Theodor Kaluza's Theory of Everything: extended

Thomas Schindelbeck, Mainz, Germany
schindelbeck.thomas@gmail.com
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Abstract

Using a Kaluza-type of model, describing the laws of electromagnetism within the formalism of differential geometry, provides a coherent, comprehensive and quantitative description of phenomena related to particles, including the values for electroweak coupling constants, a convergent series of quantized particle energies with limits given by the energy values of the electron and the Higgs vacuum expectation value. The geometry of the solutions for spin 1/2 define 6 lepton-like and 6 quark-like entities with their corresponding electric charges and allow an accurate calculation of the magnetic moments of baryons.
Electromagnetic and gravitational terms will be linked by a series expansion, the corresponding relation suggests the existence of a cosmological constant in the correct order of magnitude.
The model can be expressed *ab initio*, necessary input parameters are the electromagnetic constants.

0.1 Introduction

This is the more adventurous version of the Kaluza model presented in [1]. It aims to give a more general and free = speculative approach emphasizing aspects of geometry to gather new ideas and test limits of this ansatz. There will be no focus on formal rigor and neither a thorough proof reading.

The article will be divided in 2 parts:
- Euclidean-Geometry: will give simple geometric relationships for the symmetry and properties of the "elementary" particles of the standard model of particle physics;
- Non-Euclidean-Geometry / Kaluza model: will give ab initio values for the free parameters needed in the standard model;

0.1.1 5D-photon

The basic background will be a flat 5D-world with coordinates of time, space and the 5th coordinate, representing energy, W [1], expressed as length parameter, λ [m] ~ 1/W, to fit in to the other dimensions' units. The sign of the latter remains undecided for the time being, $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 +/- d\lambda^2$. Since the 5th-coordinate will correspond to terms of the stress-energy-tensor in 4D, $T_{\alpha\beta}$, flat 5D is equivalent to curved 4D. In the following only energy density, i.e. T_{00} will be considered.
The only inhabitants of this 5D world are supposed to be 5D-photons. Their flat 4D-version may be characterized by symmetry U(1) representing a rotation of E and B fields in a xy-plane around the axis of propagation (z), which is a straight line in 3D-space. Their particle cousins represent localized states of symmetry SO(3) or the related SU(2), attributed to fermions, where the photon is self-trapped in space curved by the energy density of its own fields.

The most basic particle properties may be deduced from a very simplified model:

A circular polarized photon with its intrinsic angular momentum interpreted as having its E- and B-vectors rotating around a central axis of propagation will be transformed into an object that has the - still rotating - E-vector constantly oriented to a fixed point, the origin of a local coordinate system. The vectors E, B and C of the propagation velocity[2] are supposed to be locally orthogonal [3] and subject to standard Maxwell equations. This has the following qualitative consequences:

1) Such a rotation is related to the group **SO(3)** (and **SU(2)** as important special case). In the following a

1 The reasons to do so are essentially the same as in space-time-matter theory, though the more general term "energy" is preferred as "matter" = mass will be attributed to a special case of symmetry.
2 In the limit $r \to r_n$ („particle radius") => $C \to c_0$;
3 Referred to as "EBC-triple" in the following;

quaternion ansatz will be used for modeling the respective rotations.
2) E-vector constantly oriented to a fixed point implies **charge**.
3) A local coordinate system = rest system implies:
3a) **mass;**
3b) in case of any lateral extension of the E-field, for r -> 0 the overlap of a rotating E-vector implies rising energy density, resulting in rising curvature of space-time according to GR or its modification as of equ. (3);
4) The orthogonal vectors E, B and C can be given in 2 different **chiral** states (left- right-handed). Switching direction of the fields and chirality will result in corresponding antiparticles.

The crucial point for turning this into a quantitative model in agreement with experiments is 3b). The appropriate tool for calculations will be based on Kaluza’s theory. Spin will be an explicit boundary condition though it might be considered to be implicit in Kaluza's ansatz as well, since electrodynamics allows solutions for circular polarized light.

0.1.2 Kaluza model

Theodor Kaluza in 1919 developed a unified field theory of gravitation and electromagnetism that produced the formalism for the field equations of the general theory of relativity (GR) and Maxwell's equations of electromagnetism (EM) thus unifying the major forces known at his time. His 5-dimensional model [2] is not suited to give properties related to particles, a problem addressed by Oskar Klein [3] who introduced the idea of compactification and attempted to join the model with the emerging principles of quantum mechanics. Therefore the theory is mainly known as Kaluza-Klein theory today. This version became a progenitor of string theory. The classical Kaluza model was developed further as well [4], Wesson and coworkers elaborated a general non-compactified version to describe phenomena extending from particles to cosmological problems. The equations of 5D space-time may be separated in a 4D Einstein tensor and metric terms representing mass and the cosmological constant, Λ. Particles may be described as photon-like in 5D, traveling on time-like paths in 4D. This version is known as space-time-matter theory [5]. Both successor theories give general relationships rather than providing quantitative results for specific phenomena such as particle energy.

The model described in the following does not attempt to give a complete solution for a 5D theory but to demonstrate that Kaluza's ansatz provides very simple, parameter-free and in particular quantitative solutions for a wide range of phenomena. Basic equations from the existing literature may be used, with one significant *simplification*:

Kaluza discovered that Maxwell’s equations may be described within the formalism of GR. To get both these *and* the Einstein field equations (EFE) he needed an additional dimension and had to introduce the constant of gravitation in his metric. He chose a gravitational term to keep the electromagnetic potential terms in the metric dimensionless, a rather unfitting combination [4]. If one settles for electromagnetic phenomena as first approximation there is no place for the gravitational constant term. This does not give a unification of EM and GR, however, it is a suitable ansatz to “unify” EM and particle physics. Gravitational terms can be recovered via a simple series expansion of the electromagnetic equations and such a proceeding may actually reflect the huge difference in order of magnitude of both phenomena better than the more linear original approach.

Curvature of space-time based on an electromagnetic version of the field equations of GR will be strong enough to localize a photon in a self trapping kind of mechanism, yielding energy states in the range of the particle zoo. Circular polarized light is part of conventional electromagnetic theory, in the following this feature will be treated equivalently with the terms angular momentum or spin as intrinsic property of a photon and will be a necessary boundary condition in the equations used. In particular, unless noted otherwise, it is assumed that particles posses half-integer spin or are composed of half-integer spin components (e.g. mesons).

The basic proceeding will be as follows:

Kaluza’s equations for flat 5D-space may be arranged to give [5, chapter 6.6]
1) Einstein-like equations for space-time curved by electromagnetic and scalar fields (equ. (5)),

4 In the closing remarks of [2] Kaluza suggests to reconsider „die etwas fragwürdige Gravitationskonstante“ –„the somewhat questionable constant of gravitation“.

2) Maxwell equations where the source depends on the scalar field,
3) a wave-like equation connecting the scalar Φ with the electromagnetic tensor (equ. (6)).

Solutions for Φ of 3) in a flat 5D-metric will be used as general ansatz in a 4D-metric. This is considered to be a proof of concept only, a more thorough ansatz has to be expected to incorporate angular momentum/spin into the field equations appropriately.

The solution for Φ gives $\Phi \sim \exp(-(\rho/r)^3)$ and may be seen as representing curvature of space. Due to the derivation from a Kaluza ansatz coefficient ρ is a function of the electromagnetic potential, A, in the static approximation of this work the electric potential, $\rho \sim \rho_0 = A_{el} = e/(4\pi\varepsilon r)$. The only other parameter entering ρ will be a function of the fine-structure constant [5], α, which enters the equations through the boundary condition half integer spin, requiring a relationship between A_{el}, i.e. the values of elementary charge and electric constant, and $\hbar/2$, see chpt. I 2.1 [6]. Since a geometric interpretation allows to give α in terms of Γ-functions ρ may be given in terms of A_{el} and mathematical constants only.

Based on this the model yields absolute particle energies with limits given by the energy values of the electron and the Higgs vacuum expectation value. Part of the α-terms included in ρ are identical with the ratio of electron and Planck energy, $\alpha_{Pl} = W_e/W_{pl}$, see chpt. I 7.1.1 [7], which is convenient for the series expansion of the exponential used in the metric to recover an appropriate term for gravitation. With this ansatz additional minor terms in the field equations will be in the correct order of magnitude for the cosmological constant, Λ.

In 1st approximation for calculations of energy the boundary condition of half integer spin may be sufficiently dealt with by settling for a fit of the integration limit of the relevant integral involved. A more detailed analysis of the model parameters requires minor additional assumptions, improving accuracy by roughly one order of magnitude (chpt. I 2).

Focusing on the angular momentum aspects of the model, in chpt. II 1, II 4 the rotation of a set of orthogonal E, B, C-vectors, attributed to the electromagnetic fields and the propagation with the speed of light, will be modeled via quaternions. This gives 3 possible solutions for trajectories of E, B for spin 1/2 enveloping 3 spherical cones corresponding to partial charges 2/3, 1/3 and 1/3 and their complementary 3D-ball sections, corresponding to partial charges 1/3, 2/3 and 2/3. Each spherical cone and its complement by definition give back a sphere if angular momentum and chirality match, and may be identified with leptons. Combinations of such solutions with differences in angular momentum and chirality, implying nodal surfaces and higher energy states, may represent hadrons. Combining appropriate total spin and using the results of this model for energy gives *ab initio* values for magnetic moments for all $J = 1/2$ baryons of the uds-octet of the standard model of particle physics (SM).

