McCoy et al., 2019). This is particularly relevant when not all DEIS schools offer higher-level Mathematics and when students can be discouraged from considering such an option, even when expressing an interest in pursuing the subject at higher level (Ní Shuilleabháin et al., 2020).

Addressing Socioeconomic Inequity in Mathematics in Ireland

Dietrichson et al. (2017) suggest that targeted interventions have the potential to improve the educational achievement of students from low socioeconomic backgrounds. Their systematic review of the literature demonstrated that tutoring, feedback and progress monitoring, and cooperative learning lead to large and robust average effect sizes on student achievement. We consider two such examples in the Irish context.

Firstly, Project SUMS is an initiative which seeks to increase students' potential for positive academic achievements through the provision of extra supports and guidance to students. These services are provided free of charge and are designed to develop a positive attitude towards maths learning in students by instilling confidence in students' abilities. Project SUMS involves a multitude of resources based on students' preferred approaches and incorporates a combination of group and individual work. Secondly, the Maths Sparks engagement programme aimed to support post-primary students from DEIS schools with weekly out-of-school, extra-curricular workshops that were designed and delivered by undergraduate mathematics students. The activities consisted of a mixture of mathematical games, problems and puzzles, with the aim of providing students with the experience of non-traditional exercises. A focus of workshops was to develop mathematical thinking and an appreciation for not being able to immediately find a solution. The programme was designed with the aim of encouraging participating senior post-primary pupils from DEIS schools to continue their studies of mathematics at a higher level and to consider pursuing mathematics or STEM-based courses in higher education. Participating in the programme was found to positively impact students' attitudes towards, enjoyment of, and self-confidence in mathematics, and encouraged students to consider continuing in their Mathematics and STEM education (Ní Shuilleabháin et al., 2020).

While student-based interventions have the potential to improve students' mathematical achievement, Dietrichson et al. (2017) also suggest that interventions focusing on strengthening teachers' knowledge and practices are impactful. Indeed, Gersten et al. (2009) found that providing teachers with an opportunity to reflect on and receive feedback on their teaching had significant effects on mathematics outcomes. The current study builds on this work and that of Lewis et al. (2022) by focusing on school-wide interventions to impact student achievement successfully and positively.

Potential Challenges and Solutions in Educational Reform

The difficulty in addressing and scaling up educational improvements is one of the greatest challenges in research around teaching and learning (Fullan, 2001). Initiating top-down reforms, often through in-service forms of professional development, can fail to capture the buy-in of teachers, thereby losing necessary knowledge, leadership, and motivation to cultivate change (Ní Shuilleabháin & Seery, 2017). Lesson study, an approach to teacher professional development which originated in Japan, offers one potential way to combine pedagogical research with teachers' expertise and ameliorates the theory-practice divide so often noted in reviews of traditional, expository ways of teaching and learning (Lewis &

Perry, 2017). Research from Lewis et al. (2022) based in historically underserved populations, including students from low-income families, suggests that Lesson Study can be embedded in schools as a sustained and effective form of teacher learning.

Teachers, their knowledge, skills, and interests, are core to Lesson Study, which provides groups of teachers with opportunity to research their own practice with a view to improving student learning (Lewis & Perry, 2017). Lesson Study consists of a cycle of four phases in which a group of teachers engage in careful discussions around planning, conducting and reflecting on a research lesson. Discussions within the cycle specifically focus on student learning, with reference to specific tasks, actions, and communication which may arise within the lesson (Takahashi & McDougal, 2016). Teacher knowledge is a key factor in influencing students' mathematical achievement (Baumert et al., 2010) and Lesson Study has been demonstrated as a way of improving teachers' content and pedagogical content knowledge (Lewis & Perry, 2017). Furthermore, research has also demonstrated that teachers' participation in Lesson Study can lead to improvements in students' mathematical knowledge (Lewis & Perry, 2017). Most importantly for this project, Lesson Study provides teachers with a vehicle within which they can trial new classroom practices with the support of their colleagues (Ní Shuilleabháin & Seery, 2017). As an additional gain, Lesson Study has been demonstrated to develop teacher community beyond their participation in the model (Lewanowski-Breen et al., 2020), a key feature of educational reform.

In introducing professional development to a school, it is important to explicitly acknowledge the situated context of the learning (Lave 1988). This has been highlighted in various success and failure stories of Lesson Study (Bjuland & Mosvold, 2015). In this project, we look to learn from Mathematics teachers in DEIS schools who might act as positive examples of teaching and learning and from whom we might learn and share findings specifically relevant to DEIS schools. The design of the research is therefore outlined below.

Research Design

The overarching aim of the project is to support the pedagogical practices of Mathematics teachers in post-primary DEIS schools. To achieve this, a two-stage project was devised which would investigate best practice in post-primary mathematics education through comprehensive case studies in four DEIS schools. The outcomes of this will inform the design of an intervention, which is intended to support an additional cohort of 10 DEIS schools across a school year to improve their practice in a sustainable way. This structure allows for many of the traits of successful mathematics initiatives mentioned in the previous section to be incorporated. It also allows for clear and concise research questions which align with the research aim to be posed. The research questions (RQs) are as follows:

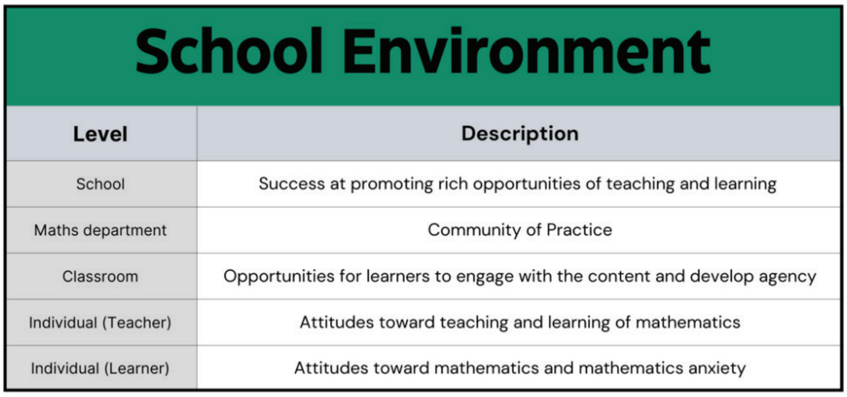
1. How are positive mathematical learning experiences cultivated and supported in post-primary DEIS schools?
2. How effectively can Mathematicians inform the teaching and learning of post-primary mathematics through Lesson Study?
3. How are the mathematical knowledge and attitudes of teachers and students impacted by participation in school-based Lesson Study?

4. How can Lesson Study be sustainably incorporated in post-primary DEIS schools?

As written, RQ1 aligns with the initial phase of the project, and RQ2-4 relate to the second phase. For the purposes of this paper, we will focus on RQ1 by detailing the recruitment and collaboration with the four case study schools. In attempting to investigate the mathematics classrooms and cultures which exist in our case study schools, a multi-layered description of the post-primary school environment was developed (see Fig. 1).

Figure 1

Description of the school environment


School Environment

Level	Description
School	Success at promoting rich opportunities of teaching and learning
Maths department	Community of Practice
Classroom	Opportunities for learners to engage with the content and develop agency
Individual (Teacher)	Attitudes toward teaching and learning of mathematics
Individual (Learner)	Attitudes toward mathematics and mathematics anxiety

The importance of multiple stakeholder groups, and their interrelations, are a significant feature of the description, which translated directly to the participants that were recruited and the methods of data collection utilised. Data were gathered pertaining to each of these aspects through a combination of classroom observations (video, audio, and written field notes), student focus groups and surveys, teacher interviews, and interviews with school leadership. When the instruments and protocols were developed, ethical approval from UCD was sought and obtained, and a call for participants began.

Case Study Schools

The project was advertised to post-primary DEIS schools through email lists, attendance at teacher events, collaboration with relevant professional bodies, and via a targeted social media campaign. This resulted in seven expressions of interest which were shortlisted after satisfying various thresholds for participation (minimum of three teachers and one member of management were required to participate). The case study schools (detailed in Table 1 in a non-identifiable manner), were then selected from the shortlist based on the contents of the application and with an effort to attain a diversity of representation across school sector, enrolment (sex and number of students), and location. The case studies were conducted between January and March 2023, with each site visit lasting one week.

Table 1

Case study schools and data collection

	Enrolment		Location	Classes Observed	Surveys Completed	Focus Groups Conducted	Interviews Conducted
	Gender	Number					
1	Co-Ed	200-250	East Coast	3	53	3	4
2	Girls	800-1000	Dublin	8	136	7	9
3	Co-Ed	800-1000	Midlands	3	51	3	4
4	Co-Ed	600-700	Midwest	3	39	3	4

Data Analysis

A large quantity of qualitative and quantitative data was gathered during each site visit (see Table 1), and multiple independent paths of analysis are currently underway to inform the understanding of school and classroom practices, mathematical anxiety, and the opinions of students, teachers, and school leadership across all four schools. The data in Table 1 amounts to 14 hours of classroom observations, three hours of focus groups, and 10 hours of interviews. Given the range of data sources and subject matters, a multitude of theoretical frameworks will be leveraged across the various sources as appropriate to the data source. Though the analysis is only beginning, we include some initial takeaways in the final section.

Phase 2 Outline

The case studies inform the intervention, which will involve collaboration with 10 post-primary DEIS schools. In addition to the learnings from Phase 1, a significant component of the intervention will be introducing the schools to Lesson Study which they will practice across the academic year. A key aspect in the planned use of Lesson Study will be the inclusion of research Mathematicians as the 'knowledgeable other' to provide support for the teachers in the area of Mathematical Content Knowledge (MCK). Gaps in MCK have been found to hinder the development of mathematically rich lessons (Lewis & Perry, 2017), and the preliminary analysis suggests it may be an issue in this study.

The pilot schools will participate in interviews and student surveying in a similar manner to the case study schools so that comparable data can be gathered across both phases of the project. This data will be gathered across the entirety of Phase 2, and will inform RQ2-4 which, in turn, will describe the success of the intervention.

Initial Findings and Discussion

Though much of the formal analysis is still underway, some of the data has yielded preliminary findings and potential lines of enquiry, which we include below.

- Mathematics anxiety is prevalent in post-primary schools to varying degrees. Sources and mitigators are being identified and targeted by schools with reasonable levels of success, but much more remains to be done. Of note was the success that schools are having with co-curricular activities and targeting the dispositions of parents.
- Myriad external factors seem to adversely impact pedagogical approach. Time pressures are used to justify streaming (and even banding) after first year. Schools with 1-hour lessons seemed to reference time pressure less, though this is subject to further exploration.

- There is variance in the use of the whiteboard and of technology in classrooms. There was very little use of IT beyond simple screen mirroring; however, several teachers made excellent use of IT to improve the accessibility of and engagement in their lessons.

We reiterate that these findings are emerging as the data analysis is ongoing; however, they are supported by subsets of data across all four case study schools. We anticipate being able to describe a more complete picture as the analysis progresses, in addition to distinct findings relating to attitudes toward mathematics.

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Mathematics Curriculum Reform in Irish Primary Schools: Encouraging Faithful Implementation

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Ireland's Primary Mathematics Curriculum is undergoing a critical reformation emphasising developing students as mathematicians (National Council for Curriculum and Assessment, 2020). Students will be called upon to problem-solve, analyse, and synthesise information. To understand how students are progressing through learning these valuable 21st-century skills, teachers must use formative assessment to inform and adjust instruction (Burkhardt & Schoenfeld, 2019). Teachers will require additional support through continued professional development (CPD) to transition to this new curriculum and increased focus on formative assessment (Spillane & Thompson, 1997). This paper provides an overview of Ireland's developing primary mathematics curriculum, discusses how assessment must progress alongside it, and links a curriculum's success to effective CPD. Finally, this paper reports on a current research study aiming to develop a toolkit that supports teachers in asking higher-level thinking questions in primary mathematics lessons. Teachers who effectively assess students' higher-level thinking will be better positioned to adjust instruction to help students develop as mathematicians.

