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A FUZZY PRODUCTION INVENTORY MODEL WITH RANDOM DETERIORATION RATE AND DEMAND RATE USING REGULAR WEIGHTED POINT TECHNIQUE

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Abstract:

This paper deals, a fuzzy production inventory model with two warehouse have been considered. In this model the deterioration rate and demand rate considered as random and the production rate depends directly on demand rate. The lotus petal fuzzy number is defined and its properties are given. The parameters involved in this model are represented by lotus petal fuzzy number. The average total cost is defuzzified by the regular weighted point technique. The analytical expressions for expected inventory level in temporary warehouse and permanent warehouse at time t₂, expected deterioration level, maximum inventory level and average total cost are derived for the proposed model by using nonlinear programming technique. A numerical example is presented to illustrate the results.

Kev Words: Lotus Petal Fuzzy Number & Regular Weighted Point Technique

1. Introduction:

Inventory problems are common in manufacturing, service and business operations in general. Some inventory models were formulated in a static environment where the demand is assumed to be constant and steady over a finite planning horizon. Many items of inventory such as electronic products, fashionable clothes, tasty food products etc., generate increasing sales after gaining consumer's acceptance. The sale for the other products may decline drastically due to the introduction of more competitive products or due to the change of consumer's preference. Therefore the demand of the product during its growth and decline phases can be taken as continuous time dependent function such as non-linear.

Most of the existing inventory models in the literature assume that items can be stored indefinitely to meet the future demands. However, certain types of commodities either deteriorate or become obsolete in the course of time and hence are unstable. Therefore, if the rate of deterioration is not sufficiently low, its impact on modelling of such an inventory model cannot be ignored. In this connection, inventory problems for deteriorating items have been studied extensively by many researchers [1], [4], [5] and [7] from time to time. Research in this area started with the work of Whitin [8], who considered fashion goods deteriorating at the end of prescribed storage period. Goyal and Giri [3] gave recent trends of modelling in deteriorating item inventory. Samantha and Ajanta roy work based on realistic production lot-size inventory model for deteriorating items is given in [6].

In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. However, in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory introduced by Zadeh [9] is applicable. A. Faritha Asma and E.C. Henry Amirtharaj analized multi objective inventory model of deteriorating items with two constraints using fuzzy optimization technique [2].

In the real situation, at the time of production the deterioration rate and demand rate are varied. So that the deterioration rate and demand rate are considered as a random variable which follows rayleigh distribution and bounded pareto distribution respectively.

In the existing inventory models, they used only rented warehouse and own warehouse. When using the rented warehouse the transportation cost, rental and maintenance cost are all high. Here, the model newly constructed with permanent warehouse and temporary warehouse.

As warehouse is a key factor in production inventory model, so these days researcher are paying more rent in stored the goods. The temporary warehouse used for reducing the rented amount and maintenance cost.

2. Assumptions and Notations:

The following assumptions and notations are used throughout this paper:

Assumptions:

- ✓ Demand rate is a random variable which follows bounded pareto distribution
- \checkmark Production rate is demand dependent that is p = cd, 0 < c < n, n is a finite number.
- ✓ Deterioration rate is a random variable which follows reyleigh distribution
- ✓ First, the goods are stored in permanent warehouse then the remaining goods are stored in temporary warehouse.
- ✓ The goods of permanent warehouse are consumed only after consuming the goods kept in temporary warehouse.
- ✓ The permanent warehouse has a fixed capacity of I_P units.
- ✓ The inventory level depleted due to demand and deterioration.
- ✓ Shortages are not allowed.

Notations:

 $I_1(t)$ - the level of inventory at time t, $0 \le t \le t_1$

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 $I_2(t)$ - the level of inventory in temporary warehouse at time t, $t_1 \le t \le t_2$. $I_3(t)$ - the level of inventory in permanent warehouse at time t, $t_1 \le t \le t_3$

I_P – fixed capacity for permanent warehouse at time t₁.

I_m – expected maximum inventory level at time t₁. (Decision variable)

I_T – expected maximum inventory level of temporary warehouse at time t_{1.} (Decision variable)

I_{P1} - expected inventory level of permanent warehouse at time t₂. (Decision variable)

p – production rate.

d – demand rate.

 θ - deterioration rate.

 θ_I - expected deterioration level. (decision variable)

h_c – holding cost per unit per unit time.

S_c – setup cost per cycle.

T_{ws} – total space for temporary warehouse.

TC – expected average total cost per cycle.

 \widetilde{S}_{c} - fuzzy setup cost per cycle.

 \tilde{h}_c - fuzzy holding cost per unit per unit time.

