Katarina Maksimović[1], Ivana Vasović[2], Mirko Maksimović[3], Dragi Stamenković[4], Dragana Turnić

OPTIMAL DESIGN OF AIRCRAFT STRUCTURAL COMPONENTS

OPTIMIZACIJA ELEMENATA VAZDUHOPLOVNIH KONSTRUKCIJA

Originalni naučni rad / Original scientific paper
UDK /UDC: 629.735.01
Rad primljen / Paper received: 9.08.2017

Adresa autora / Author's address:
1) City Administration of the City of Belgrade, Serbia, e-mail: katarina.maksimovic@beograd.gov.rs
2) University of Belgrade, Innovation Centre of the Faculty of Mechanical Engineering
3) Belgrade Waterworks and Sewerage, Belgrade, Serbia
4) IDS GmbH Oberhausen, Germany
5) University of Niš, Faculty of Civil Engineering, Niš, Serbia

Keywords
- optimal design
- minimum weight design
- stress constraints
- finite element method

Ključne reči
- optimalno projektovanje
- minimizacija mase strukture
- ograničenja naponskog stanja
- metod konačnih elemenata

Abstract

The subject of this investigation is focused on developing a computation procedure for optimal design of aircraft structural components with respect to stress, displacement and technology constraints. Attention is focused on minimal weight design of plates with a reinforced hole. The optimal design procedure is based on combining the finite element method (FEM) for structural analysis with using optimality criteria (OC) method in the optimisation process. Special attention is focused on optimal design of structural components with reinforced holes. The effect of the shape of reinforcements around the hole on the optimal design is analysed. Some practical design examples are used to illustrate the capability of this procedure.

Izvod

Predmet istraživanja u radu usmeren je na optimalno dimenzionisanje elemenata avionskih konstrukcija sa aspekta naponskog stanja, pomeranja i tehnoloških ograničenja. Pažnja je usmerena na minimizaciju mase strukturalnih elemenata tipa tanke ploče sa ojačanjem oko otvora. Procedura optimalnog projektovanja zasniva se na kombinaciji metode konačnih elemenata (MKE) za strukturalnu analizu sa primenom kriterijuma optimalnosti (OC) u procesu optimizacije. Posebna pažnja je usmerena na optimizaciju elemenata konstrukcija tipa polja oplata/rebara sa ojačanjima oko otvora. Uticaj oblika ojačanja oko otvora na minimizaciju mase je razmatran. Neki praktični primeri su uključeni da ilustruju proces optimizacije.

INTRODUCTION

Extensive literature research has developed the optimisation of structures and structural elements whose shape are defined by cross-section and thickness variables. A more important problem, from the point of view of mechanical/ aircraft design is the determination of the shape of two- and three-dimensional structural elements subject to constraints involving displacements and stress fields in the elements. Structural optimisation using mathematical programming (MP) or numerical optimisation techniques is extensively developed over the past twenty years. Damage tolerance analysis (DTA) is considered in the global design optimisation of an aircraft wing structure. Residual strength and fatigue life requirements, based on the damage tolerance philosophy, are investigated as new design constraints. The global/local finite element approach allowed local fatigue requirements to be considered in the global design optimisation. AFGROW fatigue crack growth analysis provided a new strength criterion for satisfying damage tolerance requirements within a global optimisation environment.

Composites are widely used in aerospace structures. However, most designs do not exploit the inherent tailorability of composites. Optimisation methods can be used to design composite structures by tailoring the ply angles and stacking sequences. Typical objectives include minimum weight, and constraints include strength, buckling, displacements and frequencies. This paper covers some of the literature in this area, with the objective of understanding the historical evolution of the field. Some ideas for future research are provided. It is our hope the review shall provide new researchers with a basic idea about the growth of the field and point towards useful references for further information.

An efficient computation procedure is considered for optimal design of plate and shell structures when the design variables are continuous or discrete. The design variables can be sizing or shape variables. Attention in this work is focused on practical sizing optimization problems. Optimal design of structures can be posed as a mathematical programming problem in which the objective function reflecting the weight or cost is minimised while the design constraints are satisfied. The objective and the constraints are expressed in terms of design variables. Examples of design variables are the thickness and cross section dimensions. The design constraints are bounds on stresses, displacements, etc.

