THE LANDSCAPE OF QUANTUM SENSING

LEGEND

- Involves Heisenberg scaling
- Involves Machine Learning / Variational approaches
- Well understood theoretically & experimentally accessible
- Well understood theoretically
- Experimentally accessible

QUANTUM ERROR CORRECTION

- By correcting for dominant errors, one can extend the lifetime of the system & achieve a sensing advantage & potentially recover Heisenberg limit [37]
- Works when QEC protocol faster than noise rate, which is more feasible & can correct for high frequency noise, which dynamical decoupling can't. [38]
- Usually performed with ancilla modes. The new quantum resource are the fidelities of the QEC protocol & these determine the timescales where Heisenberg scaling extends till.
- Hamilton-not-in-Lindblad span (HNLS): As long as the sensing Hamiltonian H is not in span $\in \{1, L_i, L_j^{\dagger}, L_i^{\dagger}L_j\}$ where L_i are noise jump processes, then one can achieve Heisenberg scaling [39]

MAXIMAL ENTANGLEMENT

- Cononical sensing task of phase: Ramsey intereferometry [1-3]
- Heisenberg scaling in number of modes
  GHZ state $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle^{\otimes N} + |1\rangle^{\otimes N}]$.
  The scaling can cease due to the noise also being enhanced by N
- Transducers allow for sensing of physical parameters
  - Magnetic Field: Zeeman shift of NV centers, Flux of SQUIDS
  - Electric Field: Pockels effect imprinting phase on light
  - Dominated by 1/f noise & hence not quantum limited

CRAMER-RAO & FISHER INFO

- Provides a bound on maximum information obtainable by a sensor. Based on locally unbiased measurements.

$$\underbrace{\Delta^2\Theta_N}_{\text{Estimated Variance}} \geq 1/\underbrace{N}_{\text{Samples}}\underbrace{F(P_\Theta)}_{\text{Fisher Info}}$$

- The Fisher Info is based on a given measurement protocol & bounds are obtained by optimizing over all possible protocols

$$F(\underbrace{P}_{\text{probability}}{}_{\underbrace{\Theta}_{\text{sensing parameter}}}) = \sum_{x \in \Omega} \frac{1}{P_\Theta(x)}\left[\frac{\partial P_\Theta(x)}{\partial \Theta}\right]^2$$

- Maximum possible Fisher Info can be analytically derived for simple tasks such as phase sensing, but needs to be numerically optimized for multi-parameter scenarios.

HEISENBERG LIMIT

- In the standard quantum limit (SQL), the error in prediction scales via the central limit theorem. Therefore given N qubits all unentangled, we have $\sigma \sim 1/\sqrt{N}$. Given one qubit for interrogation for time T, we have $\sigma \sim 1/\sqrt{T}$.
- In Heisenberg scaling, we go beyond this regime; only limited by quantum mechanics /uncertainty principle. We obtain $\sigma \sim 1/N$ or $\sim 1/T$.

MULTI-PARAMETER SENSING

- This involves performing sensing based on more than one variable [27-28]
- For commuting variables, one can achieve Heisenberg scaling and a common sensing protocol works for all
- Can use variational methods [29]
- For non-commuting variables, a general protocol doesn't exist, but bounds based on the Holevo Cramer Rao bound found by numerically optimizing the Quantum Fisher Information matrix. While usually a convex optimization problem, can be computationally intractable to optimize over protocols unless for specific symmetries/conserved quantities in the system. [30,31]

NON-HERMITIAN SYSTEMS

- Non hermitian physics can be explored by introducing gain & loss; both in classical & quantum contexts.
- Such methods introduce bifurcation points in spectrum, sensors operating around this regime will be more sensitive to parameters causing perturbations to the eigenvalues [18]
- Topological systems: PT symmetry can be embedded in dynamics of Hermitian systems. Truly a quantum setting
  - Can show exponential enhancement in sensitivity over modes [19]

DISPLACEMENT SENSING

- Single Mode: since position & momentum don't commute; there is no Heisenberg scaling. Optimal states include compass states, GKP [10]
  For single axis displacement, squeezed modes give Heisenberg scaling in squeezing strength [11]
- Two Mode: squeezing in orthogonal directions for each modes gives Heisenberg scaling since the displacement on different modes commute
- Dark Matter Detection: An application for displacement sensing where the displacement corresponds to a dark matter mode. Figure of merits include scan rate & dynamic range [12,13]

