Exploring the Duality of Space and Place through Formal Geo-Concepts

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Places are initially perceived through the senses, and the fidelity between these senses and their mental representations is imperfect. At the same time, it is impossible to know what platial memories a place name may evoke in the recipient. In contrast, spaces can be reasoned about, but encompass endless continua that cannot be fully comprehended. In this paper, we explore the thesis that the interplay between spaces and place representations parallels the duality between extent, the things a class ranges over, and intent, the range of descriptive qualities of a class. We discuss how a duality may manifest between spaces and representations of senses of places. In doing so, we use definitions from the mathematical basis of Formal Concept Analysis and we introduce the notion of a geo-concept, which is a matching of a space and a place representation. We conclude with a short outlook on implications and directions for future work.

Keywords: place description; spatial information; duality; geographical information; Formal Concept Analysis; geo-concept

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1 Introduction

The recent availability of rich data about places has reinvigorated interest in a problem troubling many geo-information (GI) scientists: what is a place? The problem is particularly prominent because geodata contributed through social media and other sources are predominantly *platial*, while GI-systems are designed to work primarily with spatial data. Translation between platial and spatial data is, to the very least, non-trivial: many places, such as the best spot on the beach or the territory of a nation, have inherently fuzzy and subjective geometric qualities, and for many spaces it is impossible to describe in full the places over which they extend.

The lack of success in integrating place into existing GI-systems – which, due to their dominant focus on spatiality, may as well be called spatial information (SI) systems – has motivated some scholars to pursue a whole new branch of GI, namely that of platial information (PI) systems (Goodchild, 2011; Mocnik, 2022; Purves et al., 2019). The idea is that instead of requiring that place be confined to space, places are first and foremost understood through qualitative representations, such as place names and place descriptions. However, this merely swaps the roles of the initial problem's two participants. How should spaces then be represented in a PI-system? There is a need for an information system that consolidates both the platial and the spatial aspects of geo-information, without one among place and space being subordinate to the other. In order to achieve this, the essence of the relation between representations of place and space must first be understood.

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There seems to be an intrinsic trade-off between place and space. Whenever a place description, which we interpret as being the set, sum, or concatenation of platial qualities, becomes more precise, there are fewer spaces that meet the description and, whenever a space is expanded, there are fewer place descriptions that can truthfully label it. In this essay, we consider the thesis of an order-reversing duality between place representations and spaces. More precisely, we explore whether and how the relation between place representations and spaces corresponds with the duality between formal notions of extent and intent, where extent is that which a class – a collection of things with some aspect or rule of sameness – ranges over, and intent is that which a class is characterized by. This place–space correspondence can be exploited to develop a sophisticated form of GI-systems, composed of concurrently-operable SI and PI systems.

We first expand upon our view of extents and intents, which we base on how they are understood in the theory of formal concept analysis (FCA; Wille, 1992). We do so because FCA offers a precise interpretation of what extents and intents are and an explicit formalization of the relation between them. We then outline our preliminary view on the things that are presented to subjects as place and space and we consider the interplay between extent and intent in conjunction with representations of places and spaces. We end by reflecting on practical implications of our arguments.

During these exercises, we touch on existing literature and philosophical ideas, but we leave a thorough review of the literature, a formalization of our proposed theory, and scrutiny of the latter using the former for future work. Also, we provide examples for many of our statements, but we do not make use of one overarching example. We found that, because of the topic's high level of abstraction, a single comprehensive example could easily be misinterpreted and lead to confusion. Therefore, we opted for multiple smaller examples, which allow to approach the subject matter from multiple viewpoints and thereby a triangulation of the relation between place representations and spaces. Finally, we do not mean to make any definitive claims about places themselves. We are interested in the relations between their representations and spaces, not in their sources. We acknowledge that place and sense of place may not be the same thing, but when we speak of a representation of place, we generally mean a symbol by which a sense of place is represented in communication. The reader may notice that we generally do not mention representations of spaces. This is because we feel that we do not need to represent space in order to specify a duality between notions of space and place. Rather, we claim there is a duality between spaces themselves and qualitative descriptions of places.