Mathematically, an optimisation problem can be stated as follows:

$$F(X) \quad (1)$$

$$g_j(X) \le 0 \quad j = 1, m \quad (2)$$

$$X_i^l \le X_i \le X_i^u \quad i = 1, n \quad (3)$$

where: F(X) and $g_j(X)$ are the objective function and constraints, respectively; X is the vector of design variables; X_i^l and X_i^u are the lower and upper bounds of the design variable X_i; m is the number of constraints; and n is the number of variables.

The problem given by Eqs.(1)-(3) is, in general, a nonlinear programming problem (NMP) or optimality criterion (OC) and there are various techniques to solve this optimisation problem /1, 2/. Most optimisation algorithms require that an initial set of design variables, X^0, be specified. From this starting point, the design is updated iteratively. Most common form of this iterative procedure is given

$$X^k = X^{k-1} + \alpha S^k \quad (4)$$

where: k is the iteration number and S is the vector of search direction. The scalar quantity α defines the distance that we wish to move in direction S to improve the design. There is a wide variety of methods for determining the search direction, S, as well as for finding the value of α. In numerical optimisation techniques, these methods require evaluation of the objective and constraint functions as well as their gradients. As the overall iteration process is iterative, thus to reach the optimum solution, we often require hundreds of function evaluations and gradient calculations. Dealing with large scale optimisation problems, a great number of finite element analyses of the structure is required to complete the process, thereby making the process very inefficient.

In order to solve the problem efficiently, an attempt should be made to create a high quality approximation to the design problem and solve the approximated problem completely, without actually performing any finite element analyses. Because it is an approximation, it must be repeated so that at least a few detailed finite element analyses will be needed. The key to efficiency is the creation of high quality approximation, thus reducing the number of structural analyses. In the past, attempts have been made to reduce the computational burden by introducing some approximation concepts. The number of the design variables is reduced by linking. This idea is reasonable as in practice, some of the variables are the same. The number of constraints is also reduced by considering only the critical or the near critical constraints at each iteration. A first order Taylor series expansion is used to generate the approximation forms of the constraints in terms of design variables (intermediate variables) or their reciprocals. The reason for using the reciprocal variables is due to the fact that structural response quantities such as stress and displacement are approximately linear with respect to the reciprocal variables. A second generation approximation technique achieves the highest quality approximation. Implicit structural responses such as forces, displacements, frequencies, etc., appearing in the optimisation problem are first approximated. By substituting these approximate functions into the original problem, a nonlinear explicit problem is created, the solution of which, often requires less than ten analyses of the structure. This method is very robust and efficient for large structures, where the computational cost of the analysis is high. This work deals with structural analysis and minimum weight design of structural components that are modelled by shell finite elements.

APPROXIMATION CONCEPTS

Few approximate concepts are presented here. The concept of selecting the functions to be approximated and the intermediate variables to be used to create a high quality approximation is fundamental to the overall efficiency and reliability of the optimisation process, /3-7/. Consider a simple rectangular beam element of width B and height H. These are the physical design variables to be determined in the optimisation problem. The maximum stress used in evaluating a stress constraint is

$$\sigma = \frac{M \cdot c}{I} \pm \frac{P}{A} \quad (5)$$

where: c = H/2; I = BH^3/12; and A = BH are simple functions of B and H. M is the bending moment and P is the axial force. A traditional linearization would be to create a Taylor series approximation to stress as

$$\bar{\sigma} = \sigma^0 + \frac{\partial \sigma}{\partial X}(X - X^0) \quad (6)$$

where: X^T = [B, H]. However, it is clear that the stress is highly nonlinear in the design variables, B and H, so the approximation of the stress given by Eq.(6) is not accurate, and a very small move limit would be necessary during the solution of the approximate problem.

Consider how the stress might approximate better. First A and I are considered as intermediate variables. Next, the gradients of M and P (intermediate responses) are calculated with respect to A and I, and M and P are approximated as

$$\bar{M} = M^0 + \frac{\partial M}{\partial A}(A - A^0) + \frac{\partial M}{\partial I}(I - I^0), \quad (7)$$

$$\bar{P} = P^0 + \frac{\partial P}{\partial A}(A - A^0) + \frac{\partial P}{\partial I}(I - I^0). \quad (8)$$

When we need the value of stress, first A and I are calculated explicitly as functions of B and H. Then, the approximate member forces, $\bar{M}$ and $\bar{P}$ are evaluated. Finally, the stress and constraint are recovered in the usual fashion.

