# Design of symmetric lossless LC filter with analysis of critical element changes

**Article** · October 2011

DOI: 10.1109/TELSKS.2011.6112046



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# Design of Symmetric Lossless LC Filter with Analysis of Critical Element Changes

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*Abstract* – **The paper presents the design of analog filter based on a new class of all-pole approximation. The approximating function is derived using Legendre orthogonal polynomial with the appropriate weights and the usage of two Legendre multiplication factors (at the origin and at the passband edge frequency). The design procedure is illustrated on LC ladder filter (inductors L, capacitors C) and analysis of the critical element changes of the 11th-order filter. The whole process of the design and optimization is automated using software Mathematica.**

*Keywords* – **Approximation, Returned loss, Tolerance analysis.**

## I. Introduction

Passive filters are realized with only passive elements and are implemented with resistors (R), inductors (L), and capacitors (C). The magnitude function for a well-designed passive filter has low sensitivity for variations in the element values, such as when component's values are affected by temperature and by aging [1].

A doubly resistively terminated LC filter that has been designed for maximum power transfer has a very low sensitivity for changes in element values. If there are many frequencies with maximal power transfer in the passband, the magnitude sensitivity to reactive element values will be zero at those frequencies. Therefore the sensitivity will be very low throughout the passband because only the frequency can be changed by L's and C's small value changes, but the maximum power transfer from the source to load stay the same at those frequencies in the passband. This reasoning is called Orchard's argument [2]. Due to very low sensitivity in the passband, the passive LC filters are used as prototypes for the design of many other selective filters, such as continuous-time active filters or digital filters [3].

It is therefore important at an early stage in the design process to estimate the effect of small and large element values changes, effect of component tolerances of the most sensitive – critical element, or ratio of the maximal and minimal values of elements of the same type.

In general, fully symmetrical filters, with symmetrical structure and symmetrical element values, have very good sensitivity properties [1]. Experience indicates that errors in symmetrically placed circuit elements tend to have a small effect on the magnitude function [1], and small element changes mainly cause a frequency shift. Thus, the doubly resistively terminated symmetric ladder structures are the best filter structures from a sensitivity point of view.

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In this paper, we start the filter design with known filter structure, the doubly resistively terminated symmetric LC ladder structure, and derive some properties using CAS (computer algebra system). Next, we identify the influence of the critical element. In order to exploit the good properties of those filter structure, we define a new approximation, we derive filter coefficients using some optimization criteria, and we present the analysis of critical element changes using CAS.

## II. Symmetric Lossless LC Filter

### *A. Schematic*

Computer algebra systems (CAS) such as Mathematica [4] and SchematicSolver [5] can be used for drawing schematics of continuous-time systems or discrete-time systems in such a way that schematic description has all details for solving the systems [6]. A detailed procedure for drawing and solving doubly resistively terminated LC filters is presented in [7, 8].

The describing of the filter structure can be automated in CAS. It is sufficient to specify only the filter order, and enter the knowledge that describes the filter structure. For example, the knowledge that contains graphical description of filter elements is obtained using GUI (graphical user interface) formerly coded in Matlab [9]. The commands for drawing filters in Matlab are translated into command for drawing in Mathematica [3], and saved as special package. The package for drawing can be loaded into Mathematica notebook in the initial cells. Using CAS commands, all variable names can be generated automatically as a function of filter order. The input and output part of the filter structures can be drawn using GUI [5], where element names can be changed at any stage of drawing. The part that depends on filter order (or on number of nodes in filter structure) can be generated using **Do** command, in such a way to generate a list of elements

```
LCnodes = {}; LCground = {}; LCseries = {}; LCparallel = {};
Do [LCnodes = Join[LCnodes,
    {LCNode[x[[x0 - 3 + i * 5]], y[[y0 + 5]], n[[i]], 1, 1, F]}],
  {i, 2, nNodes}]
Do [LCseries = Join[LCseries,
    {LCInd[x[[x0 - 3 + 5 i]], y[[y0 + 5]],  x[[x0 + 2 + 5 i]], " ",
      Z[[i]], 0, ds / 2, F]}], {i, 1, nNodes - 1}]
Do [LCparallel = Join[LCparallel,
    {LCCap[x[[x0 - 3 + 5 i]], y[[y0]], y[[y0 + 5]],  " ", Y[[i]],
      3, ds / 2, F]}], {i, 1, nNodes}]
Do [LCground = Join[LCground,
    {LCGrnd[x[[x0 - 3 + 5 i]], y[[y0]], 0, ds / 2]}], {i, 1, nNodes}]
Schematic = Join[LCinout, LCnodes, LCseries, LCparallel, LCground]
```

