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Existence and Convergence of Gamma Matrices and their Application to Infinite Series

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Abstract :

In this paper we have develop a new class of matrices giving conservative transformations is sequence to series, together with its regular sub-class in pseudo-algebra of γ-matrices by introducing the new-mean and the term product of existence and convergence of Gamma Matrices and their application to infinite series.

1.1. Introduction & Preliminaries :

In the beginning of 19^{th} century, it was found that there were several series in which the concept of ordinary convergence was clearly failed so it becomes necessary to consider generalized convergence methods see (Aasma,1994), Boos, 2000) with the appearance of Cauchy's monumental work course 'd'Analysis Algebrique in 1821 and Abel's (see Nigam, 2013) researches on the binomial series in 1826, the old hazy notion of convergence of infinite series was put on sound foundation. It was, however, observed that there were certain non-convergent series, which particularly in Dynamical Astronomy furnished nearly correct results. A theory of divergent series was formulated explicitly for the first time in 1890 when Cesàro (see Bataineh, 1999) published a paper on multiplication of series. Since then the theory of series whose sequences of partial sums oscillate, has been the centre of creative activity for most of leading mathematicians. It was only towards the closing decade of 19^{th} and in early years of the 20^{th} century that satisfactory methods were devised, so as to associate with Cauchy's concepts of convergence, certain values which may be called their sums in a less direct way (see [Bataineh, 2017], [Sarigöl, 2011], [Jarrah, 2003], [Malkowsky, 2017]). Such processes of summation of series, which were formally tabooed as divergent, have given rise to modern rigorous theory of summability. Summability itself has became an independent branch of mathematics. The concept of convergence having seen generalized, it was but natural to enquire if the notion of absolute convergence was capable of similar generalization. The answer to this has been found in affirmative and infact, just as the notion of convergence was instrumental to be the development of various summability methods, so also, big analogy, the idea of absolute summability (Kransniqi, 2012), (Yildiz, 2019), (Aasma, 2011). In the same manner, by analogy, the idea of uniform summability can be formulated as a generalization of notion of uniform convergence (see [Sahani, 2020, 2021]).

In this paper, we use the following German abbreviation.

FF	for	sequence to sequence	(1.1.1)
RF	for	series to sequence	(1.1.2)
RR	for	series to series	(1.1.3)

Let $P = (p_{ab}), (a, b = 1, 2,...)$ be a given matrix and consider the transformation

$$u_n = \sum_{k=1}^{\infty} p_{ab} v_k \qquad (1.1.4)$$

then the matrix P provided FF, RF or RR transformation according as it transform a sequence $Z = \{v_b\}$ into the sequence $r = \{4_a\}$, the series $\Sigma\, v_a$ into the series $\Sigma\, u_a$, provided that each of the series (1.1.4) is convergent. A corresponding to FF-transformations each be made applicable with obvious changes to RF and RR transformation.

Summability transformations help us to generalize the concept of limit of a sequence or series. It provide us a method to assign limits even to sequences which are divergent. These transformations or methods can be calculated or classified into two ways:

(i) Sequence-to-sequence transformations.

(ii) Sequence to function transformations.

Sequence to sequence transformations is accomplished using infinite matrices. We consider an infinite matrix $C = (c_{lr})$ and a sequence $\{S_l\}, l = 0, 1, 2,...$ we form a new sequence $\{t_l\}$ defined by

$$t_l = \sum_{g=0}^{\infty} c_{lg} s_g.$$

We shall assume that the series converges for every l.

If $\{t_l\}$ converges to t, then t is called the c-limit of $\{S_l\}$. A transformation C is called a regular summability transformation if it preserves limit in the case of convergent sequences. Silverman-Toeplitz theorem gives necessary and sufficient conditions for a matrix to represents a regular method and thus help us to construct regular transformations. This theorem can be stated as the necessary and sufficient conditions that the matrix $C = (c_{lg})$ represents a regular transformation are (see [Verma's, 1946]).

(i) $\sum_{g=0}^{\infty} |c_{lg}| < N$ for some N and $l = 0, 1, 2,...$

(ii) $\lim_{l\to\infty} c_{lg} = 0$ for each $g = 0, 1, 2,...$

(iii) $\lim_{l\to\infty} \sum_{g=0}^{\infty} c_{lg} = 1$

1.2 Main results :

In this research note, we prove some specific theorems relating to the product of matrices which define either convergence preserving or limit preserving transformation of series or a sequence into either a series or a sequence. The actual nature of sequence to sequence, series to sequence and series to series transformation have been studied by several researchers such as Dienes, Cooke, Hill (see (Verma's,1946)). The products, taken two by two, of sequence to sequence series to sequence and series to series matrices have been studied by Verma's [1946]. In this research note, we extend Verma's (1946) works:

Theorem 1 : If $f_{lg} = m_{1g} + m_{2g} + ... + m_{lg}, (l, g \geq 1)$ (1.2.1)
then $F = (f_{lg})$ is a γ_A-matrix iff $M = (M_{lg})$ is α_A-matrix.