Now the same approximate strategy is applied to the optimal design of plate and shell structures, considering a four-node plate element with 6 degrees of freedom per node (3 translations and 3 rotations). Thus, we have 24 nodal forces in the element. Only the 6 components of the forces at the centre of the element (element forces) are approximated. Then the approximate stresses can be calculated using these approximate forces. At the starting point of each iteration, the nodal displacements of the structure are known from the finite element analysis. The vector of element forces, P^0, at the starting point is determined as

$$P^0 = CSu^e \quad (9)$$

where: C is the element material matrix in the element coordinate system; S is the strain-displacement matrix; and u^e is the nodal displacement vector of the element. We are now able to approximate the element forces using a Taylor series expansion in the intermediate design variables. The intermediate design variables for plate structures are shape design variables, plate thickness t, and bending stiffness D, where $D = t^3/12$.

Finally, the approximate principle, maximum shear, and Von-Mises stresses can be calculated. For example, the approximate Von-Mises stress is

$$\sigma_{VM} = \sqrt{\bar{\sigma}_x^2 + \bar{\sigma}_y^2 - \bar{\sigma}_x\bar{\sigma}_y + 3\bar{\tau}_{xy}^2} \quad (10)$$

This can be used to establish the approximate stress constraint as

$$\bar{g}(\mathrm{X}) = \frac{\sigma_{VM} - \sigma_a}{\sigma_a} \le 0 \quad (11)$$

where: σ_a is the allowable stress.

GRADIENT CALCULATION

In this approach, the gradients of element forces with respect to intermediate variables are required. First the gradient of nodal displacements is evaluated by

$$KU = F \quad (12)$$

where: K is the global stiffness matrix; U is the displacement vector; and F is the external load vector. Differentiation of this equation with respect to intermediate variables, Y, yields

$$\frac{\partial u}{\partial \mathrm{Y}_i} = K^{-1}\left[\frac{\partial F}{\partial \mathrm{Y}_i} - \frac{\partial K}{\partial \mathrm{Y}_i}u\right] \quad (13)$$

Now the gradients of element forces with respect to Y are easily calculated from the relationship

$$P^e = CSu^e = Mu^e \quad (14)$$

as

$$\frac{\partial P^e}{\partial \mathrm{Y}_i} = \frac{\partial M}{\partial \mathrm{Y}_i}u^e + M\frac{\partial u^e}{\partial \mathrm{Y}_i} \quad (15)$$

where: M is a known matrix and its derivatives can be evaluated; and $\partial u^e/\partial \mathrm{Y}_i$ are the gradients of nodal displacements associated with this element and are recovered from Eq.(13).

At the end it is very important that in the practical design problem, there is a great number of constraints involved and a large percentage of these constraints may never be critical during a design cycle, and so could be excluded from the constraint set in that cycle. This is done to decrease the cost of sensitivity analysis (gradient calculation) and to reduce the size of the approximate optimisation problem. In addition, if a number of constraints are active in one region of the structure, say near a stress concentration, only a small number of most critical constraints in that region is retained.

DEFINITION OF THE MODEL

All models examined in this work have similar geometry. The only difference between them is in the size and shape of the reinforcement around the hole. The basic model is 200 mm long; 100 mm width; 1 mm thick, and with a hole of diameter D_1 = 30 mm in the centre of the model. The model is made of aluminium alloy. Mechanical properties of the used material are given in Table 1.

Table 1. Material properties.

Isotropic aluminium alloy - dural	
Young's modulus [E]	E = 74000 N/mm^2
Poisson's ratio [ν]	ν = 0.3
Mass density [ρ]	ρ = 2.8·10^{-6} kg/mm^3
Limit stress (tensile)	σ = 400 N/mm^2

NUMERICAL OPTIMISATION

To illustrate the optimal design process of the shell-type structure, here are several examples are included. The following example represents a FE model of plate with a circular reinforcement hole, and clamped at one edge, and with applied continuous tension load on the opposite edge. The plate is made of aluminium alloy.

