CVaR-based Robust Beamforming Framework for Massive MIMO LEO Satellite Communications

Madyan Alsenwi, Eva Lagunas, Hayder Al-Hraishawi, Symeon Chatzinotas

Interdisciplinary Centre for Security, Reliability and Trust (SnT), University of Luxembourg
Luxembourg, L-1855, Luxembourg
Email: madyan.alsenwi@uni.lu, eva.lagunas@uni.lu, hayder.al-hraishawi@uni.lu, symeon.chatzinotas@uni.lu

***Abstract*—This paper proposes a robust beamforming algorithm for massive multiple-input multiple-output (MIMO) low earth-orbit (LEO) satellite communications under uncertain channel conditions. Specifically, a Conditional Value at Risk (CVaR)-based stochastic optimization problem is formulated to optimize the hybrid digital and analog precoding aiming at maximizing the network data rate while considering the required Quality-of-Service (QoS) by each ground user. In particular, the CVaR is used as a risk measure of the downlink data rate to capture the high dynamic and random channel variations of the satellite network, achieving the required QoS under the worst-case scenario. Utilizing the decomposition and relaxation optimization techniques, an alternating optimization algorithm is developed to solve the formulated problem. Simulation results demonstrate the efficacy of the proposed approach in achieving the QoS requirements under uncertain satellite channel conditions.**

***Index Terms*—6G, LEO satellites communication, NTN, massive MIMO, CVaR, digital precoding, analog beamforming.**

I. Introduction

The next-generation wireless networks are expected to offer ubiquitous uninterrupted connectivity to everywhere, everything, and everyone with ultra-high reliability and extremely-high data rate. However, the deployment of terrestrial base stations (BSs) with wired backhaul infrastructure in sparsely populated remote areas is costly and unprofitable. These limitations of terrestrial wireless systems have steered the focus towards non-terrestrial networks (NTNs) as a pivotal technology in next-generation wireless communications [1], [2]. NTNs present an opportunity to overcome several shortcomings associated with traditional terrestrial networks, such as coverage gaps and high throughput demands. To support the integration of NTN technologies into 5G and beyond, the 3rd Generation Partnership Project (3GPP) has undertaken numerous studies as part of its Release 15 and Release 16 [3]. Among the diverse range of NTN technologies, low Earth orbit (LEO) satellite systems stand out as a promising candidate to meet future wireless network requirements in terms of global coverage and ubiquitous connectivity. LEO satellites are typically positioned at altitudes ranging from $500-2000$ km above the Earth's surface, enabling the utilization of focused beams that result in faster communications and reduced signal attenuation compared to conventional geostationary orbit (GSO) satellite systems. Notably, recent launches of LEO satellite constellations, such as Starlink, Telesat, and OneWeb, have aimed to deliver seamless and high-capacity wireless services, driving the realization of the envisioned future wireless network landscape.

In recent years, numerous terrestrial communication technologies have emerged to enhance wireless connectivity. Among these advancements, massive multiple-input multiple-output (MIMO) transmission has gained significant attention as a promising technology for 5G networks, offering significant performance improvements and expanded coverage. Extensive research has been conducted to investigate the implementation of massive MIMO in terrestrial wireless networks [4]. However, the application of massive MIMO in satellite communication systems is still in its nascent stages and requires further exploration. This work aims to harness the potential of massive MIMO technology in LEO satellite communication systems by integrating large antenna arrays into LEO satellites. In particular, most LEO satellite systems are designed to operate within the Ku and Ka frequency bands, which correspond to the mmWave band in terrestrial networks. The use of these higher frequency bands enables a significant reduction in antenna size, facilitating the integration of a large number of antennas within a confined space.

Massive MIMO technology can significantly enhance spectral efficiency due to its intrinsic feature of exploiting multiple spatial degrees of freedom, making it an optimistic approach for future satellite communication systems [5], [6]. Specifically, the precoding technique plays a crucial role in massive MIMO to exploit the spatial dimension and enhance the overall system performance by enabling efficient beamforming, interference mitigation, and increased spectral efficiency. However, the effectiveness of the precoding in massive MIMO depends on the availability of channel state information (CSI), which is hard and sometimes infeasible to obtain in satellite systems [7]. In practice, the swift movement of satellites relative to ground users can introduces rapid variations in the channel conditions. Consequently, the integration of massive MIMO into satellite communication systems emerges as a challenging research direction. More specifically, the fundamental challenge is how to design reliable precoding that can ensure link quality in uncertain and dynamic wireless environments.

