THE PROOF: GOLDBACH'S CONJECTURE

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Abstract. In this paper, I want to present the proof to 'The Goldbach's Conjecture'. I have used fundamental concepts of number theory and proving techniques like contradiction to accomplish my goal.

1. Introduction

Conjecture:- *Every even natural number greater than 2 is the sum of two prime numbers.*
Author's Words:- I, Aniket Bhattacharjee, had been working on this proof for a few months when I came across the Goldbach function g(E) which is defined for all even integers $E > 2$ to be the number of different ways in which E can be expressed as the sum of two primes. This function helped me to prove the conjecture using fundamental concepts of prime numbers and number theory. I hope my efforts will serve as an aid for future generations.

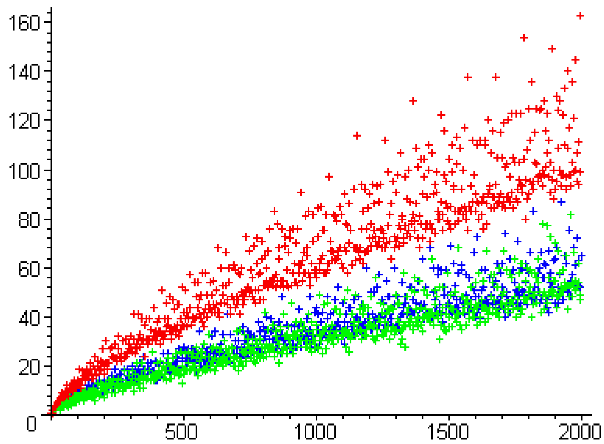

Figure 1. Plot of Goldbach function g(E) (Source:- Wikipedia)

Proof. Assuming the above mentioned conjecture to be false, there must be any even integer greater than 2 which cannot be expressed as the sum of 2 prime numbers.

Let the even integer be of the form 2m ($m \in N$, $m > 1$). Therefore, to show that our asumption is true, we need to show that the sum of 2 primes is not equal to atleast one value of $2m$:-

$$p + p' \neq 2m(p, p' \in prime) \tag{1}$$

- Prime numbers are both odd as well as even(i.e. 2) but as we know that sum of an odd and even number yields to odd number, we shall consider 2 cases.

Case I:- (Even Prime + Even Prime)

- Let $p, p' = 2$. (as 2 is the only even prime.)

$$p + p' = 2 + 2 = 4 = 2m \tag{2}$$

- As 4 is an even number, it is of the form $2m$.

Case II:- (Odd Prime + Odd Prime)

- **Any odd prime is of the form $4k \pm 1$. ($k \in N$)**

Proof. Let n be any odd prime number($n \in N$).If we divide any n by 4, we get, $n = 4q + r$ ($q, r \in Z$) where $0 \leq r < 4$ i.e., $r = 0, 1, 2, 3$. Clearly, 4n is never prime and $4n + 2 = 2(2n + 1)$ cannot be prime unless n=0 (since, 4 and 2 cannot be factors of an odd prime). Thus, An odd prime n is either of the form $4q + 1$ or $4q + 3$ (or $4q' - 1$ where $q' = q + 1$) i.e. it is of the form $4q \pm 1$.

- Let p = $4k \pm 1$ and p' = $4k' \pm 1(k, k' \in N)$.

  Therefore, when two odd primes are added, 4 cases can arise.

$$p + p' = 4k \pm 1 + 4k' \pm 1 \tag{3}$$

Case A:-

$$p + p' = 4k + 1 + 4k' + 1 = 2(2k + 2k' + 1) \tag{4}$$

Case B:-

$$p + p' = 4k - 1 + 4k' - 1 = 2(2k + 2k' - 1) \tag{5}$$

Case C:-

$$p + p' = 4k + 1 + 4k' - 1 = 2(2k + 2k') \tag{6}$$

Case D:-

$$p + p' = 4k - 1 + 4k' + 1 = 2(2k + 2k') \tag{7}$$

- In all the cases (both Case I and Case II), the outcome is always of the form 2m i.e. it is even (Contradiction) and thus no values of 2m exists which follows our assumption, thus our assumption was wrong. Therefore, every even natural number greater than 2 can be represented as the sum of two prime numbers.
- Thus,the above mentioned conjecture (Goldbach's conjecture) is proved.

□

References

[1] M. Burton, *Elementary Number Theory*, McGraw-Hill, 2010. [1]

[1]Date: September 1, 2023