Typical accuracy of the calculations is in the order of 0.0001 [8]. The deviation of calculated results from the experimental values will be in the range 0.01 - 0.001, consistent with a variation of input parameters related to elementary charge in an order of magnitude of QED corrections, which are not included in this model.

0.2 System of natural units

The approach sketched in the introduction requires the use of an electromagnetic unit system appropriate for the general formalism of GR. It is common to define natural electromagnetic units by referring them to the value of the speed of light. The same will be done here, thus subscript c will be used. Retaining SI units for length, time and energy the electromagnetic constants may be defined as:

$$c_0^2 = (\varepsilon_c \mu_c)^{-1} \qquad (1)$$

with $\varepsilon_c = (2.998E+8\ [m^2/Jm])^{-1} = (2.998E+8)^{-1}\ [J/m]$
$\mu_c = (2.998E+8\ [Jm/s^2])^{-1} = (2.998E+8)^{-1}\ [s^2/Jm]$.

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307E-28$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671E-36\ [J^2]$. In the following $e_c = 3.110E-18$ [J] and $e_c/(4\pi\varepsilon_c) = 7.419E-11$ [m] may be used

5 The relation of the masses e, μ, π with α was noted first in 1952 by Nambu [6]. MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [7].
6 The coefficient of angular moment may be interpreted as either σ, which will in general indicate the integration limit, $(r/\rho)^3$ for calculating the incomplete gamma function, or its main component $\alpha_{lim} \approx 1.5/\alpha$, see chpt. I 3.3.
7 Giving $\rho^3 \approx \sigma\, \alpha_{Pl}\, A_{el}^3$;
8 Including e.g. errors due to the numerical approximation of incomplete Γ-functions.

as natural unit of energy and length [9].

With the unit system above the 00-component for an electromagnetic stress-energy-tensor, $T_{\alpha\beta}$, of the field equation in an electrostatic approximation will simply be $T_{00} = E^2/2$ [m^{-2}]. In the case of T_{00} referring to energy density the constant G/c_0^4 [m/J] in the Einstein field equations (EFE) will be replaced by:

$$(8\pi)G/c_0^4 \quad => \quad \approx \frac{1}{\varepsilon_c} \tag{2}$$

in an accordingly modified field equation:

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = \frac{1}{\varepsilon_c}T_{\alpha\beta} \tag{3}$$

I Terms based on Non-Euclidean Geometry
Kaluza theory and corresponding integrals

I 1 Kaluza theory

I 1.1 Basic equations

Kaluza theory is an extension of general relativity to 5D-space-time with a metric given as [5, equ. 2.2]:

$$g_{AB} = \begin{bmatrix} (g_{\alpha\beta} - \kappa^2 \Phi^2 A_\alpha A_\beta) & -\kappa\Phi^2 A_\alpha \\ -\kappa\Phi^2 A_\beta & -\Phi^2 \end{bmatrix} \tag{4}$$

In (4) roman letters correspond to 5D, Greek letters to 4D. κ corresponds to the constant in the field equation (2)f, A is the electromagnetic potential. In the context of the electrostatic approximation of this model A will be assumed to be represented by the electric potential, $A_{el} = e_c/(4\pi\varepsilon_c r) = \rho_0/r$ [-]. Assuming 5D space-time to be flat, i.e. $R_{AB} = 0$, gives for the 4D-part of the field equations [5, equ. 2.3]:

$$G_{\alpha\beta} = \frac{\kappa^2\Phi^2}{2}T^{EM}_{\alpha\beta} - \frac{1}{\Phi}\left(\nabla_\alpha(\partial_\alpha\Phi) - g_{\alpha\beta}\Box\Phi\right) \tag{5}$$

I 1.2 Scalar Φ

From $R_{44} = 0$ follows:

$$\Box\Phi = -\frac{\kappa^2\Phi^3}{4}F_{\alpha\beta}F^{\alpha\beta} \tag{6}$$

In the following only derivatives with respect to r of a spherical symmetric coordinate system will be considered. Equation (6) will be used to obtain an ansatz for a metric to get a solution of the 00-component in (3). A function Φ_N

$$\Phi_N \approx \left(\frac{\rho}{r}\right)^{N-1} e^{\nu/2} = \left(\frac{\rho}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{2r}\right)^N\right) \quad ^{10} \tag{7}$$

yields solutions for an equation of general type of (6), where the term of highest order of exponential N, given by $\Phi'' \sim \rho^{3N-1}/r^{3N+1}$, may be interpreted to provide the terms for $A'(r) \sim e_c/(4\pi\varepsilon_c r^2) \sim \rho_0/r^2$:

$$\Phi_N{}'' \sim \left(\frac{\rho^{3N-1}}{r^{3N+1}}\right)e^{\nu/2} \sim \boldsymbol{\Phi_N^3}\ e^{-\nu}(\boldsymbol{A_0}')^2 \approx \left[\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r}}\right)^{\boldsymbol{N-1}} \boldsymbol{e^{\nu/2}}\right]^{\boldsymbol{3}} e^{-\nu}\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r^2}}\right)^{\boldsymbol{2}} = \left(\frac{\rho}{r}\right)^{3N-3} e^{\nu/2}\left(\frac{\rho}{r^2}\right)^2 \tag{8}$$

$R_{44} = 0$ does not have to be obeyed strictly and is secondary to condition $R_{AB} = 0$. The significance of (7)f lies in providing the relationship of exponential and pre-exponential terms and first of all in the requirement to contain powers of $A_{el} \sim (\rho_0/r)$ in the exponent of Φ_N, to be used in the following.

9 The term $e_c/(4\pi\varepsilon_c)$ might be a rough measure for the transition between classical and QM descriptions.

10 Below occasionally the notation $f(r,N) = e^{\nu(r,N)} = \varphi_N(r)$ will be used.

I 1.2.1 Solutions for Φ

The solutions for the scalar Φ depend on the complete metric used. The main problem to obtain $R_{44} = 0$ is to eliminate the terms of lowest order in ρ, which lack coefficients in their terms enabling an easy cancellation of them. The easiest method to get a solution of order N is to use spherical coordinates of dimension N+1. Using e.g. the line element for a 4D metric of [5, equ. 6.76]

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - e^\mu r^2 \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right) \tag{9}$$

and $A_\alpha = (A_{el}, 0, 0, 0)$ gives as solution for equ.(6) (cf. [5, equ. 6.77], prime corresponds to derivatives with respect to r):

$$\Phi'' + \left(\frac{\nu' - \lambda' + 2\mu'}{2} + \frac{2}{r}\right)\Phi' - \frac{1}{2}\Phi^3 e^{-\nu}\left(A_{el}'\right)^2 = 0 \tag{10}$$

This can be solved with function (7) for N = 2:

$$\Phi_2' = \left[-\left(\frac{\rho}{r^2}\right) + \left(\frac{\rho^3}{r^4}\right)\right] e^{\nu/2} \tag{11}$$

and

$$\Phi_2'' = \left[2\left(\frac{\rho}{r^3}\right) - 4\left(\frac{\rho^3}{r^5}\right) + \left(\frac{\rho^5}{r^7}\right)\right] e^{\nu/2} \tag{12}$$

The ρ^1 terms cancel in (10), the ρ^3 terms can be eliminated by appropriate choice of ν', λ' and μ', a remaining factor in the ρ^5 term has to be compensated by assuming a corresponding factor in A_{el}. For N = 3 hyperspherical coordinates with the line element

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - e^\mu r^2 \left(d\psi^2 + \sin^2\psi\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right) \tag{13}$$

may be used. A more complex metric may be used as well to solve equation (8) [11].

I 1.3 Metric

I 1.3.1 General solution N = {1; 2; 3}

This article has a focus on a solution of (7) with N = 3. However, all solutions in a 5D space-time according to chpt. I 1.2.1 with N = {1; 2; 3}, might be used for the ansatz of a metric such as

$$g_{00} = \sum_{N=1}^{3} \left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^N\right) \tag{14}$$

With **r** in the exponential limited to $e_c/(4\pi\varepsilon_c)$, σ ≈ 1, α_{Pl} not included in ρ, see I 7.1, this may be given as

$$g_{00} = \exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)\right) + \left(\frac{\rho_0}{r}\right)\exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)^2\right) + \left(\frac{\rho_0}{r}\right)^2 \exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)^3\right) \tag{15}$$

The 3rd term corresponds to the case discussed below, resulting in terms giving the square of the E-field in G_{00} and eventually particle energy as well as an equivalent term for gravitation from the series expression. The second term is the linear version and might be used to construct a Schwarzschild-like solution.

The first term would represent a general vacuum solution, i.e. without presence of any field ρ_0/r. A series expansion would give the 1 for flat space, while the minor terms of G_{00} could give Λ-like orders of magnitude, see chpt. I 8.1.

To comply to the boundary condition $J_Z = 1/2$ the angular terms of φ and ϑ should be related appropriately to yield the relationships given by the quaternion model of chpt. II 1.2.

I 1.3.2 Example for metric, point charge energy

A general metric using solutions for Φ according to chpt. I 1.2.1 and only diagonal components will be:

$$g_{\alpha\alpha} = \left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(-\left(\frac{a\rho}{r}\right)^N\right),\ -\left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(-\left(\frac{b\rho}{r}\right)^N\right),\ -r^2,\ -r^2\sin^2\vartheta \tag{16}$$

11 Using terms of Φ_N for canceling of similar terms of other $R_{\alpha\beta}$ components may in fact increase the resources to obtain a specific solution.