Keywords: primary mathematics, teacher questioning, professional development

Introduction

Mathematics curriculums worldwide are being examined and revised to assist students in developing skills which prepare them for 21st-century challenges (Australian Curriculum Assessment and Reporting Authority, 2022; Ministry of Education, 2020; National Council of Curriculum and Assessment [NCCA], 2020). Although student achievement in mathematics has increased recently, students simultaneously fall short in demonstrating higher-level thinking skills such as problem-solving, analysing, and synthesising (Mullis et al., 2020). Therefore, curriculum developers and educational institutions have recognised a considerable need in mathematics classrooms and have begun to act through curriculum reform.

This paper aims to give insight into mathematics curriculum reform currently happening in Ireland. We highlight that primary teachers in Ireland may need to make significant changes to their approaches to teaching mathematics and will need continued professional development (CPD) opportunities to do so. To support this viewpoint, we give an example of the role of formative assessment in Ireland's Draft Primary Mathematics Curriculum (DPMC). In addition, we also give a brief overview of an in-progress research study on designing a teacher toolkit for implementing higher-level questioning into mathematics lessons with the following research questions:

(a) Can we collaboratively develop a toolkit for primary teachers on effectively incorporating questioning in mathematics?
(b) Will the toolkit positively impact teachers' instruction, and to what degree? Will it act as a form of continued professional development, expanding teachers' knowledge of teaching mathematics?

(c) How can this research contribute to education theories on mathematics education and continued professional development?

As this research is in its formative stages, this paper does not intend to provide in-depth answers to the research questions but rather spark a discussion around faithful curriculum implementation during periods of change and give meaning and purpose to an ongoing study, discussed in the second half of the paper.

Primary Mathematics Curriculum Reform in Ireland

Similar to countries worldwide, Ireland is undergoing periods of curriculum reform. The primary mathematics curriculum is being revised to reflect the needs of students and teachers to increase academic achievement (NCCA, 2020). The NCCA has released numerous reports in the last eight years which review the current state of primary mathematics education and recommend specific steps to increase students' academic progress (Burke, 2014; Dooley, 2019; NCCA, 2016). The DPMC was subsequently released in 2022.

The DPMC focuses on shaping students into mathematicians with mathematical proficiency. Mathematical proficiency is developed through learning to connect information, communicate effectively, construct and justify arguments, and analyse solutions (Ball, 2003). Therefore, the draft curriculum concentrates on procedural fluency, strategic competence, adaptive reasoning, productive disposition, and conceptual understanding (NCCA, 2022). While incorporating these skills into mathematics education is not new, the draft curriculum emphasises students developing these skills by refining how mathematics is taught.

Role of Formative Assessment

The DPMC intertwines formative assessment as a critical pedagogical practice to ensure students' academic success (NCCA, 2022). While summative assessments, such as end-of-unit exams or standardised tests, remain a primary method for measuring academic achievement, formative assessments are becoming integral to curriculums worldwide (Ministry of Education, 2020; New York State Education Department, 2017). Formative assessment can employ various approaches, but all methods aim for teachers to understand students' progression daily and make appropriate adjustments to instruction frequently (Wiliam, 2011).

Incorporating effective questioning methods into mathematics lessons is one method of using formative assessment (Palm et al., 2017). Integrating dialogue into mathematics lessons, including asking higher-level thinking questions, can help teachers identify specific student learning needs (Zack & Graves, 2002). As a result, Irish teachers must include higher-level questioning in their lessons to fully implement the new primary mathematics curriculum. However, effectively using questions in mathematics lessons is complex (Dillon, 2004) and often a challenging pedagogical skill to master (Fennema et al., 1996). Teachers should plan lessons with questions to ask students that span all cognitive levels (Shahrill, 2013). Unfortunately, teachers often stick to asking questions of lower cognitive levels and fail to fully understand how to interpret student responses and react accordingly (Moyer &

Milewicz, 2002). Therefore, to fully implement the new curriculum as written, Irish teachers must further develop questioning skills to incorporate formative assessment fully.

Adjustments to Teacher Practice in Coordination with Curriculum Implementation

Effective implementation of a new curriculum requires teachers to understand the curriculum's components fully and have the necessary pedagogical skills (Spillane & Thompson, 1997). Each teacher possesses various skills, pedagogical knowledge, and beliefs about teaching (König et al., 2015). Therefore, the changes to instruction each teacher may need to make during curriculum reform may differ between individuals.

The NCCA consulted with primary teachers to understand their perceptions of the draft curriculum and identify the skills teachers may need support with once a new curriculum is enacted (Bryne et al., 2023). Questionnaire results indicated that around half of teachers were unsure or did not believe the draft curriculum would help students develop mathematical proficiency (Bryne et al., 2023). Teachers who disagree with a curriculum's aims and goals will lack buy-in, and how it is implemented may be negatively affected. Without teachers understanding the underpinning research which supports a curriculum, there is a risk of it being implemented incorrectly or incompletely (Broadhead, 2001). In addition, the questionnaire aimed to understand how formative assessment fits with teachers' current approaches to teaching mathematics. Only 35% of teachers identified that formative assessment corresponds with their current practices, while the majority said it only matched to some degree (Bryne et al., 2023). While these questionnaire results may not indicate the needs of every primary teacher, it signals that many teachers may need to adjust their approach to teaching mathematics.

Role of Continued Professional Development Alongside Curriculum Reform

Continued professional development is essential to help teachers teach any curriculum (Radford, 1998). The structure of CPD which occurs alongside curriculum reform, should be considered early on. Despite various approaches to CPD, commonalities exist among methods, such as ensuring the content connects to teachers' practice, considers time and capacity barriers, and meets the individual needs of teachers (Hargreaves, 2014; Kennedy, 2014; Loucks-Horsley et al., 2009). To construct CPD that meets the diverse needs of teachers, a necessary first step is to evaluate the current approach to professional learning. This process should include understanding teachers' experiences with CPD and their recommendations for future CPD opportunities (King, 2014).

Current Approach to Continued Professional Development in Ireland

In Ireland, CPD is provided mainly through the Professional Development Service for Teachers (PDST), funded by the Teacher Education Section of the Department of Education. Through the PDST, numerous supports are available, including online and in-person workshops for teachers, school support, and access to various publications written by the PDST. In addition, the Teaching Council in Ireland released Cosán: Framework for Teachers' Learning in March 2016, guiding teachers' professional learning and growth during all stages of an individual's teaching career (The Teaching Council, 2017). This framework identifies core principles for professional learning, such as teachers being autonomous learners, flexibility in education, and access to quality CPD (The Teaching Council, 2017).

To aid in writing this framework, the Teaching Council consulted teachers in 2014 through online surveys and in-person feedback. This consultation identified that teachers desire high-quality CPD connected to their classrooms while offering CPD choices (The Teaching Council, 2016), mirroring teachers' sentiments worldwide (Organisation for Economic Co-operation and Development, 2019). Notably, the data analysis from the Teaching Council also revealed that the time required for CPD, access to workshops, cost, and required teacher capacity were of great concern to teachers (The Teaching Council, 2016). A striking theme emerging from the data is that teachers felt their concerns and suggestions would be discounted (The Teaching Council, 2016). Teachers being treated as professionals and with respect is indicative of a successful education system (Sachs, 2016). Structuring CPD opportunities and guidelines should consider teachers' perspectives, concerns, and suggestions to meet teachers' needs.

Methodology

The remaining discussion centres around an educational design-based research study conducted in Galway, Ireland. The researcher collaborated with six primary teachers to design a toolkit to guide teachers through using questioning effectively in lessons. All DEIS primary schools within Galway City were invited to participate, with a goal to recruit 6-10 teachers. The University of Galway provided ethical approval, and informed consent was received by all participants at the start.

Three design cycles were carried out to develop the toolkit. Teachers used the toolkit in their classrooms for six weeks during each cycle, followed by focus groups to analyse the toolkit and identify necessary adjustments. Observations of teacher instruction, researcher memos from teacher conversations, and additional anonymous questionnaires were also used. The questionnaires asked teachers to provide insight into their experiences using the toolkit and any desired changes. Data from focus groups were analysed with guidance from Galletta and Cross' interactive data analysis process, which included: becoming familiar with the qualitative data from the interview transcripts, generating and assigning codes to the text, looking for thematic patterns, and synthesising the codes into categories (2013). Analysis from the second design cycle's focus group interviews, completed in 2023, guides the discussion below.

Findings

Through focus group analysis, themes emerged from teachers' insights on resources that would help them teach mathematics. During the interview, the conversation diverted from the toolkit's development and toward the draft mathematics curriculum. It should be noted that these teachers indicated only basic knowledge of the draft curriculum. Therefore, in analysing teachers' responses on what might assist them in enacting the draft curriculum, it is essential to recognise teachers' limited knowledge of it.

Teachers made it clear that the mathematics textbook is integral to teaching mathematics. Teachers view the textbook as a method to keep them on track with teaching all the required content. One teacher commented:

Tadgh: If there isn't structure of the book, you do end up kinda going down a rabbit hole on a topic and going, 'Oh! This is great, and we're having a great time doing this; jeez, that two weeks gone on something that should have taken half an hour.

As a follow-up to this teacher's comment, the moderator asked the teachers if they believed resources, such as the toolkit being developed, should be designed to be used alongside textbooks. Teachers agreed that the textbook should come first and then additional resources used to support instruction. Another teacher indicated they use the textbooks to extend learning, stating:

Brian: And if you have three or four different maths textbooks and the topic is finished and you go here…we're going to try this one. And then, the guy in the middle is struggling; at least you have something to work with.

Teachers indicated that differentiating was easier with textbooks and that if the new curriculum focuses more on single in-depth tasks, this would be a significant challenge in the classroom. In addition, teachers indicated in this case that they would still utilise textbooks. The same teacher commented:

Brian: So, yes, we have the tasks, and we have that, but we're doing it in conjunction with the book. Because, when you have, like, you know, in my class, I have to differentiate all the time.

As part of this research project, the researcher travels to the primary school one day per week to work alongside the teachers. In the four mainstream classrooms, textbooks were used 100% of the time when the researcher supported the teachers with their lessons, which mirrors the teachers' comments in the interview.

Discussion and Conclusion

As the NCCA develops a new primary mathematics curriculum and decides how to support teachers through its implementation, it is critical to investigate teachers' current experiences and perspectives. The success of CPD can be increased if teachers see the learning as directly connected to their classrooms and consider their learning preferences (Loucks-Horsley et al., 2009). During this research, teachers indicated strong dependence on a mathematics textbook. The new draft curriculum focuses on students developing mathematical proficiency, including problem-solving, analysing, and synthesising skills. Yet, reports indicate that textbooks often fall short in aiding students in developing these higher-level thinking skills (Dooley, 2019). Similarly to the cohort of teachers in this research project, data across Ireland indicated that 91.5% of sixth-class teachers utilised a textbook daily (Dooley, 2019). If the new primary mathematics curriculum is to depend less on textbook use, the NCCA needs to examine how teachers can be supported in making this transition.

The NCCA indicates an upcoming shift in teaching mathematics to develop students as mathematicians with critical thinking skills (NCCA, 2022). This pathway may require rethinking the use of textbooks, which primary teachers commonly use in daily instruction. As a result of the focus group interviews, it was clear that teachers needed resources that would provide some structure to lessons while also allowing for differentiating instruction. While the toolkit developed in this research project aims to aid teachers in asking higher-level thinking questions, it is not a textbook or lesson plans. CPD will be necessary to educate teachers on developing lesson plans that utilise formative assessment methods. It is crucial for those developing and facilitating CPD alongside the new curriculum's adoption to use teacher' voice to guide professional learning (Gozali et al., 2017). If CPD facilitators cannot obtain teacher buy-in, the curriculum may fail to be fully implemented, ultimately resulting in students' academic success suffering.

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Addressing the Professional Development Needs of Adult Numeracy Practitioners in Ireland

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The development of a numerate society is an international and national priority in education. Given such importance, there is a need for increased attention on adult education, and particularly to the availability and quality of adult numeracy education. While there has been considerable focus on mathematics teaching in general, there is a dearth of research and resources in relation to teaching adult numeracy. Consequently, there is an unmet demand for the professional development of adult numeracy practitioners with many looking for opportunities to network and further develop their practice. This study aimed to design, implement, and evaluate a professional development model that supported adult numeracy practitioners in developing the necessary skills to support their students. After an initial needs analysis, a series of six online 'Numeracy-Meets' were designed and implemented between February and May 2022. After all the Numeracy-Meets had taken place, five practitioners took part in individual semi-structured interviews to evaluate their experiences. This paper details the design, implementation, and evaluation of these Numeracy-Meets.

