A solution to Collatz's conjecture

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Abstract.

A simple and innovative approach is presented to solve Collatz's conjecture, based on an equivalence relation and its corresponding equivalence classes. As a consequence, it is demonstrated that the union of all equivalence classes forms the set of odd numbers.

Keywords: Collatz conjecture; equivalence classes.
MSC2020: 11B99, 65Q30

1 Introduction

The Collatz's conjecture, also known as the $3\,x+1$ problem, is a famous unsolved problem in mathematics which consists of the following: if a positive integer x is even, it is divided by two. If it is odd, it is multiplied by three and one is added. This operation can be expressed as:

$$f(x) = \begin{cases} x/2 & x \equiv 0\ (mod\ 2) \\ 3\,x+1 & x \equiv 1\ (mod\ 2) \end{cases}$$

In this way, by repeatedly applying the function $f(x): \mathbb{N}+1 \to \mathbb{N}+1$, with $\mathbb{N} := \{0,1,,2\ldots\}$, to any positive integer, we invariably reach the number 1, regardless of the initial chosen number. This process occurs within a finite number of steps. Let $n \in \mathbb{N}+1$, we use the definition of Collatz orbit given in [1], $Col^{\mathbb{N}}(n) := \{n, f(n), f^2(n)\ldots\}$. Let $Col_{min}(n) := min\ Col^{\mathbb{N}}(n) = inf_{k\in\mathbb{N}} Col^k(n)$, then the Collatz conjecture is expressed as $Col_{min}(n) = 1$ for every $n \in \mathbb{N}+1$.

If the previous statement were false, it would mean that there exists a number $m \in \mathbb{N}+1$ that generates a cycle where the number 1 does not belong to its cycle, i.e., $Col_{min}(m) \neq 1$. This would imply that the sequence enters a cycle that does not contain the number 1. Furthermore, the cycle could potentially increase indefinitely.

For example, if we start with $x = 7$ and apply the function repeatedly, we obtain the following sequence of numbers until reaching 1: $7 \to 22 \to 11 \to 34 \to 17 \to 52 \to 26 \to 13 \to 40 \to 20 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$. Thus, its orbit would be $Col^{\mathbb{N}}(7) = \{7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1\}$.

In [1], the function $Aff_n: \mathbb{R} \to \mathbb{R}$ is defined such that for every positive integer $n \in \mathbb{N}\backslash\{0\}$, we have $Aff_n(x) = \frac{3x+1}{2^n}$, where n is the largest natural number such that 2^n divides $3x+1$, resulting in an odd number if x is odd. In this paper, we will always consider x to be an odd number. Thus, we can define the Collatz orbit for this new function as follows: $Col^{2\mathbb{N}+1}(x) := \left\{x, Aff_{n_1}(x),\ Aff_{n_2}\left(Aff_{n_1}(x)\right) \ldots\right\}$. This corresponds to the formulation known as Syracuse, so the Collatz conjecture can be expressed as $min\, Col^{2\mathbb{N}+1}(x) = 1$ for every $x \in 2\mathbb{N}+1$. This Syracuse orbit $Col^{2\mathbb{N}+1}(x)$ corresponds to the Collatz orbit but for odd numbers. Thus, in the previous example, we obtain the sequence $Col^{2\mathbb{N}+1}(7) = \{7, 11, 17, 13, 5, 1\}$.

From the direct observation of the odd numbers that reach 1 after applying the function only once Aff_n, it is observed that there is recursion between them. For example, consider the odd numbers $x = 1, 5, 21, 85, 341 \ldots$ then $5 = 4 \times 1 + 1$; $21 = 4 \times 5 + 1$; $85 = 4 \times 21 + 1$; $341 = 4 \times 85 + 1$; in general $x_{k+1} = 4x_k + 1$, obtaining the general term of the series as $4^k + \frac{1}{3}(4^k - 1) \in 2\mathbb{N}+1$.

