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Lattice QCD developments in flavor physics

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Abstract

The recent progress in the lattice QCD studies of heavy flavored multiquark states is reviewed. The rapid development of numerical techniques permits the investigation of the nature of newly observed exotic hadrons and the underlying dynamics through the hadron-hadron scatterings on the lattice. We focus on the recent lattice QCD calculations on $T_{cc}^+(3875)$ and its bottomed counterpart T_{bb} , P_c relevant $\Sigma_c D(D^*)$ scatterings, etc. We also introduce the lattice QCD results on charmoniumlike bound states and resonances.

1 Introduction

The lattice QCD study on flavor physics has two categories. The first category is the precise calculation of Standard Model (SM) parameters and hadronic matrix elements. This sector provides the indispensable nonperturbative QCD inputs and serves the precision test of the Standard Model (SM) and the search for new physics beyond SM. The light flavored quantities includes light quark masses, the decay constants of pion and kaon, the $K \rightarrow \pi$ semileptonic decay form factors, the neutral kaon mixing factor, etc. The heavy flavored quantities are related to leptonic and semileptonic D - and B - meson decays and mixings. These light and heavy flavored quantities are very relevant to the determination of CKM matrix elements and the CKM unitarity test. There are also many other hadronic matrix elements relevant for new physics search have been calculated to a high precision in lattice QCD. The cutting edge development in this sector is that, for many precisely determined quantities, the isospin breaking and QED effects have been taken into account. The topics in this sector extend to a very broad scope, and we refer the readers to the comprehensive review article by Flavour Lattice Averaging Group (FLAG) [1].

The other category of the lattice QCD developments in flavor physics is heavy flavored hadron spectroscopy. Ever since the discovery of $X(3872)$, a large number of charmoniumlike structures, called XYZ particles, have been observed by various experiments [2, 3]. Apart from charmonium-like states, LHCb observes several P_c states in $J/\psi p$ final states and a doubly charmed structure $T_{cc}^+(3875)$. Almost all of these particles are close to the open-charm (bottom) thresholds, and are good candidates for hadron molecules and their properties are worthy of investigations in depth. Lattice QCD plays an important role in the spectroscopic study of these new particles. The state-of-art lattice approach is the study of the related meson-meson(baryon) scatterings. The major consideration behind is that, for a given quantum number, the finite volume energy levels E_n of the two-hadron system are connected with the scattering amplitude in the Minkowski space-time through the Lellouch-Lüscher's quantization condition $\det[F^{-1}(E_n, L, \mathbf{P} + \mathcal{M}(E_n))] = 0$ (see Ref. [4] for a review), where \mathbf{P} is the center-of-mass momentum of the system, L is the spatial extension of the lattice, $F^{-1}(E, L, \mathbf{P})$ is a mathematically known function, and $\mathcal{M}(E)$ is the scattering amplitude. For a proper parametrization of $\mathcal{M}(E)$, one can obtain the explicit expression of $\mathcal{M}(E)$ from E_n by data analysis, whose pole singularities shed lights on the nature of the state considered. So the key task of LQCD calculation is to extract E_n as precisely as possible. In doing so, one first builds a large enough interpolating operators set and calculates their correlation matrix, then solve the generalized eigenvalue problem to it to derive E_n . For a S -wave ($l = 0$) single-channel scattering, the scattering amplitude is expressed in terms of the phase shift δ_0 as $t_0 \propto (p \cot \delta_0(p) - ip)^{-1}$ with $p = p(E)$ being the scattering momentum. For low energy scatterings, the effective range expansion (ERE) gives

$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 p^2. \quad (1)$$

In the following, we will focus on the recent lattice QCD studies on heavy flavored multiquark states, such as $T_{cc}^+(3875)$ and its bottomed counterparts, P_c states, heavy flavored dibaryons and some charmoniumlike resonances.