Keywords: adult numeracy, practitioners, professional development, Numeracy-Meets

Background to the Study

It is well documented that numeracy skills are critically important for the adult population to support active citizenship in the economic, social, and community spheres (Goos et al., 2023). However, despite such recognised importance, findings from the Organisation for Economic Co-operation and Development (OECD)Programme's for the International Assessment of Adult Competencies (PIAAC), suggest that large numbers of adults have low or very low numeracy skills (OECD, 2019). The PIAAC numeracy proficiency scale is divided into six proficiency levels: Levels 1 to 5 and below Level 1. When Ireland took part in PIAAC (2012), it was found that over one-quarter (25.3%) of adults scored at or below Level 1 on the numeracy scale (OECD, 2013). This score ranked Ireland 19th out of 24 participating countries and suggested that 754,000 Irish people struggle with everyday numeracy (NALA, 2017). This concerning issue has been reflected in a continued national policy focus over the past decade which has culminated in the publication of the first Adult Literacy for Life (ALL) Strategy for Ireland in 2021. This cross-Government strategy is underpinned by one simple vision: "An Ireland where every adult has the necessary literacy, numeracy and digital literacy to fully engage in society and realise their potential" (Government of Ireland, 2021, p.33).

In Ireland, the Education and Training Boards (ETBs) are the national providers of post-compulsory Further Education and Training (FET), alongside a smaller range of adult and community education groups. ETBs offer a range of accredited and unaccredited courses of learning, and they have responsibility for providing adult education and training, and youth work programmes in Ireland. In terms of adult numeracy education, the provision across this sector remains diverse and little is known about the delivery of courses, those who attend them, and those who teach on them. For example, the National Adult Literacy Agency (NALA) estimate that there are currently over 40,000 adults attending literacy courses mainly provided by the ETB Adult Literacy Service in Ireland (NALA, 2023). However, because adult numeracy is considered to be a component of adult literacy, there is no data available to ascertain how many adults access or persist with numeracy-specific provision (Goos et al., 2023). In 2021, SOLAS (a State agency of the Department of Further and Higher Education, Research, Innovation and Science) commissioned a research study that sought to "…capture and document standalone and integrated adult numeracy activity in the Education and Training Board (ETB) context, in order to develop good practice guidelines and inform future development of adult numeracy policy and practice" (SOLAS, 2021, p. 6). It was envisaged that such an endeavour would provide a contemporary overview of numeracy provision in Ireland and inform a series of guidelines that would help shape the future of numeracy provision for adults. There were four main guidelines set out by SOLAS, the last of which focused on 'Supporting and developing adult numeracy tutors'. This fourth guideline proposed that ETBs "Plan for adult numeracy tutors' professional development" and "Create networking opportunities for adult numeracy tutors" (SOLAS, 2021, p. 11). With these recommendations in mind, this study[1] sought to address the following research question: How can the needs of adult numeracy practitioners in Ireland be addressed through the design, implementation, and evaluation of a professional development model?

The Study

The research team for this study brought together expertise in the field of numeracy, teacher education, and adult numeracy education. The researchers had worked together on previous adult numeracy projects and felt well placed in targeting Guideline 4 of the SOLAS (2021) report. They also sought the support of NALA which is an independent charity committed to supporting people with literacy and numeracy difficulties in Ireland. Since its establishment in 1980, NALA has developed and delivered professional development resources and opportunities for adult numeracy practitioners in Ireland. The researchers felt that the organisation's knowledge and networks in the area would be an invaluable addition to the project team.

Ethical approval for the study was granted in January 2022 by the Social Research Ethics Committee in University College Cork (Log 2021-226). The research was carried out in three main phases which took place over a five-month period between February and June 2022. Phase 1 was a needs analysis of a sample of adult numeracy practitioners in Ireland and the subsequent design of a professional development model. This model was implemented and evaluated in Phase 2 and Phase 3, respectively. The write up of this paper will be structured using each of these sequential Phases to guide the reader through the study.

[1] This paper is based on Prendergast, M., Forester, A., O'Meara, N., O'Sullivan, K., and Faulkner, F. (2023). Numeracy-Meets: An innovative professional development model for adult numeracy practitioners in Ireland. *Irish Educational Studies.* https://doi.org/10.1080/03323315.2023.2209854

Phase 1 – The Design

The first phase of the research was a needs analysis of a sample of adult numeracy practitioners in Ireland. This was done through an online questionnaire using an instrument adapted from the Teaching and Learning International Survey (TALIS) 2018 Teacher Questionnaire Professional Development section. There were three parts to the questionnaire, namely:

1. Background information
2. Attitude towards and perception of teaching
3. Professional development needs.

The 16-item questionnaire was circulated via a Microsoft Forms link using non-probability sampling through NALA adult numeracy networks in February 2022. It was completed by 33 respondents. The quantitative data was recorded and transferred into an SPSS (version 25) file for statistical analysis. The data from any open-ended questions were transcribed into a Microsoft Word for content analysis in relation to the professional development needs of adult numeracy practitioners. Respondents ranged in age from 30 – 69 and the majority were female (79%). Prendergast et al. (2023) provide a detailed description of the analysis of the data. In summary, in terms of professional development, 87% had engaged in online courses or seminars in the previous 12 months. However, they identified specific areas in which they had a need for Continuous Professional Development (CPD). For example, 65% of practitioners indicated that they had a moderate to high level of need for CPD in pedagogical competencies in teaching numeracy. Furthermore, 75% of practitioners revealed a moderate to high level of need for CPD in Information and Communication Technology (ICT) skills for teaching numeracy. In an open-ended question on how the CPD needs of adult numeracy practitioners might be best supported, the main findings were centred on workshops, communities of practice, or collective forums where ideas could be shared. Participants also noted that shorter inputs which did not involve a long commitment would be most suitable.

Taking the findings from the needs analysis into account, the research team decided to design a series of online Numeracy-Meets “to best address practitioners' needs”. The term Numeracy-Meets was adapted from the Teach-Meet model of professional development which has been referred to as ‘guerrilla CPD’ (Bennett, 2012). According to Blanchett (2014), Teach-Meets “provide a nice informal atmosphere to share ideas and good practice with a chance for everyone to have their say” (p. 5). Many consider Teach-Meets to be based on the community of practice (CoP) model of CPD (Amond et al., 2018). According to Wenger (1998), it is necessary to consider three defining characteristics in order for a CoP to emerge: a focus on a shared interest and domain; involvement in joint activities, discussions and sharing of information; and the development of a shared repertoire of resources. After considering each of these characteristics, the authors were satisfied that the Numeracy-Meets model would meet the criteria of a CoP. The shared focus would be adult numeracy; there would be discussions of practice and sharing of planning and instructional approaches; and there would be a shared repertoire of resources gathered after each Numeracy-Meet.

Given the dispersed nature of numeracy practitioners in Ireland, the Numeracy-Meets needed to be a nationwide professional development strategy for remote practitioners to network and further develop their practice. According to Wenger et al. (2002), CoPs can exist both physically and virtually. Karam et al. (2018) noted that online CoPs afford promising alternatives for overcoming the absence of collocated peers and so it was decided that the Numeracy-Meets would take place virtually.

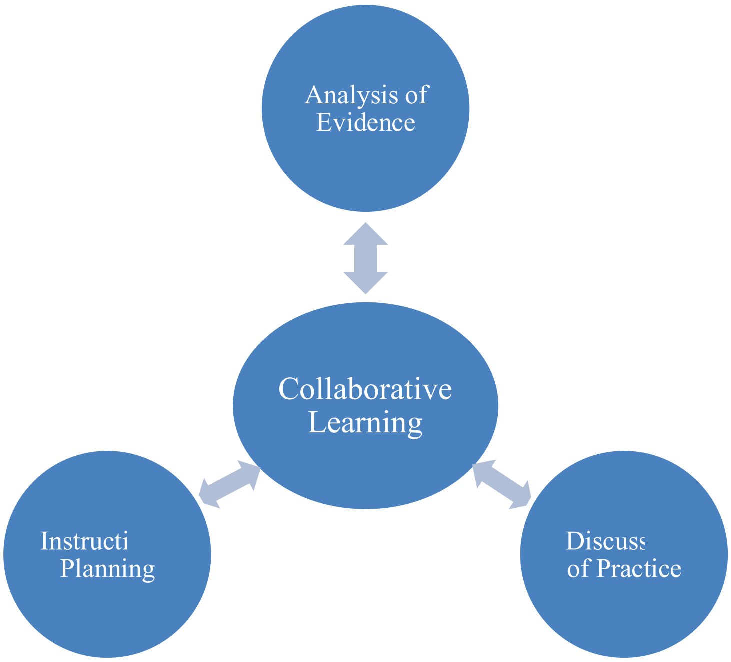
With all of this in mind, six online Numeracy-Meets were designed with the aim of providing practitioners with professional development in which learning could occur in an informal way, primarily through social interaction.

Phase 2 – Implementation

In terms of structure, the underlying theory of each Numeracy-Meet was an adaptation from Farley-Ripple and Buttram's (2014) work on developing learning communities in the United States (see Figure 1). The central assumption of any community is that collaboration helps and that teaching and learning is more effective if practice and experiences are openly shared and discussed (Bolisani et al., 2020). Thus, there were three important components to the planned structure of each Numeracy-Meet. As evidenced from Figure 1, analysis of evidence, discussion about teaching practice, and instructional planning were at the heart of effective collaboration between the numeracy practitioners.

Figure 1

Underlying Theory of Each Numeracy-Meet



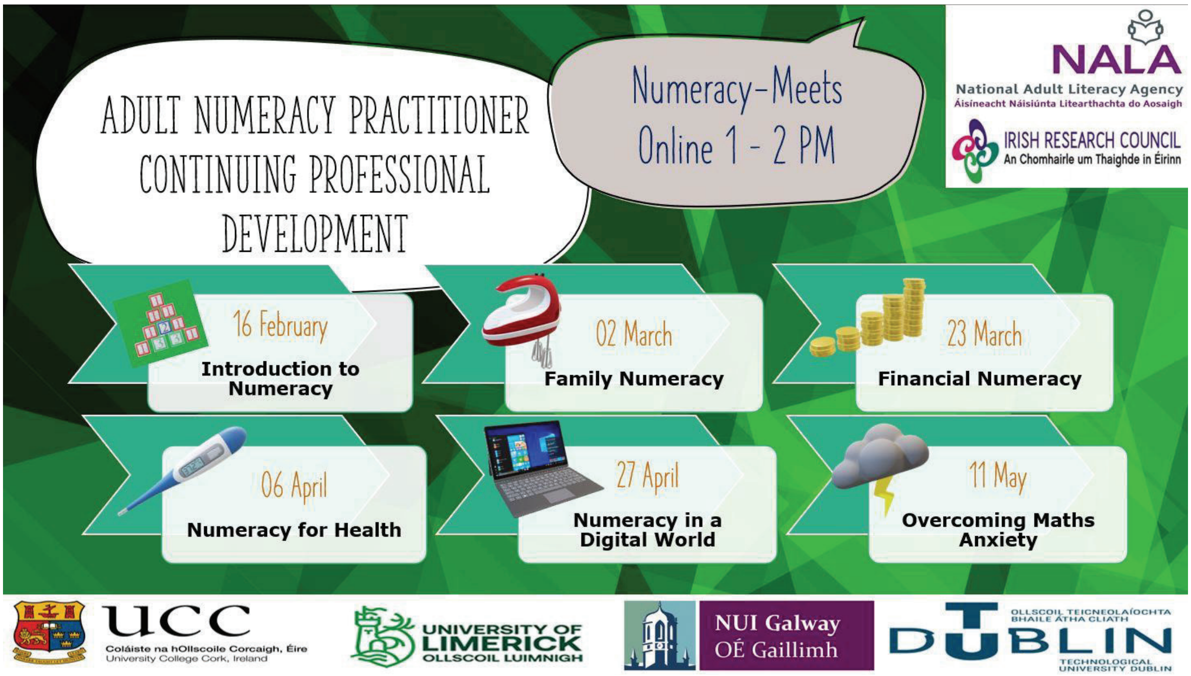
The content focus of the Numeracy-Meets centred on specific life domains (e.g., financial, health, digital, etc) and related uses and practices of numeracy in the context of work, family, and everyday life. The common format of each Meet generally involved a twenty-minute input on a specified topic from an invited speaker followed by discussion and contributions from several attending practitioners. The resources from each of the Numeracy-

Meets were shared with all participants and are available at https://epistem.ie/numeracy-meets/.