DISTRIBUTED QUANTUM SENSING

- The goal is to estimate a global property of multiple parameters [22]
- Useful when there are classical correlations in signal
- Applications in dark matter detection with multiple entangled sensors [12]
- SLEAN (Supervised Learning Assisted by an Entangled Sensor Network): Machine learning task such as classification and regression based on optimizing variationally. [23,24]

VARIATIONAL METHODS

- The encoding & decoding layers can be parameterized which are numerically optimized
- Optimization performed close loop with parameter update performed by classical computers (hybrid methods)
  - Takes into account native operations in hardware
  - Robust to device specific noise processes
- Goal to reduce user defined cost function
  - Can take into account finite shot analysis
  - Can take into account dynamic range / apriori distribution [20,21]

RADIO-FREQUENCY SENSING

- Here the goal is to sense properties of AC signals eg amplitude & frequency [1,14-16]
- Echo sequences & Carr Purcell multipulse sequence can be used to modify where the system is most sensitive.
- Such methods are also used for dynamical decoupling to reduce the effect of slow varying noise and in characterizing T2*
- Originally developed in NMR settings
- Cryogenic LNA: by using higher Fock states of qudit can help characterize AC signals inside the fridge: figure of merit include sensitivity & dynamic range [17]

HAMILTONIAN ENGINEERING

- Time dependent sensing: sense a single parameter describing a time dependent signal. With time dependent control one can show "super-heisenberg" scaling of $1/T^2$ [7-9]
- Dephasing noise (which commutes with Hamiltonian) cannot be error-corrector. This can destroy Heisenberg scaling. [40,41]
- Hamiltonian engineering, based on pulse control, interactions, disorder can enhance sensitivity [42-44]

BAYESIAN METROLOGY

- Beyond Cramer-Rao bounds in a finetly sampled regime [20,21]
- Amenable for adaptive methods which take into account current knowledge
- Probably Approximately correct metrology [47]

JOINT-DETECTION RECEIVERS

- Using entanglement can help increase sensitivity in detecting classical signals
- Idea of quantum belief propagation [48-50]

ENTANGLEMENT-FREE

- Single/few mode sensors [4,5]
- Heisenberg scaling for phase sensing
  - 1/T: Kitaev's phase estimation
  - 1/N: single mode bosonic states
    $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |N\rangle]$
    : NOON states $|\psi\rangle = \frac{1}{\sqrt{2}}[|N0\rangle + |0N\rangle]$
  - Sensitive to noise
- Adaptive methods which update protocols based on measurement history
- Classical correlations among sensors to infer higher order moments of sensing signal: eg in NV centers [6]

QUANTUM RECEIVER

- The goal is to have a scaling advantage wrt "signal strength" [32]
- Limited by Helstrom bound for state discrimination. Given discriminating $|\psi_1\rangle, |\psi_2\rangle$ received with probability η_1, η_2 the upper bound in probability is:
  $P = \frac{1}{2}\left[1 - \sqrt{1 - 4\eta_1\eta_2 |\langle\psi_1|\psi_2\rangle|^2}\right]$ [33]
- Can be combined with adaptive methods & ML approaches: Quantum Receiver enhanced by adaptive learning (QREAL) [34]
- Can be used in a variational context and applied generally to discrimination of quantum phases of matter [45,46]

ADAPTIVE METHODS

- These are methods which determine the next sensing protocol based on measurement history [35]
- Can achieve Beyond SQL scaling
- RL based approaches can outperform analytical solutions & constrain policy search based on Markovian assumptions (to avoid exponentially increasing search space) [36]

RESERVOIR SENSING

- The goal is to use a configuration of entangled modes as a reservoir for sensing advantage
- This can be useful for the decoding layer whose goal is to extract high dimensional features from sensing state
  - Quantum Neuromorphic platform [25]
- Theory of expressive capacity of systems at finite sampling
- Hierarchy of eigenfunctions based on noise resilience
- Expressive capacity sample for all L-qubit states at infinite samples $= 2^L$
- Expressive capacity usually higher for entangled states compared to product states at finite sampling [26]

SENSOR COMPLEXITY (ie the difficulty of realizing experimentally)

TASK COMPLEXITY (ie the complexity of parameters/task to be sensed)