The conventional digital precoding approach leads to high power consumption as many radio frequency chains are required due to the numerous antennas in the massive MIMO design. To reduce power consumption in massive MIMO, the hybrid precoding architecture has been proposed by leveraging the low-dimension digital precoding and analog precoding technology [8]. The hybrid precoding design in terrestrial networks has been well investigated in several recent studies. For instance, the work in [9] studied the hybrid beamforming design in millimeter wave communication systems. The authors in [10] proposed a hybrid beamforming framework for massive MIMO wireless systems considering accurate CSI estimation. Some recent works have studied the transmit design of massive MIMO LEO satellite communications. The work in [11] proposed different network architectures for distributed massive MIMO LEO satellite systems. A joint power and handover management approach was proposed to optimize the power allocation and handover using artificial intelligence technology. The authors in [12] analyzed the throughput of multi-user MIMO multibeam LEO satellite systems with four-color frequency reuse. In [13], a closed-form tight upper-bound approach is applied to obtain an approximate data rate. The authors then proposed an algorithm to calculate the hybrid precoding considering fully and partially connected architecture scenarios. The study in [14] developed a novel activation strategy for LEO satellite nodes by utilizing the cell-free massive MIMO communication scheme in order to enhance system energy efficiency. The problem was formulated as a binary integer linear optimization, and its optimal solutions were obtained using the quantum approximate optimization algorithm (QAOA).

Unlike the previous works, this paper proposes a risk-aware robust precoding approach for massive MIMO LEO communication systems that considers the random and dynamic behavior of wireless channels. Specifically, we formulate a stochastic optimization problem based on the Conditional Value at Risk (CVaR) to optimize the digital and analog precoding at the LEO satellite side. The formulated problem aims to improve the network data rate while considering the required transmission reliability. The motivation to use the CVaR is its capability to define the tail distribution of random variables, capturing the worst-case scenario. Thus, the provided CVaR-based optimization problem can ensure achieving the required data rate by each ground user under the worst-case scenario of the wireless channels, which is anticipated to efficiently suit the turbulent and dynamic propagation environment of LEO satellite systems. To solve the formulated problem, we introduce an alternating optimization approach that simplifies the optimization problem using relaxation and decomposition techniques and obtains a near-optimal solution. *To our knowledge, this work is the first to propose a risk-aware optimization framework based on the CVaR for massive MIMO LEO satellite networks.*

II. System Model and Problem Formulation

Consider a LEO satellite serving a set $\mathcal{K} = \{1, 2, \ldots, K\}$ of ground users over the Ka-band, as depicted in Fig. 1. The LEO satellite connects with a central terrestrial data server via ground gateway stations distributed in different locations due to satellite movement. The satellite covers a region of interest using a set of potential beams following the scenarios

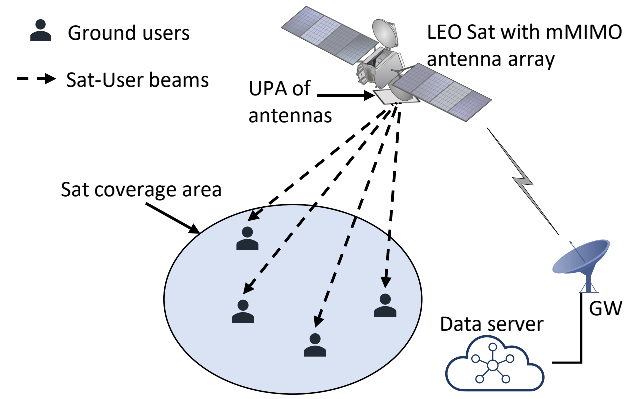

Figure 1. Considered system model.

in the 3GPP technical reports on solutions for NR to support non-terrestrial networks [3]. Uniform planar arrays (UPAs) of active digital antennas are installed at the satellite and ground users, where the phase of each antenna element can be digitally controlled [15]. Specifically, the LEO satellite is equipped with a large-scale UPA composed of $N_s = N_s^x \times N_s^y$ antenna elements where N_s^x is the number of elements on the x-axis, and N_s^y is the number of elements on the y-axis. In the considered scenario, the x-axis is the direction of the satellite movement, while the y-axis is the orthogonal direction of the movement. A UPA consists of $N_u = N_u^{x'} \times N_u^{y'}$ omnidirectional elements is installed at each ground user in the x′-axis and y′-axis.

A. Communication Model

We adopt the multi-path channel model for the downlink LEO communication system. Using the ray-tracing model, the downlink channel response $\boldsymbol{H}_k(t,f)$ between the satellite and the k^{th} ground user at time t and frequency f can be defined as [5], [16]

$$\mathbf{H}_k(t,f) = \sum_{l=0}^{L_k-1} g_{k,l}(t,f) \cdot \exp\left\{\jmath 2\pi \left[\nu_{k,l} - f\tau_{k,l}\right]\right\} \cdot \boldsymbol{\mu}_{k,l}, \quad (1)$$

where L_k is the number of propagation paths of the ground user k, and $g_{k,l}$ denotes the channel gain between user k and the LEO satellite. $\nu_{k,l}$ and $\tau_{k,l}$ are the Doppler shift and the propagation delay, respectively. $\boldsymbol{\mu}_{k,l}$ represents the array response vector associated with the l^{th} path of the k^{th} user which is expressed as follows:

$$\boldsymbol{\mu}_k = \boldsymbol{\mu}_k^{\mathrm{x}} \otimes \boldsymbol{\mu}_k^{\mathrm{y}} = \boldsymbol{\mu}_{\mathrm{x}}\left(\phi_k^{\mathrm{x}}\right) \otimes \boldsymbol{\mu}_{\mathbf{y}}\left(\phi_k^{\mathrm{y}}\right), \quad (2)$$

where $\boldsymbol{\mu}_k^{\mathrm{x}}$ and $\boldsymbol{\mu}_k^{\mathrm{y}}$ denote the response vector with angles measured from the x-axis and y-axis, and defined as

$$\boldsymbol{\mu}_k^{\mathrm{x}} = \frac{1}{\sqrt{N_{\mathrm{s}}^x}} \left[1 \exp\left\{-\jmath\pi\phi_k^{\mathrm{x}}\right\} \cdots \exp\left\{-\jmath\pi\left(N_{\mathrm{s}}^x - 1\right)\phi_k^{\mathrm{x}}\right\}\right]^T, \quad (3)$$

$$\boldsymbol{\mu}_k^{\mathrm{y}} = \frac{1}{\sqrt{N_{\mathrm{s}}^y}} \left[1 \exp\left\{-\jmath\pi\phi_k^{\mathrm{y}}\right\} \cdots \exp\left\{-\jmath\pi\left(N_{\mathrm{s}}^y - 1\right)\phi_k^{\mathrm{y}}\right\}\right]^T. \quad (4)$$

In the space domain, the propagation properties of wireless channels depend on the space angles ϕ_k^{x} and ϕ_k^{y} defined as $\phi_k^{\mathrm{x}} = \sin\theta_k^{\mathrm{y}}\cos\theta_k^{\mathrm{x}}$ and $\phi_k^{\mathrm{y}} = \cos\theta_k^{\mathrm{x}}$, respectively. The statistical properties of the channel gain term $g_k(t,f)$ are determined by the ground user locations. Furthermore, the LEO satellite system is usually operated with Line-of-Sight (LoS) propagation [17]. Thus, the term $g_k(t,f)$ can be modeled as a Rician fading distribution with factor υ_k and power $\mathbb{E}\left\{|g_k(t,f)|^2\right\}$.

In the considered network model, the transmitter of the LEO satellite is based on the hybrid architecture beamforming with M RF chains, such that $K \leq M \leq N$. Let the vector $\boldsymbol{x} = [x_1, x_2, \dots, x_K]^T \in \mathbb{C}^{K\times 1}$ be the transmit symbols. The vector $\boldsymbol{x}$ is first precoded with a digital precoder and then processed by an analog precoder. Let $\boldsymbol{W} \in \mathbb{C}^{M\times K}$ be the digital precoding matrix and $\mathbf{V} \in \mathbb{C}^{N\times M}$ denote the analog precoding matrix. We consider that the analog precoder is designed with phase shifters that adjust the signal phase only, i.e., $|\mathbf{V}_{i,j}| = 1$. The phase shifters are implemented by a uniform quantizer with c-bits resolution and step size $\Delta = 2\pi/2^c$. Furthermore, a discrete phase shifter with a finite set of resolutions is considered in this work. To this end, the set of values of the analog precoder $\mathbf{V}$ satisfy

$$\mathcal{V} = \left\{ \mathbf{V} \mid \mathbf{V}_{n,m} = \exp\left\{ \jmath \left(\frac{2\pi}{2^c}\iota + \frac{\pi}{2^c} \right) \right\}, \quad \forall n, m, \iota = 0, 1, \dots, 2^c - 1 \right\} \tag{5}$$

Thus, the received signal at the ground user k at time slot t can be modeled as

$$y_k(t) = \mathbf{H}_k^H(t)\boldsymbol{V}(t)\boldsymbol{w}_k(t)x_k(t) + \varsigma_k(t), \tag{6}$$

where $\varsigma_k \sim \mathcal{CN}\left(0, \sigma^2\right)$ is the additive Gaussian white noise variable of the k^{th} ground user with a zero mean and variance σ^2. In particular, advanced signal processing techniques, such as interference alignment and interference cancellation, can be employed to mitigate inter-beam interference [18]. Accordingly, the Signal-to-Noise Ratio (SNR) of the k^{th} user can be given by

$$\gamma_k(t) = \frac{\left|\boldsymbol{V}(t)\boldsymbol{w}_k^H(t)\boldsymbol{H}_k(t)\right|^2}{\sigma^2}, \tag{7}$$

and the obtained downlink data rate of the ground user k is given by $r_k(t) = F\log\left[1 + \gamma_k(t)\right]$, where F is the downlink bandwidth.