Basic model optimisation

The basic model is examined first. The plate is clamped on one side and continuous tension load is applied at the other side. Load intensity is F = 12 000 N. The purpose of these examples is to present an efficient FE method with adequate optimisation algorithms to optimisation process. Optimum design structure in this case is subjected to minimizing of weight, while the design constraints are satisfied. The design variables are the thicknesses and design constraints are bound on stresses. Msc/*NASTRAN* and *FEMAP* software are used for support in this optimisation process.

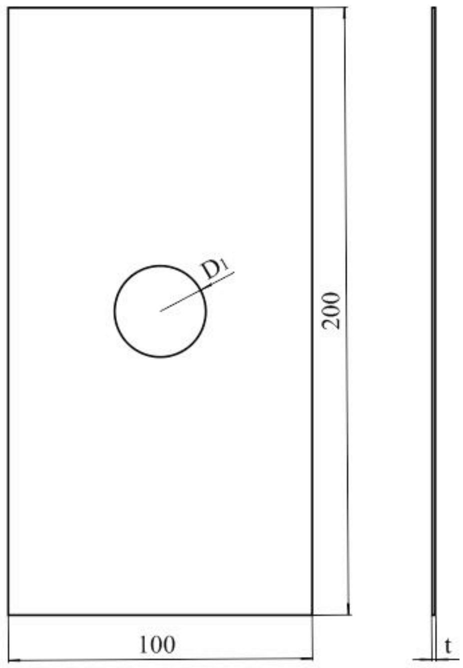

Figure 1. Basic model.

The design constraint is presented by maximal Von Mises stress σ = 400 N/mm^2. The design objective is to minimize the weight, and the design variable is the model thickness. The optimisation process is done in 5 cycles and the minimal weight of the model, in the given conditions, is obtained and presented in Table 2.

Table 2. Basic model optimisation results.

Plate thickness (mm)	0.977
Weight (kg)	0.053

Results obtained for the basic model will be used for comparison with the optimisation results of other models.

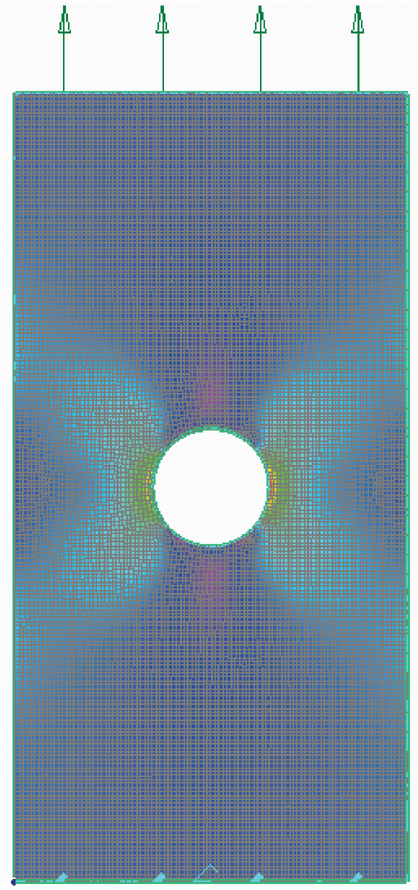

Figure 2. Stress distribution of the optimised model.

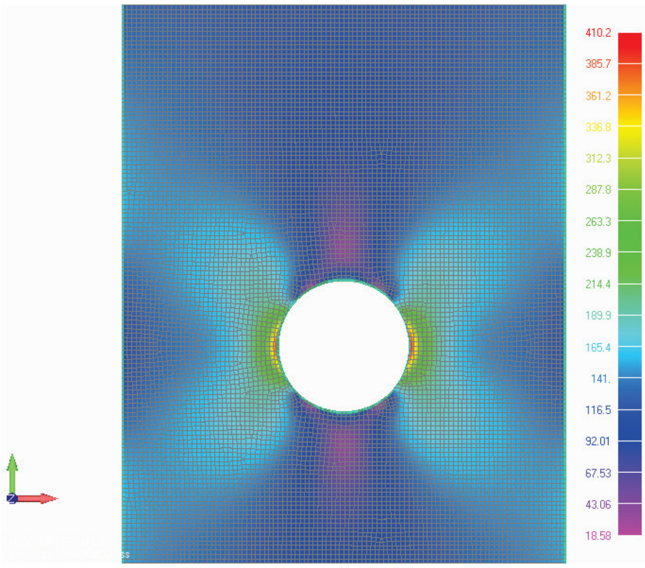

Figure 3. Stress distribution around the hole.