2 Heavy flavored multiquark states

2.1 Lattice studies of $T_{cc}^+(3875)$

In 2021, LHCb observed the first doubly charmed $T_{cc}^+(3875)$ with a minimal quark configuration is $cc\bar{u}\bar{d}$. Many phenomenological studies interpret it as a $D^0 D^{*+}$ molecule (see Ref. [3] for a review). There are also a series of lattice QCD studies on T_{cc} that give consistent results. In Ref. [5], the finite volume energies are calculated for the $I = 0$ S -wave DD^* system, from which

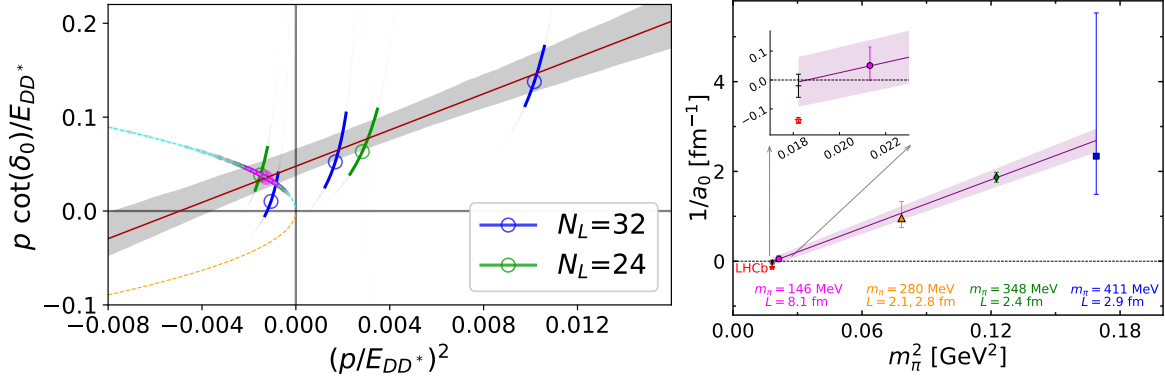


Figure 1: **Left panel:** The S -wave $DD^*(I=0)$ scattering phase with respect to the scattering momentum p [5]. The grey band illustrate the ERE of $p \cot \delta_0 = 1/a_0 + r_0 p^2/2$, while the cyan/orange lines indicate $ip = \mp|p|$. The intersection point signals a virtual bound state. **Right panel:** The chiral extrapolation of the inverse scattering length $1/a_0$ [6]. The pink, yellow, green, and blue points are the results of Ref. [6], [5], [7] and [8], respectively. The value at the physical m_π is consistent with that of a bound state.

the scattering phase shift δ_0 is derived, and is plotted versus the scattering momentum squared p^2 in the left panel of Fig. 1. The intersection point (pink point) satisfies the pole condition of the scattering amplitude $p \cot \delta_0 = ip$ with $p = -i|p|$ and signals a virtual bound state pole. It is possible that this virtual state can develop into a bound state when the u, d quark mass decreases. The right panel of Fig. 1 indicates this tendency [6], where the data points in pink, yellow, green, and blue are the results of $1/a_0$ from Ref. [6], [5], [7] and [8], respectively. The value at the physical m_π is consistent with that of a bound state. The isospin dependence of the DD^* interaction [7] is discussed in Ref. [7], where the contribution of the different components of the correlation functions is analyzed. It is turned out that the attractive (repulsive) interaction in the $I=0$ ($I=1$) channel is mainly due to the charged vector meson ρ^\pm exchange on the hadron level. This study provides a new aspect to explore the hadron-hadron interactions and the result is in qualitative agreement with phenomenological studies [9, 10]. Note that the lattice study in Ref. [6] takes the HAL QCD method [11] to calculate the Nambu-Bethe-Salpeter wavefunction of the $I=0$ DD^* at a pion mass $m_\pi = 146.4$ MeV and then to extract the DD^* interaction potential, through which the scattering phase shift is obtained by solving the Schrödinger equation. To summarize, the existing lattice QCD results relevant to $T_{cc}^+(3875)$ are consistent with each other, and support the existence of a DD^* bound state.

2.2 Doubly bottomed counterpart of $T_{cc}^+(3875)$

It is intriguing if there exist bottomed counterpart T_{bb} of T_{cc} . The potential between two bottom mesons (BB) was first studied in the static quark limit [12, 13]. Let r_{bb} be the distance between the two b quarks in the BB system, the r_{bb} dependence of the BB energy is interpreted as the BB potential. Then the binding energies can be obtained by solving the related Schrödinger equation. A bound state is observed to exist in the $I(J^P) = 0(1^+)$ channel with a binding energy $E_B = -90^{+43}_{-36}$ MeV, while no binding is observed in the $I(J^P) = 1(1^+)$ channel [12]. When the heavy spin and the $BB^* - B^*B^*$ coupling are considered, the $I(J^P) = 0(1^+)$ bound state still exists but the binding energy is lowered to be $E_B = -59^{+30}_{-38}$ MeV [13]. A later study also observes a $\bar{b}b u d$ tetraquark bound state in the $I(J^P) = 0(0^+)$ channel [14]. Recently, the HAL QCD method is applied to the study of the $BB^* - B^*B^*$ coupled channel potential in the $I(J^P) = 0(1^+)$ channel at different quark masses [15]. The solution of the Lippmann-Schwinger equation with the derived potential turns out that there exists a $\bar{b}b u d$ bound state at each pion mass. The chirally extrapolated binding energy is determined to be $E_B^{\text{single}} = -154.8 \pm 17.2$ MeV