2 Conceptualizing Space and Place

The notions of extent and intent are well-established in logic as what a term designates and what it means, respectively. A quintessential example is how the terms *morning star* and *evening star* both designate Venus, even though the terms have different meanings. Extent and intent are generally used as metalogical terms to describe aspects of logic. For instance, extensional and intensional definition are two opposing approaches to imbuing meaning into terms that work by means of, respectively, a list of examples and a list of necessary and sufficient conditions.

In FCA, extents and intents are formalized as the two constituents of concepts. FCA was introduced by Wille (1992) as a structural framework for developing conceptual hierarchies. Through the specification of ordered pairs called *incidences*, a *context* emerges. This context is formalized as a structure (G, M, I), where the incidence relation I is a subset of $G \times M$. Elements in G are referred to as *objects* and elements in M as *attributes*. It should therefore be noted that these terms have specific meaning in relation to FCA. We only intend to refer to notions of FCA if it is clear from context like in this subsection or if we signify them by adding the prefix 'FCA'. For example, an 'object' is not necessarily a 'FCA-object', unless we specify otherwise. Extents are subsets $A \subseteq G$ and intents are subsets $B \subseteq M$. The use of the so-called *derivation* operators $\uparrow$ and $\downarrow$ makes possible to derive extents from intents, and vice versa:

$$A\uparrow = \{m \in M \mid \forall g \in A\colon gIm\} \qquad \text{(extent to intent)}$$
$$B\downarrow = \{g \in G \mid \forall m \in B\colon gIm\} \qquad \text{(intent to extent)}$$

Together, $\uparrow$ and $\downarrow$ form a Galois connection, i.e., $x\uparrow \leq y$ if and only if $x \leq y\downarrow$, where $\leq$ denotes a partial ordering relation. Therefore, it is postulated that if an extent and an intent derive each other, they form a *concept*. Furthermore, an order is formed through subset relations. If one concept's extent is a subset of another concept's extent and if one concept's intent is a superset of another concept's intent, the former concept may be called a sub-concept of the latter concept, which then is also a super-concept of the former. This sub-concept relation forms a lattice, and more specifically a concept lattice. A key property of a lattice is duality between its two operators $\wedge$ and $\vee$. Formally, this means that the two partial orders are dually-isomorphic. Informally, it means that the structure can be flipped and flipped back, i.e., the order relation can be reversed without loss of information (for more on lattice theory and FCA, see Davey and Priestley, 2002).

FCA is a contextualist framework, meaning it is operated on the supposition that any and all knowledge is context-dependent. Depending on which aspects are taken into account, concepts come into and go out of being. It is also structure-oriented in that symbolic meaning is ignored and conceptual differences are explained in terms of relational discrepancies. Finally, the framework does not allow negation of the incidence relation, which means there can be truth and lack of truth, but no falsehood. FCA is used in a variety of fields, such as ontology engineering, data mining, and chemistry (Poelmans et al., 2013a). Also, multiple extensions have been introduced, such as fuzzy FCA, relational FCA, and pattern structures (PS; Poelmans et al., 2013b). Furthermore, a variety of lattice generation algorithms have been developed, predominantly versions of the InClose and Close-by-One algorithms (Konecny and Krajča, 2021).

In the following subsections, we first draw a parallel between extents and representations of places. Specifically, we argue that representations of place are generalized, which means platial reasoning is comparable to the derivation of intents from extents. We then equate intents to spaces. We argue that spatial partitioning is similar to the derivation of extents from intents. Finally, we consider the correspondence between place representations and spaces, observing that a more precise description of places is accompanied by a reduced list of instances that have these qualities and that, as the list of instances grows, the list of descriptions that apply to all of them shrinks.

2.1 Place and Its Representations

In this subsection we make the case that there is a resemblance between representations of places and extents. According to a popular school of thought, places appear through the senses (Tuan, 1979). Beyond spatial awareness, the overall sense of place connects to some of a place's affordances and restrictions (Jordan et al., 1998), i.e., the ways in which the subject believes they may or may not interact with a place, and some of its memory cues, those things that cause memories of past experiences to resurface. This is at least in part why sense of place is subjective; it manifests in perception dependent on the subject's potential agency and experiential history.