Below the metric of (16) with N = 3 will be used.

$$g_{\mu\mu} = \left(\frac{\rho_0}{r}\right)^2 \exp\left(-\left(\frac{\rho}{\mathbf{r}}\right)^3\right),\ \ -\left(\frac{\rho_0}{r}\right)^2 \exp\left(\left(\frac{\rho}{\mathbf{r}}\right)^3\right),\ \ -r^2,\ \ -r^2\sin^2\vartheta \tag{17}$$

This is based on the following considerations:
1) flat 5-D-space-time; terms including the 5th-coordinate will correspond to terms of the stress-energy-tensor in 4D, $T_{\alpha\beta}$; in the following only energy density, i.e. T_{00} will be considered;
2) the limit in absence of electromagnetic fields will not be given by a component $g_{\alpha\beta}$ related to gravitational effects as given in equ. (4), gravitational terms will be recovered by a series expansion of the exponential terms of (17), see chpt. I 7.1;
3) coefficient ρ_0 in the pre-exponential terms ensures Coulomb terms as limit cases;
4) Equation (17) is an approximation not only in neglecting contributions of the magnetic potentials but also in not considering spin, a necessary boundary condition for particles which is not represented in Kaluza's equations either. Thus some modification in the metric of (4) has to be expected. A metric according to (17) will give correct quantitative particle related results. However, only the exponential part of Φ_3, $e^{v/2}$, will be squared in the metric terms, giving $e^v A_\alpha A_\beta$ instead of $e^v(A_\alpha A_\beta)^2$. This is somewhat ad hoc and considered a proof of concept only [12].

The exponential part represents $\Phi^2 \sim e^v$ in the metric. The variable r is marked bold if originating from the exponential term to facilitate a discussion of the implications of its restricted range of validity.

$\Gamma_{01}{}^0 = \Gamma_{10}{}^0 \quad = -1/r^1 + 3/2\ a\ \rho^3/\mathbf{r}^4 \qquad \Gamma_{00}{}^1 = -1/r^1\ e^{(a-b)v} + 3/2\ a\ \rho^3/\mathbf{r}^4 e^{(a-b)v}$
$\Gamma_{11}{}^1 \quad = -1/r^1 + 3/2\ b\ \rho^3/\mathbf{r}^4$
$\Gamma_{12}{}^2 = \Gamma_{21}{}^2 = \Gamma_{13}{}^3 = \Gamma_{31}{}^3 \quad = +1/r^1 \qquad \Gamma_{22}{}^1 = -r^3/\rho_0{}^2\ e^{-bv} \ = \Gamma_{33}{}^1/\sin^2\vartheta$
$\Gamma_{23}{}^3 = \Gamma_{32}{}^3 = \cot\vartheta \qquad \Gamma_{33}{}^2 = -\sin\vartheta\cos\vartheta$

$R_{00} = e^{(a-b)v}\ [-1/r^2 + 3\ (a-b)\ \rho^3/(r\mathbf{r}^4) + 6\ a\ \rho^3/\mathbf{r}^5 - 9/2\ a\ (a-b)\ \rho^6/\mathbf{r}^8) + 2(\Gamma_{01}{}^0\ \Gamma_{00}{}^1) - \Gamma_{00}{}^1\ (\Gamma_{10}{}^0 + \Gamma_{11}{}^1 + 2\Gamma_{12}{}^2)]$
$= e^{(a-b)v}\ [-1/r^2 + 3\ (a-b)\ \rho^3/(r\mathbf{r}^4) + 6\ a\ \rho^3/\mathbf{r}^5 - 9/2\ a\ (a-b)\ \rho^6/\mathbf{r}^8) - \Gamma_{00}{}^1\ (-\Gamma_{10}{}^0 + \Gamma_{11}{}^1 + 2\Gamma_{12}{}^2)]$
$= e^{(a-b)v}\ [-1/r^2 + 3\ (a-b)\ \rho^3/(r\mathbf{r}^4) + 6\ a\ \rho^3/\mathbf{r}^5 - 9/2\ a\ (a-b)\ \rho^6/\mathbf{r}^8) + 1/r^2 + 3\ a\ \rho^3/(r\mathbf{r}^4) + 9/4\ a^2\ \rho^6/\mathbf{r}^8 - 1/r^2 + 3/2\ a\ \rho^3/(r\mathbf{r}^4) + 3/2\ b\ \rho^3/(r\mathbf{r}^4) - 9/4\ ab\ \rho^6/\mathbf{r}^8 + 2/r^{2-} - 3\ a\ \rho^3/(r\mathbf{r}^4)]$
$= e^{(a-b)v}\ [+1/r^2 + (9/2a - 3/2b)\ \rho^3/(r\mathbf{r}^4) + 6\ a\ \rho^3/\mathbf{r}^5 - 9/4\ (a^2 - ab)\ \rho^6/\mathbf{r}^8)]$

$R_{11} = [+1/r^2 - 6a\ \rho^3/\mathbf{r}^5 - 2/r^2 + \Gamma_{10}{}^0\ \Gamma_{01}{}^0 + \Gamma_{11}{}^1\Gamma_{11}{}^1 + 2\Gamma_{12}{}^2\ \Gamma_{21}{}^2 - \Gamma_{11}{}^1\ (\Gamma_{10}{}^0 + \Gamma_{11}{}^1 + 2\Gamma_{12}{}^2)]$
$= [+1/r^2 - 6a\ \rho^3/\ \mathbf{r}^5 - 2/r^2 + \Gamma_{10}{}^0\ \Gamma_{01}{}^0 + 2\Gamma_{12}{}^2\ \Gamma_{21}{}^2 - \Gamma_{11}{}^1\ (\Gamma_{10}{}^0 + 2\Gamma_{12}{}^2)]$
$= [+1/r^2 - 6a\ \rho^3/\ \mathbf{r}^5 - 2/r^2 + 1/r^2 - 3\ a\ \rho^3/(r\mathbf{r}^4) + 9/4\ a^2\ \rho^6/\mathbf{r}^8 + 2/r^2 - 1/r^2 + 3/2\ a\ \rho^3/(r\mathbf{r}^4) + 3/2\ b\ \rho^3/(r\mathbf{r}^4) - 9/4\ ab\ \rho^6/\mathbf{r}^8 + 2/r^2 - 3\ b\ \rho^3/(r\mathbf{r}^4)]$
$= [+3/r^2 - 3/2(a+b)\rho^3/(r\mathbf{r}^4) - 6a\ \rho^3/\ \mathbf{r}^5 + 9/4(+a^2 - ab)\rho^6/\mathbf{r}^8]$
$R_{22} = -1 + e^{-bv}\ [+3\ r^2/\rho_0{}^2 - 3b\rho^3\ r^3/(\rho_0{}^2\mathbf{r}^4) + 2(\Gamma_{21}{}^2\ \Gamma_{22}{}^1) - \Gamma_{22}{}^1\ (\Gamma_{10}{}^0 + \Gamma_{11}{}^1 + 2\Gamma_{12}{}^2)]$
$= -1 + e^{-bv}\ [+3\ r^2/\rho_0{}^2 - 3b\rho^3\ r^3/(\rho_0{}^2\mathbf{r}^4) - \Gamma_{22}{}^1\ (\Gamma_{10}{}^0 + \Gamma_{11}{}^1)]$
$= -1 + e^{-bv}\ [+3\ r^2/\rho_0{}^2 - 3b\rho^3\ r^3/(\rho_0{}^2\mathbf{r}^4) + r^3/\rho_0{}^2\ (-2/r^1 + 3/2\ (a+b)\ \rho^3/\mathbf{r}^4)]$
$= -1 + e^{-bv}\ [+3\ r^2/\rho_0{}^2 - 3b\rho^3\ r^3/(\rho_0{}^2\mathbf{r}^4) - 2r^2/\rho_0{}^2 + 3/2\ (a+b)\ \rho^3 r^3/(\rho_0{}^2\ \mathbf{r}^4)]$
$= -1 + e^{-bv}\ [+r^2/\rho_0{}^2 + 3/2\ (a-b)\ \rho^3 r^3/(\rho_0{}^2\mathbf{r}^4)]$

$g^{00}R_{00} = e^{-bv}\ [+1/\rho_0{}^2 + (9/2a - 3/2b)\ \rho^3\ r/(\rho_0{}^2\ \mathbf{r}^4) + 6\ a\ \rho^3\ r^2/(\rho_0{}^2\ \mathbf{r}^5) - 9/4\ (a^2 - ab)\ \rho^6\ r^2/(\rho_0{}^2\ \mathbf{r}^8)]$
$g^{11}R_{11} = -e^{-bv}\ [+3/\rho_0{}^2 - 3/2(a+b)\ \rho^3\ r/(\rho_0{}^2\ \mathbf{r}^4) - 6\ a\ \rho^3\ r^2/(\rho_0{}^2\ \mathbf{r}^5) + 9/4(+a^2 - ab)\ \rho^6\ r^2/(\rho_0{}^2\ \mathbf{r}^8)]$
$g^{22}R_{22} + g^{33}R_{33} = +2/r^2 - e^{-bv}\ [2/\rho_0{}^2 + 3\ (a-b)\ \rho^3 r/(\rho_0{}^2\mathbf{r}^4)]$

In the following b = -a.

