Classical Stochasticity in Quantum Mechanics and Multiple Solutions to Pressure Balance

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We have argued in previous notes that quantum formalism emerges from the one dimensional photon reflection-refraction scenario which is readily observable in the classical world. What about the situation for a particle with rest mass? In previous notes, we have considered the case of a constant E,p and V1 which changes at xo to E,p2,V2 (all again constants with E remaining unchanged). This problem is then associated with an incident and reflected particle for x<xo and a particle with p2 for x>-xo.

In a classical world, however, how does one obtain reflection and refraction together? If there is a strong wall, a particle may reflect with -p and the same kinetic energyE, but it does not pass through the wall. If there is a ramp at xo leading to a horizontal floor a little higher than that for x<xo, then one has a conserved total E, but a new p2 and V2. The ramp, however, is not a sudden change at xo, but occurs over a range of x. As a result, it seems the quantum scenario, rather than leading to the classical case in a high energy limit (as in bound states), is associated with fluctuations and non-sharp boundaries. It is fine to have a sharp wall at x=xo, but in quantum mechanics, one has atoms with electrons moving in space according to probability distributions. Furthermore, there is movement due to temperature. One does not have a sharp edge or ramp scenario alone, but fluctuations which could lead to both. This suggests stochasticity at a point of discontinuity, but also stochasticity in the regions V1 and V2. In other words, V1, V2 and the sharp discontinuity points are all approximations (averages).

If one accepts the fluctuations at xo (ramp and sharp wall), then classically one might impose a pressure balance equation at an idealized xo, i.e.: AAp -BBp=p2CC where AA represents a positive number etc. AA represents the incident particle, BB the reflected and CC the particle which moves up the ramp. Nonrelativistically and in the photon case, the pressure balance equation is also equivalent to a conservation of particle number or energy equation. Thus one has two unknowns (if A is known), i.e. B and C, and one equation. Here, however, we wish to stress a different, but related issue. The pressure balance equation allows for multiple solutions because it only exists at xo. For example, BB=0 is a solution as is BB=.5, CC=.5. If p2 is almost equivalent to p, one might expect that BB=0. In other words, there is a specific solution which goes beyond pressure balance, i.e. here seems to be a p (momentum) biased solution. This forces one to go beyond classical mechanical ideas, even if one introduces a fluctuating wall-ramp scenario at xo. The fluctuating scenario suggests fluctuations in V1 and V2 as well, suggesting one may have some kind of dynamical equilibrium in these regions. Would this be enough to create a momentum bias, linked say to a continuity of a function of p,x and its first derivative at xo? This extra momentum bias seems to be the extra feature which emerges in a quantum treatment of the problem, which only occurs because one introduces fluctuations at the xo point. We suggest that a dynamical equilibrium approach solves the problem if one maximizes Shannon's entropy subject to a the constraint of a constant average classical action written as:

-P(t) iEt + ipx P(x) with a Lagrange multiplier of 1.

"i" is introduced to a have a periodic/dynamic probability, i.e. one that does not grow.

Thus we argue that quantum mechanics for a particle with rest mass emerges because a classical pressure balance equation for a fluctuating wall-ramp picture at a V1-V2 junction allows for multiple solutions. Ifr V1 is almost equal to V2, one has a biased or selective solution which should force BB to be 0. This may be done through continuity, but there must be a physical basis for the function used and we suggest that this is maximization of Shannon's entropy subject to an average classical action introduced to make the probability periodic due to the dynamical nature of the problem. Thus there is not just stochasticity at the discontinuity point xo, but throughout x.

Classical Mechanics

It is well-known in the classical world that a steady stream of light (photons) may reflect and refract at the same time. Classically, one may draw sharp boundaries, but in reality there are atoms which move due to temperature and have moving electrons. The photons interact with these atoms and so the sharp boundary is a simplification. In reality there is stochasticity at the interaction boundary.

What is the analogue of the reflection-refraction picture for a nonrelativistic particle with rest mass? It is the constant V1-V2 (potential) interface at x=xo with E (total energy being constant) for x<xo and x>-xo, i.e. one has E=pp/2m + V1 for x<xo and E=p2p2/2m + V2 for x>=xo.
In such a case, one may introduce a reflected particle with momentum -p as well as the particle with p2 moving in x>xo.