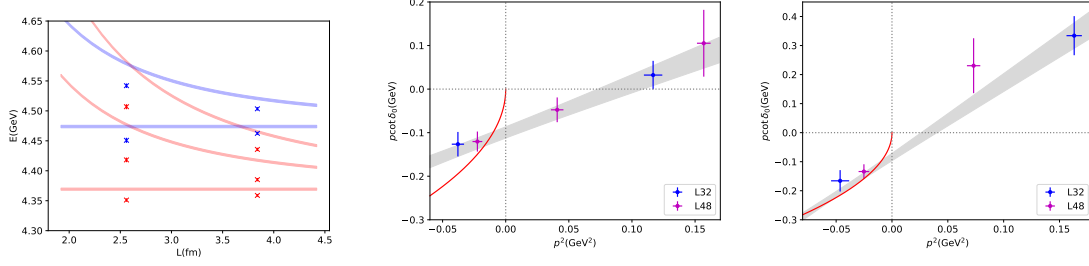


Figure 2: **Left panel:** The energy levels of S -wave $\Sigma_c D$ (red points) and $\Sigma_c D^*$ (blue points) scatterings on two lattices. **Middle panel:** The $\Sigma_c D$ scattering phase $p \cot \delta_0$ vs. scattering momentum p^2 . The red curve illustrates the bound state condition $ip = -|p|$, while the grey band shows the ERE of $p \cot \delta_0$ in Eq. (1). The intersection point indicates a bound state pole. **Right panel:** Similar to the middle panel, but for the $\Sigma_c D^*$ scattering.

for a single BB^* channel and $E_B^{\text{coupled}} = -83.0 \pm 10.2$ MeV for the $BB^* - B^*B^*$ coupled channel case. The lattice studies using the Lellouch-Lüscher formalism can be found in Refs. [16, 17], where E_B at the physical m_π is determined to be $E_B = -128 \pm 24 \pm 10$ MeV for the $0(1^+) \bar{b}b u d$ system and $E_B = -86 \pm 22 \pm 10$ MeV for $J^P = 1^+ \bar{b}b u s$ tetraquark. Obviously, all the existing lattice QCD studies support the existence of a deeply bound $T_{bb}^+(0(1^+))$ state.

2.3 P_c states and $\Sigma_c D(D^*)$ scattering

The P_c states $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, and $P_c(4457)$ are observed in the $J/\psi p$ system, therefore their minimal quark configuration must be $uudc\bar{c}$. In phenomenological studies, they are interpreted as hadron molecules made up of a charmed baryon and a $\bar{D}^{(*)}$ meson (see Ref. [3] for a review). Recently, the P_c relevant $\Sigma_c D$ and $\Sigma_c D^*$ scatterings are explored in lattice QCD [18]. The energy levels on two lattice volumes are shown in the left panel in Fig. 2. The phase shifts of the $\Sigma_c D$ and $\Sigma_c D^*$ scatterings are plotted with respect to p^2 (the grey band illustrates the ERE fit) in the middle and right panels, respectively, where the red curves indicate the bound state condition $p = i|p|$. The intersection of the red curve and the grey band indicates the existence of a bound state in either the $\Sigma_c D$ or $\Sigma_c D^*$ scattering. By solving the pole equation of the scattering amplitude, the binding energy E_B is determined to be $-6(2)(2)$ MeV for $\Sigma_c D$ system and $-7(3)(1)$ MeV for $\Sigma_c D^*$ system, respectively. Note that, the $J/\psi p - \Sigma_c D(D^*)$ coupled channel effect has not been considered yet in this study. This coupled channel effect could be important, since P_c states are observed in the $J/\psi p$ invariant mass spectrum.

