

A method to calculate excess mortality

Hans Lugtigheid
Amsterdam

hanslugtigheid.nl

hans@hanslugtigheid.nl

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Abstract

With the pandemic we have seen that excess mortality can be calculated in different ways. In this article we give a method that calculates the excess mortality per year. This method takes into account that excess mortality in one year can influence the expected mortality, and hence the excess mortality, in subsequent years. We also show possible definitions of excess mortality.

1 Preliminaries

The expected mortality usually is calculated on the base of the past. The excess mortality is calculated as actual mortality minus the expected mortality.

When there is excess mortality in a single year, for instance because of a flu outbreak, than mortality in the next year is lower than normally expected. Usually the expected mortality is not adjusted, but one explains the lower than expected mortality by the flu the year before. With the corona pandemic we had a phenomenon that spread over multiple years. Than the excess mortality gets less clear.

An example:

In a country 150K people die every year. In a year there is a strong flu and 10K elderly people die who would otherwise have died the next year. Mortality in the first year is 160K. So it is to be expected that in the next year 140K people die, 10K less than normal. Suppose that in the second year 152K people die. Then excess mortality for the second year is usually calculated as $152K - 150K = 2K$. When we adjust the expected mortality on January 1 to 140K the excess mortality becomes 12K. A significant difference. See table 1.

Table 1: Mortality in second year

Mortality (x 1.000)	Usual	Adjusted
Real mortality	152	152
Expected mortality	<u>150</u>	<u>140</u>
Excess mortality	2	12

On January first of the second year we have information on the pandemic in the previous year. We can use this information to adjust the ‘normal’ expected mortality. Then we get the expected mortality given the number of people that have died a year earlier. Then we get a more accurate picture of excess mortality. When the pandemic stretches over multiple years we must adjust expected mortality every year on January first. For the expected mortality in the third year we have people who have died in the first year and would have died in the third year without pandemic. There are also people who died in the second year who would have died in the third year without the pandemic. We have to adjust the expected mortality in the third year for both these numbers. Etcetera.

2 The model

Excess mortality is calculated as actual mortality minus expected mortality. So we have to determine the expected mortality for every year.

We assume a pandemic. At the beginning of every year we calculate/estimate for every year the expected mortality given the excess mortality in previous years.

In order to calculate/estimate the expected mortality for every year we have to distribute the excess mortality in every year over the subsequent years in which people would have died without the phenomenon/pandemic.

We determine the expected mortality and the excess mortality in four steps.

Step 1: Year 0

We start with year 0. See table 2.

Table 2: Excess mortality year on year

Year	Excess mortality	Would have died in year:												
		1	2	3	4	5	6	7	...	M-1	M	M+1	...	∞
0	A_0	a_{01}	a_{02}	a_{03}	a_{04}	a_{05}	a_{06}	a_{07}	...	a_{0M-1}	0	0	0	0

Explanatory notes with table 2:

Column 1	Relevant year
Column 2	Excess mortality in the year. A_0 = excess mortality in year 0.
Columns #	In those columns we note the number of people who died in year 0 but without the pandemic would have died in that year. Or: would have lived until that year. So with year 5 a_{05} is the number of people that has died in the year 0 and without the pandemic would have died in year 5.
Column M	M is the maximum age one can reach. If someone is born in the year 0 then one can live maximally until the year M-1. Then the value from M and up in this row equals 0.
Column ∞	It is not known how many years an event will occur. Hence the table goes to ∞ . In practice when applied one shall work with a finite number of years.

We have the following relations:

$$A_0 = \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{M-1} a_{ij}$$

Step 2: Other years

We add the subsequent years. See table 3.

Table 3: Excess mortality year on year

Year	Excess mortality	Would have died in year:												
		1	2	3	4	5	6	7	...	M-1	M	M+1	...	∞
0	A_0	a_{01}	a_{02}	a_{03}	a_{04}	a_{05}	a_{06}	a_{07}	...	a_{0M-1}	0	0	0	0
1	A_1		a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	...	a_{1M-1}	a_{1M}	0	...	0
2	A_2			a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	...	a_{2M-1}	a_{2M}	a_{2M+1}	...	0
3	A_3				a_{34}	a_{35}	a_{36}	a_{37}	...	a_{3M-1}	a_{3M}	a_{3M+1}	...	0
4	A_4					a_{45}	a_{46}	a_{47}	...	a_{4M-1}	a_{4M}	a_{4M+1}	...	0
5	A_5						a_{56}	a_{57}	...	a_{5M-1}	a_{5M}	a_{5M+1}	...	0
:	:	:	:	:	:	:	:	:	:	:	:	:	:	0
N-1	A_{N-1}								...	a_{N-1M-1}	a_{N-1M}	a_{N-1M+1}	...	0
N	A_N								...	a_{NM-1}	a_{NM}	a_{NM+1}	...	0
Total	T_A	T_1	T_2	T_3	T_4	T_5	T_6	T_7	...	T_{M-1}	T_M	T_{M+1}	0	0

Explanatory notes with table 3

See also notes table 2.

A_i	Excess mortality in year i.
a_{ij}	Number of people that died in year i that would hav died in year j without the phenomenon. So a_{37} is thenumber of people that died in year 3 en without the phenomenon would have died in the year 7. These numbers are unknown. They have to be estimated for every year.
T_j	Total number of people that would have died in year j but because of the phenomenon died in earlier years. T_j is the amount with which we adjust the expected mortality for year j.
M	At year 0 one lives maximally M years until the year M-1 and its value from year M and up equals 0. From year 1 one lives maximally M years until the year M and its value from year M+1 and up equals 0. Etcetera.
N	Number of years a phenomenon takes and/or has influence.