$g^{00}R_{00} = e^{av}\ [+1/\rho_0{}^2 + 6\ a\ \rho^3\ r/(\rho_0{}^2\ \mathbf{r}^4) + 6\ a\ \rho^3\ r^2/(\rho_0{}^2\ \mathbf{r}^5) - 9/2\ a^2\ \rho^6\ r^2/(\rho_0{}^2\ \mathbf{r}^8)]$
$g^{11}R_{11} = -e^{av}\ [+3/\rho_0{}^2 - 6\ a\ \rho^3\ r^2/(\rho_0{}^2\ \mathbf{r}^5) + 9/2\ a^2\ \rho^6\ r^2/(\rho_0{}^2\ \mathbf{r}^8)]$
$g^{22}R_{22} + g^{33}R_{33} = +2/r^2 - e^{av}\ [+2/\rho_0{}^2 + 6\ a\ r\rho^3/(\rho_0{}^2\mathbf{r}^4)]$

The solution for R will be:
$R = +2/r^2 + e^{av}\ [(-4/\rho_0{}^2 + 12\ a\ \rho^3\ r^2/(\rho_0{}^2\ \mathbf{r}^5) - 9\ a^2\ \rho^6\ r^2/(\rho_0{}^2\ \mathbf{r}^8)]$

G_{00} will be:
$G_{00} = e^{2av}\ [+1/r^2 + 6\ a\ \rho^3/(r\mathbf{r}^4) + 6\ a\ \rho^3/\mathbf{r}^5 - 9/2\ a^2\ \rho^6/\mathbf{r}^8)] - e^{av}\ \rho_0{}^2/r^4 + e^{2av}\ [2/r^2 - 6\ a\ \rho^3/\ \mathbf{r}^5 + 9/2\ a^2\ \rho^6\ r^2/(\rho_0{}^2\ \mathbf{r}^8)] =$
$-e^{av}\ \rho_0{}^2/r^4 + e^{2av}\ [3/r^2 + 6\ a\ \rho^3/(r\mathbf{r}^4)]$

---

12 A solution of (7) with N = 1 could be inserted directly in Kaluza's term $\Phi^2 A_\alpha A_\beta$ of (4). Using the coefficients of this work this would yield the energy of the electron, however, not the energy relation (24)ff, etc.

Using the parameters and terms as given below, volume integrals over the ρ^n/r^{n+2} terms will yield energy results $\varepsilon_c \int e^{av}\ \rho^n/r^{n+2}\ d^3r \approx \varepsilon_c\, \rho \approx$ 1E-22 [J] compared to the term $\varepsilon_c \int e^{av}\ \rho_0^2/r^4\ d^3r \approx \varepsilon_c\, \rho_0^2\ \rho^{-1} \approx$ 1E-13 [J] (both with coefficients for the electron, $\sigma_0\alpha_{Pl}$; $\rho_0 = e_c/(4\pi\varepsilon_c r)$), giving negligible contributions to particle energy within the parameter range discussed here. This leaves the first term as leading order. With a =1:

$G_{00} = -\ e^v\ \rho_0^2/r^4$

In all parts of this work energy will be expressed as positive value, giving equ. (35).

I 1.4 Euler Integrals

Solutions for integrals over e^v, with v according to (7), times some function of r can be given by:

$$\int_0^{r_n} \exp(-(\rho_n/r)^N)\, r^{-(m+1)} dr = \Gamma(m/N, (\rho_n/r_n)^3)\ \frac{\rho_n^{-m}}{N} = \int_{(\rho_n/r_n)^3}^{\infty} t^{\frac{m}{N}-1} e^{-t} dt\ \frac{\rho_n^{-m}}{N} \qquad (18)$$

valid for N = {3; 4}, m = {-2; -1; 0; +1;+2}. The term $\Gamma(m/N, (\rho_n/r_n)^3)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind. Euler integrals yield positive values, the sign convention of Γ-functions gives negative values for negative arguments. The abbreviation $\Gamma_{-1/3}$ will be used for $|\Gamma(-1/3)|$; in $\Gamma_{+1/3}$ the "+" sign will be used if needed for clarity. In the range of values relevant in this work, for m/N ≥ 1 the complete gamma function $\Gamma_{m/N}$ is a sufficient approximation, for m/N ≤ 0 the integrals have to be calculated numerically, requiring an integration limit, see I 5.1.

An important relation used below will be [8]:

$$\Gamma(+x)\,\Gamma(-x) = \pi/(x \sin(\pi x)) \quad => \quad \Gamma_{1/3}\,\Gamma_{-1/3} = 3\pi/\sin(\pi/3) = 3^{0.5}\, 2\pi \qquad (19)$$

I 2 Angular momentum, coefficient σ

I 2.1 calculation of σ from integral boundary

The integral limits required for Euler integrals of (18) with m/N ≤ 0 are r_n („particle radius" of state n) in integrals over e^v and $(\rho_n/r_n)^3$ in the Euler integrals. The latter will be expressed via a constant defined as 8/σ [13]:

$$(\rho_n/r_n)^3 = 8/\sigma \qquad (20)$$

whose value may be derived from the condition for angular momentum J_z = 1/2 [ħ].

In 1st approximation: using the term for energy of (39) and the Compton wavelength, λ_C, according to equ. (62) below including the term $\sigma^{1/3}$ for r_2, requires $\sigma^{1/3}$ to be of order of the inverse fine-structure constant α^{-1}: $1/c_0 \int w(r)\, dr * \int dr \approx b_0/\rho_n * \sigma^{1/3}\rho_n/c_0 \equiv \hbar/2 \quad => \quad \sigma^{1/3} \approx \alpha^{-1}$.

In the following a simple relation with angular momentum J_z for spherical symmetric states will be given by applying a semi-classical approach:

$$J_z = r_2\, x\, p(r_1) = r_2 W_n(r_1)/c_0 \equiv 1/2\ [\hbar] \qquad (21)$$

Using term $2b_0$ of equ. (39) as constant factor and integrating over a circular path of radius $|r_2| = |r_1|$, equation (18) will give for m = 0:

$$J_z = \int_0^{r_n}\int_0^{2\pi} J_z(r,\varphi)\, d\varphi\, dr = 4\pi \frac{b_0}{c_0} \int_0^{r_n} e^v r^{-1} dr = 4\pi\alpha\hbar \int_0^{r_n} e^v r^{-1} dr = \frac{4\pi}{3}\alpha\hbar \int_{8/\sigma}^{\infty} t^{-1} e^{-t}\, dt \equiv 1/2\ [\hbar] \qquad (22)$$

To obtain J_z = 1/2 [ħ] the integral over $e^v r^{-1}$ of (22), *has to* yield $\alpha^{-1}/8\pi$.

$$\int_0^{r_n} e^v r^{-1} dr = 1/3 \int_{8/\sigma}^{\infty} t^{-1} e^{-t} dt \equiv \frac{\alpha^{-1}}{8\pi} \approx 5.45 \qquad (23)$$

Relation (23) may be used for a numerical calculation of the integration limit, 8/σ, giving a value of σ_0 for spherical symmetry, σ_0 = 1.810E+8 [-]. Assuming the coefficient $\Gamma_{-1/3}/3$ according to (18) has to be part of the expression for σ_0 [14] this results in $\sigma_0 \approx 8\,(1.5\alpha^{-1}\Gamma_{-1/3}/3)^3$.

To get a more detailed description of σ in a range of 1 percent precision is difficult since there are several options available and in this range of accuracy QED and other minor effects may be expected which might be amplified due to the non-linear nature of the gamma functions involved. A factor ≈ 3/2 appears in several

---

13 Chosen to give coefficient σ in the exponent of e^v, see I 2.4.

14 Since according to (59)ff $\sigma^{1/3}$ is proportional to a length parameter, r_n, which according to (18) includes $\Gamma_{-1/3}/3$.

terms such as $\sigma_0 \sim 1.5\alpha^{-1}$ of (), the ratio of electron and muon energy =1.5088, $\Gamma_{-1/3}/\Gamma_{1/3}$ =1.516 and the irregular electron coefficient in the power series that is part of α_{Pl} as well. The following discusses some relevant aspects with a focus on identifying possible underlying relationships while minimizing assumptions about the term ≈ 3/2 in particular.

Analogous to the postulate for neutral particles to be composed of volume elements of opposite charge, particles with J = 0, J ≥ 1 may be assumed to be composed of a combination of half integer contributions of angular momentum J = ± 1/2, adding up accordingly, formally implying appropriate multiples for the relation of $|r_2|$ and $|r_1|$ in (21).

I 2.2 σ, geometric interpretation

A coefficient representing geometry, with a value very close to the numerical one, would be:

$$\sigma_0 \approx 8\,(1.5\,\alpha^{-1}\,\Gamma_{-1/3}/3)^3 \approx 8\left(\frac{4\pi\,\Gamma_{-1/3}{}^3}{3}\right)^3 = 1.772\text{E+8 [-]} \tag{24}$$

As a consequence a dimensionless volume-like term appears in the denominator of the energy expression (53)ff for spherical symmetry, approaching the limit of a one-dimensional term, $2\Gamma_{-1/3}/3$, for higher angular states according to chpt. I 3.3. Expression (24) is closely related to the value of α and will be used in this context for the calculation of α in chpt. I 5.