2.4 Dibaryon $\Omega_{bbb}\Omega_{bbb}$ from lattice QCD

It is expected there exist two-baryon bound states (dibaryons) other than deuteron in nature. Fully heavy $\Omega_{QQQ}\Omega_{QQQ}$ systems with $Q = b, c$ provide unique probes to the baryon-baryon interaction insensitive to the chiral dynamics. The HAL QCD method has been applied to study the interaction potential between two Ω_{ccc} baryons and a weakly attractive interaction is observed [19]. However, the authors claim that this interaction is attractive enough to form a shallow 1S_0 bound state of $E_B \approx -6$ MeV. In view of lattice results that T_{bb} is bound much more deeply than T_{cc} , it is interesting to check if this also the case for the fully heavy dibaryons. Ref. [20] reports the lattice QCD investigation of the ground state of fully bottomed dibaryon formed by two Ω_{bbb} baryons. A deeply bound $\Omega_{bbb}\Omega_{bbb}$ dibaryon in the 1S_0 channel is clearly observed with a binding energy $E_B = -81_{-16}^{+14}$ MeV. It could be the most deeply bound heaviest dibaryon in nature.

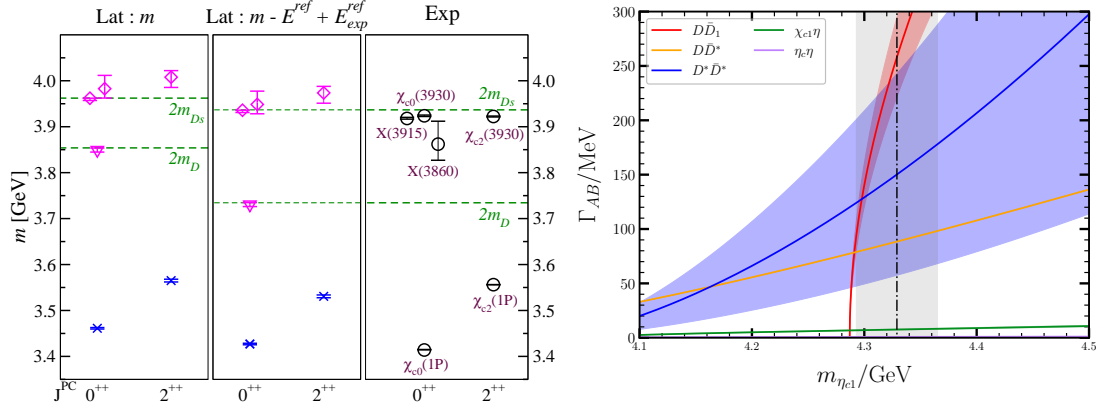


Figure 3: **Left panel:** Energy spectrum of $(0, 2)^{++}$ charmoniumlike system from lattice QCD compared with experiment [21]. The energies in the middle plot are shifted from those in the left plot by $E_{exp}^{ref} - E^{ref}$ where E^{ref} is $2m_D(2m_{D_s})$ for pink points and the $1S$ spin averaged mass value of charmonium. **Right panel:** The $m_{\eta_{c1}}$ dependence of the partial decay widths [22]. The vertical dashed line and the grey band illustrate the lattice result of $m_{\eta_{c1}}$ with errors, and the colored bands show the partial decay widths varying with respect to $m_{\eta_{c1}}$ in the range from 4.1 to 4.5 GeV.