The final list that describes the filter schematic is obtained by joining all lists, that we are calling schematic specification. In order to draw a schematic, the element names in the schematic specification should be replaced with commands for drawing and the command for presenting the graphics, Fig. 1.

```
Show[Schematic /. subsdrawYZ /. {ds → 5, F → 10}]
```

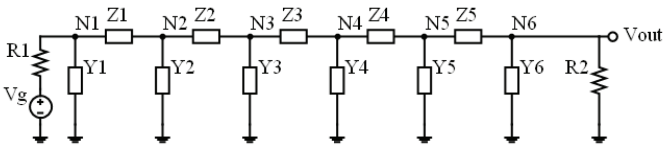


Fig. 1. Schematic of the filter generated by CAS

The same specification can be used for high-quality schematic, using the replacement rules, Fig. 2.

```
Show[Schematic /. subsdrawLC /.
    Thread[Y → c] /. Thread[Z → l] /.
   {Vg → "Vg", Vout → "Vout", "C6" → "C1", "C1" → "C1",
    "C2" → "C2", "C5" → "C2", "C3" → "C3", "C4" → "C3",
    "L1" → "L1", "L5" → "L1", "L3" → "L3", "L2" → "L2",
    "L4" → "L2", R1 → "R1", R2 → "R2"} /.
  {N1 → "", "N2" → "", "N3" → "", "N4" → "", "N5" → "",
   "N6" → ""} /. {ds → 5, F → 10}]
```

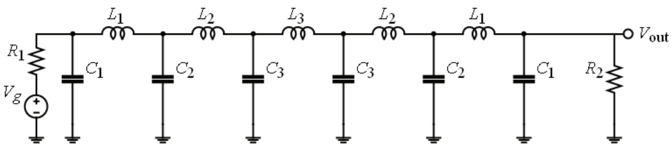


Fig. 2. Schematic of symmetric lossless LC filter generated by CAS

Replacement rules are used, firstly, to change symbols, for example from Y to C, and secondly, to change the appearance on the element values on figure, for example, changing from C6 to $C_1$. This way a drawing of the schematic of the doubly resistively terminated symmetric LC ladder structure is automated for arbitrary filter order (or number of nodes). The symmetric structure is a special case obtained by using replacement rules for element values.

### *B. Solving Systems*

Using the similar procedure for automated generation of system description, a system of equations that describe the system can be automatically generated. Firstly, by using modified nodal analysis, the equation of the input node is specified

```
CircuitEquations =
  {v[[1]] * Y[[1]] + (v[[1]] - Vg) / R1 +
     (v[[1]] - v[[2]]) / Z[[1]] == 0};
```

Next, the equations of all other nodes of the system are generated:

```
Do [CircuitEquations = Join[CircuitEquations,
   {v[[i]] * Y[[i]] + (v[[i]] - v[[i - 1]]) / Z[[i - 1]] +
      (v[[i]] - v[[i + 1]]) / Z[[i]] == 0}],
 {i, 2, nNodes - 1}]
```

Finally, the equation of the output node is defined:

```
CircuitEquations =
 (Join[CircuitEquations,
      {v[[nNodes]] * Y[[nNodes]] + v[[nNodes]] / R2 +
        (v[[nNodes]] - v[[nNodes - 1]]) /
         Z[[nNodes - 1]] == 0}] /. subsV /. subsZ /.
   subsY) // FullSimplify
```

