Lesson Module:
Drawing the sky: the position of celestial bodies

Topic 1: Angles as a proxy for lengths

In our everyday life we often deal with the measure of length.
What if we need to deal with lengths not so easy to measure, as vertical length (=height)? Geometry can help us.

Let's consider a **right-angled triangle** (*Figure 1*).

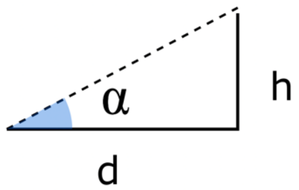


Figure 1: right-angled triangle

We notice that, if we double the measure of one of the catheses, keeping constant the angle α, the other cathetus will double as well (*Figure 2*).

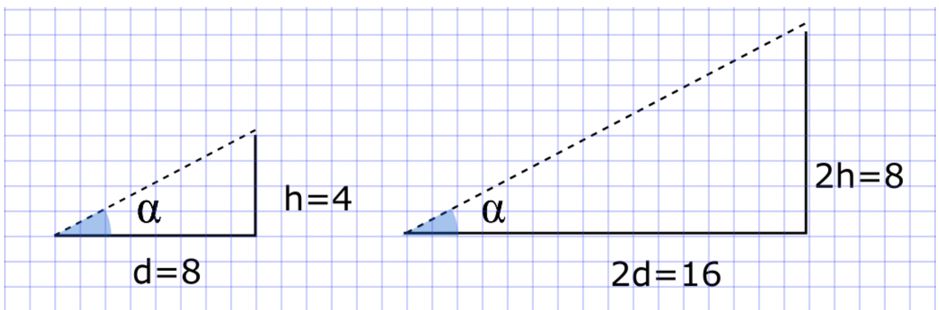


Figure 2: triangles in proportion

You can see that $\frac{h}{d} = \frac{2h}{2d} = \frac{4}{8} = \frac{8}{16} = 0{,}5$.

We can thus construct a series of triangles, all with the same angles but with different sides. We are anyway certain that the **ratio between the sides remains the same** (mathematicians call this ratio the *tangent* of the angle).

$$tan\ (\alpha)\ = \frac{h}{d}$$

We can use this proportionality to easily measure the height of objects, avoiding climbing on them.

Exercise 1a: How high is...?

- Choose the object to be measured and place yourself at a given distance from it. You could modelize the situation as in the figure below. The angle α is called the *view angle* (*Figure 3*).

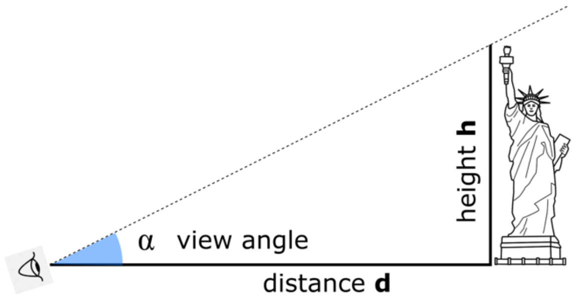


Figure 3: view angle

- First measure the distance d:

  Use a (roller) meter (*Figure 4*) to measure the distance directly or measure one step of a test person, stepping down the distance.

  d = ……………………………………

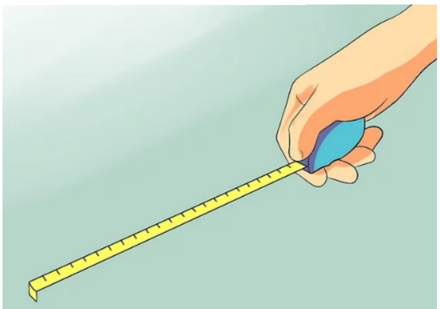


Figure 4: roller meter

- Using the *coordinates measurements device*, measure the *view angle* of of the chosen object (*Figure 5*).