This fact reveals a structure or pattern that is generated by the repeated application of the function Aff_n. Clearly, the numbers in this series satisfy that $Aff_{2(k+1)}\left(4^k + \frac{1}{3}(4^k - 1)\right) = 1$.

Due to this, we can consider that the series of nodes thus defined belongs to the same branch. See figure 1.

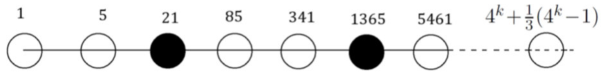

Figure 1. Representation of a branch with its nodes. Nodes that are multiples of three are marked with a black circle.

Thus, the distribution of odd numbers is ordered in the branch according to the number of 2^{2k} required to belong to it. For example, for $Aff_2(1) = 1$ with 2^2, $Aff_4(5) = 1$ with 2^4, $Aff_6(21) = 1$ with 2^6, $Aff_8(85) = 1$ with 2^8, so the number 2^{2k} varies but it is clear that $1, 5, 21, 85$ reach 1 in one step (a single application of Aff_n), hence they belong to the same branch.

In general, and as a consequence of this result, we can consider all numbers $x, y \in 2\mathbb{N}+1$ that satisfy the following equation $Aff_{n_x}(x) = Aff_{n_y}(y)$, which means that $\frac{3x+1}{2^{n_x}} = \frac{3y+1}{2^{n_y}}$. If we solve for x, we obtain $x = 2^{n_x - n_y}\, y + \frac{1}{3}(2^{n_x - n_y} - 1)$. Since x and y are odd numbers, it can be deduced that $n_x - n_y$ must be a natural and even number, that is, $n_x - n_y = 2\,k$ with $k \in \mathbb{Z}$. Rewriting it, we obtain $x = 2^{2k}\, y + \frac{1}{3}(2^{2k} - 1)$. The sign of k will be determined by the relation between x and y. If $x < y$, then this implies that $k > 0$. On the other hand, if $x > y$ then necessarily $k < 0$ since both x and y have to belong to $2\mathbb{N}+1$. For example, if $x = 1$ and $y = 5$ we have that $1 = 2^{-2}\, 5 + \frac{1}{3}(2^{-2} - 1)$, in this case $k = -1$.

In this way, we can establish the following relation: let $x, y \in 2\mathbb{N}+1$ be given. Then, $x \sim y$ if $Aff_{n_x}(x) = Aff_{n_y}(y)$ for suitable n_x and n_y. This relation is one of equivalence relation since it is reflexive, symmetric and transitive, and its proof is trivial.

This equivalence class allows us to regroup the elements into equivalence classes and consider subsets or equivalence classes accordingly.

2 Nodes and Branches

Definition 2.1. The equivalence classes, which we will call branches, are defined as $b(m) := \left\{x \in 2\mathbb{N}+1 \mid x = 4^k m + \frac{1}{3}(4^k - 1),\ k \in \mathbb{Z}\right\}$. The number $m \in 2\mathbb{N}+1$ is taken as the initial node of the branch.

Clearly the equivalence relation holds for any $x \in b(m)$, since $Aff_n(x) = Aff_n\left(4^k m + \frac{1}{3}(4^k - 1)\right) = Aff_{n-2k}(m)$.

As mentioned earlier, the initial node of the equivalence class can be any element from it. The only thing to consider is the sign of k so that all members of the class belong to $2\mathbb{N}+1$.

Proposition 2.1. Let $m \in b(n)$ then $n \in b(m)$ and vice versa. Therefore, $b(n) = b(m)$.