I 2.3 σ, expression with Γ-terms

Alternatively a term may be used that is related to angular momentum coefficients, is expressed using coefficients considered essential for yielding basic quantities such as e_c itself and corresponds to the 3rd power structure of the equations best.

If the equations above are used for the integral over the point charge value only, the result is expected to yield e_c. Since Γ(+1/3)/3 is required to appear as a term in $W(e_c)$ due to the Euler integral, a counter term *must* be part of ρ in (49)f:

$$W(e_c) = \frac{e_c^2}{4\pi\,\varepsilon_c}\int \exp\left(\frac{-\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\,\varepsilon_c}\right)^3 r^{-2}dr = \frac{e_c^2}{4\pi\,\varepsilon_c}\frac{\Gamma_{+1/3}}{3}\left(\frac{\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\,\varepsilon_c}\right)^{-1} = e_c \tag{25}$$

For r_c follows, considering the basic coefficients only, using (19), (46)

$$r_c \sim 3^{1.5}\frac{\Gamma_{-1/3}}{3}\int \exp-\left(\frac{\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\,\varepsilon_c}\right)^3 dr \sim \frac{\Gamma_{-1/3}\Gamma_{+1/3}}{3^{0.5}}\;\frac{e_c}{4\pi\,\varepsilon_c} = \frac{e_c}{2\,\varepsilon_c} \tag{26}$$

again removing all coefficients that are not part of a Coulomb-expression and suggesting an additional term of 2π in the denominator of ρ.

Looking only at the basic mathematical coefficients entering the equation (61)ff (i.e. σ -> $2\Gamma_{-1/3}/3$) an additional term $1/\alpha_{lim} \sim (2\pi)^{-1}\Gamma_{+1/3}/\Gamma_{-1/3}$ (bold in (27)) in ρ would cancel redundant $\Gamma_{-1/3}/3$ terms in the length expression as well:

$$\lambda_C \sim 3^{1.5}\frac{\Gamma_{-1/3}}{3}\;\frac{\sigma^{1/3}}{2}\rho \sim 3^{0.5}\Gamma_{-1/3}\frac{\Gamma_{-1/3}}{3}\frac{2\Gamma_{-1/3}}{3}\;\frac{\boldsymbol{\Gamma_{+1/3}}}{\boldsymbol{2\pi\Gamma_{-1/3}}} = \frac{2\Gamma_{-1/3}}{3} \tag{27}$$

The term $(2\pi)^{-1}\,\Gamma_{1/3}/\Gamma_{-1/3}$ consists of components related to angular momentum and seems to be a suitable replacement for $1/(2\alpha_{lim})$ e.g. in (78) and may thus be used in expressions such as (54)f [15].

$$\sigma_0 = \left[\frac{1}{4}\left(\frac{\Gamma_{-1/3}2\pi}{\Gamma_{+1/3}}\right)^3\frac{2\Gamma_{-1/3}}{3}\right]^3 = \left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3 = 2.008\text{E+8 [-]}\;^{16} \tag{28}$$

The highly nonlinear incomplete Γ-functions, possible QED corrections, etc. complicate the selection of a final version. Thus energies calculated with both the numerical value (close to (24)) and with (28) will be presented in table 1.

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15 The need of $\Gamma_{+1/3}/\Gamma_{-1/3}$ to appear in (25)ff and its more pronounced relationship with angular terms is the reason to prefer $\alpha_{lim}^{-1} \approx 2\,(2\pi)^{-1}\,\Gamma_{+1/3}/\Gamma_{-1/3}$ over $\alpha_{lim}^{-1} \approx (2\pi)^{-1}\,2/3$ in spite of the latter term being almost identical to the value of σ_0 fitted to J_z.

16 Approximating $\Gamma_{-1/3}/\Gamma_{1/3}$ by 3/2 would give σ_0=1.821E+8[-], i.e. a term very close to that of the numerical one or (24).

I 2.4 Coefficient σ as component in ρ

The exponential term, $\exp(-\rho^3/r^3)$, together with the r^{-2} dependence of the field of a point charge define a maximum of particle energy near $r_{W(max)} \approx \rho$, rapidly approaching 0 for $r_{W(max)} > \rho$, effectively allowing to calculate energy terms without using a specific upper integration limit ("particle radius") [17]. On the other hand the weaker r-dependence of angular momentum, ~1/r results in the calculated values being completely dominated by an integration limit, r_n. The limit of the Euler integral is given by ρ_n^3/r_n^3, a constant which will be denoted 8/σ in this work.

A general exponential function of radius featuring a limit radius may be given in 1st approximation as (cf. chpt. I 6):

$$e^{\nu'} = \exp\left(-\left(\frac{ß\rho'^3}{2r^3} + \left[\left(\frac{ß\rho'^3}{2r^3}\right)^2 - 4\frac{\rho'^3}{2r^3}\right]^{0.5}\right)/2\right) \tag{29}$$

ß being some general coefficient. At the limit r_n of the real solution (29)

$$\left(ß\rho'^3/r_n^3\right)^2 = 8\rho'^3/r_n^3 \quad => \quad ß = 8\left(\frac{r}{\rho}'\right)^3 = \sigma \tag{30}$$

holds. Within the parameter range of this work the function $e^{\nu'} \approx \exp(-ß\rho'^3/r^3))$ is a very good approximation of an equation of the kind of (29) and consequently coefficient σ will be part of the exponential, $\rho^3 \sim \sigma\rho_0^3$.

I 2.5 Ground state coefficient, α_0

In this model elementary charge may be given as $b_0\int\exp(-(e_c/(4\pi\varepsilon_c r))^3)r^{-2}dr \approx e_c$, cf. (25), the corresponding radial distribution of energy has its maximum at $r_{c,max} \approx e_c/(4\pi\varepsilon_c)$. It has to be expected that the energy of a (partially-) charged particle lies above e_c, $W_n > e_c$, with a characteristic length, interpreted as the Compton wavelength, being below r_c, $\lambda_C < r_c$. Setting angular momentum $J_z = \hbar/2$ as boundary condition results in (cf. equ (61)ff) $\lambda_C \approx \sigma_0^{1/3}\rho$ and since $\rho \sim \sigma^{1/3} \sim \alpha^{-1}$, $\lambda_C \sim \alpha^{-2}\, \alpha_0^{\,+1/3}\, e_c/(4\pi\varepsilon_c)$. For above inequalities to hold, an additional, ground state coefficient, $\alpha_0 \leq \alpha^6$ is required. Assuming that according to the relation given by the fine-structure constant a basic photon-like state assigned to hc_0 should be α^{-1} higher in energy than a pure point charge state would result in $\alpha_0 \approx \alpha^9$, a value that except for the irregular coefficient 2/3 for the energy of the electron seems to be a good approximation for the relationship of particle energies [18].

This gives $\rho^3 \sim \sigma\, \alpha_0\, \rho_0^3$. According to the relationships given below, an angular contribution will have to be part of the exponent as well, giving

$$\rho^3 \approx \sigma_0\, \sigma\, \alpha_{lim}^{-1}/2\, \alpha_0\, \rho_0^3 = \sigma\, \alpha_{Pl}\, \rho_0^3 \tag{31}$$

This reasoning does not apply to intrinsically neutral particles as neutrinos are assumed to be, hence no need for a factor α_0, see chpt. I 3.8.3.

I 2.6 Relation r_n – $r_{n,m}$ electron – muon

The r at the maximum of W(r), see fig.1, may be given as $r_m \approx \Gamma_{-1/3}\sigma^{1/3}\rho_n/3)$, thus:

$$r_e \approx 1.5\, \alpha^{-1}\, \Gamma_{-1/3}\sigma^{1/3}\rho_e/3 \tag{32}$$

$$r_\mu \approx 1.5^{-1}\, \alpha^{+1}\, [1.5\, \alpha^{-1}\, \Gamma_{-1/3}\, \sigma^{1/3}\rho_e/3\,] = \Gamma_{-1/3}\sigma^{1/3}\rho_e/3 = r_{m,e} \tag{33}$$

The factor 1.5 α^{-1} in the ratio of electron to muon energy is identical to the factor needed to describe the correct spin 1/2.

I 2.7 1.5088 of the ratio W_μ/W_e

Factor 1.5088 of the ratio W_μ/W_e may be subject to a 3rd power relationship of the same kind as the α coefficients:

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17 For an upper limit $r_n \geq 10\rho$ other limitations supersede the attainable precision.

18 Such a reasoning for the ground state term might require additional or modified terms of σ_0 in place of α only, to enter α_0 and the power series. In addition it might be terms close to α^{-1}, e.g. such as 2/3 $\Gamma_{-1/3}/\Gamma_{1/3}$ $4\pi\Gamma_{-1/3}\Gamma_{1/3}$ of (66) that should be used in place of α, though this is opposed to a reasoning involving the definition of α as given above and could hardly improve the values of table 1.

$$\left(\frac{1.5133}{1.5088}\right)^3 = \left(\frac{1.5133}{1.5}\right) \tag{34}$$

indicating that the particle specific term of ρ_n and the components of σ are not correctly separated yet even in the case of spherical symmetric states.
The limit of a corresponding partial product in the energy expression is given by 1.5133 $\Pi_{k=0}^{\infty}$ (1.5/1.533)^1/3^k ≈ 1.5066.

