STRESSES AROUND A HORIZONTAL CIRCULAR OPENING PASSING THROUGH TWO ISOTROPIC LAYERS

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ABSTRACT: The article examines the question of determining the stresses in a layered rock mass around a horizontal circular opening. The rock mass consists of homogeneous isotropic layers. The boundary planes between them are steeply inclined relative to the axis of the opening. Moreover, they are parallel to the horizontal axis of its cross-section. The influence of the stresses due to pushing the opening extends into the cubic domain. The specified class of tasks is solved with the complex potential theory and an approach from the mechanics of layered media.
The two layers in the rock mass are replaced by an equivalent layer. The expressions for its physical and technical characteristics and for the stresses on the excavation boundary are given for it. The expressions for the stresses in the layers are derived.
The results are applied to a real rock mass. The physical and technical constants in the equivalent rock mass were calculated. The stresses in both the equivalent rock mass and the two layers were obtained. Three tangential normal stress diagrams for points on the excavation boundary are given. The first diagram shows the stresses in the equivalent rock mass. The stresses in the two layers are shown in the second and third diagrams.

Key words: complex potential theory, mechanics of layered media.

НАПРЕЖЕНИЯ ОКОЛО ХОРИЗОНТАЛНА КРЪГОВА ИЗРАБОТКА, ПРЕМИНАВАЩА ПРЕЗ ДВА ИЗОТРОПНИ ПЛАСТА

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РЕЗЮМЕ: В статията се разглежда въпроса за определяне на напреженията в напластен масив около хоризонтална кръгова изработка. Масивът се състои от хомогенни изотропни пласта. Граничните равнини между тях са стръмно наклонени спрямо оста на изработката. Освен това те са успоредни на хоризонталната ос на нейното напречното сечение. Влиянието на напреженията, дължащи се на прокарването на изработката, се простира в кубична област. Указаният клас задачи се решава с комплексна потенциална теория и подход от механика на напластените среди.
Двата пласта в масива се заменят с еквивалентен пласт. За него са дадени изразите за физическите и техническите му характеристики и за напреженията около отвора. Изведени са изразите за напреженията в пластовете.
Резултатите са приложени за реален масив. Изчислени са физическите и технически константи в еквивалентния масив. Получени са напреженията както в еквивалентния масив, така и в двата пласта. Дадени са три диаграми на тангенциалните нормални напрежения за точки по границата на изкопа от контура на изработката. На първата диаграма са представени напреженията в еквивалентния масив. Напреженията в двата пласта са изобразени на втората и третата диаграми.

Ключови думи: комплексна потенциална теория, механика на напластените среди.

Introduction

Modern horizontal opening often cross inclined strata. Each of these layers has different physical and technical characteristics. This non-uniformity of the medium is accounted for by methods for solving layered linearly deformed media. On the other hand, the stress state around the opening is determined by complex potential theory (Muskhelishvili, 1953; Bulichev, 1982).

Solutions are known for two types of layer placement cases. In the first case, the layers are parallel and thin, and in the second case, they are steep and thick. For the first type of problems, there are solutions in the plane-strained and generalised plane-strained state (Trifonova-Genova, 2012; 2016; 2017). When the medium consists of steep and thick layers with boundary planes that are parallel to the fabrication axis, the authors have proposed solutions in previous works (Trifonova-Genova, 2018a, 2018b; Trifonova-Genova et al., 2022).

The aim of the present work is to expand the described cases of layer placement. Cases will be considered where the boundary planes of the layers are inclined to the axis of fabrication and parallel to the horizontal axis of the cross-section of the hole.

Methods

1. Formulation of the problem

A horizontal circular opening with a radius R has been driven to a great depth H. A part of it passing through isotropic layers given in Figure 1 and Figure 2 is considered. The influence of the hole extends to the stresses in the array encloses a cube of side equal to $12R$.