As evidenced in Figure 2, the implementation of the six Numeracy-Meets took place between 16th of February 2022 and 11th of May 2022. They were facilitated online using the Microsoft Teams platform and were scheduled on Wednesdays between 1pm and 2pm. Practitioners were asked to register their interest to attend the Numeracy-Meets via a Microsoft Forms link which was circulated through NALA's adult numeracy networks.

Figure 2

Marketing Poster with Overview of the Numeracy-Meets



In total, 60 adult numeracy practitioners registered their interest. There was a wide range of diversity in the background of the practitioners who registered. For example, 14 of the 16 ETBs in Ireland were represented, along with adult numeracy practitioners from the Irish Prison Service, third level, and other non-profit organisations. There was a range of 14 – 23 attendees across the six Meets and the median number of attendees who attended each Meet was 20. Of the 60 adult numeracy practitioners who initially registered their interest, 40 attended at least one of the Numeracy-Meets. There was a core group of 15 practitioners who attended at least four of the six Numeracy-Meets.

Phase 3 – Evaluation

The third and final phase concerns the evaluation of the Numeracy-Meets model. The data with which to evaluate the intervention was gathered using semi-structured interviews. After the six Numeracy-Meets had taken place, five participants (pseudonyms used - Abbi, Brian, Chloe, Deirdre, and Eimear) were purposively selected from the core group of 15 practitioners who had attended at least four of the six Numeracy-Meets. These five participants were purposively selected to ensure that there was a proportional gender balance (one male and four female practitioners) and that there was representation from five different

ETB's and adult education settings. Once selected, an email invitation was sent to take part in a one-to-one interview to evaluate their experience of the Numeracy-Meets. This email contained an information sheet explaining the key points and procedures of the study. All five participants consented to take part and the interviews were conducted via Microsoft Teams in June 2022. The interviews were semi-structured, with questions divided around three main areas, namely participants' perceptions of Numeracy Meets, the effectiveness of the CPD, and the future sustainability of the model. The data from each interview was transcribed into a Microsoft Word document and content analysis was performed by two of the authors. The application of content analysis ensured a systematic examination and interpretation of the qualitative data which informed our overall findings.

Overall, the data from the interviews indicated that practitioners found the intervention useful and felt that the sessions positively affected their practice. Interviewees liked the combination of theory and practice and the mutual sharing of practical ideas which took place at the Numeracy-Meets.

Deirdre: I found it very good. It went through theory and then it related back to our everyday practices. It went back over past research that had been done and applied it and probably showed where it was going for the future.

Practitioners were asked about their overall perception of Numeracy-Meets as a professional development model, in terms of what they most and least liked. All participants contended that the Numeracy-Meets provided relevant content, useful ideas for the classroom, and insightful information. They enjoyed the participation of invited speakers from different areas and the adult education focus. Chloe noted that she would share the research findings discussed at Numeracy-Meets with coordinators at her education centre so that the research could inform decision making at the centre.

Chloe: The resources that were made available as well the presentations and the recordings I think they are a great tool for us maths tutors to have to try to influence strategy at centre level if we can.

On the other hand, participants suggested that further practitioner participation in Numeracy-Meets sessions would be desirable.

Abbi: I would like to see even more tutors. I know every day we had tutors give a talk you know. I think I'd like to see that maybe, more of that…

They contended that more opportunities for discussion would have been beneficial and would have provided participants with further chances to interact with each other and be honest about the challenges they face.

In terms of structure, all interviewees liked the flexibility of the one-hour Numeracy-Meets sessions, which were conducted online via Microsoft Teams. In comparison to one-day, face-to face CPD sessions, Numeracy-Meets were short and continuous, giving practitioners time to absorb what they learned.

Deirdre: It was nice and condensed. I liked the topics. I liked the way it was run. You got a little bit of everything in a short period of time.

Abbi felt that online CPD was easier than face-to-face sessions, which are time-consuming and expensive due to travel and childcare costs. Deirdre concurred that face-to-face CPD is time-consuming and requires the practitioner to travel but noted that it does provide opportunities for practitioners to engage in informal chats to share ideas. Overall, all interviewees agreed that Numeracy-Meets were a very good model of CPD and expressed a desire for them to continue in the future.

Abbi: The only suggestion would be to continue having similar meetings as they were great support for those of us working in adult numeracy. It can sometimes feel that the issues we encountered are unique to our situation when in fact it's very similar in all adult education areas.

Conclusion

Given the vision of the ALL Strategy, the research outlined in this study is timely and topical and provides useful insights into the professional development needs and wants of adult numeracy practitioners in Ireland. The short implementation and evaluation of the model suggests that the Numeracy-Meets provided an effective, flexible, and cost-effective model for supporting and developing adult numeracy practitioners in Ireland. The participants developed a sense of being part of a larger community with a common goal to improve adult numeracy provision. However, despite the positive feedback, the authors are also aware that there is still much work to be done for Numeracy-Meets to become a sustainable professional development model going forward. While there was huge enthusiasm amongst the core participants for the series to continue, the longer-term viability and wider scale needs to be considered and this is more difficult in adult education in Ireland given the diverse and dispersed nature of practitioners. Going forward, further research is needed to create a solid foundation for practical interventions which support the development of adult numeracy practitioners in Ireland. However, Numeracy-Meets offer a useful starting point for the establishment of a national adult numeracy learning community to support practitioners in sharing resources and local good practice. They also provide a basis for the design, implementation, and evaluation of future research-informed professional development to support adult practitioners.

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Addressing (Inadequate) Mathematics Teacher Supply and Teacher (Mis)assignment: Considering Ireland's past, present and future

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This paper frames the issue of out-of-field mathematics [OOF] teaching, highlighting the practice as a common solution to mathematics teacher shortages and a product of the organisational constraints of staffing schools. Two approaches that have sought to address OOF teaching are documented: the provision of professional development in Ireland that increases mathematics teacher supply; and a policy initiative in the US that sought to govern teacher assignment in schools. The paper concludes with a turn to the advancements in defining OOF teaching and offers some considerations, as Ireland seeks to ensure that, where possible, secondary school students are taught by competent mathematics teachers.

Keywords: Teacher supply; out-of-field teaching; mathematics education

Introduction

Teacher supply is a significant issue facing the many state, national and international bodies tasked with governing their respective teacher workforce. In Ireland, secondary school leaders are reporting significant difficulties staffing their schools due to teacher shortages (Harford & Fleming, 2023). This issue of teacher shortages is not confined to Ireland, with reports in Australia (Peace, 2023), the US (Sutcher et al., 2019), and England (Jones, 2023) documenting a substantial shortfall in the supply of suitably qualified teachers. Furthermore, although the issue of teacher shortage is currently receiving attention in the public domain, it has been an enduring educational challenge (Sutcher et al., 2019). Mathematics teacher shortages have been omnipresent and are still being experienced worldwide (Behrstock-Sherratt, 2016). For example, in Sweden, school authorities encounter acute difficulties recruiting certified mathematics teachers (Boström, 2023); while in Estonia the existing shortage of qualified mathematics teachers could be exacerbated due to an ageing workforce and low supply of potential successors (Täht et al., 2023). Ireland, too, is experiencing mathematics teacher shortages, with school leaders identifying shortfalls in the availability of certified mathematics teachers (ASTI, 2023). However, despite the shortage of qualified teachers, mathematics classrooms are required to be staffed, and so, in such cases where a certified teacher is not available, alternative arrangements must be put in place. One such arrangement is to assign a teacher to teach a subject outside their field of academic preparation, a practice termed *out-of-field* [OOF] teaching (Conant, 1963). It is this practice of teacher misassignment that is at the core of this paper. What follows then is an account of the evolving definitions of OOF teaching and the solutions implemented – a professional development programme in Ireland and a policy initiative in the US – that have sought to eradicate its existence. These solutions were chosen to be reported on in this paper given their uniqueness – the governance of OOF teaching through policy has only been implemented in the US, while the professional development programme in Ireland is novel due to the large number of graduates. This paper also documents the most recently available figures on OOF mathematics teaching in Ireland and projected teacher shortages. The paper concludes with considerations for the future, as Ireland, like many other jurisdictions, continues to wrestle with the enduring issues of mathematics teacher shortages and OOF teaching.

OOF Mathematics Teaching in the US

For as long as mathematics teacher shortages have been an issue, OOF mathematics teaching has been the modus operandi. Reports in the US from the 1960s (e.g., Mills, 1961) highlighted that up to a third of teachers who were teaching mathematics were underqualified. Research by Robinson (1985), which set about uncovering the incidence of OOF mathematics teaching across US, found that the misassignment of teachers to mathematics classes was common practice. In 1999, drawing on national survey data, Ingersoll identified that about a third of all secondary school mathematics teachers did not have a major or minor in mathematics, mathematics education or a related discipline. More recently, Van Overschelde and Piatt (2020) found that, in Texas between the years 2011 and 2018, 24.4 percent of mathematics classes were taught by an OOF teacher, and Shah and colleagues (2019), drawing on national survey data from 2017, concluded that 58 percent of mathematics teachers were OOF. In the case of the latter, 28 percent of these OOF teachers had either a major/ minor degree or in-field certification in mathematics, but not both. The various definitions of an OOF mathematics teacher employed in these surveys highlights a further challenge, the complexity of measurement.

Measuring OOF Mathematics Teaching

Ingersoll (2002) remarks that the way in which one defines and measures OOF teaching impacts on the amount of OOF teaching one finds. For Ingersoll, measures of OOF teaching are not concerned with the quantity and quality of training, education and experience the teacher brings to the job, rather they focus on whether the teacher is *qualified* in each of the fields they are assigned to teach, once employed in a school. The nature of, and requirements for, being qualified to teach in a subject area differ across jurisdictions, and so measures vary according to the teacher quality standards held in a particular context. Teachers typically find themselves teaching more than one subject, and therefore, may be teaching both in-field and OOF as part of their employment. As a result, measures may differ regarding whether they are focused on those teaching OOF only or both in-field and OOF, and in the case of the latter, decisions are required about the amount of teaching a teacher must be doing in an OOF subject to be counted. Concomitantly, measures of OOF teaching may differ regarding focus, for example, the number of teachers teaching OOF, the number of classes being taught OOF, or the number of students being taught by OOF teachers. In Ireland, the research on OOF mathematics teaching has reported on the former – the number of mathematics teachers who are OOF –, using mathematics teacher certification, acquired in line with the criteria mandated by the governing body, The Teaching Council, as the determining factor.

OOF Mathematics Teaching in Ireland

In Ireland, research in late 2000s uncovered the scale of OOF mathematics teaching, with 48 percent of mathematics teachers surveyed uncertified to teach the subject (Ní Ríordáin & Hannigan, 2009). This research indicated that OOF teachers of mathematics were most likely to be under the age of 35 (60%) and assigned to lower secondary school classes. In a follow-up study by Goos and colleagues (2021) it was discovered that 25 percent of mathematics teachers were OOF, and akin to the previous study, a large proportion of these teachers were teaching at lower secondary school level. More recently, research from the Teacher Supply Data Working Group [TSDWG], on behalf of the Department of Education [DoE] (2022), found that 22.6 percent of mathematics teachers (1,438 of 6,353) are OOF. The reduction in Ireland's incidence of OOF mathematics teaching since 2009 is primarily due to the introduction of a professional development programme, *The Professional Diploma in Mathematics for Teaching* [PDMT], which enables teachers to acquire mathematics teacher certification.