Model with reinforced hole, case 1

After obtaining results of optimal design for the basic model, several different models with a hole reinforcement are examined. By using the benefits of finite element modelling and design optimisation, the goal is to find the minimal weight design that shall meet the terms. The characteristic measures of Case 1 model are given in Table 3.

Table 3. Characteristic measures of Case 1 model.

Hole diameter	$D_1 = 30$ mm
Reinforcement diameter	$D_2 = 35$ mm
Plate thickness	$t_1 = 1$ mm
Reinforcement thickness	$t_2 = 2$ mm

When the model is defined, the optimisation process is performed in Msc/Nastran software code, /8/. The design objective and design constraints for all cases and the basic model are the same. Optimisation results are obtained after 5 design cycles, and the best design is accomplished in cycle 4. Results are presented in Table 4 and Fig. 4.

Table 4. Case 1 model optimisation results.

Plate thickness t_1 (mm)	0.55
Reinforcement thickness t_2 (mm)	2.42
Weight (kg)	0.031

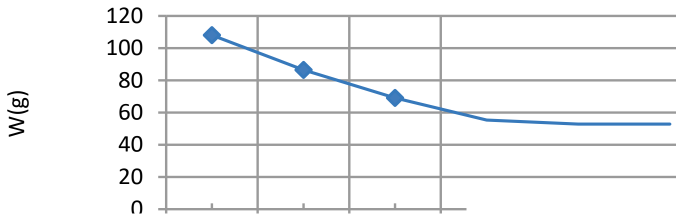

Figure 4. Weight reduction during the optimisation process.

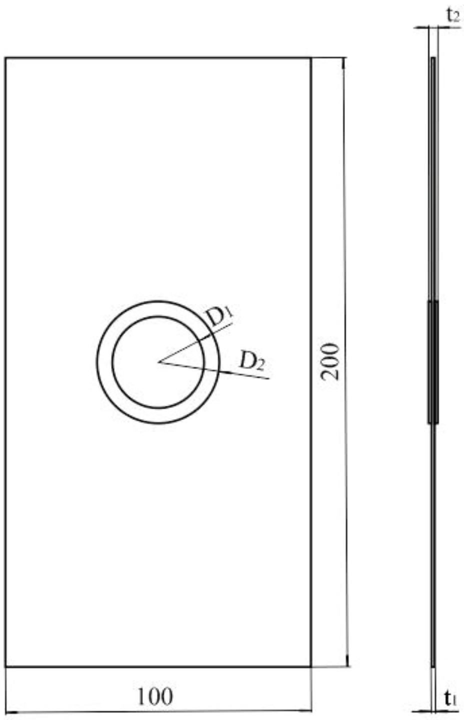

Figure 5. Model with reinforced hole.

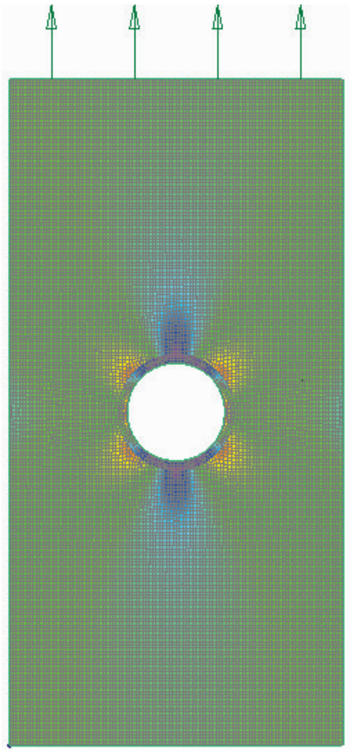

Figure 6. Stress distribution in Case 1 optimised model.