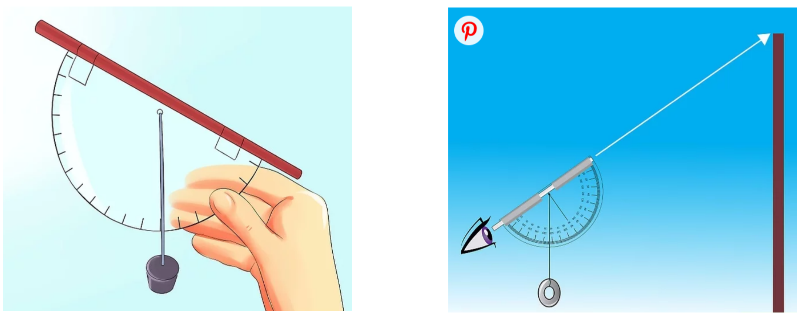


Figure 5 - Coordinate Measurement Device (paper version) and its use

- Now, using the grid here below (*Figure 6*), choose a convenient scale and report, accordingly to it, the measured distance from the object.

Figure 6 - space to reproduce and calculate the heigh of the chosen object

- Construct now a triangle with the **same view angle** 〈 that you measured (use a goniometer)
- Measure now on your drawing the vertical side and determine the height of the object.

  Conclusion: h = ……………………………………………

Tip: if you want to use a more direct method, you can remember by trigonometry that $tan\ \alpha = \frac{h}{d}$ and calculate the height using the relation $h = \alpha$ (you can use a scientific calculator to find the value of $tan\ tan\ \alpha$).

  Calculation: h = ……………………………………………

Exercise 1b: How far is…?

The same model can be inverted to measure, given the height of an object, how far you are from it.

- Choose the object to be measured, of which you want to know the distance.
- Measure the height of the object using a (roller) meter (*Figure 4*):

h = ……………………………………………

- Using the *coordinates measurements device*, measure the *view angle* of the chosen object (*Figure 5*).

  α = ……………………………………………

- Now, using the grid here below (*Figure 7*), choose a convenient scale and report accordingly to it, the measured height of the object.