The Professional Diploma in Mathematics for Teaching

The PDMT provides a route for OOF teachers of mathematics to add the subject to their teacher certification. The DoE provided €7 million in funding for version one of the PDMT (2012-2020), resulting in 1,068 teachers (4 cohorts) graduating from the programme, and committed a further €1 million for version two of the programme. It is intended that three cohorts will complete the latest version, leading to 600 teachers graduating by 2025. Research on the impact of the programme for the participants has shown that upskilled teachers developed self-efficacy beliefs akin to those of in-field mathematics teachers (Goos & Guerin, 2022), documented high levels of job satisfaction and affective commitment (Ní Ríordáin et al., 2022), and indicated a greater sense of belonging as a mathematics teacher, with validation for their practice provided by their teacher certification (Quirke, 2022). However, the latter research raised concerns about the teaching practices employed by the upskilled teachers, in addition to documenting that the teachers believed their practice was unaltered by the programme. Professional development programmes, such as the PDMT, address teacher supply as the root of OOF teaching, and as shown in Ireland, have the capacity to positively impact the issue. However, despite the progress Ireland has made in alleviating OOF mathematics teaching, it is still an ongoing challenge, particularly in the context of projected teacher shortage.

Ireland's Projected Teacher Shortage

Sutcher et al. (2019) note that teacher shortage typically refers to an insufficient production of new teachers, in response to the number of student enrolments and teacher retirements. These authors argue, however, that teacher shortage is more complex, given that it is affected by numerous factors, such as teacher turnover, changes in educational programmes, pupil-teacher ratios, and the appeal of the teaching profession both generally and in specific locations. In Ireland, inadequate teacher supply is a main cause for concern, with secondary schools experiencing a surge in enrolments, expected to peak in 2024. In their technical report, the Teacher Supply Steering Group, on behalf of the DoE (2021), projected that secondary school enrolments will have increased by 30,000 from 2019 to 2024, resulting in an additional 2,612 full-time equivalent teaching posts, taking the pupil-teacher ratio to remain constant at 12.3:1. When estimated retirements and resignations are considered, the group predicts teacher demand to exceed supply by 6,141 full-time equivalent posts. These projections do not account for teachers leaving the country to teach elsewhere, which appears to be an ever-increasing trend (O'Brien, 2023). In any case, the projections highlight the imbalance between demand and supply, further heightening the potential for OOF teaching. Teacher shortages can also vary in accordance with location (Sutcher et al. 2019), as the projections for Ireland demonstrate: Dublin is expected to experience the largest increase in secondary enrolments by 2024 (an increase of 13.3 percent), while the Mid-West region[1] is expected to experience minimal growth (2.3 percent). Additionally, subject specialisms result in teacher shortages varying across disciplinary areas, thus leading to OOF teaching at the local level even in cases where teacher supply surpasses demand (Sutcher et al. 2019). To account for these complexities associated with teacher shortage, Sutcher and colleagues (2019) generate the following definition:

> teacher shortages emerge in different fields and locations when there is an imbalance between the number of teachers demanded and the number of qualified teachers willing to offer their services to fill these demanded positions. (p. 4)

Addressing OOF teaching in different fields arising from teacher shortages is challenging; however, the issue is further complicated when the occupational and administrative characteristics of staffing schools are considered. For instance, the TSDWG research highlights that 646 teachers registered to teach mathematics are not currently teaching the subject, despite being deployed in schools. This points to other factors residing outside of the realm of teacher shortage that contribute to OOF teaching. These alternative factors derived from how schools are managed and how teachers are utilised once employed are situated within the organisational and occupational perspective on OOF teaching (Ingersoll et al., 2004).

Organisational and Occupational Perspective

In contrast to the rigorous regulations surrounding teacher certification, once a teacher is employed, school management has autonomy regarding the subjects they are assigned to teach (Ingersoll et al., 2004). This teacher assignment process for school leaders is complex; decisions are mediated by industrial legislation governing working conditions, teacher seniority issues, class-size guidelines, and contractual obligations. Additionally, teacher assignment is impacted by the need to provide a broad range of subject choice despite limitations to resources, budget, and staff, while provision must also be made for future retirements and reform efforts. Thus, the assignment of teachers at the local level constrained and impacted by a myriad of factors leads to OOF teaching. In the US, the policy initiative *No Child Left Behind* [NCLB] set about governing practices at the local level as a solution to OOF teaching.

No Child Left Behind

In the US, the NCLB policy was a unique educational reform that sought to eradicate OOF teaching, mandating that all classes were taught by 'highly qualified' teachers (McConney & Price, 2009). However, the complexities of sourcing highly qualified teachers, particularly in rural areas, resulted in schools acquiring waivers to employ teachers who did not meet the specified criteria. In line with the policy, these schools informed parents that

[1] The Mid-West region refers to the counties of Clare, Tipperary, and Limerick.

their children were not being taught by a highly qualified teacher, which consequently led to the district superintendents reassuring parents that a high-quality education was still being provided (Reeves, 2003). In 2015, the NCLB policy was replaced by the *Every Student Succeeds Act* [ESSA], abolishing the requirement of a highly qualified teacher for every class. Instead, under ESSA, each state determines teacher certification requirements and the criteria for being deemed highly qualified (ASCD, 2015). The ESSA, however, appears to be exacerbating the issue of OOF teaching, with a greater number of teachers being assigned to teach OOF (Van Overschelde & Piatt, 2020).

The approach of policy initiatives, such as the NCLB, sets out to eradicate teacher misassignment at the local level through regulation, yet teacher shortages, particularly in hard-to-staff areas, lead to the impracticalities of such a solution. Alternatively, the approach of professional development bolsters the available workforce through the provision of certification. Yet, the administrative and occupational constraints make it difficult, if not impossible, to eliminate OOF teaching in schools. Thus, although both solutions made varying levels of progress in reducing OOF teaching, neither approach has completely eradicated its existence. This highlights the complexity of the issue at hand and the need to develop innovative and sustained solutions to, perhaps not eradicate the issue, but reduce and manage its prevalence on an on-going basis. This calls for re-framing OOF teaching, and Hobbs and colleagues have done significant work in this area.

Reframing OOF Teaching

Hobbs et al. (2020) explain that an agreed definition of OOF teaching is required to inform policy, practice, and research. Subsequently, these authors distinguished six criteria which focus on numerous aspects of the phenomenon and serve to identify dimensions of 'suitable alignment' between a teacher and the subject/s they are assigned to teach: (1) Qualification; (2) Workload; (3) Capability; (4) Identity; (5) Structures; (6) Pathways. The first three criteria are located within the cluster, *Measurable Criteria*, with *Qualification* viewing OOF teaching as the misfit between teacher allocation and teacher qualification; *Workload* examining the allocation that maximises teacher effectiveness; and *Capability* drawing attention towards teacher expertise and experience. It is the perspective of Qualification that has dominated the Irish research landscape in terms of measuring OOF teaching, but perhaps future work should investigate both Workload, the proportion of the teacher's load that is OOF, and Capability, the degree to which an OOF teacher has engaged with professional learning and repeated practice of teaching the subject. Cluster 2, comprising *Identity and Structures*, draws attention to the self-reported criteria paramount to OOF teaching. In terms of *Identity*, teachers mediate OOF-*ness* in a variety of ways, spanning from high levels of personal and professional commitment to low levels of compliance. Research on teachers undertaking the PDMT programme elucidates this point, with some teachers pursuing the programme primarily for job security while others demonstrated high personal interest in the subject (Quirke, 2022). *Structures* accounts for the ways in which school contextual factors mediate OOF-ness. Research has shown that this is experienced in different ways by different teachers. In the Quirke (2022) study, one teacher was supported by her experienced colleague, whom she referred to as her "maths support worker" (p. 279), through the sharing of resources and feedback, while another teacher refused to ask colleagues for assistance, highlighting that this may be viewed as a sign of weakness. The final cluster, *Longitudinal Criteria*, encompassing *Pathways*, addresses the mechanisms by which an OOF teacher can become in-field. These mechanisms can be subdivided into qualification upgrade – the approach offered in Ireland through the PDMT, professional development concentration – which may be a precursor to micro-credentialing, and experience – be it sustained or temporary. How the OOF teacher then experiences the pathways implicates their identity, in the sense that they may accept the subject with extended identity, accept the subject but their identity is not extended to include the subject, or the teacher may not accept the subject, reluctantly teach it, and not acknowledge it as part of their identity. And so, this extensive work of Hobbs and colleagues points towards the lack of homogeneity across OOF teachers and their experiences, indicating that a one-size fits all approach may not best cater for their variety of needs.

Discussion

The definition for OOF teaching has evolved since the work of Conant (1963), Robinson (1985), and Ingersoll (1999, 2002). Being defined as OOF must now account for how the teacher feels; how the teacher views oneself and is viewed by others; along with their academic preparation, certification, expertise, and experience. Thus, the existing measures used for researching OOF teaching based on certification alone may not be capturing the phenomenon as it is now defined. There is a need then to develop more nuanced approaches for researching OOF teaching that reflect the theoretical advances. Furthermore, OOF teaching remains an under-documented and under-researched area. The reluctance to openly discuss and research OOF teaching is unsurprising, particularly given the possibility that this may jeopardise the teaching profession. Additionally, OOF teaching serves an important role in addressing teacher shortages and the unequal distribution of teachers both geographically and with respect to their subject-specific qualifications (Hobbs et al., 2020). Therefore, discussing and reporting on OOF teaching is a sensitive issue; it requires the complexities of the issues at hand to be accounted for and the provision of respect and acknowledgement for the work of OOF teachers. Moreover, often it is the case that these OOF teachers are subjected to misassignment as part of the recourse adopted by school leaders in response to broader, system-level challenges.

In Ireland research is required on where the PDMT teachers are now, investigating if they are still teaching the subject, and if not, the reasons for this. This would inform future iterations of the PDMT and other upskilling programmes such as those in Physics and Spanish teaching that have recently been introduced. Consideration must also be given to evolving the PDMT, acknowledging that teacher shortages coupled with organisational constraints inevitably lead to some level of OOF teaching. This advancement of the PDMT may consider the contributing factors to OOF teaching in terms of being *long-term* and *short-term*. From a long-term perspective, history informs us that mathematics teacher shortages have been ever-present, and so ongoing work is required to address this issue. The PDMT is suitably designed to continue in this respect. Short-term contributory factors occur at the local level, whereby school management misassigns teachers typically for a short period of time, often, in their view, for the betterment of the school in the long term. These decisions may, in some cases, be made due to succession planning and/or to increase subject choice for students. Short-term solutions are required for teachers who find themselves in such a position; this may not be in the form of acquiring formal certification, but it may offer some credentials that are acknowledged if subsequent study for teacher certification is pursued. There is also a need for greater awareness amongst school management of the consequences of misassigning teachers. In Texas, for instance, the tracking of student performance on courses taught by OOF teachers highlighted the detrimental effect the practice had on learning (Van Overschelde, 2022). Perhaps similar research could take place in the Irish context to inform future decisions made at the local level. Ireland has made significant progress in reducing OOF mathematics teaching; however, considering the projected increase in teacher shortages and the organisational constraints of staffing schools, Ireland must continue to be innovative and progressive dealing with the issue.

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Nurturing the Next Generation of Women in STEM

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The need to encourage greater female participation in Science, Technology, Engineering and Mathematics (STEM) is globally recognised as an education priority. The gender gap is most pronounced in mathematics-intensive fields including physics, computer science, engineering and mathematics. Such a gap is influenced by a range of complex and nuanced factors that combine to deter female students from considering STEM careers. These deterrents are in place from early childhood and are encountered across all spheres of influence in girls' lives, including individual, family, school, and wider society. Effective change requires a multi-faceted approach targeting all stages of girls' education. Additionally, a societal and cultural shift is required to help female students overcome existing barriers and ensure greater equality in STEM participation. This paper considers the perspectives of female STEM students who have overcome barriers to entry by choosing to study mathematics-intensive STEM in higher education. Informed by a qualitative research study, the paper explores how students' expectations for success, interests, and sense of belonging in STEM were nurtured during school. The objective is to provide a deeper understanding of how the school environment encourages STEM choices with the aim of informing potential avenues for further research into measures addressing the gender gap.