The automated procedure generates the system of equations that describes doubly resistively terminated ladder filter:

$$\left\{\frac{V1-Vg}{R1}+V1\,Y1+\frac{V1-V2}{Z1}==0,\ V2\,Y2+\frac{-V1+V2}{Z1}+\frac{V2-V3}{Z2}==0,\ V3\,Y3+\frac{-V2+V3}{Z2}+\frac{V3-V4}{Z3}==0,\ V4\,Y4+\frac{-V3+V4}{Z3}+\frac{V4-V5}{Z4}==0,\ V5\,Y5+\frac{-V4+V5}{Z4}+\frac{V5-V6}{Z5}==0,\ V6\left(\frac{1}{R2}+Y6\right)+\frac{-V5+V6}{Z5}==0\right\}$$

Using the command **Solve**, the transfer function can be obtained in closed form in terms of symbolic values of filter elements. The final result of the 11th-order filter is too long to be presented in this paper.

The basic idea of this paper is to find the transfer function of the symmetric filter. Using replacement rules for element values, similarly to that presented in Fig. 2, the transfer function becomes

```
Hs = H /. {C6 → C1, C5 → C2, C4 → C3, L5 → L1, L4 → L2} // Factor
```

$$2\Big/\big((1+C1\,s+C2\,s+C3\,s+C2\,L1\,s^2+C3\,L1\,s^2+C3\,L2\,s^2+C1\,C2\,L1\,s^3+C1\,C3\,L1\,s^3+C1\,C3\,L2\,s^3+C2\,C3\,L2\,s^3+C2\,C3\,L1\,L2\,s^4+C1\,C2\,C3\,L1\,L2\,s^5)\,(2+2\,L1\,s+2\,L2\,s+L3\,s+2\,C1\,L1\,s^2+2\,C1\,L2\,s^2+2\,C2\,L2\,s^2+C1\,L3\,s^2+C2\,L3\,s^2+C3\,L3\,s^2+2\,C2\,L1\,L2\,s^3+C2\,L1\,L3\,s^3+C3\,L1\,L3\,s^3+C3\,L2\,L3\,s^3+2\,C1\,C2\,L1\,L2\,s^4+C1\,C2\,L1\,L3\,s^4+C1\,C3\,L1\,L3\,s^4+C1\,C3\,L2\,L3\,s^4+C2\,C3\,L2\,L3\,s^4+C2\,C3\,L1\,L2\,L3\,s^5+C1\,C2\,C3\,L1\,L2\,L3\,s^6)\big)$$

The transfer function is all-pole. The denominator is a product of two polynomials in s, where the fifth-order polynomial does not depend on L3. This means that changes of L3 will affect only every second pole of the transfer function.

Notice that CAS finds factored form of the transfer function that will be practically impossible for manual derivation.

## III. APPROXIMATION

### *A. Approximating method*

The general form of the lowpass continuous-time prototype all-pole filter transfer function is:

$$H_n(s)=\frac{K}{\prod_{r=1}^{n}(s-s_r)} \tag{1}$$

The filter order is $n$, $K$ is a constant to specify the attenuation at $s$=0 (for example 0 dB), and the poles of the transfer function are $s_r=\sigma_r+j\omega_r,\quad r\in(1,2,3,\ldots n)$

The squared magnitude response is

$$H_n(j\omega)\, H_n(-j\omega) = \frac{1}{1+\varepsilon^2\, A_n^2(\omega^2)} \tag{2}$$