B. Problem Formulation

The objective is to design a robust precoding technique considering the dynamic channel variations. To achieve that, we formulate an optimization problem that maximizes the sum data rate of all users while considering transmission reliability. In this work, transmission reliability is defined based on the minimum data rate satisfaction of each ground user. In particular, uncertainty in channel variations due to the dynamic behavior of the satellite networks exacerbates the obtained throughput by each ground user, impacting transmission reliability. In such a case, formulating an optimization problem based on the average quantity of data rate without considering the uncertainty of channel conditions may violate the data rate constraint of each ground user.

This work uses the CVaR as a risk measure for data rate. In particular, the CVaR can efficiently define the tail distribution of random variables, capturing the worst-case scenario. Specifically, the CVaR gives the average potential loss that exceeds the Value-at-Risk (VaR). In general, the CVaR of a random variable Z is defined as [19]

$$\mathrm{CVaR}_\alpha(Z) := \inf_{\delta\in\mathbb{R}} \left[\delta + \frac{1}{1-\alpha}\mathbb{E}\left[(Z-\delta)^+\right] \right], \tag{8}$$

where $\alpha \in (0,1)$. Thus, the data rate constraint of each ground user can be formulated based on the CVaR as follows

$$\mathrm{CVaR}_\alpha[r_k(t)] \geq r_{\min}, \ \forall k \in \mathcal{K}, \tag{9}$$

where $r_{\min}$ defines the minimum data rate threshold of the ground users.

Lemma 1. *Constraint* (9) *can guarantee the minimum required data rate with a probability higher than* $1-\alpha$*.*

Proof. The α-percentile (Value at Risk) of a random variable, i.e., the value for which the likelihood of a random variable is less than or equal to it is at least α, is given by [19]:

$$\mathrm{VaR}_\alpha(Z) = \arg\inf_{\delta}\{\delta : \Pr(Z \leq Z_{\min}) \leq \alpha\}, \tag{10}$$

where $\Pr(\cdot)$ defines the probability. Thus, $\mathrm{VaR}_\alpha[r_k(t)] \leq r_{\min}$ is equivalent to following probability constraint:

$$\Pr[r_k(t) \geq r_{\min}] \geq 1-\alpha. \tag{11}$$

Since the VaR of distribution of $r_k(t)$ is a minimizer of the right-hand side in (8), $\mathrm{CVaR}_\alpha[r_k(t)] \geq \mathrm{VaR}_\alpha[r_k(t)]$ always holds. Therefore, the constraint (9) defines an approximation of the chance constraint (11). ■

Accordingly, we formulate the following stochastic optimization problem

$$\underset{\boldsymbol{V},\boldsymbol{W}}{\text{maximize}} \quad \sum_{k\in\mathcal{K}} \mathbb{E}_t[r_k(t)] \tag{12a}$$

$$\text{subject to} \quad \mathrm{CVaR}_\alpha[r_k(t)] \geq r_{\min}, \ \forall k \in \mathcal{K}, \tag{12b}$$

$$\sum_{k\in\mathcal{K}} ||\boldsymbol{V}\boldsymbol{w}_k||^2 \leq P_{\max}, \tag{12c}$$

$$\boldsymbol{V}_{m,n} = \exp\left\{ \jmath \left(\frac{2\pi}{2^c}\iota + \frac{\pi}{2^c} \right) \right\}, \quad \iota = \{0, 1, \dots, 2^c - 1\}, \ \forall m \in \mathcal{M}, n \in \mathcal{N} \tag{12d}$$

where $P_{\max}$ is the maximum transmit power of the LEO satellite. Problem (12) aims to obtain the optimum analog and digital precoding matrices $\boldsymbol{V}^*$, $\boldsymbol{W}^*$, respectively, that maximize the average total data rate of all ground while keeping the required transmission reliability. The constraint (12b) is set to achieve the data rate required by each ground user regardless of the channel variations. Constraints (12c) and (12d) define the feasibility regions of the optimization variables.

III. Proposed Solution Approach

We first reformulate problem (12) by using the CVaR definition in (8) as follows

$$\underset{\boldsymbol{V},\boldsymbol{W},\delta\in\mathbb{R}}{\text{maximize}} \quad \sum_{k\in\mathcal{K}}\mathbb{E}_t[r_k(t)] \tag{13a}$$

$$\text{subject to} \quad \delta+\frac{\mathbb{E}[(r_k(t)-\delta)^+]}{(1-\alpha)}\geq r_{\min},\ \forall k\in\mathcal{K}, \tag{13b}$$

$$\sum_{k\in\mathcal{K}}||\boldsymbol{V}\boldsymbol{w}_k||^2\leq P_{\max}, \tag{13c}$$

$$\boldsymbol{V}_{m,n}=\exp\left\{\jmath\left(\frac{2\pi}{2^c}\iota+\frac{\pi}{2^c}\right)\right\}, \quad \iota=\{0,1,\dots,2^c-1\},\ \forall m\in\mathcal{M},n\in\mathcal{N}. \tag{13d}$$

The formulated risk-aware stochastic problem (13) is a non-convex mixed-integer programming problem that poses significant challenges to solve it. To tackle this complexity, a decomposition approach is employed to simplify the problem into two subproblems. Specifically, we decompose problem (13) into the following subproblems: 1) Analog beamforming problem and 2) Digital precoding problem. Subsequently, appropriate relaxation techniques are applied to attain convex formulations.