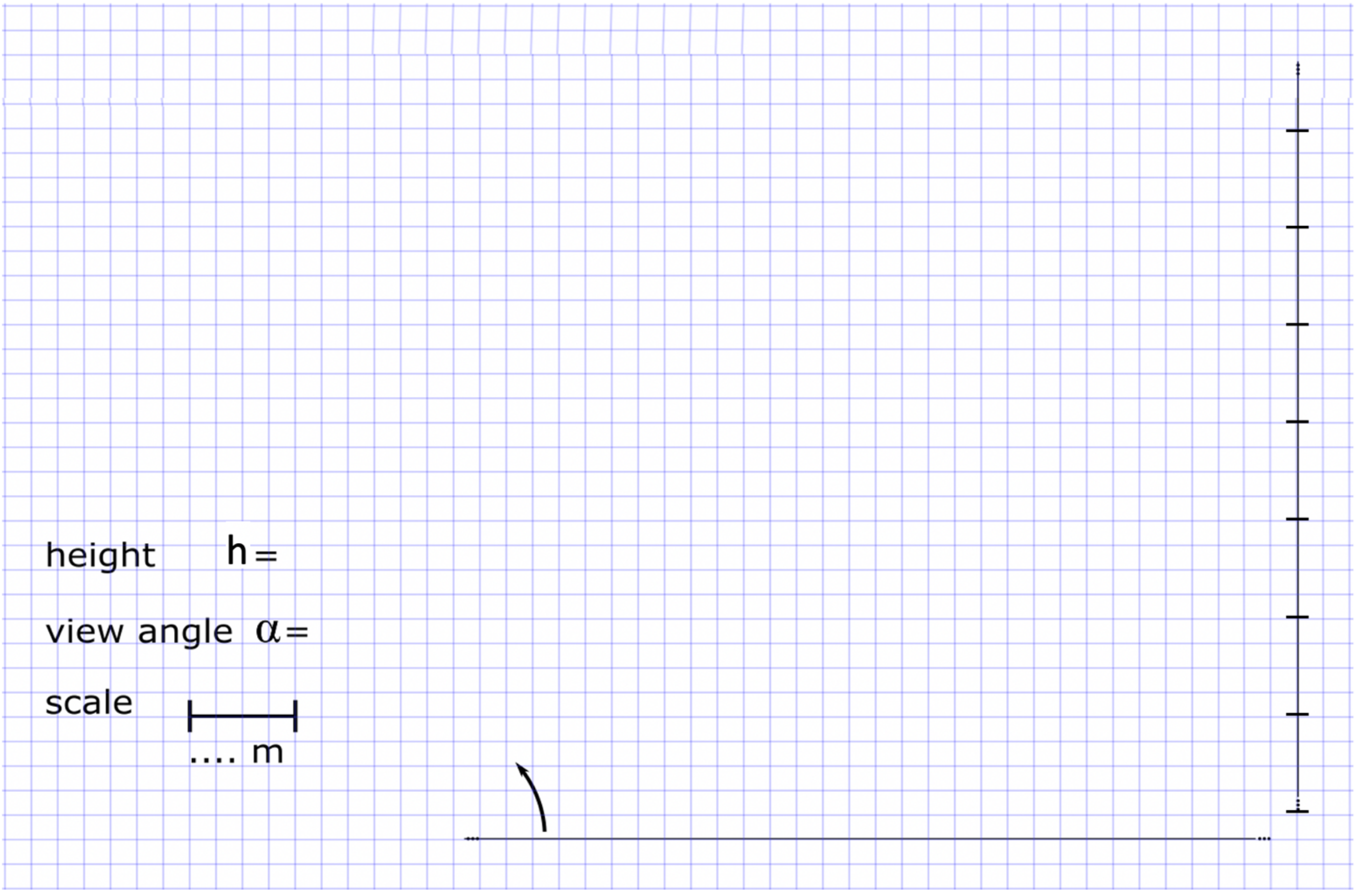


Figure 7 - Space to reproduce and calculate the distance of the chosen object

- Construct now a triangle with the **same view angle** 〈 that you measured (use a goniometer)
- Measure now on your drawing the horizontal side and rescale to determine the distance to the object.

  Conclusion: d = ……………………………………………

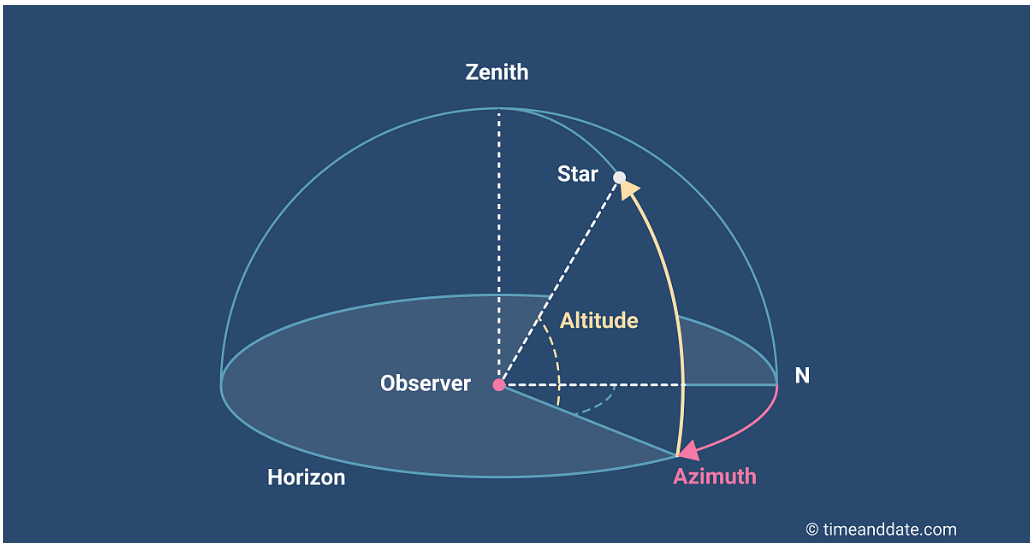
Again, If you want to use a more direct method, you can remember by trigonometry that $tan\ \alpha = \frac{h}{d}$ and calculate the distance using the relation $d = \frac{h}{tan\ \alpha}$.