It should be noted that memories of places may also trigger sensation (Tuan, 1979). After all, the memory of a place may bring along past feelings or evoke entirely new ones, like how a tranquil forest in the mind's eye may calm the stressed office worker or how the adult may feel bittersweet melancholy when reminiscing about their childhood home. Evidently, humans have mental faculties that may induce experiential imagination. Even though an imaginative place may be distinguishable from an embodied, real place, the notion of sense of place extends over both the real and the imaginative.

The senses are understood through feelings, but feelings are intangible and ineffable; it is impossible to hold onto them at will or capture their full meaning in the web of language. At most, one may train themselves to respond to their appearance and use labels to signify them. If a mixture of feelings is closest to archetypal happiness, it may be called 'happiness', even if by doing so a slight sense of, e.g., regret slips out of scope. Because of this, where a sense of place only partly represents the place itself, the qualities by which this sense could be described only partly represent this sense of place. In actuality, to say a place is beautiful is to express that a subjective sense of the place has the qualities of a subjective 'beautiful'. Thus, after the initial labeling of the sense of place, there is a trade-off between the fidelity in communication about place between subjects, and the accuracy in the qualification of a sense of place within a subject; general statements more easily find agreement between subjects and specific statements represent more precisely the subject's particular sense of place. Two subjects may agree that a place is beautiful, even when the first is inspired by the lush greenery and the other is moved by the rustic architecture.

In the relation between representations of senses of places and their descriptions, the former act as extents of the latter and the latter as intents of the former. Given a representation of a place, there is a list of qualities which describe it, and given a single quality, there is a list of representations of places that possess it. We can thus see that qualification of representations of senses of places can be understood as a kind of derivation from extents to intents.

2.2 Space

In this subsection we make the case that there is a resemblance between spaces and intents. Across disciplines, spaces are understood as continua. While in physical reality the continuum is generally assumed to spread across three dimensions, abstract spaces may have fewer (e.g., a two-dimensional cartographic map), more (e.g., in four-dimensional space-time or quantum mechanical state space), or none at all (e.g., function spaces and topological spaces). Regardless of their dimensionality and other axiomatic and theoretic properties, all these models have in common that they appear through relation between objects. Vector space permits notions of magnitude and orientation only with respect to an origin, and metric space centres around the distances between its points. In short, spaces are models formed through relation.

Representations of phenomena can be framed through the emergence of spaces. A cartographic map can be produced not because its underlying model appears at an instant to the mind's eye, but because multiple partial representations can be related to one another one-by-one to construct a greater whole. To say some place x is south of another place y is to express that when starting at y and moving directly away from the North Pole one may reach x. These relational systems may start off simple but can be expanded in principled manners to give rise to more sophisticated reference systems that may even enable quantification. This is, e.g., how metric space gains meaning: due to the combination of how some initial configuration is organized and how a set of axiomatic constraints expand this configuration, a function from all pairs of objects to a set of distances can be defined. Dimensions can also be understood in such terms, since they are generalizations of orders between spatial objects. For instance, the altitude dimension generalizes over the order of things by proximity to the Earth's surface. Axiomatic rules can be applied to expand models of empirical observations. For example, we can apply transitive induction to determine that if x is above y and y is above z, then x should be above z. According to some, places are mereological (Gilmore, 2018). Generally, this means they have a supplemented part-whole relation, which may, e.g., mean that if two spatial regions are disjointed, then they must be parts of a larger region.

So far we have considered the theoretical aspect of spaces, but the emergence of these models may be closely tied to phenomenal reality. For example, one may find that, when interacting with their surroundings, there are two glasses on a table, which may each hold up to 250ml of liquid and each seems half-filled with wine. From this it could be deduced that there must be approximately 250ml of wine on top of the table; a cognitive model of the table's space with an object of 250ml of wine is formed. However, perhaps a third half-filled glass on the table was obscured from view, or after a taste test, one of the glasses turned out to actually contain grape juice. From this, the observer may conclude that the methods they used to rationalize their senses were invalid. They may then adjust their model accordingly, and test again. By doing so, the observer can determine whether their spatial model extends over the specific things they wish to reason about (cf., Scheider and Richter, 2023).