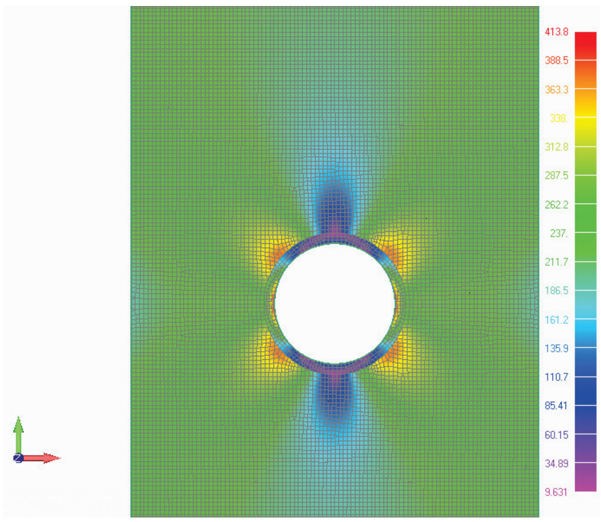

Figure 7. Stress distribution in Case 1 optimised model.

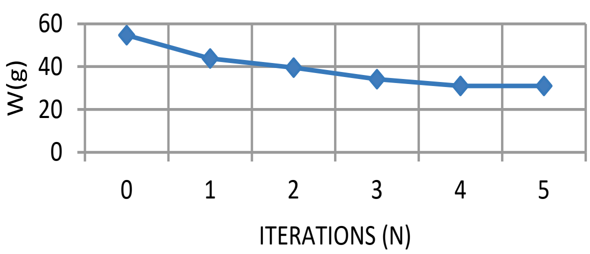

Figure 8. Weight reduction during optimisation, Case 1.

Model with reinforced hole, Case 2

In Case 2 model same measures of the model are used, but reinforcement is defined in a way that it has been divided in two properties, inner and outer reinforcement. It has been done in order to get more accurate design and lighter model. Reinforcement thickness around hole and values of diameters are given in Table 5.

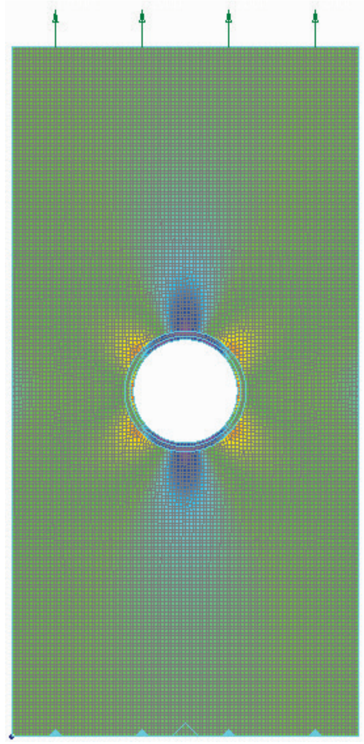

Figure 9. Stress distribution in Case 2 optimised model.

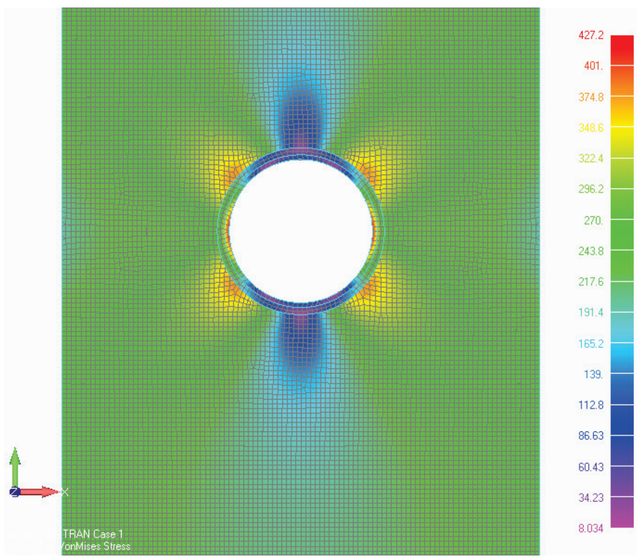

Figure 10. Stress distribution in Case 2 optimised model.

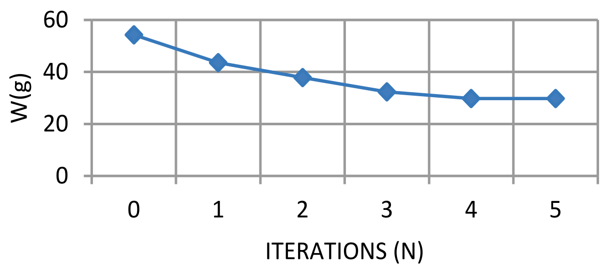

Figure 11. Weight reduction during optimisation, Case 2.