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References

[1] CL Degen et al "Quantum Sensing" Rev. Mod. Phys. 89, 035002 https://doi.org/10.1103/RevModPhys.89.035002
[2] J Barry et al "Sensitivity optimization for NV-diamond magnetometry" Rev. Mod. Phys. 92, 015004 https://doi.org/10.1103/RevModPhys.92.015004
[3] V Giovannetti et al "Quantum Metrology" PRL. 96, 010401 https://doi.org/10.1103/PhysRevLett.96.010401
[4] D Braun et al "Quantum-enhanced measurements without entanglement" Rev. Mod. Phys. 90, 035006 https://doi.org/10.1103/RevModPhys.90.035006
[5] BL Higgins et al "Entanglement-free Heisenberg-limited phase estimation" Nature 450 393-396 https://doi.org/10.1038/nature06257
[6] J Rovny et al "Nanoscale covariance magnetometry with diamond quantum sensors" Science Vol 378 Issue 6626 https://doi.org/10.1126/science.ade9858
[7] M Naghiloo et al "Achieving Optimal Quantum Acceleration of Frequency Estimation Using Adaptive Coherent Control" PRL 119, 180801https://doi.org/10.1103/PhysRevLett.119.180801
[8] Zhibo Hou et al ""Super-Heisenberg" and Heisenberg Scalings Achieved Simultaneously in the Estimation of a Rotating Field" PRL 126, 070503 https://doi.org/10.1103/PhysRevLett.126.070503
[9] LE de Clerq et al "Estimation of a general time-dependent Hamiltonian for a single qubit" Nature Communications volume 7, Article number: 11218 https://doi.org/10.1038/ncomms11218
[10] K Duivenvoorden et al "Single-mode displacement sensor" PRA 95, 012305 https://doi.org/10.1103/PhysRevA.95.012305
[11] L Maccone et al. "Squeezing metrology: a unified framework" Quantum 4, 292 https://doi.org/10.22331/q-2020-07-09-292
[12] A J. Brady et al. "Entangled Sensor-Networks for Dark-Matter Searches" PRX Quantum 3, 030333 https://doi.org/10.1103/PRXQuantum.3.030333
[13] K M Backes. et al "A quantum enhanced search for dark matter axions" Nature volume 590, pages 238–242 https://doi.org/10.1038/s41586-021-03226-7
[14] S Danilin et al "Quantum sensing with superconducting circuits" arXiv https://doi.org/10.48550/arXiv.2103.11022
[15] J Zhang et al "Single NV Centers as Sensors for Radio-Frequency Fields" https://doi.org/10.48550/arXiv.2207.05474
[16] S Magaletti et al "A quantum radio frequency signal analyzer based on nitrogen vacancy centers in diamond" Communications Engineering volume 1, Article number: 19 (2022) https://doi.org/10.1038/s44172-022-00017-4
[17] M Kristen et al "Amplitude and frequency sensing of microwave fields with a superconducting transmon qudit" npj Quantum Information volume 6, Article number: 57 (2020) https://doi.org/10.1038/s41534-020-00287-w
[18] YX Wang et al. "Non-Hermitian dynamics without dissipation in quantum systems" PRA 99, 063834 https://doi.org/10.1103/PhysRevA.99.063834
[19] HK Lau et al "Fundamental limits and non-reciprocal approaches in non-Hermitian quantum sensing" Nature Communications volume 9, Article number: 4320 (2018) https://doi.org/10.1038/s41467-018-06477-7
[20] R Kaubruegger et al. "Quantum Variational Optimization of Ramsey Interferometry and Atomic Clocks" Phys. Rev. X 11, 041045 https://doi.org/10.1103/PhysRevX.11.041045
[21] C Marciniak et al "Optimal metrology with programmable quantum sensors" Nature volume 603, pages 604–609 (2022) https://doi.org/10.1038/s41586-022-04435-4
[22] Q Zhuang et al. "Distributed quantum sensing enhanced by continuous-variable error correction" New Journal of Physics, Volume 22, February 2020 https://doi.org/10.1088/1367-2630/ab7257
[23] Q Zhuang et al. "Physical-Layer Supervised Learning Assisted by an Entangled Sensor Network" PRX 9, 041023 https://doi.org/10.1103/PhysRevX.9.041023
[24] Y Xia et al. "Quantum-Enhanced Data Classification with a Variational Entangled Sensor Network" PRX 11, 021047 https://doi.org/10.1103/PhysRevX.11.021047
[25] T Krisnanda et al. "Beating the classical phase precision limit using a quantum neuromorphic platform" PRA 18, 034011 (2022) https://doi.org/10.1103/PhysRevApplied.18.034011