The parameter $\varepsilon$

$$\varepsilon^2 = \frac{\rho^2}{1-\rho^2} \tag{3}$$

determines the attenuation at the pass-band edge frequency, while $\rho$ is the pass-band reflection factor. The characteristic function is normalized to 1 at the pass-band edge frequency $\omega_p$, $A_n(\omega_p)=1$. The approximating function can be expresses using polynomials, for example, for odd-order transfer functions

$$A_n^2(\omega) = \omega^2 \left( \sum_{r=1}^{\frac{n-1}{2}} b_{2r} P_{2r}(\omega) \right)^2 \tag{4}$$

where $P_r(\omega)$ is the $r$th-order Legendre orthogonal polynomial. This type of approximation is chosen because this is necessary condition for implementation with doubly resistively terminated symmetric ladder structures [7]. In order to find the optimal solution, one can use the minimization of the following integral [10, 11, 12, 13]

$$I_{\min}(\omega) = \int_0^1 p(\omega)\, A_n^2(\omega)\, d(\omega) \tag{5}$$

In this paper, the weight $p(\omega)$ is 1 for -1≤ω≤1. We can define a function using conditions of the proper prototype low-pass approximation

$$\begin{aligned} &\phi(b_0, b_2, b_4, b_6, \ldots b_{n-1}, \lambda_0, \lambda_1) = \\ &\int_0^1 \left( \sum_{r=0}^{\frac{n-1}{2}} b_{2r} P_{2r}(\omega) \right)^2 d(\omega) - \\ &\lambda_0 \left( \sum_{r=0}^{\frac{n-1}{2}} b_{2r} P_{2r}(0) \right) - \lambda_1 \left( -1 + \sum_{r=0}^{\frac{n-1}{2}} b_{2r} P_{2r}(1) \right) \end{aligned} \tag{6}$$

Next, we derive partial derivatives and set a system of equations that should be solved in terms of $b_0$, $b_2$, $b_4$, $b_6$, $\ldots, b_{n-1}$, $n$ is odd:

$$\begin{aligned} &\frac{\partial}{\partial b_0} \phi(b_0, b_2, b_4, b_6, \ldots b_{n-1}, \lambda_0, \lambda_1) = \\ &2 h_0 b_0 - \lambda_0 P_0(0) - \lambda_1 P_0(1) = 0 \end{aligned} \tag{7}$$

The partial derivatives can be used to set a system of equations:

$$\begin{aligned} &\frac{\partial}{\partial b_{2r}} \phi(b_0, b_2, b_4, b_6, \ldots b_{n-1}, \lambda_0, \lambda_1) = \\ &h_{2r} b_{2r} - \lambda_0 P_{2r}(0) - \lambda_1 P_{2r}(1) = 0 \\ &r = 1, 2, 3, \ldots, (n-1)/2 \end{aligned} \tag{8}$$

$$\begin{aligned} &\frac{\partial}{\partial \lambda_0} \phi(b_0, b_2, b_4, b_6, \ldots b_{n-1}, \lambda_0, \lambda_1) = \\ &\sum_{r=0}^{(n-1)/2} b_{2r} P_{2r}(0) = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} &\frac{\partial}{\partial \lambda_1} \phi(b_0, b_2, b_4, b_6, \ldots b_{n-1}, \lambda_0, \lambda_1) = \\ &\sum_{r=0}^{(n-1)/2} b_{2r} P_{2r}(1) - 1 = 0 \end{aligned} \tag{10}$$

Solving the system of equations, we can derive coefficients $b_0$, $b_2$, $b_4$, $b_6$, $\ldots, b_{n-1}$. These coefficients can be put into equation (4) and thus, the squared magnitude response is computed.

*B. Example*

The solution in CAS is a sequence of replacement rules.

$$\left\{ \mathrm{b0} \to \frac{73}{3465},\ \mathrm{b2} \to \frac{565}{9009},\ \mathrm{b4} \to \frac{789}{5005},\ \mathrm{b6} \to \frac{613}{3465},\ \mathrm{b8} \to \frac{1853}{6435},\ \mathrm{b10} \to \frac{42}{143},\ \mathrm{L0} \to \frac{512}{45045},\ \mathrm{L1} \to \frac{2}{65} \right\}$$

Using replacement rules for defined approximation function, the approximation becomes a polynomial with numeric coefficients and we can plot the function (Fig. 3).