Calculation: d = …………………………………………

Topic 2 - Angles as a measure for position: celestial coordinates

We know that, from a geocentric point of view, the celestial objects seem to be on a big, inconsistent sphere, and they appear to be moving on the surface of this sphere.
To identify position on this sphere, it is neither simple nor useful to use cartesian (i.e. plane) coordinates: they will be very difficult to measure!
Everything becomes easier if we use angles: as we are on a sphere, we will need two of them to define the position of any celestial object (*Figure 8*).
We call them:

- **HEIGH OR ALTITUDE (ALT.)**: the angle between the horizon and the object, *measured along a line perpendicular to the horizon.*
- **AZIMUTH (AZ.)**: the angle along the horizon, starting from *North and moving clockwise.*



Source: Time and Date https://www.timeanddate.com/astronomy/horizontal-coordinate-system.html

With the *coordinate measurement device* we can measure both.

Exercise 2a: Find your star

Are you able to locate the objects in the table below (*Table 1*), given the alt./az. (called ***altazimuthal*** or ***horizontal***) coordinates? Flag it when you find it!

Object	Azimuth	Altitude	Found
Polaris	0°		
Jupiter			
Algol			
MOON			

Table 1 – List of object to be found with altazimuthal coordinates

You can use the "StarWalk 2" app to verify your guess, taking the following steps:

- Hold your smartphone with your extended arm parallel to the ground in front of you
- Point your arm toward the cardinal direction North
- Now, turn on yourself clockwise of an angle equal to the azimuth of your object
- Lift your arm at an angle equal to the altitude of your object.

The object you are looking for will appear in the center of your screen!

Do you think that the altazimuthal coordinates remain fixed in time? Why?____________
____
____

Exercise 2b: Drawing the sky

Using this system we can draw a map of the sky.
We can use a different coordinate system, using two different number:

- the **RIGHT ASCENSION** (**RA**), measured in **hours, minutes and seconds**;
- the **DECLINATION** (**DEC**), measured in **degrees**.

We do not enter now into the definition of these two numbers, but you can easily find them in the "StarWalk 2" app or any other similar app.

Now try to locate the objects in the table below (*Table 2*), given their altazimuthal coordinates, as in Exercise 2a *(we use the star Deneb, in the asterism "Cignus", as an example, in Figure 9). Flag the name if you are able to find the object!*

Object	Azimuth	Altitude	Found
Deneb			√
Polaris	0°		
Vega			
Altair			
Arcturus			
Capella			
Rigil Kentaurus			
Canopus			
Pollux			
Sirius			
α Lacertae			
C/2022 E3 (ZTF)			

Table 2 – List of objects to be found with altazimuthal coordinates

Once you located the object with the altazimuthal system and the "StarWalk 2" app:

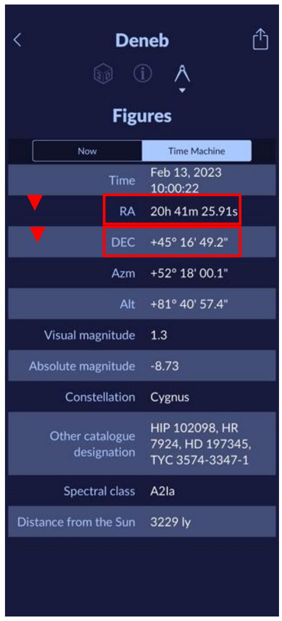
1. Click on the object

2. Click on the name that appears on the bottom of the screen;
3. Click on the compass icon that appears at the top of the screen
4. Read the **DEC** and **RA** numbers.

Step 1

Step 2

Step 3



Step 4

5. Following these coordinates, put the stars on the maps below (*Figure 10, 11*). Use different colors or symbols for stars that are above your horizon and under your horizon. P*ay attention to the* ***sign*** *of the DEC coordinate!*

What do you notice?____

____

____

Where is Polaris on the map?____

So which astronomical reference point is represented by the centers of the two maps? The centers of the maps represent the Celestial Poles