Keywords: STEM, gender, identity

Introduction

STEM education plays an ever-increasing role in today's world as we face unprecedented challenges. STEM provides the innovative solutions required to tackle challenges in areas such as healthcare, climate change and food security. Additionally, a workforce equipped with STEM skills is needed to ensure continued economic growth and national competitiveness (Dasgupta & Stout, 2014). In recent years the Irish Government, similar to countries worldwide, has placed increased focus on STEM education as Ireland aims to "*become the best education and training service in Europe by 2026*" (Department of Education and Skills, 2017, p. 3). However, statistics for Ireland show a significant gender gap in certain STEM fields where female students account for just 12% of electronics, 19% of software, 25% of physics and 32% of mathematics students (Higher Education Authority, 2023). Similar trends are seen worldwide (e.g.Cheryan et al., 2017). This research study aims to explore what motivates female students to choose STEM and how does the school environment encourage STEM choices.

Choosing STEM: Insights from Literature

Expectancy-value theory posits that achievement related choices are related to the combined influences of expectations for success and values (Eccles & Wigfield, 2020). The theory explains that, in order for female students to choose STEM, they need to believe in their ability to succeed (expectations for success) as well as having high interest (values) in pursuing STEM. Expectations for success and values develop over time and are shaped by students' previous academic achievements, individual characteristics, personal and social identity and how students perceive others' beliefs and behaviours (Eccles & Wigfield, 2020). Seeds of interest and enjoyment in STEM are set in early childhood and need to be nurtured throughout all stages of the educational journey from pre-school to higher education. At each educational stage, girls face barriers to STEM entry which cumulate from childhood to adolescence resulting in a significant gender gap. Beginning in early childhood, gender stereotypes (Banerjee et al., 2018) suggest that girls don't belong in STEM (Cheryan et al., 2017) , or have the ability to succeed in the domain (Eccles & Wigfield, 2020). In turn, this results in girls reporting less interest in and enjoyment of STEM (Eccles & Wang, 2016). Additionally, the factors that influence girls' beliefs and interests in STEM are shaped by messages they receive from home, school and wider society; i.e. the STEM ecosystem (Department of Education, 2020).

Within this STEM ecosystem, the role of the school environment in nurturing female students in Ireland to pursue STEM is underexplored. Through exploring how students' beliefs, interests and sense of belonging were shaped by their school experiences, a deeper understanding of how the school environment can encourage greater female participation in STEM is uncovered.

Research method

A qualitative research design, including semi-structured interviews, was selected as the research method since it facilitates in-depth exploration of how female students' academic beliefs, interests and sense of belonging in STEM were nurtured during school. A volunteer sampling approach (Cohen et al., 2018) was used and students from two universities in the south of Ireland were invited to participate. The criteria for selection were that the research participants were studying mathematics-intensive STEM programmes and were enrolled in, at least, the third year of undergraduate study. This cohort would have completed the Leaving Certificate examination in 2019 or earlier, prior to Covid-19 related adjustments to the examination process. Following the granting of ethical approval by UCC's Social Research Ethics Committee, one-to-one interviews were conducted with 21 students. Table 1 shows participant profiles with interviewees being identified by pseudonyms. The students' programmes of study include: S (physics or computer science); E(engineering) or M(mathematics). Where participants were studying two domains e.g., physics and mathematics, they were listed in both the S(science) and M (mathematics) categories shown in Table 1. As noted, participants were registered with two universities in the south of Ireland - 15 participants were registered with one university and six with the other.

The rationale for the research study was explained to participants at the start of the interview. Expectancy-value theory (Eccles & Wigfield, 2020) provided a guideline for investigating participants' school experiences. Questions explored academic achievements in mathematics-intensive STEM subjects; participants' confidence in their abilities; the development of participants' STEM interests and enjoyment; perceptions of how school and teachers viewed STEM; and how teachers, family and peers influenced participants' decision to choose a mathematics-intensive STEM field of study in higher education.

Table 1

Profile of Research Participants

Student Pseudonym	STEM Field of Study	Degree or year of undergraduate study	Completed TY	STEM Programmes in TY?	LC Secondary School Type	Higher Level LC subjects
Alex	S	PhD	No	n/a	Mixed	*Subjects for Secondary School:* Maths, Physics, Chemistry, Biology, Geography, Languages
Alice	S	4th	Yes	Exhibition	Mixed	Maths, Physics, Chemistry, Irish, English, French, Home Economics
Anne	SE	PhD	No	n/a	Mixed (DEIS)	Maths, Biology, English, French, German, Music, Geography
Caoimhe	E	3rd	Yes	Event promoting STEM to females	Mixed	Maths, Chemistry, Agicultural Science, Irish, English, German, Accounting
Caroline	M	4th	No	n/a	Mixed	Maths, Physics, Chemistry, Applied Maths, DCG, Irish, English, French
Clodagh	E	4th	Yes	Campus week	Girls	Maths, Physics, Chemistry, Applied Maths, Irish, English, French, Art
Emily	E	4th	Yes	Campus week for female students	Mixed	Maths, Physics, Chemistry, Applied Maths, English, Home Economics
Jade	M	4th	No	n/a	Mixed	Maths, Chemistry, Biology, Applied Maths, Irish, English, Spanish, Business
Jennifer	E	4th	Yes	Campus week	Mixed (fee paying)	Maths, Physics, Chemistry, Applied Maths, English, Accounting, Home Economics
Kathryn	E	MEng	Yes	Campus Week	Girls	Maths, Physics, Chemistry, Irish, English, German, Accounting
Leah	E	4th	Yes	n/a	Mixed	Maths, Physics, Applied Maths, Irish, English, French, History, Geography
Lola	S	4th	Yes	Campus week for female students	Mixed	Maths, Physics, Chemistry, Irish, English, French, Art
Madeline	S	3rd	Yes	Company visit	Mixed	Maths, Physics, Chemistry, Biology, English, Art
Molly	E	4th	Yes	Campus week for female students	Girls	Maths, Physics, Chemistry, Irish, English, German, Home Economics
Natalie	SM	3rd	Yes	Campus week for female students	Mixed (DEIS)	Maths, Physics, Chemistry, Applied Maths, Irish, English, French, Geography
Niamh	SM	4th	Yes	Campus week for female students	Mixed	Maths, Physics, Chemistry, Irish, English, German, Accounting
Rose	S	PhD	Yes	Campus week	Girls	Maths, Physics, Chemistry, Biology, Irish, English, Spanish
Sonia	SM	3rd	Yes	No	Mixed (fee paying)	Maths, Physics, Applied maths, Engineering, Irish, English, French
Sophie	E	4th	Yes	Event promoting STEM to females	Mixed (Fee paying)	Maths, Physics, Chemistry, Biology, Applied Maths, Irish, English, German
Úna	S	3rd	Yes	Campus week	Mixed (DEIS)	Maths, Physics, Chemistry, Biology, Irish, English, Spanish
Zoe	E	MEng	Yes	No	Mixed	Maths, Physics, DCG, Irish, English, German, Geography

Interviews were conducted online via Microsoft® Teams and were typically 50 minutes in duration. Online interviews were found to be an effective means of gathering data given that all participants were familiar with online meeting tools, having spent at least one college semester online during the Covid-19 pandemic. This format allowed students to be interviewed in an environment where they felt comfortable and relaxed. Video recordings of the interviews enabled the researcher to capture participants' unspoken responses and reactions to specific topics. The video recordings were deleted once anonymised transcripts of the interviews were created with each participant being identified via a pseudonym.

Reflexive Thematic Analysis was selected as a method of data analysis since it facilitates "exploring, interpreting and reporting relevant patterns of meanings within a dataset" (Braun & Clarke, 2022, p. 224),whilst still ensuring analysis is sufficiently rigorous to yield credible and meaningful results (Nowell et al., 2017). In this case, the anonymised interview transcripts were inputted to NVivo where the six-phase thematic analysis process was adopted. These are: dataset familiarisation; data coding; initial theme generation; theme development and review; theme refining, defining and naming and write-up (Braun & Clarke, 2022). These phases are non-linear and recursive with themes being actively generated through repeated iterations of the data coding to theme naming phases.

Following this process, three themes were generated to capture how students' school experiences impacted their decision to study STEM. These are "STEM as Appropriate" (relating to belonging); "STEM as achievable (relating to expectations for success), and "STEM as attractive" (relating to values).

Findings and Discussion

Within the STEM ecosystem, school experiences play a crucial role in encouraging female students to develop positive attitudes towards STEM, thereby countering barriers to entry that girls face. The research findings and discussion describe how the combination of three factors (themes) determined participants' decision to choose STEM in higher education. While these findings aren't intended to be generalizable across and entire population, it is intended that they provide fresh insight to inform further research for measures addressing the STEM gender gap in second level education.

STEM as Appropriate

Encouraging girls to view STEM as an appropriate career choice involves countering gender stereotypes, encouraging a sense of belonging in STEM (Dasgupta & Stout, 2014), and promoting career awareness.

Jennifer spoke of how she was always interested in STEM subjects and performed well in STEM in school. Thus, she viewed STEM as achievable and attractive. However, it was only when her teacher suggested that she might consider engineering that Jennifer began to view engineering as a "thinkable" (DeWitt & Archer, 2015) option for her. When asked how she came to choose engineering, Jennifer described a conversation with her teacher saying;

> He (*teacher*) said, well, do you like math? Have you considered engineering? And I hadn't up until then. Nobody had said it to me, it had never come across my mind because there's no engineers in my family, nobody knew any engineers, you know… and then I was like, oh well, I'm doing all the sciences. I like maths, I like applied maths. Why don't I go down that route? So that's what happened anyway (Jennifer).

In Jennifer's case, even the suggestion of engineering as an appropriate option provided sufficient impetus, alongside her existing academic beliefs and interest in STEM to lead her to choose engineering.

Teacher awareness and promotion of 'Out of School' programmes (Department of Education and Skills, 2017, p. 8) which raise awareness of STEM opportunities can positively impact girls' attitudes. Lola spoke of how her teachers always encouraged students to participate in STEM related events. She became particularly animated when describing her participation in iWish (iWish, n.d.) in Transition Year (TY).

> I loved that week..... It was actually very powerful, really kind of impactful, like really made me feel very included, very involved, very like they wanted you there. Like, they're like hello, there is space for you (Lola)

For Lola, the opportunity to meet with female role models encouraged her to view STEM as a welcoming field in which she could find a "space". The positive impact of TY programmes aimed at promoting STEM careers to female students was mentioned by 7/21 participants in this study while a further 5/21 participants spoke of how attending campus weeks piqued their STEM interests.

A notable finding of this research was that all participants spoke of high parental support for their decision to study STEM in higher education. This would suggest that STEM information campaigns need to target not just students, but must equally aim to raise awareness among parents (Department of Education, 2022, p. 9), especially for parents who don't themselves have experience of working in STEM. For example, Úna spoke of how her parents discussed STEM programmes of study with a guidance counsellor.

> So they (*parents*) didn't know what it was at the start. But once we kind of went through it with the guidance counsellor, I think rather that understanding of what the course is, they were like, oh it fits you, it kind of sounds like you, you know, and they were full steam ahead then (Úna).

Úna's mention of how her parents felt the course "fit[ted]" her and of how it "sounds like you" shows how her parents helped Úna foster a sense of belonging once they themselves gained career awareness. This example demonstrates the need to align the STEM ecosystem to ensure messages of encouragement from school are reinforced with similar message from home.

STEM as Achievable

Girls lack of confidence and self-efficacy in their mathematics abilities is frequently cited as one of the reasons for the STEM gender gap (e.g.Cheryan et al., 2017; Eccles &

Wang, 2016; O'Rourke & Prendergast, 2021). These beliefs are significantly influenced by parents and teachers, with both underestimating girls' mathematical abilities (McCoy et al., 2022). Additionally, where teachers hold low expectations for their students, this can have a negative effect on student performance (McKown & Weinstein, 2008).

When asked if she was always confident in her mathematics abilities, Alice was explicit in stating that her lack of confidence was attributable to her teachers' lack of confidence in her.

> No. In junior cycle, yes, I would have been. In leaving Cert no, because my teacher didn't have confidence in me to be honest. I had a very good class, with lots of girls actually….. Even now, like four years into a physics degree, I still think I'm bad at maths, but I'm obviously not. I still have this thing in my head that I can't do it, but then when I get into it I can (Alice)

Alice further explained that she was in a high achieving mathematics class in school and how her teacher would hand out class test and exam results in order of who achieved the highest marks. She said that mathematics was her worst result in her Leaving Cert (she got a H2) and even though she was excelling academically in University, the belief that she's "bad at maths" persisted. Although she had sufficient self-efficacy in her mathematics ability, her lack of confidence was still shaped by her senior cycle experiences.