 \tilde{d}_c - fuzzy deteriorating cost per unit per unit time.

 \widetilde{T}_{wc} - fuzzy cost for 1 square feet of temporary warehouse.

 \widetilde{T}_{ws} - fuzzy total space for temporary warehouse.

 \tilde{S} – fuzzy space per unit for temporary warehouse.

3. Mathematical Formulation:

The proposed inventory model is formulated to minimize the average total cost, which includes setup cost, holding cost, deterioration cost and temporary warehouse cost.

The rate of change of the inventory during the following periods are governed by the following differential equations

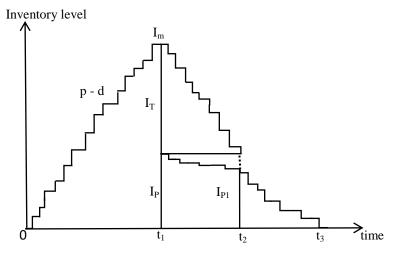


Figure 3.1:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = p - d \qquad 0 \le t \le t_1 \qquad ---- (1)$$

with boundary condition $I_1(t) = 0$ at t = 0

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d t_1 \le t \le t_2 ---- (2)$$

with boundary conditions $I_2(t) = 0$ at $t = t_2$ and $I_2(t) = I_T$ at $t = t_1$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = 0 t_1 \le t \le t_2 ---- (3)$$

with boundary conditions $I_3(t) = I_P$ at $t = t_1$ and $I_3(t) = I_{P1}$ at $t = t_2$

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$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -d t_2 \le t \le t_3 ---- (4)$$

with boundary condition $I_3(t) = 0$ at $t = t_3$

From (1)
$$I_1(t) = \frac{p-d}{\theta} \left(1 - e^{-\theta t} \right)$$
 ---- (5)

From (2)
$$I_T = \frac{d}{\theta} \left(e^{\theta (t_2 - t_1)} - 1 \right) \qquad ---- (6)$$

From (3)
$$I_{P1} = I_P e^{(t_1 - t_2)}$$
 ---- (7)

From (4)
$$I_3(t) = \frac{d}{\theta} \left(e^{\theta (t_3 - t)} - 1 \right)$$
 ---- (8)

The maximum inventory level per cycle is
$$I_m = I_T + I_P$$
 ---- (9)

Deterioration level =
$$\int_{0}^{t_3} \theta \, dt \implies \theta \, t_3 \qquad ---- (10)$$

Expected demand rate is
$$E(d) = \frac{L^{\alpha}}{1 - \left(\frac{L}{H}\right)^{\alpha}} \left(\frac{\alpha}{\alpha - 1}\right) \left(\frac{1}{L^{\alpha - 1}} - \frac{1}{H^{\alpha - 1}}\right)$$

Expected deterioration rate is $E(\theta) = \sigma \sqrt{\frac{\pi}{2}}$

Expected average total $\cos t = \frac{1}{t_3} \left[\frac{\text{setup } \cos t + E(\text{holding } \cos t)}{\text{+ } E(\text{det erioration } \cos t) + \text{temporary } \cos t} \right]$

Where, Setup $cost = S_c$

$$\begin{split} & \text{Holding cost} = \ h_c \left\{ \int\limits_0^{t_1} I_1(t) \, dt + \int\limits_{t_1}^{t_2} (I_2(t) + I_3(t)) \, dt + \int\limits_{t_2}^{t_3} I_3(t) \, dt \right\} \\ & \Rightarrow h_c \left\{ \frac{p - d}{\theta^2} \Big(e^{-\theta \, t_1} - 1 \Big) + \frac{d}{\theta^2} \Big(e^{\theta \, (t_2 - t_1)} + e^{\theta \, (t_3 - t_2)} - 2 + \theta \, t_1 - \theta \, t_3 \Big) + I_p \bigg(\frac{1 - e^{\theta \, (t_1 - t_2)}}{\theta} \bigg) \right\} \end{split}$$

Deterioration cost =
$$d_c \left\{ \int_0^{t_1} I_1(t) dt - \int_{t_1}^{t_3} d dt \right\}$$

$$\Rightarrow d_c \left\{ \frac{p - d}{\theta^2} \left(e^{-\theta t_1} + \theta t_1 - 1 \right) - d \left(t_3 - t_1 \right) \right\}$$

Temporary warehouse cost = $T_{wc} \times T_{w}$

Subject to : $SI_T \leq T_{ws}$ ---- (11)