In the relation between spaces and the objects that inhabit them, the first can be intent-to-extent derived from the second, and the second can be extent-to-intent derived from the first. A single space can be partitioned in possibly infinite ways. Conversely, a single object could be incorporated into multiple spatial models. We can thus see that partition of spaces can be understood as a kind of derivation from intents to extents.

2.3 Geo-Concepts

In Section 2.1 we argued that qualifications of places are derivations of intents from extents, and in Section 2.2 we suggested that partitions of spaces are derivations of extents from intents. Of course, the direction of derivation could just be a consequence of whether one were to partition or qualify; we would certainly argue for this to be the case. However, more importantly, it seems that reasoning about places necessitates qualification and the specification of space necessitates partition. This becomes salient when we try to do the opposite: partition of place representations and qualification of space.

Upon partitioning a place description one will find that the parts of a description are less informative than the whole, and therefore apply to more things. The parts of a 'comfortable and affordable' place are 'comfortable' and 'affordable'. However, not all comfortable places are affordable and not all affordable places are comfortable, meaning the set of both comfortable and affordable places is a subset of those that are at least one of the two. In other words, the partitioning of the description would relate to more place instances, not less. We may also consider the place representation itself. Sense of place is instantaneous; each sense of place occurs at an instant. This means the representation of place is discrete because the instance cannot be further specified. It can be argued that instances may share parthood relations, but it is impossible to disentangle these relations at an instant, and even then it may not be enough, since it can be argued that the whole is more than the sum of its parts.

In contrast, spaces are constructed models and their provenances are – or at least can be – theoretically explicit. In other words, we can identify the relations that configure them. However, to qualify them is to ignore their inner structure and view them from a metastructural level. For instance, if a topological space was described as having a distance function, then it would be specified to a metric space. However, a richer description of a space does not change the space's inner structure. If a space is qualified as 'transparent', then everything in it could also be qualified as 'transparent'; after all, if the space would contain opaque things, it would no longer be a transparent whole. This means transparency is not an effective way of discriminating some parts of space from other parts. A qualification of space may put it in contrast to things outside of that space. If an object outside of the transparent space is opaque, we may use opacity to distinguish the space and its parts from the external object. However, this is in general not the purpose of spatial reasoning, which is applied for understanding relations within space, not between spaces.

According to the philosophy of FCA, a pair of an extent and intent may together compose a formal structure called a concept. If we let a place representation be an extent and a space be an intent and – within a given context – the space derives the place representation and the place representation derives the space, we have a concept that is both platial and spatial. For the rest of the discussion, we shall call such concepts *geo-concepts*. If we have a context from which multiple geo-concepts can be generated, we may find that these are partially ordered: some geo-concepts are sub-concepts of other geo-concepts. For example, in some contexts the geo-concept of the province of Utrecht could be considered a sub-concept of the geo-concept of the Netherlands.

However, we find that geo-concepts can come in two forms, namely ones that are either more spatial and less platial, or more platial and less spatial. We shall call these respectively *spatial geo-concepts* and *platial geo-concepts*. A spatial geo-concept is one that has a minimal place representation and a maximal space. This means all qualities to specify the geo-concept's place also apply to all mereological parts of that place. For example, any mereological part of a spatial geo-concept with the FCA-attribute 'prohibited' is also prohibited. We can thus say that not just the overall place but the entire space is prohibited. Conversely, a platial geo-concept is one with a maximal place representation and a minimal space. For instance, if a geo-concept of Paris has the Eiffel tower and Champs-Élysées as qualities, then it excludes all parts of Paris that do not contain these two landmarks. If one were to list all features of Paris, then they would find that only one object in space contains all these features, namely the place of Paris itself. Note, however, that a spatial geo-concept of Paris can also be defined – even in the same context – meaning that Paris can be conceptualized from both spatial and platial points of view.