On the other hand, Sonia's experience demonstrated how teacher encouragement can raise expectations for success and subsequently performance. She said of her mathematics confidence;

> I never really visualized myself getting H1, like I was always thinking H3…..maybe a H2, if I did well. While she (*mathematics teacher*) was saying, "no you could definitely get a H1" and I was like, ok maybe I can get a H1 (Sonia)

Research participants' expectation for success in STEM was found to align to that of their teachers with students who reported having "good" teachers also reporting high academic beliefs. Students' expectations for success grew through their being faced with challenging, but achievable, tasks. Alex described their mathematics class as one where "*I really liked the fact that it was challenging, but also he (teacher) was very helpful and attentive to students' needs so he wouldn't be dismissive*" while Molly spoke of how her teacher;

> was just really clear in explaining things… she (teacher) also kind of held the class in a way that the people who were really, really good could go on and they could do extra bits and she could go over and help everybody, but she wasn't overloading the people who weren't getting it (Molly)

Teachers with high self-efficacy themselves are more likely to create a learning environment where they set challenging goals and use teaching practices that increase students' self-efficacy (e.g.Banerjee et al., 2018). Thus, ensuring that teachers have access to continuous professional development may also assist in addressing the STEM gender gap.

STEM as Attractive

There is widespread agreement in the literature that lack of interest and enjoyment in STEM is one of the primary barriers to entry for female students (Banerjee et al., 2018; Eccles & Wang, 2016).

Common amongst all the participants in this study was that their interest and enjoyment of STEM mirrored that of their teachers: *"he was so passionate about it. I think it rubbed off on all of us as well, like we were kind of excited, going into class"* (Úna). Seeing the relevance of STEM subjects in a broader context, beyond the curriculum, was also valued by students. Jade, a mathematics student, described how her mathematics teacher would:

> explain the same thing in ten different ways and he was really passionate about maths too… he was able to engage you and stimulate a genuine love of math and numbers into people… he wasn't just talking about maths but his understanding of the universe (Jade)

While Clodagh described how she loved when her teacher would "*talk about things outside the curriculum. He'd branch out a bit…. we'd always be asking him questions even if it was not relevant*".

An interesting aspect of the findings was that teacher gender didn't have any impact on how students came to view STEM as an attractive career choice. Participants were equally as enthusiastic in describing how positive learning experiences in STEM classrooms increased interest, motivation and enjoyment, regardless of whether their teacher was female or male.

Conclusions

This qualitative research study considered how female students' decision to study STEM in higher education was shaped by their school experiences. Greater female participation in STEM requires encouraging more female students to view STEM as an appropriate, achievable, and attractive career option. Positive classroom experiences nurtured students' academic beliefs and interests in STEM. Career awareness campaigns, such as transition year campus weeks, can significantly influence female students' STEM attitudes in fostering a sense of belonging and heightening STEM interests. Extending STEM career awareness campaigns to include teachers and parents would help reinforce appreciation of wide range of opportunities that STEM careers can offer.

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Do Undergraduate Medical Students Use and Value Mathematics Skills?

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Prospective medical students in Ireland typically gain high marks in mathematics but medical courses have little mathematical content. We explored medical students' usage of mathematics during their studies and their views on its usefulness and whether mathematics should be a training requirement. Participants completed an anonymous 14-item online survey. 50 students who had completed the Leaving Certificate examination (LC) and 18 A-Level students responded. 62% of LC students reported completing a mathematics problem since entering medical school. 48% used mathematics at least once or twice a month and 16 reported that they found it at least moderately useful. 58% thought mathematics knowledge helped their medical studies. 60% thought mathematics should remain a requirement for medicine. There was no significant difference from the A-level group. We found that mathematics was perceived of some use to the students but it was not regarded as being of great value.

Keywords: medical education, mathematics knowledge, mathematics usage

Introduction

Undergraduate medicine courses in Irish universities are highly competitive requiring both high examination grades and the completion of an aptitude test. Bonus points in LC higher level mathematics mean medical students typically enter with high mathematics grades and competency. There is no specific mathematical element to the curriculum in medical school.

This study explored Trinity College Dublin (TCD) medical students' usage of mathematics during their course, their perceived usefulness of mathematics, and their views on whether it should be a requirement to enter medicine. We compared the responses from students in their first two pre-clinical years with those from the senior, clinical years (years three to five). We also compared responses from LC and A-Level students.

There is evidence that mathematics can help understanding, prediction, treatment and data processing within medicine. Rolison (2020) found that lower numeracy skills were linked to misinterpreting statistical health risks. Considering the use of mathematical and numeracy skills in decision making, it would seem that having a good mathematical foundation would be beneficial to medical students.

Methodology

Following ethical approval, TCD medical students were invited in March 2023 to complete a survey by email and anonymous data were collected through an online survey tool. Demographic data were collected on gender and year of study. First year students are typically aged from 17 to 19 years. The course is 5 to 6 years in duration. The survey had 14 questions trialled for potential responses during survey development. The survey included 4 Likert scales (five-to-seven-point scales), 9 closed questions and 1 open question.

Results

Complete responses were received from 80 medical students out of 970 medical students from years one to five. Of these respondents, 50 had completed the LC and 18 had completed A-levels. The remaining students had completed a range of other qualifications varying from the International Baccalaureate to undergraduate degrees. The sample was representative in terms of gender; 55% of current medical undergraduates in TCD are female.

Of the 50 LC responders, 34 were female and 16 were male. 25 were in pre-clinical years (years one and two) and 25 were in clinical years (years three, four and five). 31 (62%) of the students reported having solved a mathematics problem since entering medical school. 24 used mathematics at least once or twice a month. 16 thought mathematics was extremely or moderately useful, 18 slightly useful and 13 useless.

When asked about situations '*where you thought having a better understanding of mathematics would help in your medical studies?*', 29 (58%) responded definitely or probably yes. Thirty of the 50 thought that mathematics should remain a LC requirement for entry.

Using Chi square tests, significant differences were found between pre-clinical and clinical students about how often they used mathematics (1-2 times per month or more frequent 44% vs 56%, X^2 =0.7, p=0.4) or whether mathematics was useful to them (72% vs 68%, X^2 =0.1, p=0.75). No difference was found between students who had taken the LC or A-levels in whether they found mathematics useful 34/50 (68%) vs 11/18 (61%), X^2 =0.3, p=0.6. There was also no difference in whether they felt mathematics should be a requirement to enter medicine 30/50 (60%) vs 10/18 (56%), X^2 =0.1 p=0.7 chi square.

Discussion

Whilst few students had any great hostility towards mathematics, only about one third felt it was more than slightly useful, a single respondent thought it was extremely useful and 5 said it definitely helped their medical studies. That acknowledged, 31 reported solving a mathematics problem since medical school entry; being used in calculation of drug doses and in situations such as calculating renal function. The majority of both LC and A Level students agreed that mathematics should remain an admission requirement.

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Creating the Links: Establishing Home School Learning of Mathematics through Paired Maths and Maths Eyes Initiatives

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In light of COVID-19 and the unprecedented interruption to education that came with it, the research school sought to address and ameliorate the consequences of the two-year disruption to pupil learning. The research school looked to implement and evaluate a Paired Maths and Maths Eyes initiative at a whole school level. The aim of these chosen initiatives are to empower parents to get involved in their child's mathematics learning, improve and foster home school links in a curricular area, improve parent-child relationships, parents' own learning, and make learning through mathematics fun. These initiatives aim to promote active learning, improve concentration and help pupils develop problem-solving skills. The activities involved in the Paired Maths initiative allow for repetition and consolidation of concepts. Attention was also given to the reinforcement of mathematical language.

Keywords: parental involvement, primary mathematics, homework

Effectiveness of Parental Involvement in Children's Mathematics Education

Numerous researchers have sought to identify those aspects of parental involvement that are the most effective in influencing students' learning generally (Bailey, 2006) and in mathematics in particular (Cai et al, 2003). Contributing factors to children's mathematical achievement range from parental expectation, students' perception of parental involvement or influence (Cai et al, 2006); parents' beliefs in their child's competence (Aunola et al, 2003) or parental involvement in interactive homework activities (Bailey, 2006). According to Bailey (2006) increasing student learning through meaningful parent-child interaction during the completion of homework has emerged as a significant variable for improving learning outcomes of low-performing students.

Methodology

Table 1

Overview of Research Schedule

September	• Preparation and organisation/inventory of resources • Information workshop for parents
October - December	• Junior class and middle class 6 week roll out • Questionnaire for Parents/Pupils/Teacher Focus Group interviews with pupils • Evaluate findings • Feedback to staff to inform phase 2
January - March	• Roll out of Paired Maths in middle and senior classes for 6 weeks

	Questionnaire for Parents/Pupils/Teachers Focus Group interviews with Pupils Evaluate findings Share findings with staff and school community
March -Maths Eyes	Whole school initiative A week in March was identified by whole staff where homework was Maths Eyes and working as Maths Detectives Information shared with parents explaining Maths Eyes initiative A mixture of qualitative and quantitative data methods were used through questionnaires and focus group interviews

Findings and Discussion

Parents welcomed the alternative approach to mathematics homework with many acknowledging that mathematics does not have to be confined to the mathematics textbook for it to have meaning and value. Questionnaires and focus group interviews found that pupils were challenged more in a meaningful way, they were engaged with the tasks, and displayed greater intrinsic motivation to learn and solve problems. The Maths Eyes allowed pupils to take on the role of Maths Detectives where they looked at the world around them through a mathematical lens. It allowed them the freedom to be creative and imaginative. Pupils challenged each other to problem solve, make meaning, ask questions, prove their solutions to their peers, be open to the different solutions of others and learn alternative methods in which problems could also be solved. Pupils were communicating using appropriate mathematical language, making connections and reasoning. Pupils also enjoyed the autonomy of being a Maths Detective.

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Challenging Discontinuities in Early Years Math Pedagogy: a participatory, cross-sectoral approach to improving math outcomes for lower SES children in Ireland

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Research findings point to strong links between socioeconomic disadvantage and educational outcomes in mathematics. This paper presents an overview of some of the literature and the methodology underpinning the 'Count Me In' project which aims to improve mathematics outcomes and pedagogical discontinuities for children experiencing socioeconomic disadvantage through establishing cross-sectoral professional learning communities made up of educators from pre compulsory and compulsory school sectors. Early findings from the project suggest that while participatory methods are complex and messy, they provide scope for improved teacher agency, create a forum for depersonalised collective enquiry, foster the use of evidence-aware practices in math, and address transition discontinuities via a focus on math pedagogy.

Keywords: early mathematics education, socioeconomic disadvantage, cross curricular collaboration, transition, participatory research

Theoretical Perspective

There is a strong link between socioeconomic disadvantage and educational outcomes in mathematics. In Ireland, at primary school level, there have been attempts to mitigate this negative relationship, primarily through the DEIS Scheme (Delivering Equality of Opportunity in Schools). Whilst the scheme has seen some improvement in student outcomes in mathematics, the 2018 PISA report highlights that Irish pupils from non DEIS schools continue to 'significantly and substantially' outperform those from DEIS schools in math, with students from non DEIS schools performing above the OECD average and those from DEIS schools scoring significantly below it (OECD, 2019). While the reasons for this are complex and, notwithstanding structural injustices, schools are institutions that can be seen to maintain existing cycles of advantage and disadvantage rather than acting to disrupt them. Several studies have shown that well-planned mathematical programmes in early education settings can work toward reducing the achievement gap between lower SES children and more middle SES groups (e.g. Anthony and Walshaw, 2009).

The transition from pre-school to primary school is increasingly seen as a dynamic process of continuity and change (ETC, 2011) rather than an event reliant on maturational constructs of preparedness (Boyle, Petriwskyj and Grieshaber, 2018). Internationally, research suggests that children from lower socio-economic backgrounds are on average more likely to experience difficult transitions to school, stemming in part from curriculum and pedagogical discontinuity and lack of practice sharing.

Maths in Early Childhood

International research consistently points to the need for high expectations of young children's potential as learners of mathematics where effective math pedagogy is built on the premise that all students are powerful mathematics learners (Anthony and Walshaw, 2009). With the advent of neuroscience relating to mathematics learning, there is a growing awareness that mathematics is for all and that young children can access powerful mathematical ideas regardless of age or background (Perry and Dockett, 2008).