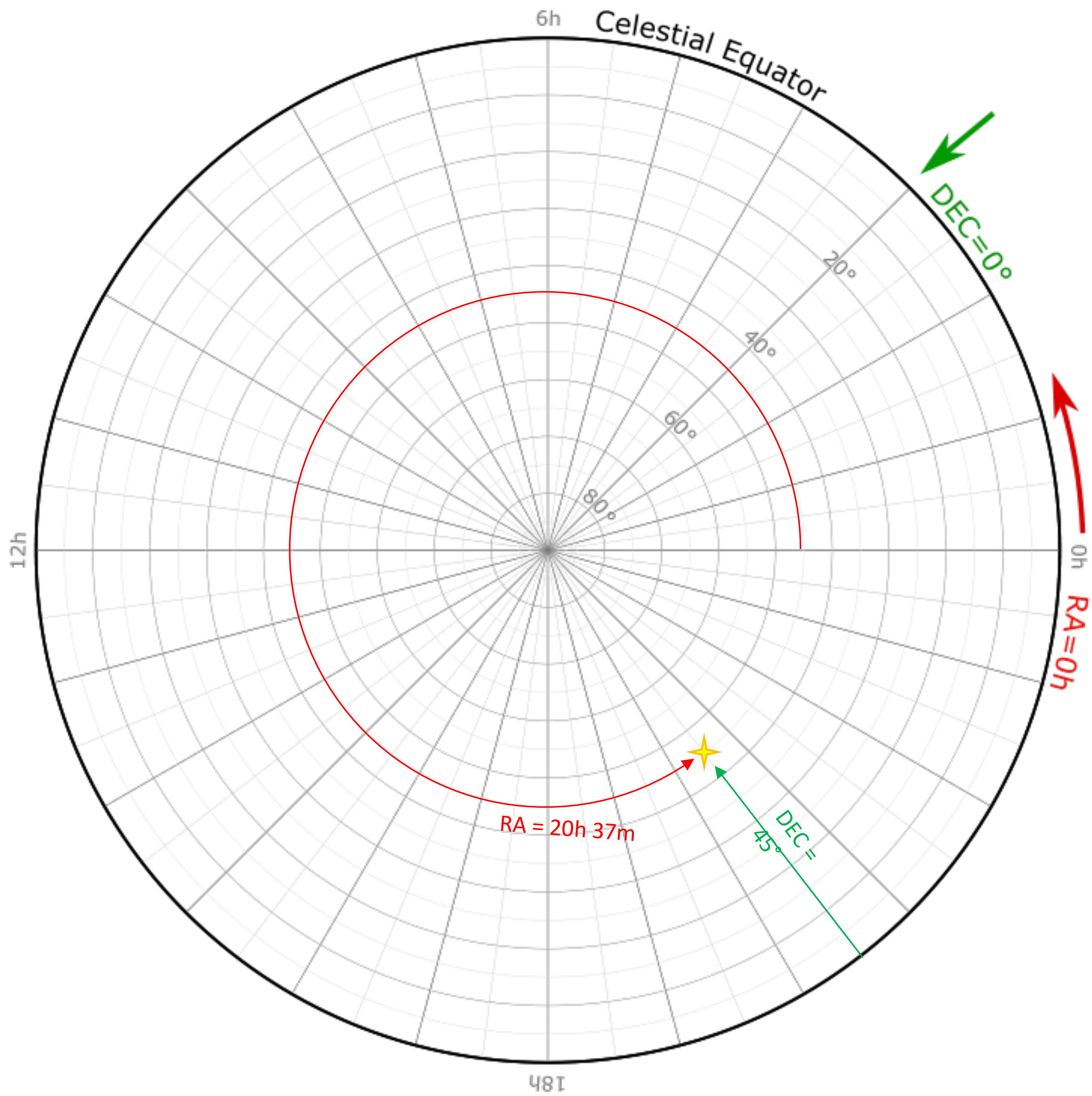
And which astronomical reference line is represented by the rim of the maps? The rim of each map represents the Celestial Equator

So how would you define the DEC coordinate for an object? The DEC coordinate is the angular distance of an object from the Celestial Equator, taken along a circle perpendicoular to the Celestial Equator itself