A. Analog Beamforming Problem

The analog beamforming problem is derived by imposing a fixed value for the digital precoding variable in (13). This subproblem focuses on optimizing the analog beamforming matrix to enhance the total network data rate while considering the data rate constraint required by each ground user. Thus, the analog beamforming problem can be given by

$$\textbf{P1:}\quad \underset{\boldsymbol{V},\delta\in\mathbb{R}}{\text{maximize}} \quad \sum_{k\in\mathcal{K}}\mathbb{E}_t[r_k(t)] \tag{14a}$$

$$\text{subject to} \quad \delta+\frac{\mathbb{E}[(r_k(t)-\delta)^+]}{(1-\alpha)}\geq r_{\min},\ \forall k\in\mathcal{K}, \tag{14b}$$

$$\sum_{k\in\mathcal{K}}||\boldsymbol{V}\boldsymbol{w}_k||^2\leq P_{\max}, \tag{14c}$$

$$\boldsymbol{V}_{m,n}=\exp\left\{\jmath\left(\frac{2\pi}{2^c}\iota+\frac{\pi}{2^c}\right)\right\}, \quad \iota=\{0,1,\dots,2^c-1\},\ \forall m\in\mathcal{M},n\in\mathcal{N}. \tag{14d}$$

We can notice that problem **P1** is an integer non-convex programming, which poses challenges for obtaining tractable solutions. In order to address this, we employ a relaxation technique to simplify **P1** and transform it into a convex optimization problem. The relaxation method involves treating the integer variable $\boldsymbol{V}$ as a continuous variable constrained within the interval $[\exp(j\frac{\pi}{2^c}),\exp(j\frac{(2^c-1)\pi}{2^c})]$. The term $\exp(j\frac{\pi}{2^c})$ corresponds to setting $\iota=0$ in (14d), and the upper bound is obtained by setting $\iota=2^c-1$. Furthermore, we replace the term $\mathbb{E}[(r_k(t)-\delta)^+]$ by introducing a new variable β and imposing the constraints $\beta\leq\mathbb{E}[r_k(t)-\delta]$ and $\beta\geq 0$. Thus, we can achieve the following optimization problem:

$$\tilde{\textbf{P}}\textbf{1:}\quad \underset{\tilde{\boldsymbol{V}},\boldsymbol{\beta},\delta\in\mathbb{R}}{\text{maximize}} \quad \sum_{k\in\mathcal{K}}\mathbb{E}_t[r_k(t)] \tag{15a}$$

$$\text{subject to} \quad \delta+\frac{\beta_k}{(1-\alpha)}\geq r_{\min},\ \forall k\in\mathcal{K}, \tag{15b}$$

$$\beta_k\leq\mathbb{E}[r_k(t)-\delta],\ \beta_k\geq 0,\forall k\in\mathcal{K} \tag{15c}$$

$$\sum_{k\in\mathcal{K}}||\tilde{\boldsymbol{V}}\boldsymbol{w}_k||^2\leq P_{\max}, \tag{15d}$$

$$e^{j\frac{\pi}{2^c}}\leq\tilde{\boldsymbol{V}}_{m,n}\leq e^{j\frac{(2^c-1)\pi}{2^c}},\forall m\in\mathcal{M}. \tag{15e}$$

The optimization problem (15) consists of a convex objective function with respect to the variable $\tilde{\boldsymbol{V}}$, along with convex constraints. Furthermore, constraints (15b) and (15c) are linear functions with respect to β. Thus, we can achieve a convex optimization problem with respect to each variable in (15) by fixing the other two variables. A convex optimization solver can be used to solve the problem (15) iteratively, i.e., obtaining one variable at each iteration, such as CVXPY [20]. Finally, it is important to note that the relaxation approach introduces a gap between the relaxed solution and the optimal integer solution of the original problem. To bridge this gap, a rounding technique is applied to transform the fractional values of V back into the original discrete set, thus restoring the integrity of the original problem formulation.

B. Digital Precoding Problem

We hold the analog beamforming variable in (13) constant and focus on optimizing the digital precoding matrix to obtain the digital precoding problem. Furthermore, we replace the term $\mathbb{E}[(r_k(t)-\delta)^+]$ by introducing a new variable ϱ in a similar way to $\tilde{\textbf{P}}\textbf{1}$. The digital precoding problem can be given as follows

$$\textbf{P2:}\quad \underset{\boldsymbol{W},\boldsymbol{\varrho},\delta\in\mathbb{R}}{\text{maximize}} \quad \sum_{k\in\mathcal{K}}\mathbb{E}_t[r_k(t)] \tag{16a}$$

$$\text{subject to} \quad \delta+\frac{\varrho_k}{(1-\alpha)}\geq r_{\min},\ \forall k\in\mathcal{K}, \tag{16b}$$

$$\varrho_k\leq\mathbb{E}[r_k(t)-\delta],\ \varrho_k\geq 0,\forall k\in\mathcal{K} \tag{16c}$$

$$\sum_{k\in\mathcal{K}}||\boldsymbol{V}\boldsymbol{w}_k||^2\leq P_{\max},. \tag{16d}$$

Similar to $\tilde{\textbf{P}}\textbf{1}$, problem (16) is a convex optimization problem for each variable; thus, we can use convex optimization tools to obtain a solution for each variable and fixing other variables iteratively in a similar way to the analog beamforming problem.