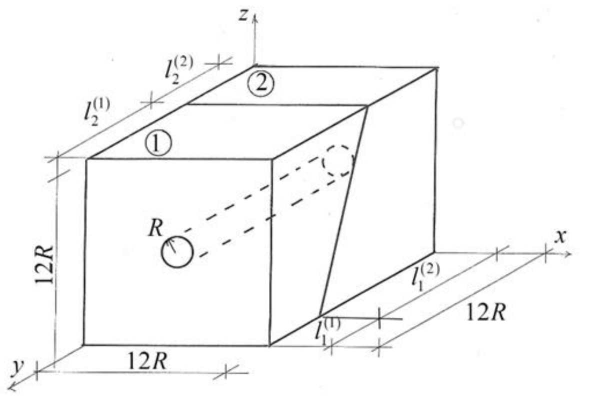

Fig. 1 Opening passing through two layers

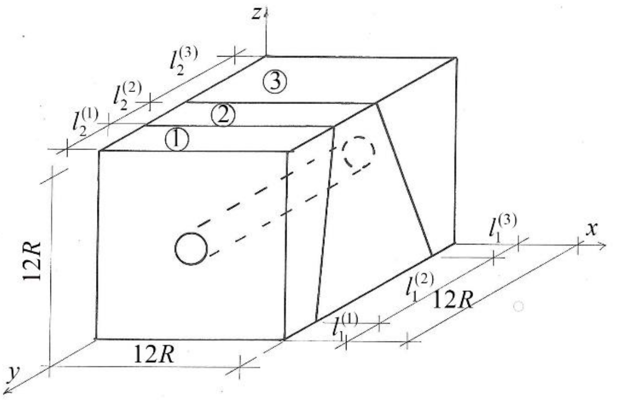

Fig.2 Opening going through three layers

2. Method for determining ~~voltages~~ stresses

The following two-stage approach is used to determine the stresses in each layer.

In the first stage, the isotropic layers are replaced by an equivalent uniform homogeneous isotropic layer. It has the following characteristics (Trifonova-Genova, 1991):

$$E^{(o)} = \frac{\sum_{k=1}^{n} E^{(k)} V_k.}{\sum_{k=1}^{n} V_k} ;\ \mu^{(o)} = \frac{\sum_{k=1}^{n} \mu^{(k)} V_k E^{(k)}}{\sum_{k=1}^{n} V_k E^{(k)}} ;$$

$$\gamma^{(o)} = \frac{\sum_{k=1}^{n} \gamma^{(k)} V_k.}{\sum_{k=1}^{n} V_k} ; \qquad k = 1 \div n . \tag{1}$$

Here, the characteristics for layer k are: $\gamma^{(k)}$ is volumetric weight, $E^{(k)}$ is Young's modulus, $\mu^{(k)}$ is Poison's ratio. In the homogeneous isotropic layer, the characteristics are: $\gamma^{(o)}$ is volumetric weight, $E^{(o)}$ is Young's modulus, $\mu^{(o)}$ is Poison's ratio. The total number of layers is n . For figure 1, this number is 2, and for figure 2 it is 3.

In dependencies (1), the volumes of the layers are determined by the following expression:

$$V_k = 72\left[l_1^{(k)} + l_2^{(k)}\right] r^2 ;\ k = 1 \div n . \tag{2}$$

To determine the stresses in the uniform layer, an area around the hole shaped of a square of size $12R$ is used (Fig.3). The vertical load on the calculation scheme is Q , and $\lambda^{(o)} Q$ is the horizontal load. The stresses in the medium are plotted in a polar coordinates system ($Or\theta$). An angle θ is measured from axis to axis z to axis x . The radius vector of a point from the field is r .

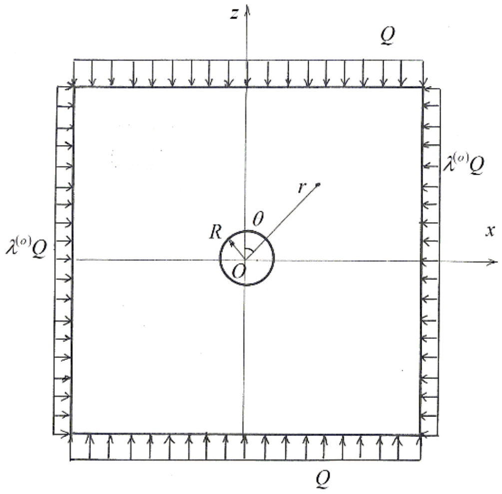

Fig.3 Calculation scheme

In the second stage, the stresses in each layer of two conditions are determined. The first condition describes the balance of forces, and the second equalises the relative deformations of the contact between the two layers. For a rock mass consisting of two layers, the conditions are (Trifonova-Genova, 2012):

