Binomial Geometric Series

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Abstract: This paper introduces a binomial geometric series in this article. This new idea can enable the scientific researchers for further involvement in research and development.

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1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management and its applications. In this article, a new geometric series [1-3] is constructed for application [4] of computational science and engineering.

2. Binomial Geometric Series

The binomial geometric series is given below:

$$\sum_{k=0}^{n-1}(x+y)^k = \frac{(x+y)^n - 1}{(x+y)-1} = \frac{\sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k - 1}{(x+y)-1}, (x+y) \neq 1. \qquad (1)$$

$$\text{Here, the binomial series is } (x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}. \qquad (2)$$

Theorem 2.1: $\sum_{k=0}^{n-1} 2^k = \sum_{k=0}^{n}\binom{n}{k} - 1 = 2^n - 1.$

Proof.

By substituting $x = y = 1$ in the binomial geometric series (1), we get

$$\sum_{k=0}^{n-1}(1+1)^k = \frac{(1+1)^n - 1}{(1+1)-1} = \frac{\sum_{k=0}^{n}\binom{n}{k}(1)^{n-k}(1)^k - 1}{(x+y)-1}.$$

By simplifying this numerical expression, we obtain

$$\sum_{k=0}^{n-1} 2^k = 2^n - 1 = \sum_{k=0}^{n}\binom{n}{k} - 1. \qquad (3)$$

Hence, theorem is proved.

3. Conclusion

In this article, a binomial geometric series has been introduced for mathematical and computational application. Also, this idea can enable the researchers for further involvement in the scientific research

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