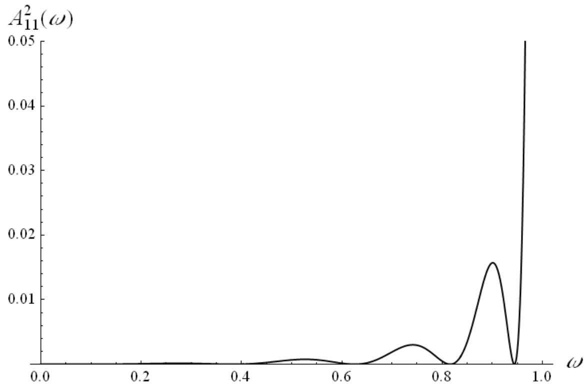


Fig. 3. Approximation

The poles of the transfer function are from the left-half-plane. The poles are shown in Fig. 4. Two polynomials can be created, one using every second pole starting from the real pole $p_1(s)$, and the second polynomial using all other poles, $p_2(s)$. The critical element can be computed directly from these polynomials,

$$L3 = 2\frac{p_1(0)}{p_2(0)} \tag{11}$$

The closed form expressions in terms of symbolic values can be used for simple derivation of the sensitivity to any element of the filter, as well as to the critical element, $L_3$,

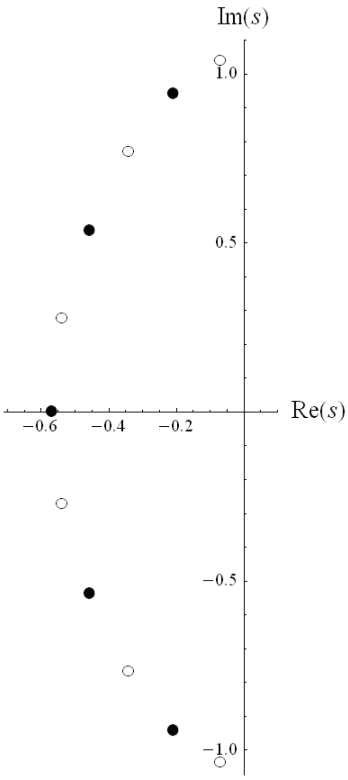


Fig. 4. Transfer function poles: poles ○ are changing when L3 is changing, poles ● are insensitive to L3 changes

Fig. 5 shows that the summed sensitivity is very small in the whole pass-band, and that the sensitivity to the critical element is slightly larger than the summed sensitivity.

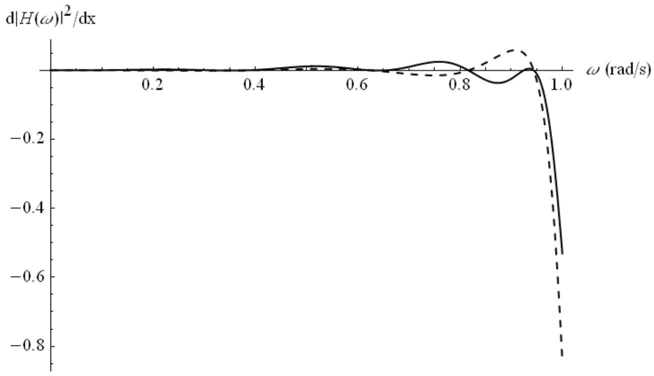


Fig. 5. Summed sensitivity (solid line) and sensitivity of the critical element (dashed line)

By moving the passband edge frequency of the designed filter to the transition band (so that the frequency of maximal sensitivity is at unit normalized frequency), the larges magnitude deviation can be reduced in the passband.

Even more, the tuning of the critical elements can be used for correcting large element changes because only half of the poles are influenced by the critical element value. Some other optimization methods are shown in [14].

## IV. Conclusion

Computer algebra systems (CAS) can be used for automated analysis, design, and optimization of continuous-time filters. Some known feature of passive filter, such as very low magnitude sensitivity to element value changes can be re-discovered and employed in optimization.

## Acknowledgement


This work was partially supported by the Ministry of Science of Serbia under Grants TR-32023 and TR-32048.


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