# A Novel Approach to Computation of Multiple Geometric Series

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**Abstract:** Nowadays, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. In this article, a novel summation of geometric series is introduced for mathematical and computational applications.

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**Keywords:** computing, geometric progression, summation

## 1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management and its applications. In this article, a new geometric series [1-9] is constructed for application [10] of computational science and engineering.

## 2. Novel Geometric Series

The author of this article has already introduced the following geometric series in his previous papers [1,2,9].

$$\sum_{i=0}^{n-1} x^{pi} = \frac{x^{pn} - 1}{x^p - 1}, p \neq 0. \tag{1}$$

Now, let us construct the novel summations of multiple geometric series using (1).

**Lemma 2.1**: $\left(\sum_{i=0}^{p-1} x^i\right)\left(\sum_{j=0}^{n-1} x^{pj}\right) = \sum_{k=0}^{np-1} x^k = \frac{x^{np} - 1}{x - 1}, x \neq 1.$

*Proof.*

$$\left(\sum_{i=0}^{p-1} x^i\right)\left(\sum_{j=0}^{n-1} x^{pj}\right) = \left(\frac{x^p - 1}{x - 1}\right)\left(\frac{x^{np} - 1}{x^p - 1}\right) = \frac{x^{np} - 1}{x - 1}, x \neq 1. \tag{2}$$

$$\sum_{k=0}^{np-1} x^k = \frac{x^{np} - 1}{x - 1}, x \neq 1. \tag{3}$$

$$\therefore \left(\sum_{i=0}^{p-1} x^i\right)\left(\sum_{j=0}^{n-1} x^{pj}\right) = \sum_{k=0}^{np-1} x^k = \frac{x^{np} - 1}{x - 1}, x \neq 1. \tag{4}$$

**Lemma 2.2**: $\left(\sum_{\mathrm{i}=0}^{p_1-1} \mathrm{x}^{\mathrm{i}}\right)\left(\sum_{j=0}^{p_2-1} x^{jp_1}\right)\left(\sum_{k=0}^{p_3-1} x^{kp_1p_2}\right) = \sum_{l=0}^{p_1p_2p_3-1} x^l = \frac{x^{p_1p_2p_3}-1}{x-1}, x \neq 1.$

*Proof.*

$$\left(\sum_{\mathrm{i}=0}^{p_1-1} \mathrm{x}^{\mathrm{i}}\right)\left(\sum_{j=0}^{p_2-1} x^{jp_1}\right)\left(\sum_{k=0}^{p_3-1} x^{kp_1p_2}\right) = \left(\frac{x^{p_1}-1}{x-1}\right)\left(\frac{x^{p_1p_2}-1}{x^{p_1}-1}\right)\left(\frac{x^{p_1p_2p_3}-1}{x^{p_1p_2}-1}\right) = \frac{x^{p_1p_2p_3}-1}{x-1}.$$

$$\therefore \left(\sum_{\mathrm{i}=0}^{p_1-1} \mathrm{x}^{\mathrm{i}}\right)\left(\sum_{j=0}^{p_2-1} x^{jp_1}\right)\left(\sum_{k=0}^{p_3-1} x^{kp_1p_2}\right) = \sum_{l=0}^{p_1p_2p_3-1} x^l = \frac{x^{p_1p_2p_3}-1}{x-1}, x \neq 1.$$

**Corollary 2.1**:

$$\left(\sum_{\mathrm{i}=0}^{p_1-1} \mathrm{x}^{\mathrm{i}}\right)\left(\sum_{j=0}^{p_2-1} x^{jp_1}\right)\cdots\left(\sum_{k=0}^{p_n-1} x^{kp_1p_2\cdots p_n}\right) = \sum_{l=0}^{p_1p_2\cdots p_n-1} x^l = \frac{x^{p_1p_2\cdots p_n}-1}{x-1}, x \neq 1.$$

## 3. Conclusion

This article provides the novel geometric series and its summations for mathematical and computational applications. Also, this idea can enable the scientific researchers for further involvement in research and development.

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