Theorem 2 : The product FM of a γ_A-matrix F and α_A-matrix M exists is a γ_A-matrix.

Theorem 3 : The product MF and α_A-matrix M and γ_A-matrix F may not exists.

Theorem 4 : The product γ_A-matrix F and an α_A-matrix M is not commutative.

Theorem 5 : The product matrix $L = FM$ exists and is a γ_A-matrix for every γ_A-matrix F iff M is an α_A-matrix MF is an α_A-matrix.

Theorem 6 : The product of two α_A-matrices is an α_A-matrix.

Theorem 7 : The product of two γ_A-matrices may not be a γ_A-matrix.

For the proof of our theorems, we need the following arbitrary definitions:

Definition 1 : An infinite matrix $F \equiv (f_{l,g})$ is a γ-matrix if it satisfies the following conditions (see [Vermas, 1947])

$$\sum_{g=1}^{\infty} |f_{l,g} - f_{l,g+1}| \leq K \,\forall l \geq 1,$$

$$f_{l,g} \to 1 \text{ as } l \to \infty \,\forall g$$

Note (1) : The elements of a γ-matrix are bounded.

Proof : From definition 1,

$$|f_{l,g}| = |f_{l,g} - f_{l,1} + f_{l,1}|$$
$$\leqq |f_{l,g} - f_{l,1}| + |f_{l,1}|$$
$$\leqq k + |f_{l,1}| \leq D$$

Note (2) : If $F^{(j)}$ are γ-matrices and $a = \sum_{j=0}^{q} \varepsilon_j \neq 0$, then the matrix $B \equiv \frac{1}{a}\sum_{j=0}^{q} \varepsilon_j F^{(j)}$ is a γ-matrix.

Proof : From definition 1, we may easily write

$$\sum_{g=1}^{\infty} |t_{l,g}^{(j)} - t_{l,g+1}^{(j)}| \leq D_j$$

consequently,

$$\sum_{g=1}^{\infty} |b_{l,g} - b_{l,g+1}| \leq \frac{1}{|a|}\sum_{j=0}^{\alpha} |\varepsilon_j|$$

$$\sum_{g=1}^{\infty} |g_{l,g}^{(j)} - g_{l,g+1}^{(j)}| \leq \frac{1}{|a|}\sum_{j=0}^{\alpha} |\varepsilon_j| D_j$$

Thus, B satisfies all the conditions of definition 1.

Definition 2 : The matrix B is as α-mean (i.e. λ-mean) of matrices $F^{(j)}$.

Note (3) : The α-mean (i.e. λ-mean) an infinity of γ-matrices is a γ-provided that

(a) $|f_{l,g}^{(j)}| \leqq D \,\forall j, l$ and g;

$$\sum_{g=1}^{\infty} |f_{l,g}^{(j)} - f_{l,g+1}^{(j)}| \leqq k \,\forall j, l, k;$$

(b) $\sum_{j=0}^{\infty} |a_j| = E$ exists and is finite,

and $\sum_{j=0}^{\infty} \varepsilon_j = a \neq 0.$

Proof : From definition 2,

$$|a| . |b_{l,g}| \leq \sum_{j=0}^{\infty} |\varepsilon_j| . |f_{l,g}^{(j)}| \leq D.E$$

and $$|a| \sum_{g=1}^{\infty} |b_{l,g} - b_{l,g+1}| \leq \sum_{j=0}^{\infty} |\varepsilon_j| \sum_{g=0}^{\infty} |f_{l,g}^{(j)} - f_{l,g+1}^{(j)}|$$

$$\leq E\,K \text{ (using above conditions).}$$

Again, the new series

$$\sum_{j=0}^{\infty} \varepsilon_j f_{l,g}^{(j)} \text{ converges uniformly by definition 2.}$$

Thus, $$\lim_{l \to \infty} b_{l,g} = \frac{1}{a} \lim_{l \to \infty} \sum_{j=0}^{\infty} \varepsilon_j f_{l,g}^{(j)}$$

$$= \frac{1}{a} \sum_{j=0}^{\infty} \varepsilon_j \lim_{l \to \infty} f_{l,g}^{(j)} = 1.$$

Definition 3 : Let $P = (p_{l,g})$ and $R = (r_{l,g})$ are two matrix. Then the new matrix $S = (s_{l,g}) = (p_{l,g} r_{l,g})$ is known as term product of A and B.

Note (4) : The term product of γ-matrix is a γ-matrix.