Proof. Assuming that $m \in b(n)$, then $m = 4^k n + \frac{1}{3}(4^k - 1)$, so if $k > 0$, solving for n, $n = 4^{-k} m + \frac{1}{3}(4^{-k} - 1)$ thus $n \in b(m)$. The same reasoning applies for $k < 0$. Therefore, $b(n) \subseteq b(m)$. By exchanging n with m, the other inclusion is demonstrated. □

For example, let $m \in b(1)$ then we have that 1 is the initial node of the branch and thus $m = 4^k + \frac{1}{3}(4^k - 1)$ and k could be $k = 0,1,2 \ldots$ such that $m \in 2\mathbb{N}+1$ and so $b(1) = \{1,\ 5,\ 21,\ 85,\ 341 \ldots\}$. If we consider the element 85 as the initial node, we obtain the same branch. Let $m \in b(85)$ then $m = 4^k\ 85 + \frac{1}{3}(4^k - 1)$ and k could be $k = -3, -2, -1, 0, 1, 2\ldots$ and so $b(85) = \{1,\ 5,\ 21,\ 85,\ 341 \ldots\}$.

By convention, the smallest element in the branch will be considered its canonical representative, meaning for $b(m)$, the odd number m will be the smallest among all elements in the branch, so $b(m) = \left\{4^k m + \frac{1}{3}(4^k - 1) \mid k = 0,1 \ldots\right\}$. Based on this convention, a node can be defined as $b_k(m) := 4^k m + \frac{1}{3}(4^k - 1)$ for $k = 0,1\ldots$ For example, for branch $b(3)$, we have the nodes $b_0(3) = 3$, $b_1(3) = 13$, $b_2(3) = 53$ and so on.

Consider the equivalence class for odd numbers $2\mathbb{N}+1$ as follows: $[m]_4 = \{x \in 2\mathbb{N}+1 : x \equiv m\ (mod\ 4)\}$

So, $2\mathbb{N}+1$ is divided into two equivalence classes corresponding to the possible two remainders when dividing any odd number by 4: $[1]_4$, $[3]_4$. It is evident that $[1]_4 \cup [3]_4 = 2\mathbb{N}+1$.

Proposition 2.2. Odd numbers in the form of $4n+1$, where n is odd, correspond to nodes $b_k(m)$ with $k>0$ from their respective branch $b(m)$, where $m \in [3]_4$ or m takes the form $m=4n+1$ with n even.

Proof. From the expression $b_k(m)=4^k m+\frac{1}{3}(4^k-1)$, it is observed that for values $k>0$, we obtain the set of numbers $4n+1$ for $k=1$; $4(4n+1)+1$ for $k=2$; $4(4(4n+1)+1)+1$ for $k=3$, and so on, which are clearly odd and of the form $4s+1$ with s odd. From the definition of the branch and following the adopted convention, it is now clear that or $b_0(m)=m$ for $m \in [3]_4$ or $m=4n+1$ with n even, because if it were otherwise, meaning $m=4n+1$ with n odd, we could simply take the first odd number in the series $m_{j+1}=\frac{m_j-1}{4}$ with $m_0=m$ that satisfies being odd and of the form $4s+1$ with s even o $m_{j+1} \in [3]_4$. In this way, the branch representative is obtained with $m_{j+1} \in [3]_4$ o $m_{j+1}=4n+1$ with n even, being the smallest element of the branch that represents it. □

For example, for $m=4053 \in [1]_4$. As 4053 is odd, we choose $m_1=\frac{m_0-1}{4}=\frac{4053-1}{4}=1013$, which means $m=4053=4\times 1013+1$. Since 1013 is odd, we choose $m_2=\frac{m_1-1}{4}=\frac{1013-1}{4}=253$, which means $1013=4\times 253+1$. As 253 is odd, we choose $m_3=\frac{m_2-1}{4}=\frac{253-1}{4}=63$, which means $253=4\times 63+1$. Since $63 \in [3]_4$ then $4053 \in b(63)$ and $b_3(63)=4053$. From this proposition, an algorithm has been developed for the repeated application of the function $f(x)$ in order to obtain the orbit of any odd number (See appendix).