Finally, we solve $\tilde{\textbf{P}}\textbf{1}$ and **P2** in an iterative manner until reaching a satisfactory desired outcome, as illustrated in Algorithm 1. By iteratively solving the two problems, the algorithm gradually improves the overall solution quality and converges towards an acceptable solution. During each iteration, the

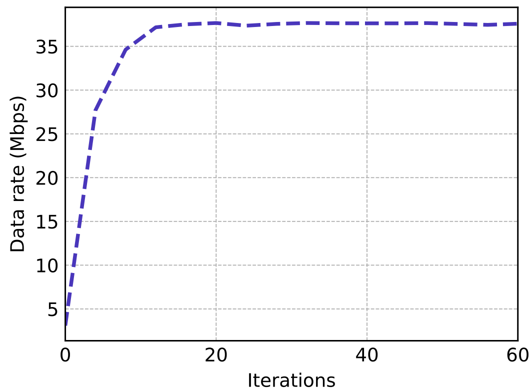

Figure 2. Convergence of the data rate over iterations.

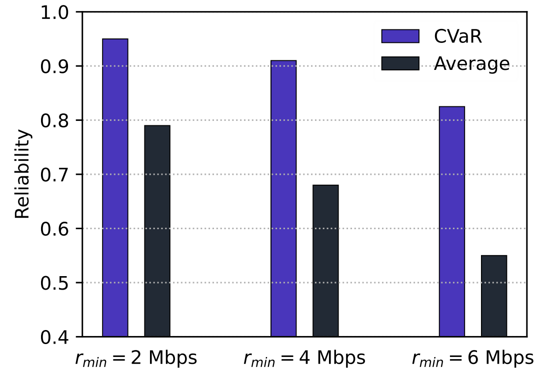

Figure 3. Percentage of ground users who achieved the minimum required data rate.

Algorithm 1 : The proposed Optimization Algorithm.

Set $j = 0$, $\beta > 0$, and initialize $\tilde{\boldsymbol{V}}^{(0)}, \boldsymbol{W}^{(\mathbf{0})}$;
Obtain **P1** and **P2** form **P**;
Obtain problem $\tilde{\mathbf{P}}\mathbf{1}$ from **P1**;
repeat
    Use $\boldsymbol{W}^{(j)}$ to solve $\tilde{\mathbf{P}}\mathbf{1}$ and get $\tilde{\boldsymbol{V}}^{(j+1)}$;
    Use $\tilde{\boldsymbol{V}}^{j+1}$ to solve **P2** and get $\boldsymbol{W}^{(j+1)}$;
    $j = j + 1$;
until $\| \frac{\mathcal{F}^{(j)} - \mathcal{F}^{(j+1)}}{\mathcal{F}^{(j)}} \| \leq \beta$;
Apply rounding to obtain $\boldsymbol{V}^{(j+1)}$ from $\tilde{\boldsymbol{V}}^{j+1}$
Then, set $\left(\boldsymbol{V}^* = \boldsymbol{V}^{(j+1)}, \boldsymbol{W}^* = \boldsymbol{W}^{(j+1)}\right)$ as the solution.

algorithm updates the solutions to $\tilde{\mathbf{P}}\mathbf{1}$ and **P2** based on the previous iteration's outcomes. This iterative refinement process continues until the stopping criterion is met, which is achieving a predefined level of accuracy defined as $\| \frac{\mathcal{F}^{(j)} - \mathcal{F}^{(j+1)}}{\mathcal{F}^{(j)}} \| \leq \beta$, where $\mathcal{F}^{(j)}$ is the obtained value of the objective at the j^{th} iteration and β is a predefined threshold.

IV. Performance Evaluation

We provide simulation analysis in this section to evaluate the proposed approach. In the simulation environment, we consider a LEO satellite orbiting at 1300 km and operating at the Ka-band with a carrier frequency of 20 GHz. At the LEO satellite side, a UPA antenna is installed composed of N_s^x antenna elements in the x-axis and N_s^y antennas in the y-axis. The values of N_s^x and N_s^y are set as $N_s^x = N_s^y = 20$. Thus, the total number of antenna elements at the satellite is $N_s = N_s^x \times N_s^y = 400$. The ground users within the satellite coverage area are generated randomly based on the Poisson process with λ arrival rate.