I 3 Particle energy

I 3.1 Energy point charge

The Einstein tensor component G_{00} will be (I 1.3.2):

$$G_{00} = \rho_0^2/r^4\, e^v \tag{35}$$

and using equ. (3) will give:

$$\frac{\rho_0^{\;2}}{r^4} e^v \approx \frac{w}{\varepsilon_c} \quad => \quad \frac{\varepsilon_c \rho_0^{\;2}}{r^4} e^v \approx w \tag{36}$$

The volume integral over (36) gives the energy of particle n according to:

$$W_n = \varepsilon_c \rho_0^{\;2} \int_0^{r_n} \frac{e^{v(n)}}{r^4} d^3 r = 4\pi \varepsilon_c \rho_0^{\;2} \int_0^{r_n} \frac{e^{v(n)}}{r^2} dr \tag{37}$$

Equation (37) will give as energy for a particle n:

$$W_{n,elstat} = 4\pi \varepsilon_c \rho_0^{\;2} \int_0^{r_n} \frac{e^{v(n)}}{r^2} dr = b_0\, \Gamma(1/3,(\rho_n/r_n)^3)\, \rho_n^{-1}/3 \approx b_0\, \Gamma_{1/3}\, \rho_n^{-1}/3 \tag{38}$$

including the integral for the energy of a point charge term modified by e^v. Particles are supposed to be electromagnetic objects possessing photon-like properties, thus it will be assumed that particle energy has equal contributions of electric and magnetic energy, i.e.

$$W_n = W_{n,elstat} + W_{n,mag} = 2W_{n,elstat} \approx 2\, b_0\, \Gamma_{1/3}\, \rho_n^{-1}/3 \tag{39}$$

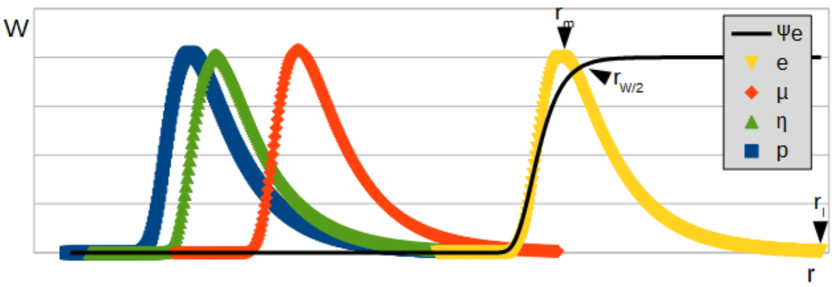

Fig 1: Example for particle energy $W_{n\,calc}$(r) (normalized) vs lg(r); $r_{W/2}$ => radius for integral of energy giving half the final value, $W_n/2$; r_m = energy at maximum W(r); $r_l \approx \lambda_C$;

I 3.2 Quantization with powers of 1/3^n over α

Most relations given here are valid for any particle energy which should be expected as there is a continuous spectrum of energies according to special relativity. However, a particular set of energies may be identified by relaxing the condition of orthogonality of different states according to quantum mechanics to requiring different states to

a) be expressible in simple terms of a ground state coefficient, α_0, in the exponent of e^v and
b) to exhibit no dependence on intermediary states.

This may be illustrated best by looking at the square of particle energy e.g. of the point charge, equ. (44)ff.
In a general case ρ_n may be given as product of $\rho_0 = e_c/(4\pi\varepsilon_c)$ [m], σ_0 and a partial product of particle specific dimensionless coefficients, α(n), of succeeding particles representing the ratio ρ_{n+1}^3 / ρ_n^3 in the exponential of

e^v as (α_0 = ground state coefficient):

$$\rho_n^3 \sim \alpha_0 \Pi_{k=1}^{n} \alpha(k) \qquad n = \{1;2;..\} \tag{40}$$

or ρ_n of the energy expression:

$$\rho_n \sim \alpha_0^{1/3} \Pi_{k=1}^{n} \alpha_k^{1/3} \qquad n = \{1;2;..\} \tag{41}$$

Inserting (40)f in the square of (38)f gives:

$$W_n^2 = \left(\frac{2\,b_0 \Gamma_{1/3}}{3\rho_n}\right)^2 \sim \frac{\alpha_0^{1/3}\alpha_1^{1/3}.....\alpha_n^{1/3}}{\alpha_0\alpha_1....\alpha_n} \tag{42}$$

The last expression of (42) is obtained by expanding the product of $\alpha_k^{2/3}$ included in ρ_n^2 of (42) with the product of $\alpha_k^{1/3}$.

All intermediate particle coefficients cancel out if a relation $\alpha_{n+1} = \alpha_n^{1/3}$ holds:

$$W_n^2 \sim \frac{\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge(n-1))}\boldsymbol{\alpha_0^{1/(3\wedge n)}}}{\boldsymbol{\alpha_0^1}\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge(n-1))}} = \frac{\boldsymbol{\alpha_0^{1/(3\wedge n)}}}{\boldsymbol{\alpha_0}} \tag{43}$$

The relationship between a photon-like object and a point charge object (of elementary charge) is based on the coefficient α, in turn related to half integer spin. This suggests a ground state coefficient $\alpha_0 \approx \alpha^9$ (cf. I 2.5). This fits the relationship of a set of fundamental particle energies with the charged particle of lowest energy, the electron, as a ground state quite well, however, requiring an ad hoc factor ≈ 3/2 for the electron itself. With W_e as ground state W_n would be given by (40)ff *relative* to the electron state as:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} \approx 3/2\,\Pi_{k=1}^{n}\alpha\wedge(-3/3^k) \qquad n = \{1;2;..\} \tag{44}$$

see table 1. The electron coefficient in the exponential of e^v and the energy term equ. (39) would be given as:

$$\rho_e^3 \sim \alpha_e \approx (3/2)^3\alpha^9 \quad \text{and} \quad W_e \sim \alpha_e^{-1/3} \approx 2/3\ \alpha^{-3} \tag{45}$$

This series is assumed to represent the simplest symmetry of particles close to a point charge, i.e. spherical symmetric solutions and index n will serve in the following as equivalent of a radial quantum number. For the angular terms of $\Phi(r, \vartheta, \varphi)$, to be indicated by index l, only rudimentary results exist, their contribution will be assigned to parameter σ, see chpt. I 3.3, I 3.4.

I 3.3 Upper limit of energy

Non-spherical particle states should exhibit lower values of σ [19]. The minimal possible value for σ is defined by the Γ-term in the integral expression for length, (59)ff, and the integers in (29) to be:

$$\sigma_{min} = (2\Gamma_{-1/3}/3)^3 \tag{46}$$

leaving a term

$$\alpha_{\lim} \approx 1.5\alpha^{-1} \approx 4\pi\,\Gamma_{-1/3}{}^2 \approx \frac{1}{4}\left(\frac{\Gamma_{-1/3}2\pi}{\Gamma_{+1/3}}\right)^3 \tag{47}$$

as variable part in σ_n (see (24)f) [20]). The maximum angular contribution to W_{max} would be:

$$\Delta W_{max,\,angular} \approx 3/2\ \alpha^{-1} \tag{48}$$

According to (44) and (48), the maximum energy will be $W_{max} \approx W_e\ 9/4\ \alpha^{-2.5}$ = 4.05E-8 [J] (= 1.03 Higgs vacuum expectation value, VEV = 246GeV = 3.941E-8 [J] [9]).

The maximum angular contribution to energy will require to modify the ground state coefficient, α_0, giving the equivalent to the ratio of electron and Planck energy, α_{Pl}, see chpt. I 7.1.1.

The expression of ρ_e^3 for the electron will be:

$$\rho_e^3 \approx 1.5^3\,\sigma_0\,\boldsymbol{\alpha_{lim}^{-1}/2\ 1.5^3\,\alpha^9}(e_c/(4\pi\varepsilon_c r))^3 \approx 1.5^3\,\sigma_0\,\boldsymbol{\alpha_{Pl}}\,(e_c/(4\pi\varepsilon_c r))^3 \tag{49}$$

For other spherical symmetric particles (n = {1;2;..}):

---

19 According to the geometric interpretation of (24) as well as higher energy $W_{n,l}$ requiring lower $\rho_{n,l}$.

20 $\sigma_0 \approx (\alpha_{lim}\ 2\Gamma_{-1/3}/3)^3$

$\rho_n^3 \approx \sigma_0\, \boldsymbol{\alpha_{lim}}^{\mathbf{-1}}\mathbf{/2\ 1.5^3}\, \boldsymbol{\alpha^9}\, \alpha^{4.5}/\alpha\text{^}(4.5/3^n)\ (e_c/(4\pi\varepsilon_c r))^3 \approx \sigma_0\, \boldsymbol{\alpha_{Pl}}\, \alpha^{4.5}/\alpha\text{^}(4.5/3^n)\ (e_c/(4\pi\varepsilon_c r))^3$ (50)
[21]

I 3.3.1 Energy value of the Higgs boson

Using the limit of σ according to (46), assuming $\Gamma_{+1/3}/\Gamma_{-1/3}$ according to (27) to replace α_{lim}, i.e. associating the maximum angular contribution of chpt. 2.8. with 2π, and using the end of the convergent series in ρ as $\alpha^{13.5}/2$ allows to give the exponential term for the Higgs boson, W_{Higgs}, in a particular simple expression:

$$\exp\left(-\left[\left(\rho_n/r\right)^3\right]\right) \approx \exp\left(-\left(\frac{2\,\Gamma_{-1/3}}{3}\right)^3 \frac{1}{2}\left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}}\right)^3 \frac{\alpha^{13.5}}{2}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right) = \left(\exp\left(-\left(\frac{\Gamma_{+1/3}}{3}\ \alpha^{4.5}\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right)\right)^2 \quad (51)$$

indicating that $\Gamma_{+1/3}/\Gamma_{-1/3}$ is indeed a necessary term in the equation. Equ. (51) inserted in (15) gives the energy of the Higgs boson as $W_{Higgs,\,calc} \approx 2^{2/3}\,\alpha^{-4.5}\,e_c = 1.016\,W_{Higgs,\,exp}$.