$$\begin{aligned}
&\sigma_r^{(1)} V_1 + \sigma_r^{(2)} V_2 = b_1 ; \\
&\sigma_\theta^{(1)} V_1 + \sigma_\theta^{(2)} V_2 = b_2 ; \\
&a_{11}^{(2)} \sigma_r^{(2)} - a_{11}^{(1)} \sigma_r^{(1)} + a_{12}^{(2)} \sigma_\theta^{(2)} - a_{12}^{(1)} \sigma_\theta^{(1)} = b_3 ; \\
&a_{12}^{(2)} \sigma_r^{(2)} - a_{12}^{(1)} \sigma_r^{(1)} + a_{11}^{(2)} \sigma_\theta^{(2)} - a_{11}^{(1)} \sigma_\theta^{(1)} = b_3 ,
\end{aligned} \tag{3}$$

where:

$$b_1 = \sigma_r^o V \ ; b_2 = \sigma_\theta^o V \ ;\ b_3 = a_{13}^{(1)} \sigma_z^{(1)} - a_{13}^{(2)} \sigma_z^{(2)} .$$

Here, $\sigma_r^{(m)}$, $\sigma_\theta^{(m)}$, and $\sigma_z^{(m)}$ are the radial, tangential normal stresses and the stresses along the axis of production in layer m , but σ_r^o and σ_θ^o are the radial and tangential normal stresses in an isotropic generalised medium.

The deformation along the axis of opening is zero, from where

$$\sigma_z^{(m)} = -\frac{a_{13}^{(m)}}{a_{33}^{(m)}}\left[\sigma_r^{(m)} + \sigma_\theta^{(m)}\right]; \to m = 1{,}2\,. \qquad (4)$$

Substitute (4) into (3) and obtain:

$$\begin{aligned}
&\sigma_r^{(1)}V_1 + \sigma_r^{(2)}V_2 = \sigma_r^o V\ ; \qquad (5)\\
&\sigma_\theta^{(1)}V_1 + \sigma_\theta^{(2)}V_2 = \sigma_\theta^o V\\
&e_2\sigma_r^{(2)} - e_1\sigma_r^{(1)} + e_4\sigma_\theta^{(2)} - e_3\sigma_\theta^{(1)} = 0\,;\\
&e_4\sigma_r^{(2)} - e_3\sigma_r^{(1)} + e_2\sigma_\theta^{(2)} - e_1\sigma_\theta^{(1)} = 0\,.
\end{aligned}$$

The following notations are adopted in these expressions:

$$e_k = a_{11}^{(m)} - \frac{\left[a_{13}^{(m)}\right]^2}{a_{33}^{(m)}}\ ;\ e_{k+2} = a_{12}^{(m)} - \frac{\left[a_{13}^{(m)}\right]^2}{a_{33}^{(m)}}\,. \qquad (6)$$

For an isotropic medium, the deformation coefficients in the above expressions are:

$$a_{13}^{(m)} = a_{12}^{(m)} = \frac{\mu^{(m)}}{E^{(m)}}\ ;\ a_{11}^{(m)} = a_{33}^{(m)} = \frac{1}{E^{(m)}}\,. \qquad (7)$$

The stresses in the equivalent generalised isotropic medium in expressions (3) and (5) have the form given in (Muskhelishvili, 1953; Bulichev, 1982; Parachkevov, 1969):

$$\begin{aligned}
&\sigma_\theta^o = -Q\left(\sigma_{\theta,1} - \sigma_{\theta,2}\cos 2\theta\right); \qquad (8)\\
&\sigma_r^o = -Q\left(\sigma_{r,1} + \sigma_{r,2}\cos 2\theta\right),
\end{aligned}$$

where:

$$\begin{aligned}
&\sigma_{r,1} = \lambda_1\left(1 - s^2\right);\ \sigma_{r,2} = \lambda_2\left(1 + 3s^4 - 4s^2\right);\\
&\sigma_{\theta,1} = \lambda_1\left(1 + s^2\right);\ \sigma_{\theta,2} = \lambda_2\left(1 + 3s^2\right);\\
&s = \frac{R}{r};\quad \lambda_1 = \frac{1+\lambda^{(o)}}{2};\quad \lambda_2 = \frac{1-\lambda^{(o)}}{2};\quad Q = \gamma^{(o)}H\ ;\\
&\lambda^{(o)} = \frac{\mu^{(o)}}{1-\mu^{(o)}}\,.
\end{aligned}$$

The solution of system (5) is performed by the Gaussian method (Petrova-Deneva et al., 1977). Thus, the stresses at a point in the rock mass are obtained. For practice, the stresses along the hole contour are of interest. Since the radial and tangential stresses are zero, system (5) takes the form:

$$\begin{aligned}
&\sigma_\theta^{(1)}V_1 + \sigma_\theta^{(2)}V_2 = \sigma_\theta^o V\,; \qquad (9)\\
&e_2\sigma_\theta^{(2)} - e_1\sigma_\theta^{(1)} = 0\,.
\end{aligned}$$