Communities of Practice

A common theme across much of the literature around effective early years practice is that of partnerships: partnerships between educators and families, educators and the community, educators, and children and increasingly, between educators themselves. It has been argued that to better understand the effective pedagogies and practices that enhance young children's learning in math, more cross sectoral research is needed (c.f. Anthony and Walshaw, 2009, Perry and Dockett, 2008). Communities of practice can act as a framework within which collective reasoning, linking lived and formal experience, and mobilising for future practice can take place (Boonstra et al., 2022) in order to improve practices and bring about sustainable, context driven change.

Methodology

This study adopts a critical participatory action research approach that builds on Habermas' notion of the public sphere (Habermas, 1992) and views research as a social practice (Kemmis et al., 2014). It arises as a critique of conventional research in acknowledging that research is a social practice that can challenge what is unproductive, irrational, and unjust in our collective practice. This collaborative approach to research values shared ownership, seeks a community-based analysis of problems and subsequent action is community orientated.

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Enhanced mathematics performance following spatial skills training using the online learning platform RIF3.0 within the mathematics classroom

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Previous studies on digital spatial training programmes have not always exhibited transfer to mathematics performance. In the present study, we investigated the effects of a digital training intervention, which targeted a variety of spatial skills, on students' performance on a variety of mathematics topics. Following a pre-post quasi-experimental design, 12-13 year-old students in an Austrian school participated in eight spatial training sessions during their mathematics classes through the online spatial thinking training platform RIF3.0, while students in the control classes received regular mathematics instructions. We measured students' arithmetical, place value, geometry, and word problem-solving skills following the Austrian mathematics curriculum guidelines. The preliminary results showed that students in the intervention classes improved their accuracy in arithmetical and geometry tasks, while students in the control classes did not show any improvement. These findings suggest that broader digital spatial training might lead to curriculum-based mathematics improvement.

Keywords: spatial ability, mathematics performance, online learning, RIF3.0

Introduction

There have been studies which demonstrate that digital spatial training transfers to mathematics performance (Hawes et al., 2022). For example, in a lab setting, Gilligan et al. (2020) found that eight-year-old students who received mental-rotation spatial training through instructional videos, improved in missing-term problems. However, in a school setting, Hawes et al. (2015) found that computerized mental-rotation training did not affect 6-8-year-old students' calculation skills. While previous studies focused on training one specific spatial skill to examine transferability in one particular mathematics topic, we aimed to train a broader range of spatial skills, based on the holistic spatial thinking model by Maresch & Sorby (2021). Our goal was to train students in a variety of spatial skills that they could subsequently utilize when solving mathematics tasks in areas such as arithmetic, place value, geometry, and other word-problem tasks. To achieve that, by use of the online spatial training platform RIF3.0 (*RIF 3.0 Home*, n.d.), we trained participants in a combination of spatial skills: visualization, form constancy, position in space, transformation in space, and object combination (Maresch & Sorby, 2021). We specifically examined whether the broad in-mathematics-class spatial training intervention transferred to mathematics performance.

Method overview

Forty-eight 2nd-grade middle-school students (M_{age}=12.31 years, *SD*=0.59) in Austria participated in the study. The intervention group consisted of 26 students (15 identified as female, 11 identified as male) while the control of 22 (16 female, 6 male). During a four-week intervention, students completed eight 15-minute spatial training sessions (two per week) in the RIF3.0 platform. Each session covered tasks from each of the following spatial skills: visualization, form constancy, position in space, transformation in space, and object combination. During that period, students in the control classes received mathematics instructions according to the mathematics curriculum. We assessed students' mathematics skills one day before and after the intervention. For the assessment, we used a collection of arithmetical, place value, geometry, and word problem tasks, which we developed based on the description of mathematics competence provided by the Austrian Ministry of Education.

Key Findings

Students from the intervention group significantly improved their accuracy in the arithmetical and geometry tasks while students at the control group did not (Table 1). However, student's accuracy in the place value and word-problems tasks, both among the intervention and the control groups, was not improved (Table 1).

Table 1

Results of paired sample t-tests comparing students' scores in the pre and post-tests

Mathematics tasks	Group	Pre-test		Post-test		*t*(df)	*p*	Cohen's *d*
		M	*SD*	*M*	*SD*			
Arithmetical	**Intervention**	**5.77**	**1.28**	**6.7**	**1.41**	**3.728(25)**	**< .001**	**0.731**
	Control	6.27	1.28	6.05	1.81	-0.561(21)	0.581	-0.120
Place value	Intervention	4.54	1.14	4.64	1.45	0.667(25)	.511	0.131
	Control	4.91	1.10	4.60	1.14	-2.037	.054	-0.434
Geometry	**Intervention**	**4.52**	**1.9**	**5.25**	**1.84**	**2.225(25)**	**0.035**	**0.436**
	Control	4.18	2.46	4.16	2.78	-0.074(21)	0.941	-0.016
Word problem	Intervention	4.50	2.02	4.04	2.41	-1.312(25)	.202	-0.257
	Control	5.18	2.50	5.27	2.23	0.222(21)	.827	0.047

Conclusion

These findings suggest that training in a combination of spatial skills may lead to improved performance in arithmetical and geometry tasks but not in place-value and word-problem tasks. Unlike Hawes et al. (2015), our within-school intervention yielded positive mathematics-related outcomes, which are in line with the lab-based findings of Gilligan et al. (2020). In our study, we demonstrated that online training in multiple spatial skills within the mathematics classroom may contribute to curriculum-based mathematics improvement.

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Investigating the use of gamification and the relationship in students' mathematical resilience

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Mathematical anxiety (MA), the feeling of discomfort, stress and fear when encountering Mathematic classes, tests, or problems, is an inevitable phenomenon in many educational settings. This anxiety can result in students avoiding mathematical situations which can cause poor academic achievements. Understanding the causes and issues involved with MA is essential for educators to develop the skill and expertise to prevent it and, in some cases, spin this feeling of anxiousness into resilience. In this paper, we explore the concept of technological gamification as a possible solution to the issue of MA and the implementation of Mathematical Resilience (MR). MR refers to the perseverance of challenges faced when encountering Mathematical problems. We examine the potential advantages of technology, and more specifically gamification as tools to encourage MR. Gamification has shown positive promising results in promoting MR although different kinds of gamification and the effectiveness in cultivating a positive stance between student and the subject need to be further explored. The paper presents findings as well as suggested practical strategies, such as opportunities for active learning, for post-primary teachers to incorporate in their classrooms.

Keywords: Mathematical Anxiety, Mathematical Resilience, Gamification, Technology

In post-primary education there is a constant challenge between educators and students about how to relate abstract concepts learned in Mathematics class to everyday situations (Schoenfeld, 1987). This can create a divide or separation between student and subject. The use of game elements in non-game contexts, known as gamification, has been increasingly applied to various domains, including education and within the classroom setting in order to enhance the academic achievement of students (Sakair and Shiota, 2016). This research paper will explore the application of gamification in Mathematics Education in addressing the issue of MA and fostering students' resilience towards Mathematic classes and tests. Ashcraft (2002) defines MA as "a feeling of tension, apprehension or fear that interferes with maths performance". Wither (1998) builds on this definition describing MA as a very serious problem that is present in many classroom environments. MR on the other hand is "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson and Suinn, 1972).

The Mathematical Anxiety Scale (Betz, 1978) and Mathematical Resilience Scale (Kooken et al., 2013)

The Mathematical Resilience Scale (MRS) (Kooken et al., 2013) is a 23-item scale consisting of a variety of both positively worded and negatively worded statements. Participants must submit a descriptor on a five-point Likert scale 1 being strongly agree, 5 strongly disagree. The score of each participant is calculated by reversing half of the scores and summing them together. The higher the score obtained the more evidence of resilience shown towards Mathematics. The Mathematical Anxiety Scale (MAS) (Betz, 1978) is a 10-item scale. The MAS has the same scoring system as the MRS. Each participant will be awarded a score that indicates their level of anxiety towards Mathematics.

Mathematical Software's and Technologies

Incorporating gamification in education has gained quite positive recognition on academic performance and achievement. An area requiring further research however, is how technology and gamification impact the MR of students. This study will explore the effects of the same by using various software platforms that are expected to increase students' resilience of Mathematics. There are numerous softwares that play a significant role in the moulding of education, including GeoGebra, MathsBot, Desmos etc., all of which will heavily influence this study. Using an Action research methodology, the study will use such technologies during an 8-week programme known as the Mathematical Resilience Enrichment Course (MREC) where the students that show a low level of resilience or a high level of anxiousness towards Mathematics will take part in various activities to foster their resilience towards Mathematics.

Conclusion.

To conclude, the use of gamification is predicted to address the issue of MA and foster MR. By integrating such tools into the Maths classroom educators should create a positive classroom experience. The Mathematical Enrichment Course will offer a practical, realistic strategy that will provide a hands-on learning experience for the participants.

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Engaging with Mathematical Modeling to Understand the Wider World: A Case Study of Primary Teachers' Encounter with Research and Implementing New Learning to the Classroom

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Mathematical modeling is a pedagogical practice that is being introduced in the draft Primary Mathematics Curriculum (NCCA, 2022). This research explores teachers' understanding of mathematical modeling as a practice that emphasises complex problem solving to understand the wider world. Five primary teachers engaged with key research articles (Doerr & English 2001; English & Watters 2004; English & Watters, 2005) over three action research cycles and implement the learning in their context. Findings suggest that reflection on pedagogical content knowledge is a lens for teacher change of current practice.

Keywords: mathematical modeling, pedagogical practices, mathematics education

Introduction

With the introduction of the revised draft Primary Mathematics Curriculum (NCCA, 2022,) mathematical modeling as a pedagogical practice is described as children forming "models through a process of testing, revising and expressing their interpretation of different mathematical ideas, experiences, problems and situations; typically posed to them as questions or challenges" (p. 29). This research looks at teachers' understanding of mathematical modeling after engaging with research articles and implementing their learning in their classrooms. Perspectives proposed by Doerr and English (2001), English and Watters (2004); English and Watters (2005) that mathematical modeling requires complex models/ artefacts or conceptual tools to solve the real world problems (Dooley, Dunphy, & Shiel, 2014). This research is part of a larger PhD but this paper will focus on one aspect of the study.

Methodology

A qualitative case study approach using action research (McNiff, 2017) was chosen as it would enable complex understanding of mathematical modeling to be captured from five participants in the field. This was a convenient sample. Three cycles of action research (McNiff, 2017) were conducted where participants engaged in an article, reflected on their key learning, implemented this learning with their class and evaluated it. This led to modification or changes they would make to their practice before engaging in another cycle of action research. In the third cycle of action research, the lesson was observed by the researcher. The data collection tools were adopted and underpinned by the Teaching for Robust Understanding (TRU) framework (Schoenfeld, 2016) which focused on; content, cognitive demand, equitable access to content, agency, ownership and identity, and formative assessment. Data collection tools include transcripts of the action research meetings, participant reflective diaries, samples of pupil artefacts, pre and post observation lesson transcripts, and an exit interview transcript that was conducted at the end of the research. Thematic analysis (Braun & Clarke, 2022) was conducted to ensure rigour across the dataset.

Findings

Through thematic analysis the overarching theme of pedagogical content knowledge as a lens for teacher change emerged. From this a number of themes arose which highlight teachers' understanding of mathematical modeling after engaging with the research and implementing it in the classroom. These include importance of the task where a context is given and there is no right or wrong answer, thematic approach to complex problem solving, the prior knowledge of the learner, the balance of power when teaching complex problem solving and agency, student voice, mathematisation, understanding the wider world, importance of class discussion, group dynamics and strategies to share learning. The data showed how participants' understanding of mathematical modeling is negotiated and evolves throughout engagement in the action research cycles. Examples of change to practise is evident throughout.

Conclusion

Teachers' understanding of mathematical modeling evolved after engaging with the research and implementing it in the classroom. The findings could contribute to implementation of complex problem solving for teachers wishing to engage in this process in their classroom. This case study demonstrates how engaging in pedagogical practices (mathematical modeling) and reflecting on learning can lead to change in how we teach mathematics.

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