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$$Min E(TC(t_1, t_2, t_3))$$

$$=\frac{1}{t_{3}}\left\{S_{c}+h_{c}\left\{\frac{\left(\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\right)\left(c-1\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\left(e^{-\left(\sigma\sqrt{\frac{\pi}{2}}\right)t_{1}}-1\right)+\frac{\left(\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\right\}\right\}$$

$$=\frac{1}{t_{3}}\left\{S_{c}+h_{c}\left\{\frac{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{(t_{2}-t_{1})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{(t_{2}-t_{2})}}-2+\left(\sigma\sqrt{\frac{\pi}{2}}\right)t_{1}-\left(\sigma\sqrt{\frac{\pi}{2}}\right)t_{3}\right\}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)}\right)\right\}$$

$$=\frac{1}{t_{3}}\left\{\frac{\left(\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\right)\left(c-1\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)}\right)\right\}$$

$$=\frac{1}{t_{3}}\left\{\frac{\left(\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\right)\left(c-1\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\right)\right\}$$

$$=\frac{1}{t_{3}}\left\{\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\left(c-1\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\right)\right\}$$

$$=\frac{1}{t_{3}}\left\{\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\left(c-1\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\right)\right\}$$

$$=\frac{1}{t_{3}}\left\{\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\left(c-1\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\right)\right\}$$

$$=\frac{1}{t_{3}}\left\{\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\left(c-1\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\right)\right\}$$

$$=\frac{1}{t_{3}}\left\{\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\right\}$$

$$=\frac{1}{t_{3}}\left\{\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}+I_{p}\left(\frac$$

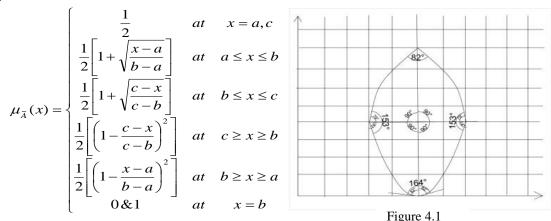
Subject to : $sI_T \leq T_{ws}$

Using Lagrange multipliers method, by using Kuhn-Tucker necessary condition. Solve the equations, the values of t_1^* , t_2^* and t_3^* are obtained. Substituting t_1^* , t_2^* and t_3^* in (6), (7), (9), (10) and (12), the optimum values I_T^* , I_{p1}^* , θ_L^* , I_m^* and TC^* are obtained.

4. Lotus Petal Fuzzy Number and its Properties:

Definition: Lotus Petal Fuzzy Number

A Lotus petal fuzzy number A described as a normalized convex fuzzy subset on the real line R whose membership function $\mu_{\widetilde{A}}(x)$ is defined as follows:



This type of fuzzy number be denoted as $\tilde{A} = [a, b, c]$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

 \checkmark $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1]

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 $\checkmark \quad \mu_{\tilde{A}}(x)$ is a convex function.

 $\checkmark \mu_{\tilde{A}}(x) = 0 \& 1 \text{ at x=b.}$

$$\checkmark \quad \mu_{\tilde{A}}(x) = \frac{1}{2} \text{ at } x = a \& c.$$

 \checkmark $\mu_{\tilde{A}}(x)$ is strictly decreasing as well as increasing and continuous on [a,b] and [b,c].

Properties:

- ✓ Left and right opposite angles are equal.
- \checkmark The horizontal and vertical diagonal bisect each other and meet at 90° .
- ✓ The lower angle is twice that of the upper angle.

5. Regular Weighted Point of Lotus Petal Fuzzy Number:

For the Lotus petal fuzzy number $\widetilde{A}=[a,b,c]$, the α -cut is $\widetilde{A}_{\alpha}=[L_{\widetilde{A}}(\alpha),R_{\widetilde{A}}(\alpha)]$ and the regular weighted point for \widetilde{A} is given by,

$$r_{w}(\widetilde{A}) = \frac{\int_{0}^{1} \frac{L_{\widetilde{A}}(\alpha) + R_{\widetilde{A}}(\alpha)}{2} f(\alpha) d\alpha}{\int_{0}^{1} f(\alpha) d\alpha}$$

$$= \int_{0}^{1} \left[L_{\widetilde{A}}(\alpha) + R_{\widetilde{A}}(\alpha) \right] f(\alpha) d\alpha$$
Where,
$$f(\alpha) = \begin{cases} \omega(1-2\alpha) & \text{when} \quad \alpha \in \left[0, \frac{1}{2}\right], \ 0 < \omega < 1. \\ (1+\omega)(2\alpha-1) & \text{when} \quad \alpha \in \left[\frac{1}{2}, 1\right] \end{cases}$$