In summary, when considering the modulo-4 equivalence classes for odd numbers, we can categorize the nodes into two types: the initial nodes of a branch and those that do not generate a new branch. In other words, initial nodes encompass all odd numbers of the form $m \in [3]_4$ or $m=4n+1$ with n being even, while the rest of the nodes fall into the category $m=4n+1$ with n being odd.

What sets branch $b(1)$ apart from the others is its unique property that its initial node remains unchanged when the Aff_n function is applied. As indicated above, $Aff_n(\mathrm{x})=1$ for any node x belonging to branch $b(1)$. This property is what precisely distinguishes it as the primary branch. In contrast, this characteristic is not met in the other branches, as applying Aff_n to any node in a branch other than $b(1)$ results in a node that does not belong to the same branch as the original node. For example, for $b(3)=\{3,13,53,213\ \ldots\}$ and $Aff_1(3)=5$, it transitions to the branch of 1. See figure 2.

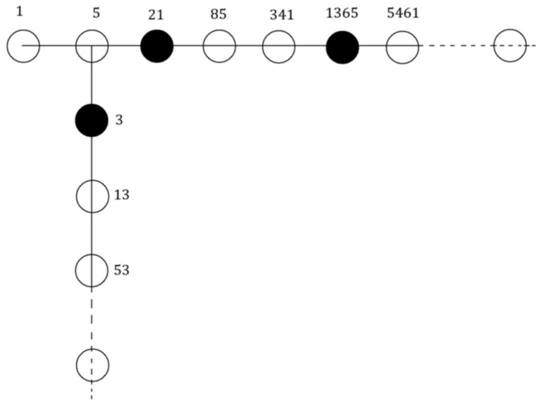

Figure 2. Representation of two branches with its nodes.

Proposition 2.3. The only node that remains invariant when applying the Aff_n function once is node 1. That is, let $x \in b(m)$ if $Aff_n(x) = m$ for some $n \in \mathbb{N}\backslash\{0\}$ then $m = 1$ and $x = 1$.

Proof. Just apply the function once Aff_n, $Aff_n(x) = Aff_n\left(4^k m + \frac{1}{3}(4^k - 1)\right) = \frac{3\left(4^k m + \frac{1}{3}(4^k - 1)\right)+1}{2^n} = \frac{4^k(3\,m+1)}{2^n} = \frac{3\,m+1}{2^{n-2k}} = m$, then $m = \frac{1}{2^{n-2k}-3}$ and since it has to be an odd number and a positive integer there is only one solution that corresponds to $n - 2k = 2$, that is, $n = 2k + 2$ and therefore $m = 1$, thus $x \in b(1)$ and since $b_0(1) = 4^0 + \frac{1}{3}(4^0 - 1) = 1$ then $x = 1$. □

When applying the function Aff_n to any number other than 1, we obtain another number that can be considered one step closer to the branch that contains the number 1. In other words, when applying the function Aff_n, it switches from one branch to another and the latter will be closer to the node 1. Similarly, we can think that the inverse function of Aff_n applied to a node results in a node that is one step further away from the node 1. Therefore, we can consider the inverse function of Aff_n defined as $Aff_n^{-1}: 2\mathbb{N} + 1 \to 2\mathbb{N} + 1$ such that $Aff_n^{-1}(m) := \frac{2^n\,m-1}{3}$.

Proposition 2.4. For every node $b_k(m) \in 2\mathbb{N} + 1$ not divisible by three with $k \geq 0$, belonging to any branch $b(m)$, another branch comes out with infinite nodes, that is, if $b_k(m) \equiv 1 \ (mod\ 3)$ or $b_k(m) \equiv 2 \ (mod\ 3)$ there exists $p \in 2\mathbb{N} + 1$ such that $Aff_n(p) = b_k(m)$, equivalent $Aff_n^{-1}\big(b_k(m)\big) = p$. And if $b_k(m) \equiv 0 \ (mod\ 3)$ no branches come from those nodes.