[26] F Hu et al. "Fundamental Limits to Expressive Capacity of Finitely Sampled Qubit-Based Systems" arXiv https://doi.org/10.48550/arXiv.2301.00042
[27] P Humphreys et al. "Quantum Enhanced Multiple Phase Estimation" PRL 111, 070403 https://doi.org/10.1103/PhysRevLett.111.070403
[28] T Proctor et al. "Multiparameter Estimation in Networked Quantum Sensors" PRL 120, 080501 https://doi.org/10.1103/PhysRevLett.120.080501
[29] R Kaunbruegger et al. "Optimal and Variational Multi-Parameter Quantum Metrology and Vector Field Sensing" arXiv https://doi.org/10.48550/arXiv.2302.07785
[30] R Demkowicz-Dobrzański et al. "Multi-parameter estimation beyond quantum Fisher information" J. Phys. A: Math. Theor. 53 363001 https://doi.org/10.1088/1751-8121/ab8ef3
[31] F Albarelli et al. "Evaluating the Holevo Cramér-Rao Bound for Multiparameter Quantum Metrology" PRL 123, 200503 https://doi.org/10.1103/PhysRevLett.123.200503
[32] K Tsujino et al. "Quantum Receiver beyond the Standard Quantum Limit of Coherent Optical Communication" PRL 106, 250503 https://doi.org/10.1103/PhysRevLett.106.250503
[33] R Han et al. "Helstrom measurement: A nondestructive implementation" PRA 101, 032103 https://doi.org/10.1103/PhysRevA.101.032103
[34] C Cui et al. "Quantum receiver enhanced by adaptive learning" Light: Science & Applications volume 11, Article number: 344 (2022) https://doi.org/10.1038/s41377-022-01039-5
[35] S Paesani et al "Experimental Bayesian Quantum Phase Estimation on a Silicon Photonic Chip" PRL 118, 100503 https://doi.org/10.1103/PhysRevLett.118.100503
[36] A Hentschel et al "Machine Learning for Precise Quantum Measurement" PRL 104, 063603 https://doi.org/10.1103/PhysRevLett.104.063603
[37] W Wang et al. "Quantum-enhanced radiometry via approximate quantum error correction" Nature Communications volume 13, Article number: 3214 (2022) https://doi.org/10.1038/s41467-022-30410-8
[38] E Kessler et al "Quantum Error Correction for Metrology" PRL 112, 150802 https://doi.org/10.1103/PhysRevLett.112.150802
[39] S Zhou et al. "Achieving the Heisenberg limit in quantum metrology using quantum error correction" https://doi.org/10.1038/s41467-017-02510-3
[40] S Huelga et al. "Improvement of Frequency Standards with Quantum Entanglement" PRL 79 3865 https://doi.org/10.1103/PhysRevLett.79.3865
[41] A Chin et al. "Quantum Metrology in Non-Markovian Environments" PRL 109 233601 https://doi.org/10.1103/PhysRevLett.109.233601
[42] P Titum et al. "Optimal control for quantum detectors" npj Quantum Information volume 7, Article number: 53 (2021) https://doi.org/10.1038/s41534-021-00383-5
[43] M OKeefe et al. "Hamiltonian Engineering with Constrained Optimization for Quantum Sensing and Control" New J. Phys. 21 (2019) 023015 https://doi.org/10.1088/1367-2630/ab00be
[44] A Bhattacharyya et al. "Disorder-induced enhancement of precision in quantum metrology" arXiv https://doi.org/10.48550/arXiv.2212.08523
[45] M Gong et al. "Quantum Neuronal Sensing of Quantum Many-Body States on a 61-Qubit Programmable Superconducting Processor" arXiv https://doi.org/10.48550/arXiv.2201.05957
[46] I Cong et al. "Quantum Convolutional Neural Networks" Nature Physics volume 15, pages 1273–1278 (2019) https://doi.org/10.1038/s41567-019-0648-8
[47] J Meyer et al. "Quantum Metrology in the finite sample regime" arXiv https://doi.org/10.48550/arXiv.2307.06370
[48] C Delaney et al. "Demonstration of a quantum advantage by a joint detection receiver for optical communication using quantum belief propagation on a trapped-ion device" PRA 106 032613 https://doi.org/10.1103/PhysRevA.106.032613
[49] S Guha at al. "On quantum limit of optical communications: concatenated codes and joint-detection receivers" ISIT 2011 6034073 https://doi.org/10.1109/ISIT.2011.6034073
[50] N Rengaswamy et al "Belief propagation with quantum messages for quantum-enhanced classical communications" npj QI 9 97 (2021) https://doi.org/10.1038/s41534-021-00422-1