The regular weighted point of a lotus petal fuzzy number is of the form $r_w(\widetilde{A}) = \omega \left(\frac{31a + 58b + 31c}{120} \right) + \left(\frac{a + 2b + c}{8} \right)$

6. Inventory Model in Fuzzy Environment:

The proposed inventory model in fuzzy environment is

 $Min E(\widetilde{T}C)$

$$= \frac{1}{t_3} \left[\widetilde{S}_c + \widetilde{h}_c \left\{ \frac{\left[\frac{L^{\alpha}}{1 - \left(\frac{L}{H} \right)^{\alpha}} \left(\frac{\alpha}{\alpha - 1} \right) \left(\frac{1}{L^{\alpha - 1}} - \frac{1}{H^{\alpha - 1}} \right) \right] \left(c - 1 \right) + \frac{\left[\frac{L^{\alpha}}{1 - \left(\frac{L}{H} \right)^{\alpha}} \left(\frac{\alpha}{\alpha - 1} \right) \left(\frac{1}{L^{\alpha - 1}} - \frac{1}{H^{\alpha - 1}} \right) \right] \left(\sigma \sqrt{\frac{\pi}{2}} \right)^2 }{\left(\sigma \sqrt{\frac{\pi}{2}} \right)^2 } \right]$$

$$= \frac{1}{t_3} \left[\widetilde{S}_c + \widetilde{h}_c \right] \left(\frac{L^{\alpha}}{\sigma \sqrt{\frac{\pi}{2}}} \right)^{(\epsilon_2 - \epsilon_1)} + e^{\left[\sigma \sqrt{\frac{\pi}{2}} \right] (\epsilon_3 - \epsilon_2)} - 2 + \left(\sigma \sqrt{\frac{\pi}{2}} \right) t_1 - \left(\sigma \sqrt{\frac{\pi}{2}} \right) t_3 \right) + I_p \left(\frac{1 - e^{\left[\sigma \sqrt{\frac{\pi}{2}} \right] (\epsilon_1 - \epsilon_2)}}{\sigma \sqrt{\frac{\pi}{2}}} \right) \right]$$

$$+ \widetilde{d}_c \left\{ \frac{L^{\alpha}}{1 - \left(\frac{L}{H} \right)^{\alpha}} \left(\frac{\alpha}{\alpha - 1} \right) \left(\frac{1}{L^{\alpha - 1}} - \frac{1}{H^{\alpha - 1}} \right) \left(c - 1 \right) - \left(e^{-\left[\sigma \sqrt{\frac{\pi}{2}} \right] t_1} + \left(\sigma \sqrt{\frac{\pi}{2}} \right) t_1 - 1 \right) \right\} + \widetilde{T}_{wc} \widetilde{T}_{ws}$$

$$- \left[\frac{L^{\alpha}}{1 - \left(\frac{L}{H} \right)^{\alpha}} \left(\frac{\alpha}{\alpha - 1} \right) \left(\frac{1}{L^{\alpha - 1}} - \frac{1}{H^{\alpha - 1}} \right) \left(t_3 - t_1 \right) \right]$$

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Subject to:
$$\tilde{s} I_T \leq \tilde{T}_{ws}$$
 ----- (13)

Where ~ represents for fuzzification of the parameters.

$$\widetilde{S}_c = (S_{c1}, S_{c2} S_{c3}), \ \widetilde{h}_c = (h_{c1}, h_{c2}, h_{c3}), \ \widetilde{d}_c = (d_{c1}, d_{c2}, d_{c3}), \ \widetilde{T}_{wc} = (T_{wc1}, T_{wc2} T_{wc3}) \ \text{and}$$

 $\widetilde{T}_{ws} = (T_{ws1}, T_{ws2}, T_{ws3})$. Now using the technique, regular weighted point of lotus petal fuzzy number, the above model is defuzzified as follows

 $Min r_w(E(TC))$

$$=\frac{1}{t_{3}}\left[r_{_{\boldsymbol{w}}}(\boldsymbol{S}_{_{\boldsymbol{c}}})+r_{_{\boldsymbol{w}}}(\boldsymbol{h}_{_{\boldsymbol{c}}})\right\} \underbrace{\left\{ \frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\right\}}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\left\{e^{-\left(\sigma\sqrt{\frac{\pi}{2}}\right)t_{1}}-1\right\} + \frac{\left(\frac{L^{\alpha}}{1-\left(\frac{L}{H}\right)^{\alpha}}\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{L^{\alpha-1}}-\frac{1}{H^{\alpha-1}}\right)\right)}{\left(\sigma\sqrt{\frac{\pi}{2}}\right)^{2}}\right\}}{\left(e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{2}-t_{1})}+e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{3}-t_{2})}-2+\left(\sigma\sqrt{\frac{\pi}{2}}\right)t_{1}-\left(\sigma\sqrt{\frac{\pi}{2}}\right)t_{3}\right)}+I_{_{\boldsymbol{p}}}\left(\frac{1-e^{\left(\sigma\sqrt{\frac{\pi}{2}}\right)(t_{1}-t_{2})}}{\sigma\sqrt{\frac{\pi}{2}}}\right)$$