Proof. Let m be any node, consider the only three possible situations

1) Suppose that $b_k(m) \equiv 0 \ (mod\ 3)$, then let p be its superior node, that is $p = Aff_n^{-1}\big(b_k(m)\big) = \frac{2^n\,b_k(m)-1}{3}$ then according to the remainder of p, we have that

a. If $p \equiv 0\ (mod\ 3)$, that is $p = 3p_0$, with $p_0 \in \mathbb{N}$, then $3p_0 = \frac{2^n\ b_k(m)-1}{3}$, which simplifies to $2^n\ b_k(m) = 3^2 p_0 + 1$; in order words, $2^n\ b_k(m) \equiv 1\ (mod\ 3)$, which is not possible.

b. If $p \equiv 1\ (mod\ 3)$, that is $p = 3p_0 + 1$, with $p_0 \in \mathbb{N}$, then $3p_0 + 1 = \frac{2^n\ b_k(m)-1}{3}$, which simplifies to $2^n\ b_k(m) = 3\ (3p_0 + 1) + 1$; in order words $2^n\ b_k(m) \equiv 1\ (mod\ 3)$, which is not possible.

c. If $p \equiv 2\ (mod\ 3)$, that is $p = 3p_0 + 2$, with $p_0 \in \mathbb{N}$, then $3p_0 + 2 = \frac{2^n\ b_k(m)-1}{3}$, which simplifies to $2^n\ b_k(m) = 3\ (3p_0 + 2) + 1$; in order words, $2^n\ b_k(m) \equiv 1\ (mod\ 3)$, which is not possible.

This make clear that if $b_k(m) \equiv 0\ (mod\ 3)$, it does not have a superior node.

2) Suppose $b_k(m) \equiv 1\ (mod\ 3)$, then let p be its superior node, which means $p = Aff_n^{-1}(b_k(m)) = \frac{2^n\ b_k(m)-1}{3}$. As $b_k(m) \equiv 1\ (mod\ 3)$, we can write $b_k(m) = 3m_0 + 1$, with $m_0 \in 2\mathbb{N}$ since $b_k(m)$ is odd. Therefore, $p = \frac{2^n(3m_0 + 1) - 1}{3} = \frac{3\ 2^n m_0 + 2^n - 1}{3} = 2^n m_0 + \frac{2^n - 1}{3} \in 2\mathbb{N} + 1$ if $n \in 2\mathbb{N}\backslash\{0\}$, as in that case $\frac{2^n - 1}{3}$ will be an odd and integer number. In this way, the branch $b(p)$ is associated with the node $b_k(m)$.

3) Suppose $b_k(m) \equiv 2\ (mod\ 3)$, then let p be its superior node, which means $p = Aff_n^{-1}(b_k(m)) = \frac{2^n\ b_k(m)-1}{3}$. As $b_k(m) \equiv 2\ (mod\ 3)$, we can write $b_k(m) = 3m_0 + 2$, with $m_0 \in 2\mathbb{N} + 1$, since $b_k(m)$ is odd. Therefore, $p = \frac{2^n(3m_0 + 2) - 1}{3} = \frac{3\ 2^n m_0 + 2^{n+1} - 1}{3} = 2^n m_0 + \frac{2^{n+1} - 1}{3} \in 2\mathbb{N} + 1$ if $n \in 2\mathbb{N} + 1$, as in that case $\frac{2^{n+1} - 1}{3}$ will be an odd and integer number. Hence, there exists $p \in 2\mathbb{N} + 1$. In this way, the branch $b(p)$ is associated with the node $b_k(m)$. □

In summary, for each node $b_k(m) \in 2\mathbb{N} + 1$ where $k \geq 0$, not divisible by three, of a branch $b(m)$, there exists a unique branch $b(p)$ whose initial value p depends on this node $b_k(m)$, where the value of p can be expressed as:

- If $b_k(m) \equiv 1\ (mod\ 3)$ then $p = \frac{2^n\ b_k(m)-1}{3}$, for $n = 2$ the value of $p = \frac{4\ b_k(m)-1}{3}$. Since $b_k(m) \equiv 1\ (mod\ 3)$, we can write $b_k(m) = 3m_0 + 1$, with $m_0 \in 2\mathbb{N}$, and therefore $p = 4\ m_0 + 1$, where m_0 is even. Thus, this associates the node $b_k(m)$ with the branch $b\left(\frac{4\ b_k(m)-1}{3}\right)$.
- If $b_k(m) \equiv 2\ (mod\ 3)$ then $p = \frac{2^{n+1} b_k(m)-1}{3}$, for $n = 1$ the value of $p = \frac{2\ b_k(m)-1}{3}$. Since $b_k(m) \equiv 2\ (mod\ 3)$, we can write $b_k(m) = 3m_0 + 2$, with $m_0 \in 2\mathbb{N} + 1$, and therefore $p = 4\ m_0 + 3$. Thus, this associates the node $b_k(m)$ with the branch $b\left(\frac{2\ b_k(m)-1}{3}\right)$.

Clearly, each associated branch is unique and distinct since all nodes are different.

For example, let's assume $b_k(m) = 341$. Since $341 \equiv 2\ (mod\ 3)$, then $p = \frac{2\, b_k(m)-1}{3} = 227$. The branch obtained is $b(227)$. Then $b_0(227) = 227$, $b_1(227) = 909$, and so on, graphically, see figure 3.

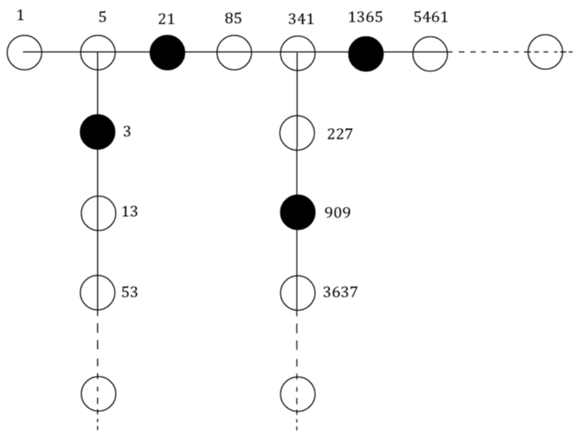

Figure 3. Representation of three branches with its nodes.

In both cases it is verified that 341 are in the orbit $Col^{2\mathbb{N}+1}(227) = \{227, 341, 1\}$ and $Col^{2\mathbb{N}+1}(909) = \{909, 341, 1\}$.

Each node m is not a multiple of 3 ($m \not\equiv 0\ (mod\ 3)$) generates a distinct equivalence class (branch) and each branch has its own nodes.

Proposition 2.5. Let $\mathrm{n}, \mathrm{m} \in 2\mathbb{N}+1$ consider the distinct branches $b(m)$ and $b(n)$ with $m \notin b(n)$ then $b(m) \cap b(n) = \emptyset$.

Proof. Assume that there exists an odd number $x \in b(m) \cap b(n)$, then $x = 4^k m + \frac{1}{3}(4^k - 1) = 4^l n + \frac{1}{3}(4^l - 1)$ with $k \neq l$; simplifying $2^{2k} m = 2^{2l} n + \frac{1}{3}(2^{2l} - 2^{2k})$ and so $m = 2^{2l-2k} n + \frac{1}{3}(2^{2l-2k} - 1)$; which implies that $m \in b(n)$ which cannot be by hypothesis, therefore there is no x that belongs to the intersection.□

3 Conclusion

Based on what has been said above, one can consider a tree-shaped structure, formed by the different branches that emerge from the nodes, with node 1 as the initial node. In other words, the main branch can be assigned to the one that contains the initial node 1, considering this branch as the tree trunk, where at each node not divisible by 3, a branch with an infinite number of nodes emerges, which in turn contain nodes from which branches arise, and so on. Each branch represents an equivalence class, and the union of all equivalence classes forms the set of odd numbers.