We have the following relations:

$$A_i = \sum_{j=1}^{\infty} a_{ij} = \sum_{j=i+1}^{M+i-1} a_{ij}$$

$$T_j = \sum_{i=0}^{j-1} a_{ij}$$

Thus the sequence becomes:

$$\begin{aligned} A_0 &\rightarrow a_{01} \rightarrow T_1 \\ A_0, A_1 &\rightarrow a_{02}, a_{12} \rightarrow T_2 \\ &\vdots \\ A_0, A_1 \dots A_{j-1} &\rightarrow a_{0j}, a_{1j} \dots a_{j-1j} \rightarrow T_j \end{aligned}$$

Step 3: Expected mortality.

We determine the new expected mortality (C_j) by correcting the old, pre-pandemic, expected mortality (V_j) with the earlier calculated adjustment (T_j).

In formula: $C_j = V_j - T_j$

Thus the sequence becomes:

$$\begin{array}{c} A_0 \rightarrow a_{01} \rightarrow T_1 \rightarrow T_1, V_1 \rightarrow C_1 \\ A_0, A_1 \rightarrow a_{02}, a_{12} \rightarrow T_2 \rightarrow T_2, V_2 \rightarrow C_2 \\ \vdots \\ A_0, A_1 \dots A_{j-1} \rightarrow a_{0j}, a_{1j} \dots a_{j-1j} \rightarrow T_j \rightarrow T_j, V_j \rightarrow C_j \end{array}$$

Step 4: Excess mortality

First we determine old excess mortality (O_j) with actual mortality (S_j) and old expected mortality (V_j).

In formula: $O_j = S_j - V_j$

Second we determine new excess mortality (A_j) with actual mortality (S_j) and new expected mortality (C_j).

In formula: $A_j = S_j - C_j$

Thus the sequence becomes:

$$\begin{array}{c} A_0 \rightarrow a_{01} \rightarrow T_1 \rightarrow T_1, V_1 \rightarrow C_1 \rightarrow C_1, S_1 \rightarrow A_1 \\ A_0, A_1 \rightarrow a_{02}, a_{12} \rightarrow T_2 \rightarrow T_2, V_2 \rightarrow C_2 \rightarrow C_2, S_2 \rightarrow A_2 \\ \vdots \\ A_0, A_1 \dots A_{j-1} \rightarrow a_{0j}, a_{1j} \dots a_{j-1j} \rightarrow T_j \rightarrow T_j, V_j \rightarrow C_j \rightarrow C_j, S_j \rightarrow A_j \end{array}$$

We use the calculated excess mortality A_j as input for table 3.

The difference between old and new excess mortality equals T_j .

In formula: $A_j = O_j + T_j$

3 Excess mortality in the same year

People also can die by a phenomenon who would have died otherwise later in the same year. Say an older person who dies from the pandemic in April but otherwise would have died in Oktober. This is premature mortality and can also be qualified as excess mortality. One can choose to add this to the earlier calculated year-on-year mortality. Then we get a new definition of excess mortality.

When we also use this year-in-year excess mortality we get table 4.

Table 4: Excess mortality year on year

Year	Excess mortality	Would have died in year:												
		0	1	2	3	4	5	6	...	M-1	M	M+1	...	∞
0	B_0	a_{00}	a_{01}	a_{02}	a_{03}	a_{04}	a_{05}	a_{06}	...	a_{0M-1}	0	0	...	0
1	B_1		a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	...	a_{1M-1}	a_{1M}	0	...	0
2	B_2			a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	...	a_{2M-1}	a_{2M}	a_{2M+1}	...	0
3	B_3				a_{33}	a_{34}	a_{35}	a_{36}	...	a_{3M-1}	a_{3M}	a_{3M+1}	...	0
4	B_4					a_{44}	a_{45}	a_{46}	...	a_{4M-1}	a_{4M}	a_{4M+1}	...	0
5	B_5						a_{55}	a_{56}	...	a_{5M-1}	a_{5M}	a_{5M+1}	...	0
:	:	:	:	:	:	:	:	:	:	:	:	:	:	0
N-1	B_{N-1}								...	a_{N-1M-1}	a_{N-1M}	a_{N-1M+1}	...	0
N	B_N								...	a_{NM-1}	a_{NM}	a_{NM+1}	...	0
Total	T_B	T_0	T_1	T_2	T_3	T_4	T_5	T_6	...	T_{M-1}	T_M	T_{M+1}	0	0

Explanatory notes (additional) with table 4:

B_i Excess mortality in year i. This includes excess mortality in the same year.

a_{ii} Excess mortality in same year i.

T_0 Excess mortality in year 0. Is equal to a_{00} .

Relations:

$$B_i = \sum_{j=0}^{\infty} a_{ij} = \sum_{j=i}^{M+i-1} a_{ij} = a_{ii} + \sum_{j=i+1}^{M+i-1} a_{ij} = a_{ii} + A_i$$

with A_i excess mortality year-on-year as in table 3.

4 Conclusion

The given model gives a way to calculate excess mortality in multiple years. The model consider the fact that the (excess) mortality in a year can influence expected and excess mortality in later years. One can use different definitions of expected and excess mortality. When using this method it is important to make very clear which definitions are used. The method gives a good overview

of expected and excess mortality as a consequence of a phenomenon that occurs over multiple years.