Figure 2 shows the convergence of the proposed algorithm, which is demonstrated through the analysis of the data rate evolution over iterations. The obtained results illustrate that the algorithm achieves convergence within a relatively small number of iterations, i.e., less than 20 iterations. The observed rapid convergence pattern validates the efficiency and effectiveness of the proposed iterative-based optimization algorithm. In particular, the demonstrated convergence within a limited number of iterations reinforces the algorithm's ability to quickly reach a near-optimal solution, ensuring efficient performance enhancement.

We assess the reliability of the proposed algorithm in terms of achieving the required QoS, i.e., the minimum data rate, at each ground user with different settings of r_{min} in Figure 3. Furthermore, we compare the obtained results by the proposed CVaR-based approach to the Average-based method, where the average data rate quantity is considered in the problem formulation instead of the worst-case scenario. The results show that the percentage of ground users who achieved the minimum data rate in the proposed CVaR-based approach is around 95% when $r_{min} = 2$, while the percentage in the Average-based method is around 79%. When setting $r_{min} = 4$, the QoS satisfaction percentage of the proposed CVaR-based algorithm reduces to around 91%, while it breaks down to 68% in the Average-based method. Finally, setting the minimum data rate threshold at 6 Mbps reduces the QoS achievement percentage of the proposed CVaR approach and the Average-based method to 82% and 55%, respectively. We can conclude that the proposed CVaR-based method enhances transmission reliability by reducing the QoS violation probability. Specifically, the obtained results illustrate that the proposed algorithm performs better than the scenario when formulating the data rate constraint considering the average quantity only, as in the related works. In particular, the proposed CVaR-based approach includes the data rate distribution when making beamforming decisions, improving transmission reliability.

Finally, Figure 4 plots the empirical cumulative distribution function (ECDF) of the sum data rate of all ground users obtained over different channel configurations. The obtained results show that the Average-based method provides a higher sum data rate than the proposed CVaR-based approach. This is because the Average-based method focuses mainly on maximizing the linear summation of the data rate and evaluates the QoS satisfaction constraint in terms of the average data rate only. However, the proposed CVaR-based algorithm considers

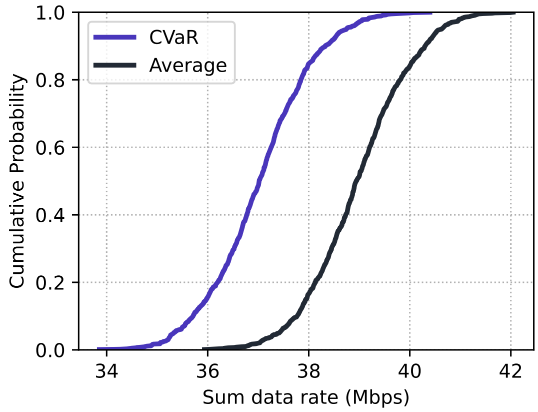

Figure 4. ECDF of the sum data rate of ground users.

the tail distribution (worst case) of the data rate in addition to the average quantity when adjusting the precoding matrices, considering users with bad channel conditions and achieving better transmission reliability. As illustrated in Figure 4, the sum data rate of the Average-based proposed approach varies between 36 to 42 Mbps with an average is around 39 Mbps. However, the sum data rate obtained by the proposed CVaR-based algorithm varies between 34 to 40 Mbps with an average data rate of 37 Mbps. In particular, Figures 3 and 4 show the trade-off between the proposed CVaR-based algorithm and the conventional Average-based method regarding the reliability and sum data rate. To conclude, the proposed algorithm provides robust and reliable precoding by considering the worst-case scenario of channel conditions for each user, while the Average-based approach focuses mainly on maximizing the sum data rate of the network without considering the risk of channel variations of each user.

V. Conclusion

The paper has discussed the precoding optimization in massive MIMO LEO satellite communication systems. A stochastic optimization problem has been formulated to optimize the hybrid digital/analog beamforming aiming at maximizing the average sum data rate while satisfying the QoS requirements of all ground users. The CVaR has been used as a risk measure to guarantee the required data rate by each ground user under uncertain channel variations. To solve the formulated problem, an alternating optimization algorithm has been developed that leverages the decomposition and relaxation optimization techniques to obtain a near-optimum solution. The obtained results have shown the efficacy of the proposed approach in keeping the QoS requirements under dynamic channel variations.

Acknowledgement

This work has been supported by the project TRANTOR, which has received funding from the European Union's Horizon Europe research and innovation program under grant agreement No. 101081983.

References

[1] H. Al-Hraishawi, H. Chougrani, S. Kisseleff, E. Lagunas, and S. Chatzinotas, "A survey on nongeostationary satellite systems: The communication perspective," *IEEE Communications Surveys & Tutorials*, vol. 25, no. 1, pp. 101–132, 2023.