Lemma 3.1. Let $m \in [3]_4 \cup \{4n+1 \text{ with } n \in 2\mathbb{N}\}$ then $\cup b(m) = 2\mathbb{N}+1$.

Proof. The inclusion $\cup b(m) \subseteq 2\mathbb{N}+1$ is obvious from the very definition of $b(m)$. Let us see that $2\mathbb{N}+1 \subseteq \cup b(m)$. Let $n \in 2\mathbb{N}+1$, according to proposition 2.2, either $n \in [3]_4$ or n takes the form $n = 4s+1$ with s even, in both cases, there exists a branch

$b(m)$ that contains it, and $n = b_0(m)$. In addition, since numbers of the form $4s + 1$ with s odd belong to $b(m)$ for some m, the inclusion holds. □

In this way, a set of infinite branches with infinitely many nodes is obtained. Nodes only belong to one branch by proposition 2.5. In addition, the branches are interconnected by the initial nodes of each branch by proposition 2.4 and the only initial node that is in its own branch is 1, which shows that there are no unconnected branches or unconnected nodes that generate another independent tree and this is because the union of all branches is the set of odd numbers. Furthermore, it is clear that the orbit of any odd number indicates the passage from one branch to another, each time the Aff_n function is applied, it jumps from one branch to a lower one. This indicates that every orbit $Col^{2\mathbb{N}+1}(m)$ contains the number 1, thus fulfilling the conjecture.

For example, consider the node $n = 1643861$; this node belongs to the branch of $b(401)$ since $1643861 = 4^k\, 401 + \frac{1}{3}(4^k - 1)$ with $\mathrm{k} = 6$, that this $b_6(401) = 1643861$. The branch of $b(401)$ starts at node 301, because $Aff_2(401) = 301$. This node belongs to branch $b(75)$, this branch starts from node 113 and finally, node 113 starts from node 85 which is in the main branch. Its orbit is $Col^{2\mathbb{N}+1}(1643861) = \{1643861, 301, 113, 85, 1\}$. See figure 4.

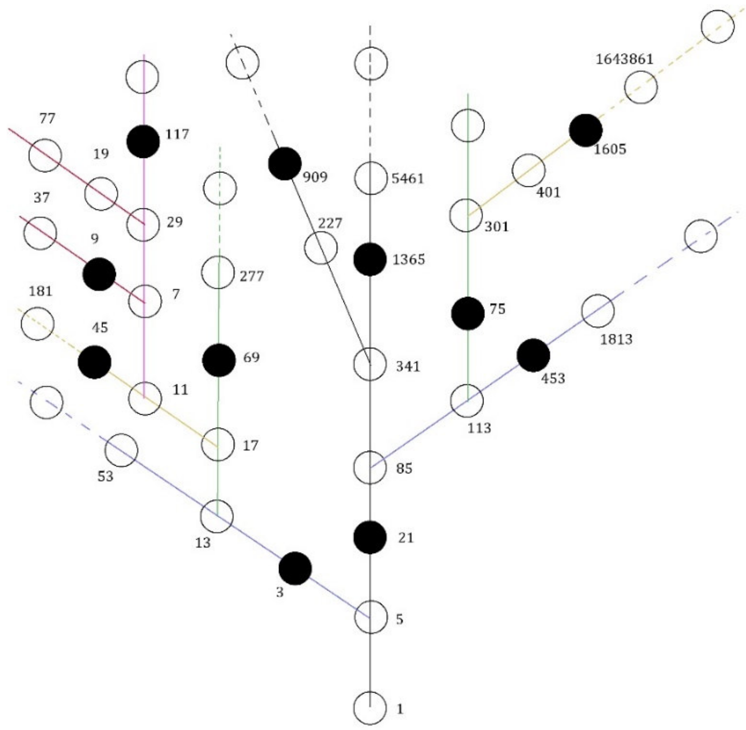

Figure 4. Representation of a part of the tree where node 1643861 and the lower branches appears

Appendix

Building upon proposition 2.2, we have devised a straightforward algorithm for generating a sequence akin to the orbit of [1]. When presented with any odd number, our objective is to ascertain its respective branch. Once the branch is identified, along with the node governing it, we proceed to compute the node from which that branch originates. This process is iteratively repeated until we arrive at the primary branch, $b(1)$.