A duality exists between partitions of space and descriptions of place. If a place description x and a space y form one geo-concept and a place description a and a space b form another, then the description of x yields a if and only if the partition of b yields y. If we let description and partition be represented by respectively the derivation operators $\uparrow$ and $\downarrow$, then we rediscover the Galois connection, i.e., $x\uparrow = a$ if and only if $y = b\downarrow$. In short, we find a co-dependence between place representations and spaces in concept lattices. In a lattice of geo-concepts, for every place there is a space, and for every space there is a place. These spaces and places can be observed or just assumed to exist. In our earlier example of a platial geo-concept of Paris, the perfectly-accurate description of Paris is practically impossible to formulate, so in practice, upon observing a spatial extent of Paris, the existence of a fitting place description is assumed rather than specified. Also, if a place representation of Paris is provided, assuming a fuzzy geometry may often already be enough for effective communication.

Because of the apparent importance of correspondence between space and place description for the formation of geo-concepts, it is worthwhile to consider under what conditions this correspondence may become manifest. In other words, when does a place representation derive a space that in return again derives the place representation? Or, when does a space derive a place description that in turn derives the initial space? These questions touch upon deep philosophical discussions and relate to dichotomies such as phenomenal versus noumenal, becoming versus being, induction versus deduction, and existence versus essence. However, we limit ourselves to a postulate that if the empirical place and the theoretical space are in agreement, then they amount to a geo-concept. We leave a philosophical grounding of the manifestation of geo-concepts out of scope.

3 Discussion and Conclusion

Geo-analysis requires both options of viewing the data as representations of places and as spaces. In some cases, data should be viewed platially (e.g., to count the number of administrative regions in a country) and sometimes spatially (e.g., to measure the size of each administrative region in a country), but their consolidation has proven difficult. Starting with FCA as a useful framework, where extents and intents form concepts, we have found that place representations and spaces can be combined in geo-concepts. These geo-concepts form a lattice, in which it becomes apparent that the description of place representations and the partition of spaces share a duality, meaning space and place representations are connected in a meaningful way.

The geo-concept seems to draw a bridge between two seemingly opposing views. On one side there are platial traditions, whose adherents are interested in the subjective experience of place, and on the other there are spatial traditions, which are more focussed on the configuration of terrestrial things. Bridging between these two paradigms has seemed – and still seems – like a near impossible task and the emergence of PI-systems that oppose SI-systems is evidence that the paradigms remain disjointed. However, the two views may be harmonized in geo-concepts. The challenge is not to determine whether spatial or platial views prevail for certain research problems, but rather how they should be combined to convert the empirical and the theoretical into conceptual knowledge. PI and SI-systems can be designed such that they work in concurrence, meaning that connections are made between place representations in PI and spaces in SI such that operation on one system informs what changes should be made in another. This could take shape as SI-systems being akin to contemporary GI-systems and PI-systems to classification structures. A result could be that a spatial intersection of two regions implies the assignment of attributes of the places associated to both regions to the intersection value. In other words, it would be possible to automatically derive place classes during spatial operations. Related to this, geometries may not be necessary to represent places. Rather, places could be related to spaces, from which geometric properties can be derived. These properties can be considered as qualities of places, although not essential to them.

After we simplified our notions of the concepts to this level, we realized that the polarity between space and place greatly resembles a dichotomy that has permeated GI-science for decades, namely that of objects versus fields, where objects are discrete spatial entities that may be counted and fields are spatially-continuous functions (Couclelis, 1992). Although the correspondences between sense of place representations and GI-objects, and between spaces and GI-fields is its entire own discussion to be had, there indubitably are commonalities. For example, both objects and places are discrete, while both fields and spaces are continuous. It is possible that our FCA-based approach to understanding space and place may also be applied to understand how objects and fields relate.

This work is still preliminary. We may have oversimplified the notions of space and place, and consequently overgeneralized some of our observations. For example, we mention mereological partitions, but it is not true that all spaces are endowed with a mereology, and we do not consider if and how mereology and FCA correspond. Our goal is not to be meticulous in our formalization, but to share an idea that there may exist a clear mathematical structure for the relation between space and place.

Future work may focus on the philosophical grounding of our interpretations of space and place and on a more rigorous formalization of the concepts. In particular, questions remain about the precedence of space and place (which one comes first?) and the definition of space (what kind of spaces are relatable to places?). Another direction is to provide a proof-of-concept. As of yet, the ideas are only hypothesized, but not implemented in FCA. Finally, it may be interesting to study human cognition to find whether a notion of geo-concepts is used in human reasoning.

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