Lorentz Factor and Time Dilation on the Special Theory of Relativity

Chinnaraji Annamalai
School of Management, Indian Institute of Technology, Kharagpur, India
Email: anna@iitkgp.ac.in
https://orcid.org/0000-0002-0992-2584

Abstract: This paper presents the derivation of the Lorentz factor and time dilation used in the Einstein's theory of special relativity.

Keywords: relativistic speed**,** mass-energy equivalence, Pythagoras theorem

1. Introduction

In Einstein's theory of special relativity [1-5], the relativistic speed is concerned with the speed of an object whose velocity approaches the speed of light. In this case, the relativistic time refers to time dilation and the relativistic time dilation denotes the observed time interval between two events.
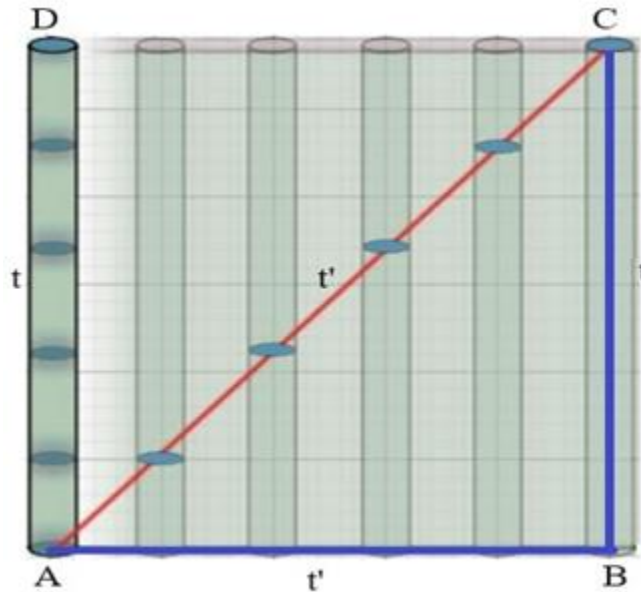
2. Lorentz Factor and Time Dilation

Fig 1. Relativistic speed of the metal tube

Let's consider that a metal tube of mass m and length h is at a stationary position and the light takes time t to travel from point A to point D.

$$AD = h = ct$$

$$\text{Also, } BC = AD = h = ct,$$

$$\text{where } c \text{ is the speed of light.}$$

Suppose the metal tube moves with velocity v horizontally and the light travels from point A along with the moving metal tube. When the light reaches point C at time t', the metal tube moves to point B at time t'.

$$AB = vt' \text{ and } AC = ct' \text{ , where } AC = AD = h = ct$$

By the Pythagorean theorem, we can derive the time dilation and Lorentz factor as follows:

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(ct')^2 = (vt')^2 + (ct)^2$$

$$t'^2 \left(1 - \frac{V^2}{C^2}\right) = t^2$$

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}, \text{ where } t' \text{ is time dilation.}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \text{ where } \gamma = \frac{t'}{t}.$$

Here, γ is the Lorentz factor.

The Einstein's mass-energy equivalence states that $E = mc^2 \text{ and } m = \gamma m_0$, where m is the relativistic mass and m_0 is the rest mass.

3. Conclusion

In this article, the Lorentz factor and time dilation have been derived used in the Einstein's theory of special relativity.

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