During the course of iteration, should we encounter a node conforming to the pattern $4s + 1$, where s is an odd integer, we will search for the node representing that branch. This node can take on the form $4s + 1$ with an even s or $4s + 3$ with any s. Consequently, the trajectory we obtain offers an alternative, yet nearly identical solution to the orbit of [1]. The difference lies in the iteration's focus on the primary nodes within the branches. Convergence is unequivocally assured due to the interconnected nature of all branches.

Next, we present the BC and C algorithms: the BC Algorithm, based on the previous description, and the C Algorithm, renowned for its applicability in orbit calculations.

Algorithm BC

```
Input: n: number odd
Output: nb: number of iterations until reaching 1
while n is not equal to 1
   if mod(n,4) is equal to 1          % n ≡ 1 (mod 4) class [1]_4
      x ← (n − 1)/4
      if mod(n,2) is equal to 0       % initial node of branch is identified
         n ← (3n + 1)/4
      else
         n ← x                        % node of branch
      endif
   else                                      % n ≡ 3 (mod 4) class [3]_4
      n ← (3n + 1)/2
   endif
      nb ← nb + 1                            % iterations
endwhile
```

Algorithm C

```
Input: n, number odd
Output: nc, number of iterations until reaching 1
while n is not equal to 1
   if mod(n,2) is equal 0
      n ← n/2
   else
      n ← (3n + 1)/2
   end
   nc ← nc + 1                               % iterations
endwhile
```

Two simple cases are presented, in the table 1, to highlight that the obtained orbits are different. In the first case, the BC algorithm passes through node 3 since it is the main node of$b_1(3) = 13$, it is worth noting that the number of iterations is lower in BC. In the second case, node $b_4(1) = 341$ belongs to the branch of 1, hence it passes through the rest of the nodes in that branch $b(1) = \{1,5,21,85,341 \ldots\}$. In this case, the number of iterations with the C algorithm doubles due to the number of times it has to divide by 2 when the number is even.

	BC		C	
	Number of iterations	Orbit	Number of iterations	Orbit
$n = 7$	7	$7, 11,17,13,3,5,1$	12	$7, 11, 17, 13, 5, 1$
$n = 909$	7	$909,227,341,85,21,5,1$	14	$909,341,1$

Table 1. Number of iterations to reach 1 and orbit for BC and C algorithms.

The behavior of both algorithms with respect to large odd numbers is reflected in Table 2. As can be seen, the number of iterations for the BC algorithm is lower in all cases.

	BC	C
$n = 2^{99} - 1$	664	937
$n = 2^{999} - 1$	5528	7841
$n = 2^{9999} - 1$	60810	86278
$n = 2^{99999} - 1$	608902	863323

Table 2. Number of iterations to reach 1 for BC and C algorithms.

In Table 3, the execution times for all odd numbers are displayed for each of the algorithms, according to the range shown. Time is measured in seconds. As can be seen, the execution time is lower for the BC algorithm.

	BC	C
$n = 1$ to $2^{20} - 1$, step 2	244.17	309.32
$n = 2^{20} - 1$ to $2^{21} - 1$, step 2	335.88	379.37

Table 3. Time in seconds for both algorithms to reach 1 within that range of numbers.

Conflict of Interest
The author declare no conflict of interest.

References

[1] T. Tao, Almost all orbits of the Collatz map attain almost bounded values, Forum of Mathematics, Pi 10 (2022).