I 3.4 Other non-spherical symmetric states

Except for the limit case of I 3.3 angular solutions for particle states are not known yet and to extend the model to such states assumptions have to be made.

Assuming the angular part to be related to spherical harmonics [22] and exhibiting the corresponding nodes would give the analog of an atomic p-state for the 1st angular state, y_1^0. With the additional assumption that $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3} \sim (2l+1)^{1/3}$ ($V_{n,l}$ = volume) is applicable for non-spherically symmetric states as well, this would give $W_1^0/W_0^0 = 3^{1/3} = 1.44$. A second partial product series of energies in addition to (44) corresponding to these values approximately fits the data, see tab. 1.

A change in angular momentum has to be expected for a transition from spherical symmetric states, y_0^0, to y_1^0 which is actually observed with $\Delta J = \pm 1$ except for the pair μ/π with $\Delta J = 1/2$.

An angular contribution relative to the electron, f(l), may be given as function of the angular quantum number l for small l, l = {1;2;3}, with a maximum value given as $4\pi\Gamma_{-1/3}^2$ according to (24) and (46):
$(2l+1)^{1/3} \leq f(l) \leq 4\pi\Gamma_{-1/3}^2$, l = {0;1;2;..}, turning (44) into :

$$W_n/W_e \approx 3/2\frac{\alpha\text{^}(1.5/3^n)}{\alpha^{1.5}}\ f(l) = 3/2\,\Pi_{k=0}^{n}\alpha\text{^}(-3/3^k)\ f(l) \qquad n = \{1;2;..\} \quad (52)$$

I 3.5 Results of energy calculation

Table 1 presents the results of the energy calculation according to (52) for y_0^0 (bold), y_1^0. Only states given in [9] as 4-star, characterized as „*Existence certain, properties at least fairly well explored*", are included, up to Σ'^0 all states given in [9] are listed. Coefficients given in col. 4 refer to (44)f, starting with the electron coefficient in W_e, including its extra term of 2/3 [23]. Exponents of -9/2 for Δ and tau are equal to the limit of the partial product of α(n), including the electron coefficient.

For comparison 2 different calculation methods for energy are given.

In col. 5 equ. (24) and (49)f are used with σ_0 according to the value of the *fit for J_Z =1/2 and α_{Pl} given by W_e/W_{Pl}* according to the experimental value of the electron and definition (76) for Planck energy.

$$W_n = 2b_0\int_0^{r_n}\exp\left(-\left(1.5^{3\delta}\sigma_0\alpha_{Pl}\frac{\boldsymbol{\alpha}^{(\mathbf{-4.5/3^n})}}{\boldsymbol{\alpha}^{\mathbf{-4.5}}}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right)\right)r^{-2}dr \quad => \quad W_\mu = \frac{2}{3}\frac{\Gamma_{+1/3}\boldsymbol{\alpha}^{\mathbf{-1}}}{\left(\sigma_0\alpha_{Pl}\right)^{1/3}}\ e_c \quad (53)$$

(n = {0;1;2;..}; 1.5^δ = extra coefficient for the electron only, δ = δ(0,n>0); bold: particle coefficient, note: electron coefficient, $(3/2\alpha^3)^3$, included in α_{Pl}; Muon given as example [24])

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21 Since non-rest mass is not restricted to the particular solutions of the model this limit does not apply to these. Energies above the value of the Higgs boson/VEV might be possible due to linear combination states or particle compounds.

22 $y_l^m = \int\int \Psi(\varphi,\vartheta)^2 \sin(\vartheta)\,d\varphi\, d\vartheta\ /\ 4\pi$ Note: the wave function will not be normalized to 1.

23 i.e. starting the series in (44) with n= 0;

24 The term for the muon is given as reference to avoid ambiguities due to extra term ≈ 3/2 of the electron.

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient in W_n $\alpha(n)^{-1/3}$ [f(l)]	W_{calc}/W_{lit} Equ.(54)	W_{calc}/W_{Lit} Equ.(56)	J
			see I 3.8	-		-
e^{+-}	**0, 0**	**0.51**	**$2/3\ \alpha^{-3}$**	**1.014**	**1.002**	**1/2**
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**1.007**	**0.996**	**1/2**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}$ [$3^{1/3}$]	1.101	1.088	0
K		495	see I 3.8			0
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**1.002**	**0.990**	**0**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ [$3^{1/3}$]	1.022	1.009	1
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ [$3^{1/3}$]	1.012	1.000	1
K*		894	see I 3.8			1
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.011**	**0.999**	**1/2**
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.010**	**0.998**	**1/2**
η'		958	see I 3.8			0
Φ^0		1019	see I 3.8			1
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**1.020**	**1.008**	**1/2**
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**1.014**	**1.002**	**1/2**
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**1.012**	**1.000**	**3/2**
Ξ		1318				1/2
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$ [$3^{1/3}$]	0.989	0.977	3/2
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$ [$3^{1/3}$]	0.982	0.970	3/2
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$ [$3^{1/3}$]	1.014	1.002	3/2
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})$ [$3^{1/3}$]	1.012	1.000	1/2
Higgs	∞,∞ *	1.25 E+5	$(\alpha^{-9/2})$ [$3/2\ \alpha^{-1}$] /2	1.042	1.066	0
VEV	∞,∞**	2.46 E+5	$(\alpha^{-9/2})$ [$3/2\ \alpha^{-1}$]	1.059	1.083	

Table 1: Particle energies; col.2: radial, angular quantum number, *, **see chpt. I 1.3; col.4: α-coefficient in W_n according to (44)f, n = {**0**;1;2;..}; col.5,6: ratio of calculated energy, W_{calc}, and literature value [9] according to σ_0 of fit J_Z =1/2 (approximately (24), giving (53)) and to σ_0 of (28) (giving (55)); col.7: angular momentum J_z [ħ];
Blanks in the table are discussed in I 3.8. The values of physical constants are taken from [9].

In col. 6 equ. (49)f is used with σ_0 according to (28), based on the considerations of I 2.3, α_{Pl} will be replaced by $\alpha_{lim}^{-1}/2$ ($3/2\ \alpha^9$) with $\alpha_{lim}^{-1}/2$ recalculated from $\alpha_{lim}^{-1} = \sigma_0^{-1/3}\, 2\Gamma_{-1/3}/3$ (cf. (54)) [25].

$$\begin{aligned}
&\exp\left(-\left[\left(\rho_n/r\right)^3\right]\right) \approx \exp\left(-\left[1.5^{3\delta}\sigma_0\,\alpha_{Pl}\,\alpha(n)\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \\
&\exp\left(-\left[1.5^{3\delta}\left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\,\Gamma_{-1/3}}{3}\right]^3\frac{\alpha(n)}{2\,\alpha_{\text{lim}}}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \\
&\exp\left(-\left[1.5^{3\delta}\left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\,\Gamma_{-1/3}}{3}\right]^3 2\left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}\,2\pi}\right)^3\left(\frac{3}{2}\right)^3\boldsymbol{\Pi_{k=0}^{n}\,\alpha\wedge\left(9/3^k\right)}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \\
&\left(\exp\left(-\left[1.5^{3\delta}\frac{\pi^2\,\Gamma_{-1/3}{}^3}{\Gamma_{+1/3}{}^2}\,\boldsymbol{\Pi_{k=0}^{n}\,\alpha\wedge\left(3/3^k\right)}\,\frac{e_c}{4\pi\varepsilon_c r}\right]^3\right)\right)^2 \qquad n = \{0;1;2;..\}
\end{aligned} \tag{54}$$

25 Expression intended to emphasize 3^{rd} power relationship, a remaining factor of 2 is attributed to $e^{v/2}$ being squared.

$$W_n = 2b_0 \int_0^{r_n} \left(\exp\left(- \left[1.5^{3\delta} \frac{\pi^2 \Gamma_{-1/3}^{\;3}}{\Gamma_{+1/3}^{\;2}} \mathbf{\Pi_{k=0}^{n}} \boldsymbol{\alpha} \wedge \left(\mathbf{3/3^{k}}\right) \frac{e_c}{4\pi\varepsilon_c r} \right]^3 \right) \right)^2 r^{-2}\, dr \quad => $$

$$W_\mu = 2 e_c \frac{\Gamma_{+1/3}}{3}\; 2^{-1/3} \left[\frac{\Gamma_{+1/3}^{\;2}}{\pi^2 \Gamma_{-1/3}^{\;3}} \boldsymbol{\alpha^{-4}} \right] = \frac{2^{2/3}}{3\pi^2} \left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}} \right)^3 \boldsymbol{\alpha^{-4}}\; e_c \qquad (55)$$

(n = {0;1;2;..}; 1.5^{δ} = extra coefficient for the electron; bold: particle coefficient; Muon given as example)