In classical physics, however, one does not seem to see the two occurring together. Reflection occurs if there is a sharp strong wall, while the p2 case may appear if there is a short ramp at xo which allows the particle to move from a horizontal floor to a higher floor. (In such a case, gravity is the potential involved.). One has one scenario or the other. How may one classically have both? One might consider a fluctuating-stochastic wall-ramp system with the xo point representing the midpoint of the x- range of the ramp. If the first floor range is say 100 km and the second also 100 km, a range of a few cm could be approximated by an xo point. Potential fluctuations could account for a reflected and refracted particle beam in a steady state picture.

Can this system then be solved classically? One might consider a wall-ramp fluctuation which gives equal weight to each, but what if one wishes to have pressure balance at xo?

In such a case, one has:

AAp - pBB = p2CC ((1)) Here AA,BB,CC represent the incident, reflected and refracted particle

AA simply shows that flux is positive. For the photon and nonrelativistic cases, ((1)) is immediately equivalent to a balance of probabilities AA/v= BB/v+CC/v2. One may notice that for AA=1, there are two unknowns BB,CC, but one equation. A related issue which we wish to stress here, is that ((1)) allows multiple solutions. For example,

BB=0 is a solution ((2a)) BB=.5, CC=.5 ((2b) is another solution

If V1 is almost equivalent to V2, then p and p2 are almost identical. One cannot have multiple solutions in such a situation, but only one solution with BB tending to 0. In other words, the pressure balance situation (which is classical) does not suffice to solve the problem, but is all one has from classical physics. It seems there is a second principle which occurs which is linked to p for x<xo being linked to p2 for x>xo. In other words, there is a kind of p bias in space. This may be achieved by introducing a function of p,x which is continuous together with its first derivative at xo. What physical reason, however, is there for such a solution, other than that it introduces the required p bias (instead of allowing BB and CC to change only at xo)?

We suggest that since the wall-ramp fluctuation exists, there should be fluctuations also in V1 and V2, as well. Thus one must deal with fluctuations throughout the whole range of x. We suggest then that one should seek an equilibrium situation, which applies to a dynamic particle, but handles all of space, not just discontinuity points.

Dynamic Equilibrium

Classical mechanics for a free particle is based on varying the classical action:

A= .5mvv t (nonrelativistic) A=-.5mo sqrt(1-vv/cc) relativistic) ((3))

If v=x/t, are: A= -Et+px ((4))

In a statistical picture, one might imagine t and x fluctuating separately such that A average is constant,iI.e. one may use the constraint: -E tP(t) +p x P(x). Maximizing Shannon's entropy:

-P(t)ln(P(t)) + P(x)ln(P(x)) subject to this constraint (multiplied by i to make it periodic (dynamic) so it does not grow) yields

P(t)=exp(-iEt) and P(x)=exp(ipx) ((5))

exp(ip) has the p bias one requires at a momentum discontinuity. It is a dynamic probability and exists in all of x space. One may consider its continuity (and the first derivative) at an xo momentum discontinuity point. As a result, a system in which there are fluctuations may require a dynamic probability for its description, rather than simply a classical flux AA value and a constant p value. At discontinuity points, one may use coefficients in front of exp(ipx) to solve the problem. It is interesting to note that p is deterministic due to energy constraints, but the coefficient (related to flux) may change). Thus the continuity function uses p which is also a physical observable linked with impulse (which is different from pressure). The two continuity equations, however, reproduce the classical pressure balance equation, i.e. one takes the complex conjugate of one and multiplies it by the other.

Discussion and Conclusion

It seems that taking a classical system and introducing fluctuations everywhere, including fluctuations at an idealized xo point, involving wall-ramp fluctuations, is not solvable using a classical pressure balance equation (which nevertheless holds). There is an extra physical idea which is required in this combined reflection-refraction situation, namely one which introduces a momentum bias. We suggest that one may introduce the idea of maximization of Shannon's entropy subject to the constraint of an average classical action, to obtain a dynamic probability which governs all of space and may be also applied at discontinuity points.Thus there is one consistent principle. This extra equilibrium physics leads to quantum mechanics for a particle with rest mass, we argue, but also reproduces the classical pressure balance equation