[2] A. Sattarzadeh, Y. Liu, A. Mohamed, R. Song, P. Xiao, Z. Song, H. Zhang, R. Tafazolli, and C. Niu, "Satellite-based Non-Terrestrial Networks in 5G: Insights and Challenges," *IEEE Access*, vol. 10, pp. 11 274–11 283, 2022.

[3] 3GPP, "Solutions for NR to support Non-Terrestrial Networks (NTN)," 2021.

[4] S. A. Busari, K. M. S. Huq, S. Mumtaz, L. Dai, and J. Rodriguez, "Millimeter-wave massive MIMO communication for future wireless systems: A survey," *IEEE Communications Surveys & Tutorials*, vol. 20, no. 2, pp. 836–869, 2018.

[5] L. You, K.-X. Li, J. Wang, X. Gao, X.-G. Xia, and B. Ottersten, "Massive MIMO transmission for LEO satellite communications," *IEEE Journal on Selected Areas in Communications*, vol. 38, no. 8, pp. 1851–1865, 2020.

[6] K.-X. Li, L. You, J. Wang, X. Gao, C. G. Tsinos, S. Chatzinotas, and B. Ottersten, "Downlink transmit design for massive MIMO LEO satellite communications," *IEEE Transactions on Communications*, vol. 70, no. 2, pp. 1014–1028, 2022.

[7] M. Vazquez, M. R. B. Shankar, C. I. Kourogiorgas, P.-D. Arapoglou, V. Icolari, S. Chatzinotas, A. D. Panagopoulos, and A. I. Perez-Neira, "Precoding, scheduling, and link adaptation in mobile interactive multibeam satellite systems," *IEEE Journal on Selected Areas in Communications*, vol. 36, no. 5, pp. 971–980, 2018.

[8] L. Liang, W. Xu, and X. Dong, "Low-complexity hybrid precoding in massive multiuser MIMO systems," *IEEE Wireless Communications Letters*, vol. 3, no. 6, pp. 653–656, 2014.

[9] A. Kaushik, E. Vlachos, C. Tsinos, J. Thompson, and S. Chatzinotas, "Joint bit allocation and hybrid beamforming optimization for energy efficient millimeter wave MIMO systems," *IEEE Transactions on Green Communications and Networking*, vol. 5, no. 1, pp. 119–132, 2021.

[10] R. Dilli, "Performance analysis of multi user massive MIMO hybrid beamforming systems at millimeter wave frequency bands," *Wireless Networks*, vol. 27, no. 3, pp. 1925–1939, 2021.

[11] M. Y. Abdelsadek, G. K. Kurt, and H. Yanikomeroglu, "Distributed massive MIMO for LEO satellite networks," *IEEE Open Journal of the Communications Society*, vol. 3, pp. 2162–2177, 2022.

[12] H. Cheporniuk, R. T. Schwarz, T. Delamotte, and A. Knopp, "MIMO throughput performance analysis in LEO communication scenario," in *2021 IEEE 94th Vehicular Technology Conference (VTC2021-Fall)*, 2021, pp. 01–06.

[13] L. You, X. Qiang, K.-X. Li, C. G. Tsinos, W. Wang, X. Gao, and B. Ottersten, "Hybrid analog/digital precoding for downlink massive MIMO LEO satellite communications," *IEEE Transactions on Wireless Communications*, vol. 21, no. 8, pp. 5962–5976, 2022.

[14] H. Al-Hraishawi, u. R. Junaid, and S. Chatzinotas, "Quantum optimization algorithm for LEO satellite communications based on cell-free massive MIMO," in *2023 IEEE International Conference on Communications (ICC)*, 2023, p. 1–6.

[15] W. Hong, Z. H. Jiang, C. Yu, J. Zhou, P. Chen, Z. Yu, H. Zhang, B. Yang, X. Pang, M. Jiang, Y. Cheng, M. K. T. Al-Nuaimi, Y. Zhang, J. Chen, and S. He, "Multibeam antenna technologies for 5G wireless communications," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 12, pp. 6231–6249, 2017.

[16] A. G. Kanatas and A. D. Panagopoulos, "Radio wave propagation and channel modeling for earth-space systems," *New York, NY, USA: CRC Press*, 2016.

[17] N. Letzepis and A. J. Grant, "Capacity of the multiple spot beam satellite channel with rician fading," *IEEE Transactions on Information Theory*, vol. 54, no. 11, pp. 5210–5222, 2008.

[18] J. Zhan and X. Dong, "Interference cancellation aided hybrid beamforming for mmwave multi-user massive mimo systems," *IEEE Transactions on Vehicular Technology*, vol. 70, no. 3, pp. 2322–2336, 2021.

[19] R. T. Rockafellar, S. Uryasev *et al.*, "Optimization of Conditional Value-at-Risk," *Journal of risk*, vol. 2, pp. 21–42, 2000.

[20] S. Diamond and S. Boyd, "CVXPY: A Python-embedded modeling language for convex optimization," *Journal of Machine Learning Research*, vol. 17, no. 83, pp. 1–5, 2016.