

Implications of Steady Stream Probabilities on Quantum Mechanics

Francesco R. Ruggeri Hanwell, N.B. Aug. 25, 2023

Steady state scenarios exist in classical physics and even though there is motion (flows) (i.e. changes in time) overall the system does not change in time. Such systems are usually not described in terms of probability. Furthermore, a single particle moving in one direction is described by $x(t)$, i.e. $x=vt$, so there is no probability and no steady stream scenario. Such a particle, however, may give rise to a steady stream scenario if it encounters a V_1 - V_2 discontinuity (V_1 , V_2 are two constant potentials) and E (energy) remains constant. In such a case, there is a probability to reflect or to pass into the V_2 region with a different p , but the same E . This situation automatically introduces probability and may be associated with a steady stream picture, even in classical physics. As a result, classical physics is not as deterministic as it may seem. If probabilities arise at the discontinuity, one may ask: Is there a steady-stream (flux) type of probability associated with a moving particle? In a steady stream situation there is no change in time, but a moving particle clearly changes in time $x(t)$. How does one associate probabilities which do not contain time to flow problems which clearly contain motion? We investigate these questions and try to argue that the quantum $\exp(ipx)$ form and the Born $W^*(x)W(x)$ wavefunction picture are associated with these questions.

What Does Probability Mean if There is Motion i.e Change in Time

If one flips a coin and it lands as heads, this seems to imply there is some time period, dt , in which the coin is not changing. It may then be flipped again. What if the coin is continuously in the process of being flipped? What does it mean to say that it is heads at some time? To measure heads takes some time, dt , and the coin is continuously changing. One would have to argue that during the dt period the coin is either heads or close to heads.

A similar situation occurs for a moving particle. It is as if it glides or translates through space, so what does it mean to talk of probabilities $P(x)$ which do not contain time? Rather a constant velocity particle is described deterministically by $x=vt$. There is no probability. One may try to introduce a probability $P(x)=1/L$ (where L is length) by arguing that the particle spends the same amount of time in each dx cell. This definition of $P(x)$, however, is completely detached from motion. It applies to any velocity and any time value. The particle, however, is moving. What does one do if one wants a steady stream probability (no time and hence no velocity) which describes motion? This almost seems like a contradiction, however, p momentum only exists if there is motion, but may be thought of strictly in terms of impulse. In other words, $p=m_1v_1=m_2v_2$ yield the same impulse, but have different motion in space-time. Thus $P(p,x)$ seems to be a steady probability which describes motion without time being present. How does $P(p,x)$ compare with $P(x)$ for constant motion? $P(x)=1/L$ because all x points carry the same weight, but no motion is described. Motion implies moving from one point to another in time, but there is no time in a steady stream picture. Furthermore, p is associated with impulse.

Description of a Steady Stream Process

A steady stream process means that if a particle leaves a cell dx , another replaces it at the same time so that there is always a particle in dx , even though there is a cyclic or periodic process of removing and adding a particle at the same time. We suggest that $P(p,x)$ should be associated with such a cyclic or periodic scenario. In other words, there may be two pieces, one for the removed particle and the other for the added particle. Given that one has a steady stream scenario with no time, one may have two probabilities, consisting of removal followed by addition, which are shifted in space so as to map to a $P(x)=1/L$ which describes the situation of a particle always being in dx . In other words, we suggest that the quantum picture of $P(x/p)$ and $P(x)$ are closely associated with steady state motion.

We suggest:

$$\{P(p,x), P(p, x+\text{shift})\} \mapsto P(x)=1/L \quad ((1))$$

$P(p,x)$ is a periodic function. A pair may mathematically be written as a complex number i.e.

$$W(x) = P(p,x) + i P(p, x+\text{shift}) \quad ((2))$$

This leads to the notion of two very different probabilities, $P(x)=1/L$ and the complex form ((2)) which describes steady state motion in terms of p , momentum associated with impulse hits, so no time appears. $P(x)$ contains no p (momentum) value so if ((2)) is to map into $P(x)$, p must disappear. Given that $P(p,x)$ and $P(p, x+\text{shift})$ are periodic functions, it is known from complex math that one may have a constant magnitude which does not depend on p . For example, if

$$P(p,x) = \cos(px) \text{ and } P(p, x+\text{shift}) = \sin(px) \quad ((3)) \text{ then Modulus} = 1 \quad ((4))$$

The modulus, however, is $W^*(x)W(x)$ i.e. a quadratic form in $W(x)$. Should one then take the square root in order to make $W^*(x)W(x)$ somewhat linear in order to equate it with $P(x)$?