Proof : We may write

$$S_{l,g} - S_{l,g+1} = p_{l,g}(r_{l,g} - r_{l,g+1}) + r_{l,g+1}(p_{l,g} - p_{l,g+1})$$

$$\Rightarrow \quad \sum_{g=1}^{\infty} |S_{l,g} - S_{l,g+1}| \leq K_1.D_2 + K_2.D_1. \text{ (by def}^{n}\text{.)}$$

Also, $S_{l,g} \to 1$ as $l \to \infty$.

Proof of theorem 1 : If F is a γ_A-matrix then using the result of Sahani and Jha [2021],

$$\lim_{l\to\infty}\left(\sum_{j=1}^{\infty} m_{jg}\right)=\lim_{l\to\infty} f_{l,g}=1 \tag{1.2.2}$$

Also,

$$\begin{aligned}|m_{lg}| &= |f_{lg} - f_{l\text{-}1,g}| \\ &\le |f_{lg}| + |f_{l\text{-}1,g}| \\ &\le k_{l\text{-}1}(F) + k_l(F) \\ &< k_l(F)\end{aligned} \tag{1.2.3}$$

$$\because\ |f_{lg}| < k_l(F) \text{ is independent of } k$$

and $$\sum_{l=1}^{\infty} |m_{lg}| = |f_{lg}| + \sum_{l=2}^{\infty} |f_{lg} - f_{l\text{-}1,g}| < D(F) \tag{1.2.4}$$

Equations (1.2.2), (1.2.3) and (1.2.4) are precise the condition for the matrix M to be an α_A-matrix.

Conversely, if the matrix $M = (m_{lg})$ is an α_A-matrix then by definition

$$m_{lg} = f_{lg} - f_{l\text{-}1,g}\,(l, g \ge 1) \text{ and}$$

$$m_{1g} = f_{1g}\,,\ (g \ge 1) \tag{1.2.5}$$

We have to show that

$$F = (f_{lg}) \text{ is a } \gamma_A\text{-matrix.}$$

It is easy to see from equation (1.2.4) that if

$$\sum_{l=1}^{\infty} |m_{lg}| < D(M) \text{ then}$$

$$\sum_{l=1}^{\infty} |f_{lg} - f_{l\text{-}1,g}| < D(F)$$

Also, using [Sahani, 2021], we may write

$$|f_{lg}| = |m_{1g} + m_{2g} + ... + m_{lg}|$$
$$\leq K_l(F) \tag{1.2.6}$$

Also, by definition,

$$\sum_{j=1}^{l} m_{jg} = m_{lg} + \sum_{j=2}^{l} m_{jg}$$
$$= f_{lg} + \sum_{j=2}^{l} (g_{jg} - g_{j-1,g})$$
$$= f_{lg}$$

Therefore,

$$\lim_{l\to\infty} f_{lg} = \sum_{j=1}^{\infty} m_{jg} = 1 \tag{1.2.7}$$

It is clear that equations (1.2.4), (1.2.5) and (1.2.6) show that the matrix $F = (f_{lg})$ in (1.2.4) is a γ_A-matrix.

This completes the proof of the theorems 1.

Proof of theorem 2 :

$\because F = (f_{lg})$ is a γ_A-matrix.

If the series $\sum V_j$ is convergent then F transform of $\sum V_j$, namely $\sum_{j=1}^{\infty} F_{li} V_i$ exists for all l and is a sequence by bounded variation (by above notes and [Sahani, 2021]).

Also, $\lim_{l\to\infty} \sum_{j=1}^{\infty} f_{li} V_i = \sum_{j=1}^{\infty} V_i$, we choose $v_i = m_{ig}$, where m_{ig} are the elements of α_A-matrix M.

Hence, by [Sahani, 2021],

$$\sum_{j=1}^{\infty} V_i = \sum_{j=1}^{\infty} m_{ig} = 1, \text{ for all } g.$$

Also, for an α_A-matrix M,

$$\sum_{j=1}^{\infty} |m_{ig}| < D(M).$$

Thus, by definition 3,

If $S = FM$ then

$$|S_{lg}| = \left|\sum_{i=1}^{\infty} f_{li} m_{ig}\right|$$

$$\leq \sum_{j=1}^{\infty} |f_{li}||m_{ig}|$$

$$< K_l(F)D(M)$$

$$< K_l(S) \tag{1.2.8}$$

Thus, the product matrix $S = (s_{lg})$ exists for all l and g.

$$\lim_{l\to\infty} S_{lg} = 1 \tag{1.2.9}$$

$$\sum_{l=2}^{\infty} |s_{lg} - s_{l-1,g}| = \sum_{l=2}^{\infty} \left| \sum_{i=1}^{\infty} (f_{li} - f_{l-1,j}) m_{jg} \right|$$

$$< \sum_{l=2}^{\infty} \sum_{i=1}^{\infty} |f_{li} - f_{l-1,i}||m_{ig}|$$

$$< D(H) \tag{1.2.10}$$

The equations (1.2.8), (1.2.9) and (1.2.10) show that $S = FM$ is a γ_A-matrix.