Table 5. Characteristic measures of Case 2 model.

Hole diameter	D_1 = 30 mm
Inner reinforcement diameter	D_2 = 32 mm
Outer reinforcement diameter	D_3 = 35 mm
Plate thickness	t_1 = 1 mm
Inner reinforcement thickness	t_2 = 1.5 mm
Outer reinforcement thickness	t_3 = 2 mm

Results of the optimisation are given in Table 6.

Table 6. Case 2 model optimisation results.

Plate thickness t_1 (mm)	0.57
Inner reinforcement thickness t_2 (mm)	1.43
Outer reinforcement thickness t_3 (mm)	2.99
Weight (kg)	0.029

Model with reinforced hole, Case 3

The difference between this model and Case 1 model is that in Case 3 model the hole reinforcement is larger. The goal is to compare optimisation results in order to accumulate more data. Case 3 model characteristic measures are given in Table 7.

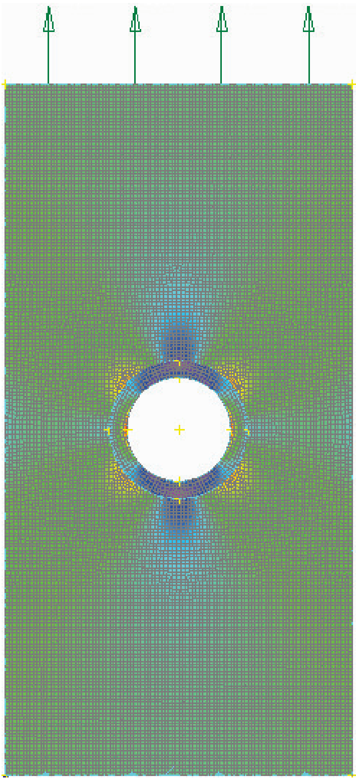

Figure 12. Stress distribution in Case 3 optimised model.

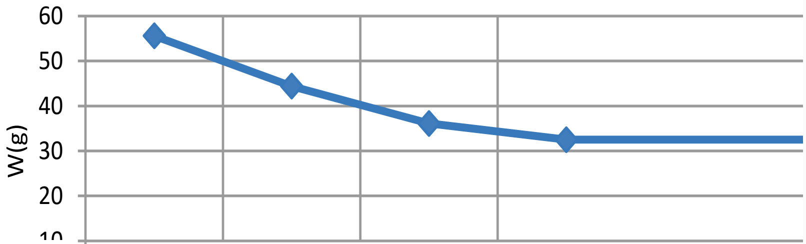

Figure 14. Weight reduction during optimisation, Case 3.

Optimisation results are presented in Table 8.

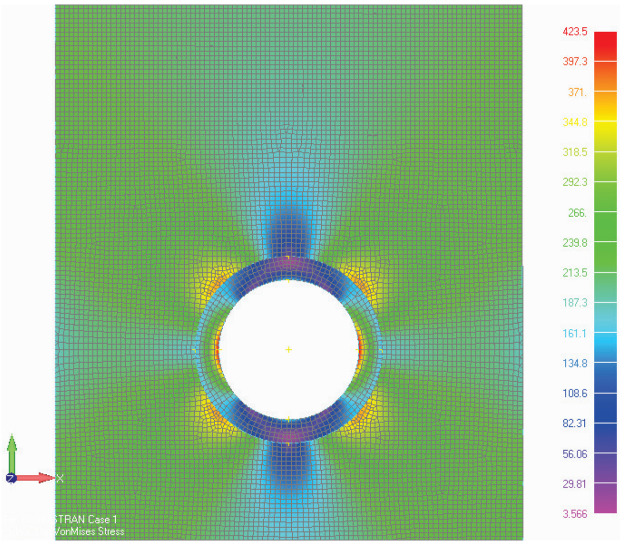

Figure 13. Stress distribution in Case 3 optimised model.

Table 7. Case 3 model characteristics.

Hole diameter	D_1 = 30 mm
Reinforcement diameter	D_2 = 40 mm
Plate thickness	t_1 = 1 mm
Reinforcement thickness	t_2 = 2 mm

Table 8. Case 3 model optimisation results.