Flux Scenario

A steady state picture depends on the idea of flux, i.e. a flow. In such a case, one may argue that $W(x)$ and dW/dx are continuous in space i.e.

$$W(x) \quad ((5a)) \text{ and } dW/dx \quad ((5b))$$

$$\text{Multiplying } W^*(x) \text{ by } dW/dx \text{ yields } = ip \text{ if } W(x)=\exp(ipx) \quad ((6))$$

One may replace d/dx with $-id/dx$ to remove 1. This leaves p . If instead one uses $A\exp(ipx)$, one has: $p AA$. In this case, $p AA$ may be associated with a physical quantity, namely p momentum times flux. Taking the square root, however, does not yield a physical quantity. Thus it is the idea of pressure, and not simply $P(x)$, which is key to the formulation of quantum mechanics.

This leads to the interesting conclusion that $\exp(ipx)$ is a kind of square root flux probability when compared with $P(x)$. In other words, $P(x)$ is really a flux probability (not a static probability). This is why it is associated with the notion that a particle spends the same dt in each dx . This idea is a flux-based argument.

In other words, we argue that the quantum mechanical $\exp(ipx)$ follows from a steady-state flux based analysis of a problem. One may ask: When would such an analysis appear in nature?

One Dimensional Reflection-Refraction

If one considers a photon moving in x to the right hitting an n_1 - n_2 index of refraction along y , it will either reflect or refract. This may be analysed in terms of steady state flux using the ideas above. One may use the $\exp(ipx)$ form and ensure its continuity together with that of its first derivative at x_0 the discontinuity point, i.e.

$$A\exp(ipx) + B\exp(-ipx) = C\exp(ip2x) \text{ at } x_0 \text{ (with } E=px) \text{ ((7a)) and}$$

$$Ap\exp(ipx) - Bp\exp(-ipx) = Cp2 \exp(ip2x) \text{ at } x_0 \text{ ((7b))}$$

Taking the complex conjugate of ((7a) and multiplying by ((7b)) removes $\exp(ipx)$ forms (i.e. p values) leaving: $p_{AA}-p_{BB}=p2CC$ which should be a pressure balance according to the above (section) arguments. Here AA is flux and it may change. Thus $P(x)$ is not the important quantity, but rather $pP(x)$ i.e. pressure.

Conclusion

In conclusion, we argue that in classical mechanics one follows a particle in time deterministically i.e. through $x(t)$. One may also, however, have steady stream (flow) scenarios, but does not describe these with probability. We ask: How may one use time independent probability to describe steady state flow such that $P(x) = \text{constant}$? We suggest that steady state motion implies a particle being removed at dx and at the same time a new one being added. $P(x)=\text{constant}$ because a particle is always present in dx . Given that two processes occur and one does not have time, but must indicate motion, we use momentum p to signify motion in a steady state probability scheme. We suggest $W(x)=\{P(p,x), P(p,x+\text{shift})\} = P(p,x)+i P(p,x+\text{shift})$ and argue it must map into $P(x)=\text{constant}$.

One way to achieve this is through: $W^*(x)W(x)=1$. This ensures p is removed. One may, however, also take the square root of W^*W and argue that this represents $P(x)$. We suggest, however, this should not be done because if one takes $W^*(x)$ and multiplies it by $-id/dx W$, one has $p W^*W$ which should be a physical quantity, namely pressure. Thus $P(x)$ is not the only consideration. pW^*W pressure is key, so W^*W represents flux. $P(x)$ is really a flux probability, which seems to make sense because $P(x)=\text{constant}$ means that a particle spends the same amount of time in each dx cell. It also means that in a steady state situation there is always a particle in dx .

These ideas may be practically applied to the one-dimensional reflection-refraction problem formulated as a steady state scenario using $\exp(ipx)$. Using continuity of an $\exp(ipx)$ expression at x_0 (the discontinuity) and its first derivative and taking the complex conjugate of the first multiplied by the second yields a pressure balance. Thus even though $P(x)$ is in the picture (and is important) so is pressure balance, and the two are closely related.