$$+ r_{_{\boldsymbol{W}}}(\boldsymbol{d}_{c}) \left\{ \begin{array}{l} \displaystyle \frac{L^{\alpha}}{1 - \left(\frac{L}{H}\right)^{\alpha}} \left(\frac{\alpha}{\alpha - 1}\right) \left(\frac{1}{L^{\alpha - 1}} - \frac{1}{H^{\alpha - 1}}\right) \left(c - 1\right) \\ \displaystyle \left(\sigma \sqrt{\frac{\pi}{2}}\right)^{2} & \left(e^{-\left(\sigma \sqrt{\frac{\pi}{2}}\right)t_{1}} + \left(\sigma \sqrt{\frac{\pi}{2}}\right)t_{1} - 1\right) \\ - \left(\frac{L^{\alpha}}{1 - \left(\frac{L}{H}\right)^{\alpha}} \left(\frac{\alpha}{\alpha - 1}\right) \left(\frac{1}{L^{\alpha - 1}} - \frac{1}{H^{\alpha - 1}}\right) \left(t_{3} - t_{1}\right) \end{array} \right) + r_{_{\boldsymbol{W}}}(T_{_{\boldsymbol{W}\boldsymbol{S}}}) r_{_{\boldsymbol{W}\boldsymbol{S}}} r_{$$

Subject to: $r_w(s)$ $I_T \le r_w(T_{ws})$ ---- (14)

Using Lagrange multipliers method, by using Kuhn-Tucker necessary condition. Solve the equations, the values of t_1^* , t_2^* and t_3^* are obtained. Substituting t_1^* , t_2^* and t_3^* in (6), (7), (9), (10) and (14), the optimum values I_T^* , I_{p1}^* , θ_L^* , I_m^* and TC^* are obtained.

7. Numerical Example:

In sugar factory, there are two warehouse (permanent and temporary). In permanent warehouse it has constant capacity, but in temporary warehouse holds its inventory in special containers, with each container occupying 50 sqft of floor space there are only 4000 sqft of storage space available. The following values of the parameter in proper unit were considered as input for the numerical result of the above problem.

L = 100, H = 1000,
$$\alpha = 0.5$$
, c = 2, $\sigma = 5$, $\pi = 3.14$, $I_p = 1000$.

$$\widetilde{S}_c = (100000, 200000, 300000)$$
 , $\widetilde{h}_c = (5000, 6000, 7000)$, $\widetilde{d}_c = (500, 600, 700)$,

$$\widetilde{T}_{wc} = (500, \, 1000, \, 1500) \;, \; \widetilde{s} \; = (50, 55, 60), \; \widetilde{T}_{ws} = (4000, \, 5000, \, 6000).$$

Using MATLAB software, the optimum values I_T^* , I_{p1}^* , θ_L^* , I_m^* and TC^* are tabulated.

Comparison Table 7.1

Model	S _c Rs	h _c Rs	d _c Rs	T _{wc} Rs	S Rs	T _{ws} Sqft	t ₃ * /month	I _T * /Ton	I _{P1} * /Ton	$ heta^*_L$ / Ton	I _m * /Ton	TC* Rs
Crisp 1	100000	5000	500	500	50	4000	1.2043	1,532	942	306	2,532	36,92,500
Crisp 2	200000	6000	600	1000	55	5000	1.3608	1,424	901	357	2,424	66,26,900
Crisp 3	300000	7000	700	1500	60	6000	1.3253	1,472	920	356	2,474	40,44,700
Fuzzy	180000	5400	540	900	49.5	4500	1.535	1,746	968	286	2,863	30,48,600

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Observation:

From the above table, it should be noted that compared to crisp model, the fuzzy model is very effective method, because of the time consuming in fuzzy analysis and the optimal results are obtained easily.

- ✓ The average total cost is obtained in fuzzy model is less than the crisp model.
- \checkmark The optimal values I_T^* , I_{p1}^* , I_m^* in fuzzy model are higher than the crisp model.
- The deterioration level in fuzzy model is less than the crisp model.

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