3 Charmonium(like) states and their decays

3.1 Charmoniumlike resonances in couple $D\bar{D}$ and $D_s\bar{D}_s$ scatterings

Regarding to $\chi_{c0}(3860)$, $\chi_{c2}(3930)$ and $X(3915)$, the coupled $D\bar{D}$ and $D_s\bar{D}_s$ scattering is investigated in the $I^G J^{PC} = 0^+(0, 2)^{++}$ channels [21]. In the energy region below the $D_s\bar{D}_s$ threshold, the single channel analysis of S -wave $D\bar{D}$ scattering results in a bound state of binding energy $E_B = -4.0^{+3.7}_{-5.0}$ MeV. This state is much higher than the conventional $\chi_{c0}(1P)$ state but lower than the $\chi_{c0}(2P)$ conventional charmonium. Experiments have not claimed this state so far. The single channel analysis of the S -wave $D_s\bar{D}_s$ scattering also observes a bound state $\chi_{c0}^{D_s\bar{D}_s}$ of a $E_B = -6.2^{+3.8}_{-2.0}$ MeV. When S -wave $D\bar{D}$ and $D_s\bar{D}_s$ scatterings are considered, two resonances are observed after a the coupled channel analysis in the energy region near the $D_s\bar{D}_s$ threshold. The broader resonance has a width of 58^{+6}_{-11} MeV and couples predominantly to $D\bar{D}$. This resonance may be related to the $\chi_{c0}(3860)$. The narrow 0^{++} resonance has a mass very close to the $D_s\bar{D}_s$ threshold and has a very small width $\Gamma = 0.27^{+2.50}_{-0.15}$ MeV. This state couples strongly to $D_s\bar{D}_s$ but very weakly to $D\bar{D}$. This state is understood as the $\chi_{c0}^{D_s\bar{D}_s}$ mentioned above, which acquires a small width due to the weak coupling to $D\bar{D}$. This state may be related to $X(3930)$ and the $X(3915)$ which lie near the $D_s\bar{D}_s$ threshold and have tiny decay rate to $D\bar{D}$. On the other hand, a single channel analysis of the D -wave $D_s\bar{D}_s$ scattering in this study comes out with a 2^{++} resonance of a mass higher than the $1S$ spin averaged mass 3.069 GeV by 905^{+14}_{-22} MeV and a width $\Gamma = 64^{+32}_{-42}$ MeV. This resonance likely corresponds to $\chi_{c2}(3930)$.

3.2 Decays of charmoniumlike 1^{-+} hybrid η_{c1}

QCD permits the existence of hybrid mesons of a $q\bar{q}g$ configuration. There exist experimental candidates for light hybrids, such as the isovector $\pi_1(1600)$ and the isoscalar $\eta_1(1855)$. So the charmoniumlike counterpart η_{c1} of η_1 should also exist. According to the lattice predictions $m_{\eta_{c1}} \sim 4.2-4.4$ GeV, η_{c1} can have the two-body decay modes $AB = D_1\bar{D}$, $D^*\bar{D}$, $D^*\bar{D}^*$, $\chi_{c1}\eta(\eta')$, $\eta_c\eta(\eta')$, and $J/\psi\omega(\phi)$. These decay modes of η_{c1} are explored in Ref. [22] using the Michael-McNeile method [23]. The lattice transition amplitude $x_{AB} = \langle AB | H_I | \eta_{c1} \rangle$ is extracted from the correlation function $\langle \mathcal{O}_{AB}(t) \mathcal{O}_{\eta_{c1}}^\dagger(0) \rangle$, where \mathcal{O}_{AB} and $\mathcal{O}_{\eta_{c1}}$ are the interpolating operators for AB and η_{c1} , respectively. From the amplitude x_{AB} one can derive the effective coupling g_{AB} ,

which can be used to predict the partial decay width $\Gamma(\eta_{c1} \rightarrow AB)$. For $m_{\eta_{c1}} = 4329(36)$ MeV determined in this work, the three largest partial decay widths are 258(133) MeV, 150(118) MeV, and 88(18) MeV for the decay modes $D_1\bar{D}$, $D^*\bar{D}^*$ and $D^*\bar{D}$, respectively. These values add up to a too large width ~ 500 MeV for η_{c1} to be detected in experiments. Actually, these partial widths of η_{c1} are very sensitive to $m_{\eta_{c1}}$, as shown in the right panel of Fig. 3. If η_{c1} has a lower mass around 4.2 GeV, then the total width will be around 100 MeV and the dominant decay modes will be $D^*\bar{D}$ and $D^*\bar{D}^*$. Especially for $D^*\bar{D}^*$, the measurement of the polarization of $D^*\bar{D}^*$ will help distinguish a 1^{-+} (the total spin of $D^*\bar{D}^*$ is $S = 1$) state from 1^{--} ($S = 0, 2$) states. It is suggested that LHCb, Belle II and BESIII to search for η_{c1} in $D^*\bar{D}$ and $D^*\bar{D}^*$ systems.

4 Summary and outlook

Lattice QCD makes a rapid progress and plays an indispensable role in the study of heavy flavored hadron spectroscopy. The numerical technique for the calculation of hadron-hadron scatterings on the lattice is becoming mature and is applied extensively to study of hadron resonances and hadronic molecules. There have been many impressive results obtained from lattice QCD on the properties of multiquark states recently, which shed lights on the nature of exotic hadron states observed in experiments. More interesting results are underway.

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