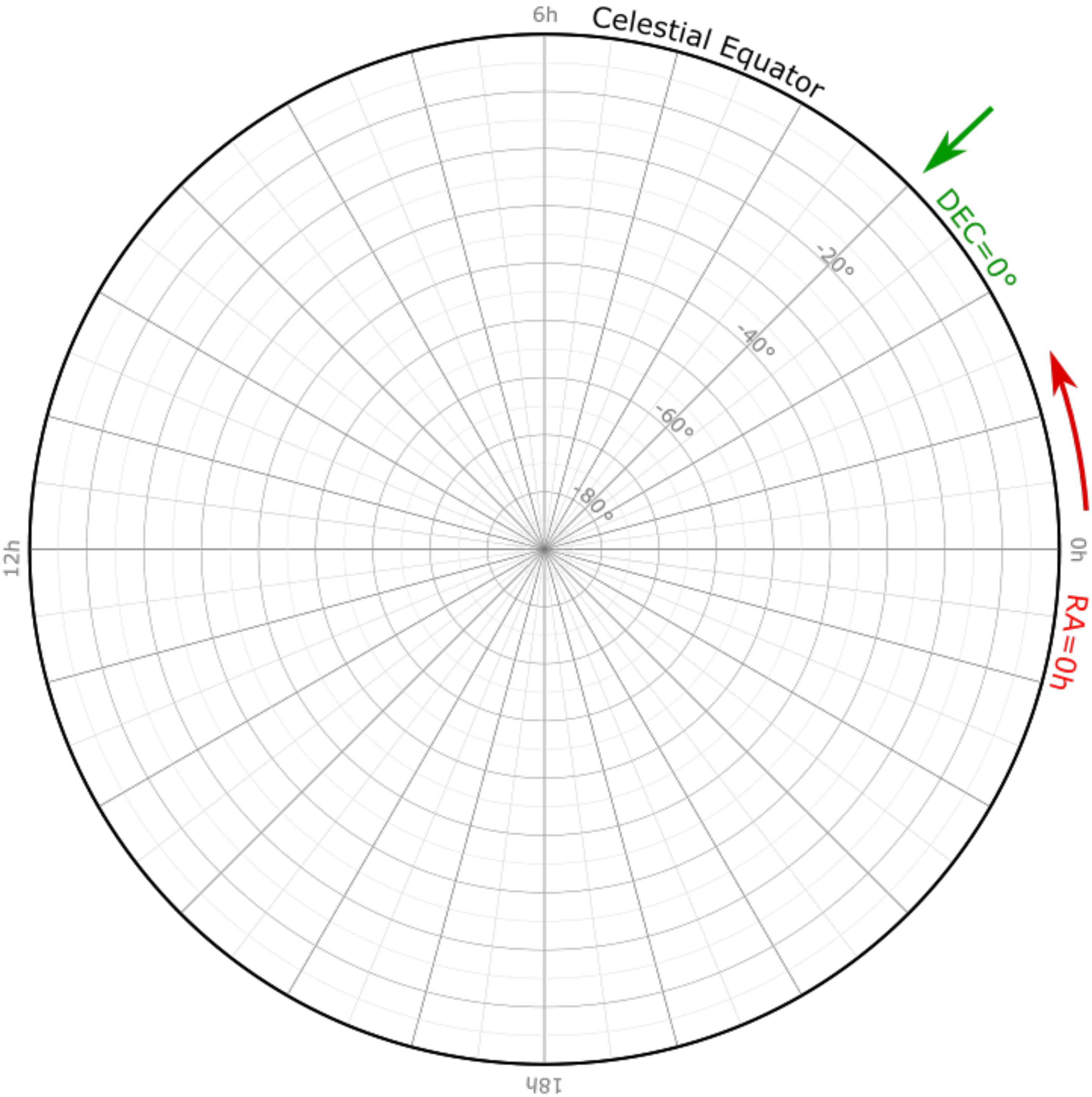
Do you think that, for the stars, the declination changes with time? Why? No, because the position of the stars with respect to the Celestial Equator does not change with time

Why are both stars from the Northern Celestial Equator and Southern Celestial Equator visible from our position?____Because our local horizon does not coincide with the Celestial Equator_

You can find more information about the definition of the equatorial coordinate in the Appendix 2. Now you are able to locate every object in the sky!