Proof of theorem 3 :

$$\text{We define } m_{lg} \begin{array}{l} = 1 \text{ for } l = 1, g = 1 \\ = 0 \text{ for } l > 1 \end{array} \Bigg\}$$

and

$$f_{lg} = 1 \quad l, g \geq 1 \tag{1.2.11}$$

These matrices M and F defined in (1.2.10) and (1.2.11) α_A and γ_A-matrices respectively.

Thus, $(FM)_{lg} = (F)_{lg}$ exists and is the γ_A-matrix F of equation (1.2.11).

$$\text{But } (MF)_{lg} = \left(\sum_{i=1}^{\infty} m_{li} f_{ig} \right)$$

$$= \sum_{i=1}^{\infty} (1+1+...)_{lg} \text{ does not exist.}$$

This completes the proof of the theorem.

Theorem 4 and sufficiency of the condition in theorem (5) follow by combining theorem 2 and theorem 3.

To prove theorem 5, we consider a γ_A-matrix F which is defined as

$$f_{lg} \begin{matrix} = 1 \text{ for } g \leq 1 \\ = 0 \text{ for } g > l \end{matrix} \Bigg\} \tag{1.2.12}$$

Then the product matrix $S = (FM)$ is

$$s_{lg} = \sum_{i=1}^{\infty} f_{li} m_{ig}$$

$$= \sum_{i=1}^{\infty} m_{ig} \tag{1.2.13}$$

hence, by theorem 1, the matrix $S = (s_{lg})$ in equation (1.2.13) is γ_A-matrix, only if F is an α_A-matrix.

Proof of theorem 6 :

Let P and Q are two α_A-matrices.

We define a new matrix

$F = (f_{lg})$ which is defined in the following way :

$$f_{lg} = p_{1g} + p_{2g} + ... + p_{lg} \ (l, g \geq 1) \tag{1.2.14}$$

By our theorem 1, (f_{lg}) is a γ_A-matrix and by definition 3, the product $(S)_{lg} = (FQ)_{lg}$ is a γ_A-matrix.

If we define

$$e_{lg} = s_{lg} - s_{l-1,g} \; (l > 1, g \geq 1) \tag{1.2.15}$$

$$\Rightarrow \; e_{lg} = s_{lg}$$

Then $E = (e_{lg})$ is an γ_A-matrix and $e_{lg} = \sum_{i=1}^{\infty} f_{li} q_{ig} - \sum_{j=1}^{\infty} f_{l-1,i}$

$$= \sum_{i=1}^{\infty} (f_{li} - f_{l-1,i}).q_{ig} = \sum_{j=1}^{\infty} p_{li}.q_{ig}$$

$$\because (E)_{lg} = (PQ)_{lg}$$

By the our assumption (1.2.15) of the matrix F.

This completes the proof of the theorem 6.

Proof of the theorem 7 :

We consider the γ_A-matrix $F = (F_{lg})$ defined in equation (1.2.15) and another γ_A-matrix $S = (s_{lg})$ in the following way :

$$S_{lg} = 1 \, \forall \, l, g \geq 1 \tag{1.2.16}$$

The product matrix $T = FS$ of the two γ_A-matrices is given by

$$t_{lg} = \sum_{i=1}^{\infty} f_{li} s_{ig}$$

$$= \sum_{i=1}^{\infty} f_{li} q_{ig} = l$$

$\because \; \lim_{l \to \infty} t_{lg} = \infty$ which doesnot equal to a multiplicative identity $\forall \; g$ and for each l.

$\Rightarrow$ T is not a γ_A-matrix.

This completes the proof of the theorem 7.

Conclusion :

The means of Cesàro, Rieŝz, Borel, Lindelöf, Mittag-Leffler are particular cases of transformation of a sequence. The three methods sequence to sequence, series to sequence, series to series of defining generalized limits by infinite matrices. The Silverman-Treplitz theorem giving sufficient and necessary conditions for the matrix $F = (f_{lg})$ to sum every convergent series to, its correct sum. The theorem similar to Silverman-Treplitz has been established by Carmichael, Perron and Bosanquet (see [Vermas, 1946]). In this note we have proved that the matrices defining convergence preserving sequence to series transformations form a Banach algebra under suitable norm. The means of Cesàro, Rieŝz, Borel, Lindelöf, Mittag-Leffler are particular cases of transformation of a sequence. The three methods sequence to sequence, series to sequence, series to series of defining generalized limits by infinite matrices. In this note, we study a new class of matrices giving conservative transformations is sequence to series, together with its regular sub-class in pseuto-algebra of γ-matrices by introducing the new-mean and the term product .

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