From the solution of (9), the stresses in the two layers are obtained:

$$\sigma_\theta^{(1)} = \frac{e_2 V}{\Delta}\sigma_\theta^{(o)};\ \sigma_\theta^{(2)} = \frac{e_1 V}{\Delta}\sigma_\theta^{(o)};$$

$$\Delta = V_1 e_2 + V_2 e_1\,. \qquad (10)$$

The tangential normal stresses in the generalised isotropic medium are obtained from (8) at $r = R$ and have the form:

$$\sigma_\theta^o = -Q\left(\sigma_{\theta,1} - \sigma_{\theta,2}\cos 2\theta\right). \qquad (11)$$

where:

$$\sigma_{\theta,1} = 2\lambda_1\,;\ \sigma_{\theta,2} = 4\lambda_2\,.$$

3. Numerical example

A horizontal circular opening with radius $R = 1{,}5m$ is considered. It is driven at a depth of $H = 300m$ and crosses two thin isotropic layers (Fig.1). The Young's modulus, Poison's ratio, and the bulk weights in the layers are given in Table 1. The sizes and volumes of the layers are given in Table 2. The characteristics of the equivalent uniform isotropic layer are obtained from equation (1). The results are shown in the last row of Table 1.

Table 1. *Characteristics of strata and homogeneous medium*

k	$E^{(k)}$	$\mu^{(k)}$	$\gamma^{(k)}$
multiplier	10^3		10^{-2}
Dimension	MPa		MN/m3
1	0.148	0.15	0.28
2	0.595	0.237	0.25
o	0.416	0.202	0.262

Table 2. *Sizes and volumes of layers*

i	l_1	l_2	V_i
Dimension	m	m	m3
1	3.6	5.0	1393.2
2	12.0	5.0	2754.0
$V \to$			5832.0

The normal tangential stresses in the uniform medium are obtained according to (11) for seven points of the first quadrant. This diagram is symmetrical about the vertical and horizontal axis of the circular section. The stresses in two layers $\sigma_\theta^{(1)}$ and $\sigma_\theta^{(2)}$ are determined according to (10). The results are referred to the stress in the undisturbed medium Q and are given in the third, fourth and fifth columns of Table 3.

Table 3. *Normal tangential stresses in the layers and in the equivalent uniform medium*

n	$\theta[\circ]$	$\sigma_\theta^{(o)}/Q$	$\sigma_\theta^{(1)}/Q$	$\sigma_\theta^{(2)}/Q$
1	0	-0.131	-0.045	-0.189
2	15	0.059	0.02	0.085
3	30	0.579	0.2	0.832
4	45	1.290	0.445	1.853
5	60	2.0	0.69	2.873
6	75	2.520	0.869	3.620
7	90	2.710	0.935	3.894

The normal tangential stress diagrams around the hole are plotted in Figure 4. Number 1 denotes the stress diagram in the uniform isotropic medium, numbers 2 and 3 – the stress diagrams in layer 1 and layer 2.

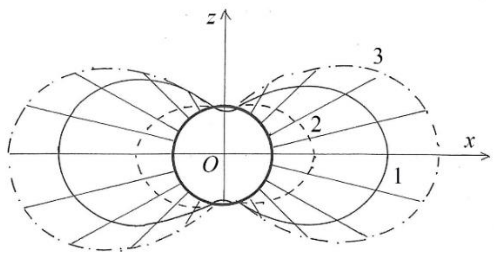

Fig.4 Diagrams of normal tangential stresses for the uniform medium and for layers 1 and 2

From the graphs obtained, the maximum difference between the stresses in the layers and in the uniform medium can be analysed. For layer 1, this difference is 65.4% smaller and for layer 2 – 30.4% larger. It can be seen from the table and figure that the stresses in layer 2 are greater than those in layer 1. This is related to the larger value of the modulus of linear deformation in the second layer. The ratio of the modules of linear deformation of the layers is 4.16.

4. Key findings

The obtained analytical expressions for the stresses in each layer are applied:

a) when the inclination of the boundary plane relative to the axis of the horizontal construction is greater than half a right angle;

b) when the ratio of the linear strain moduli is greater than 2.

Conclusion

The following guidelines for future work are established from the research done:

1. The approach for determining the stresses in an array consisting of two layers can be generalised for more layers.
2. The solution for the described class of problems can be applied to transversely isotropic layers.

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