Northern Celestial Hemisphere

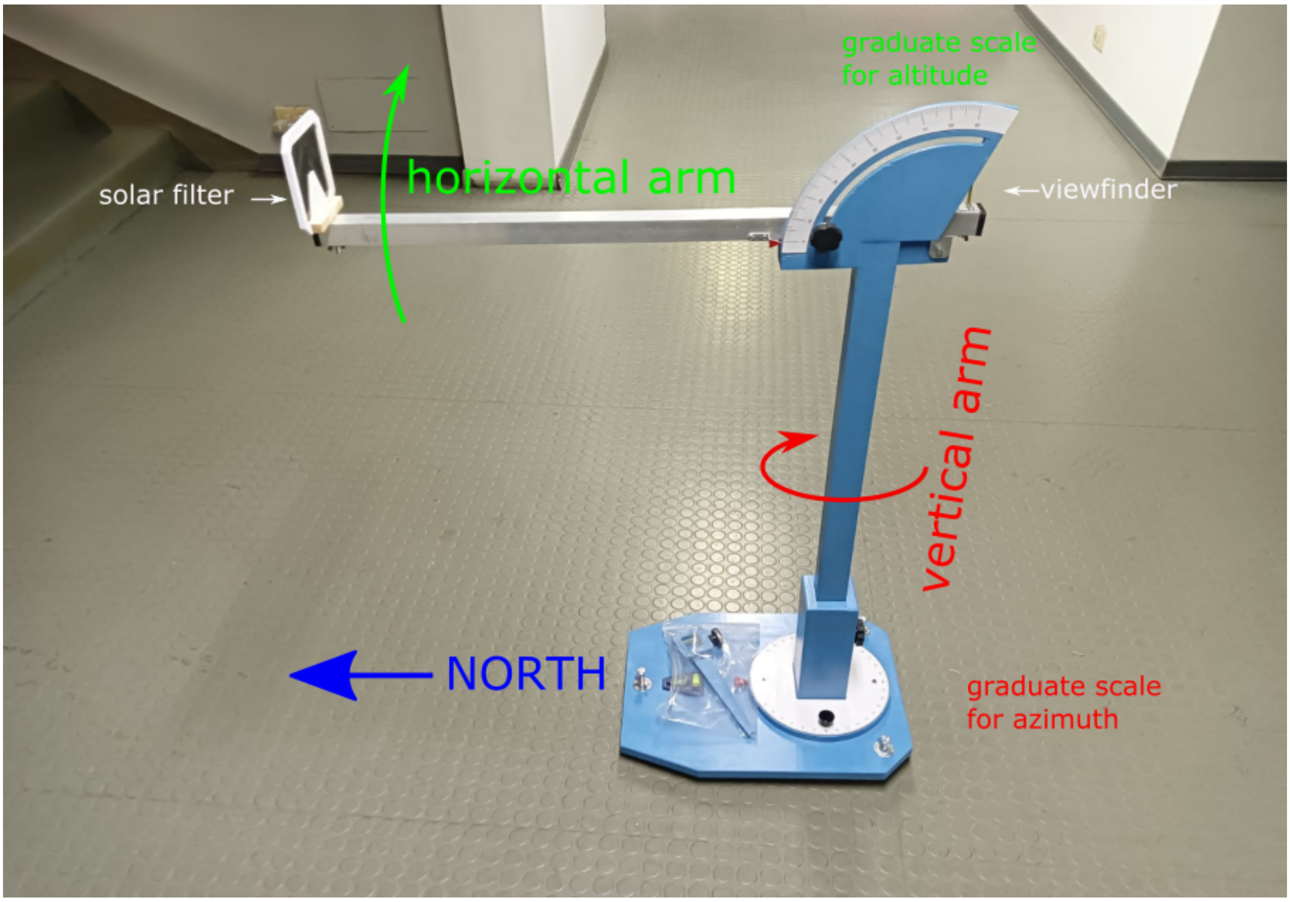


Southern Celestial Hemisphere

Appendix 1: the Coordinates Measurement Device

The **Coordinate Measurement Device** (or “CMD” for short) is an instrument that allows you to measure the *altazimuthal coordinates of an object* (both terrestrial or celestial).
It features:

- A *horizontal arm* which can be moved vertically along a graduate scale: on the scale you can read the *view angle* (for terrestrial objects) or the *altitude* (for celestial objects);
- A *vertical arm* which can be rotated around its axis, along a graduate scale: on this scale you can read the *azimuth* of the object.
- A *viewfinder*, where you have to place your eye to do the measure.