Plate thickness t_1 (mm)	0.57
Reinforcement thickness t_2 (mm)	1.75
Weight (kg)	0.032

Model with reinforced hole, Case 4

In the same way reinforcement of Case 1 model is split in two properties for Case 2 model, in Case 4 model the hole reinforcement is defined as follows (see Table 9).

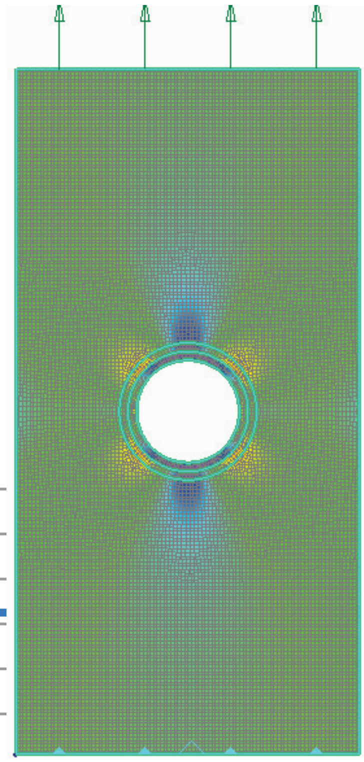

Figure 15. Stress distribution in Case 4 optimised model.

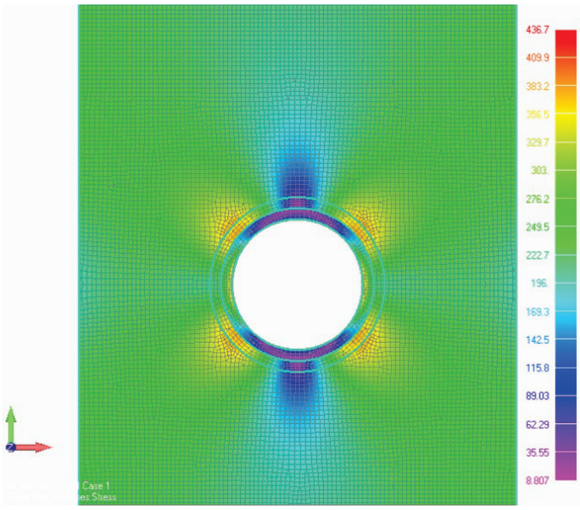

Figure 16. Stress distribution in Case 4 optimised model.

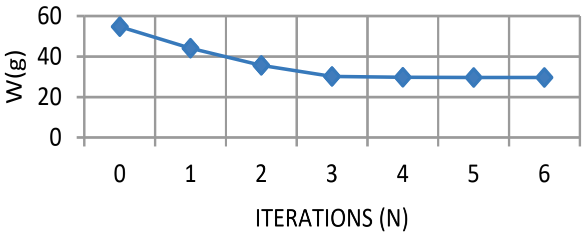

Figure 17. Weight reduction during optimisation, Case 4

Table 9. Characteristic measures of Case 4 model.

Hole diameter	D_1 = 30 mm
Inner reinforcement diameter	D_2 = 35 mm
Outer reinforcement diameter	D_3 = 40 mm
Plate thickness	t_1 = 1 mm
Inner reinforcement thickness	t_2 = 1.5 mm
Outer reinforcement thickness	t_3 = 2 mm

Table 10. Case 4 model optimisation results.

Plate thickness t_1 (mm)	0.52
Inner reinforcement thickness t_2 (mm)	0.56
Outer reinforcement thickness t_3 (mm)	2.53
Weight (kg)	0.03

CONCLUSION

Structural optimisation problems considered here consist of minimising some objective functions subject to nonlinear constraints and bounds insuring the feasibility of the structural design. Attention is focused on the minimal weight design of characteristic aircraft structural components such as panels with reinforced holes. An efficient optimisation procedure combining finite element analysis and approximation concepts can be effectively used for this purpose. The numerical examples have indicated that the approximation concept approach is a powerful and a rather general and practical approach to structural optimisation of aircraft structural components. Convergence to feasible near optimal design has been obtained after 5-8 actual iterations, depending on the problem and the move limit. It is shown that the introduction of high quality explicit approximation for the objective function and behaviour constraints improves the convergence characteristics of structural optimisation problems.

ACKNOWLEDGEMENTS

This work is financially supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia under Projects OI-174001 and TR-35045.

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