Note: to give the correct values, the CMD has to be placed:

- on a plane surface: this can be finely adjusted with the three screws on the base and checked with the bubble levels;
- with the horizontal arm pointed toward the NORTH direction (you can check it with a compass).

Appendix 2 : the Equatorial Coordinates System

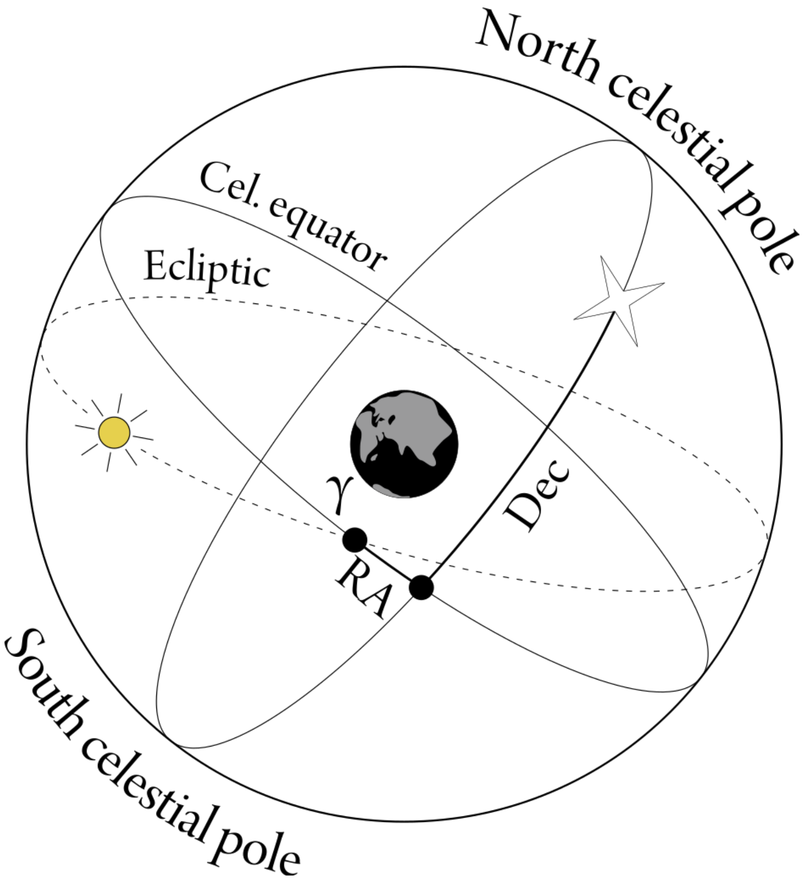
The coordinate system that you used for drawing, featuring the **DECLINATION** and **RIGHT ASCENSION** coordinates, is called the ***equatorial coordinates system.*** It is mostly used because it does not participate in Earth's rotation and it is not dependent on the observer position, so that the objects' coordinates do not change in time or according to the position of the observer.

The equatorial system is nevertheless less immediately visible than the horizontal (or altazimuthal) system. The two coordinates are defined as:

- **DECLINATION (DEC or δ)**: the angular distance of an object from the celestial Equator, measured along a perpendicular to the Equator. It is measured in degrees and can be positive (Northern Celestial Hemisphere) or negative (Southern Celestial Hemisphere);

- **RIGHT ASCENSION (RA)**: the angular distance of an object from the *vernal point (i.e. the intersection between the Celestial Equator and the Ecliptic corresponding to the spring equinox, also called "first point of Aries" or),* taken along the Celestial Equator anticlockwise. It is measured in hours, minutes and seconds and can span from 0h to 24h.

The equatorial system is the base for other celestial coordinate systems, each used in a particular situation.



Funded by the Erasmus+ Programme of the European Union

TASTE
Teaching Astronomy at Educational Level
(2020-1-IT02-KA201-079528)

These resources were created as part of the project TASTE (Teaching Astronomy at Educational Level), co-funded by the Erasmus+ Programme of the European Union, Erasmus+ project 2020-1-IT02-KA201-079528. All the TASTE resources can be found here: https://zenodo.org/communities/taste