I 3.6 Second solution for energy quantization

A second solution featuring a 3rd power relation for equation (43)f is given by $\alpha_{n+1} = \alpha_n^3$ with α_0 as reference state the next energy state relative to the electron by definition has to give the Planck energy, followed by a state of energy ratio α_0^3 relative to W_{Planck}, i.e. ~ 1.4E+72 J roughly in the order of magnitude of the estimated energy of the observable universe [26], see chpt. II 6.2 as well.
This suggests to change the particle order in table 1 according to table 2:

	n, l	$W_{n,Lit}$ [J]	α-coefficient (energy)	Calculated energy relative to electron
ν	-1,0	5.50E-21	$1/(4\pi\|\Gamma_{-1/3}\|^3/3)$	
e_c	-	3.11E-18	Reference	
e^{+-}	0, 0	8.19E-14	$\alpha^{-3}/(4\pi\|\Gamma_{-1/3}\|^3/3) \approx \alpha^{-2}$	
μ^{+-}	1, 0	1.69E-11	$\alpha^{-2}\alpha^{-1}$	
η^0	2, 0	8.78E-11	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}$	
p^{+-}	3, 0	1.50E-10	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$	
n	3, 0	1.51E-10	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$	
Λ^0	4, 0	1.79E-10	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$	
Σ^0	5, 0	1.91E-10	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$	
Δ	∞, 0	1.97E-10	$\alpha^{-2}\alpha^{-3/2}$	
Higgs	∞,∞	2.01E-08	$\alpha^{-2}(\alpha^{-5/2})$	$W_{Higgs, calc}$= 0.90 $W_{Higgs, exp}$
Planck	$W_{Higgs}/W_e\ (W_{Higgs}/W_e)^3$	1.67E+8	$\alpha^{-2}(\alpha^{-5/2})\ (\alpha^{-5/2})^3$	$W_{Pl, calc}$ = 1.28 $W_{Pl, def}$
	$W_{Pl}/W_e\ (W_{Pl}/W_e)^3$	~1.4E+72	$\alpha^{-2}(\alpha^{-5/2})\ (\alpha^{-5/2})^3\ [(\alpha^{-5/2})\ (\alpha^{-5/2})^3\ ]^3$	

Table 2: Table with particle energies, emphasizing a relationship between elementary charge and electron as well as Higgs boson and Planck energy; α-coefficients only, minor terms omitted;

I 3.7 Accuracy of energy calculation

All calculations such as the numerical approximation of Γ-functions are performed with an accuracy of about 0.0001. Agreement with experimental particle energies is typically in a range of ± 0.001 to ± 0.01. There are three major causes preventing a significant improvement of accuracy.
1) Especially in the case of particle families [27], effects on top of the relations given in this work have to play a role to explain different energy levels of differently charged particles. This limits accuracy and the possibility to precisely identify candidates for the calculated energies (e.g. both ρ^0 and ω^0 are given for 1.44 $\alpha^{-1}\alpha^{-1/3}$ in tab. 1).
If possible, particles chosen for y_0^0 in table 1 are of charge ± 1. In cases such as Σ with three energy levels, the intermediate energy level is chosen. For the y_1^0 series particles of the same charge as their y_0^0 equivalent are preferred in table 1.
2.) Principal differences in the ansatz for ρ, such as given in col. 5 and 6 of table 1.

26 Baryonic matter ≥ 1E53 kg ≈ 1E+70J. Total Energy ≈ 1E+72J.
27 Particle families, defined here as possessing the same exponent n in (44) but being different in charge, show a typical spread in energies of 3-4MeV and no dependence on total particle energy.

3) The accuracy of the calculations is already in the order of magnitude of expectable QED corrections. Since these originate from the *interaction* of particles with the vacuum they may not be included in the equations of this model yet may have some influence on experimental values.

To illustrate possible QED-Effects and the non-linearity of the Γ-functions, a calculation of σ_0 with values of (22)f varying within +/-1.00116 gives a range of energy values of +/-1.006, varying within +/-1.00116^2 gives a range of energy values of +/-1.013 compared to the values given in table 1.

I 3.7.1 Comparing with QCD results

The SM does not provide quantitative results for energy of leptons. As for comparing accuracy of the energy calculation with results from quark models, calculations of simplicity comparable to the model presented here, using constituent quarks and spin-spin interaction, yield approximately the same accuracy [10]. However, more recent lattice-QCD calculations for particle mass use the mass of current quarks as input parameter [11, 12, 13].

For particles consisting of u, d, s quarks accuracy of lattice-QCD calculations is approximately in the range of ~ 0.01 for particles with mass of the proton or higher, however, drops significantly for hadrons of lower mass. Compared to this work these calculations need a significant larger set of input parameters, consisting typically of 2-4 quark masses, coupling parameters and reference hadron mass. The light quark masses used in these calculations exhibit differences in the order of 10% and are in general not consistent in order of magnitude with those used in other QCD calculations e.g. of magnetic moments or strong decay [14, 15, 16].

I 3.8 Additional particle states

Assignment of more particle states will not be obvious. The following gives some possible approaches.

I 3.8.1 Partial products

Additional partial product series will have to start with higher exponents n in α^($-1/3^n$) giving smaller differences in energy while density of experimentally detected states is high. There might be a tendency of particles to exhibit a lower mean lifetime (MLT), making experimental detection of particles difficult [28]. To determine the factor y_l^m of higher angular states requires an appropriate ansatz for a metric / differential equation yet to be found.

One more partial product might be inferred from considering d-orbital-like equivalents with a factor of $(2l+1)^{1/3} = 5^{1/3}$ as energy ratio relative to η, giving the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV i.e. close to energy values of the first particles available as starting point, η', Φ^0. However, in general it is not expected that partial products can explain all values of particle energies.

I 3.8.2 Linear combinations

Particles supposed to be attributed to S-quaternion/s-quark states do not fit well to the simplest interpretations of this model. Approaches not in-line with the SM may be considered as well.

The first particle family that does not fit to the partial product series scheme are the kaons at ~ 495MeV. They might be considered to be linear combination states of π-states. The π-states of the y_1^0 series are assumed to exhibit one angular node, giving a charge distribution of +|+, -|- and +|-. A linear combination of two π-states would yield the basic symmetry properties of the 4 kaons as:

		+			-			-			+
K^+	+		+	K^-	-	-	K_S^o	+	+	K_L^o	+	-
		+			-			-			-

(+/- = charge)

providing two neutral kaons of different structure and parity, implying a decay with different parity and MLT values. For the charged Kaons, K^+, K^-, a configuration for different chirality equal to the configuration for charge of K_S^o and K_L^o might be possible, giving two versions of P+ and P- parity of otherwise identical particles and corresponding decay modes not violating parity conservation. The same would hold for K* particles.

A linear combination of 3 such π-states would result in an essentially spherical symmetric object which might be attributable to the η-particle.

28 Which might explain missing particles of higher n in the y_0^0 and y_1^0 series as well.

I 3.8.3 Neutrinos

Neutrinos are not part of the energy series (44)ff and thus not related to Planck energy via the coefficient α_{Pl} and the considerations of chapter I 7.1 related to gravitation, thus they need to be treated differently.
One might look for additional terms in (15) or just omit α_{Pl} in expressions such as (49)f. This would yield a particle energy of ≈ 1E-20[J] (≈0.1eV), i.e. in the order of magnitude expected for the heaviest neutrino [17].

The source of a vacuum energy, ρ_{vac}, or cosmological constant term, Λ_C, is considered to be due to an extra term in G_{00}, in this model maybe originating from the vacuum term of (15). However, there are some points concerning neutrinos and ρ_{vac} that should be noted.
The omitting of factor α_{Pl} is vaguely related to the term for vacuum energy, ρ_{vac} (5.3566E−10 [J/m^3], Planck collaboration [18]), of chpt. I 8.1. Looking at the energy density of a state according to relation (53), omitting α_{Pl}, one gets an energy of 6.5E-21[J] [29] and a Compton wavelength, λ_C = 3.1E-5[m] giving an energy density w = 2.2E-7. Less heavy neutrinos might be closer to ρ_{vac}. If the states of this model would be considered to be excitations of a single field, the lowest neutrino state would be the lowest excitation and might be viewed to correspond to the zero energy of the vacuum.

I 4 Magnetic moment [30]

Within this model particles are treated as electromagnetic objects principally enabling a direct calculation of the magnetic moment M from the electromagnetic fields.
The magnetic moment M_e of the electron is given as product of the anomalous g-factor, g_a = 1.00116, Dirac-g-factor, g_D = 2, and the Bohr magneton, μ_B = e ħ/(2m_e), times the quantum number for angular momentum J = 1/2:

$$\boldsymbol{M}(e) = g_a g_D \mu_B/2 = g_a \frac{2ec_0^2}{2W_e}\frac{\hbar}{2} = g_a 9.274\text{E-}24\,[\text{Am}^2] \tag{56}$$

The factor g_a arises from the interaction of the electron with virtual photons as calculated in quantum electrodynamics and should not be part of a calculation of the magnetic moment from the field of the electron itself. Within this model the factor 2 of g_D originates from the fact that particle energy is supposed to be equally divided into contributions of the electric and magnetic field, W_{el} = W_{mag} = W_n/2 and only the magnetic field, i.e. W_{mag} contributes to the magnetic moment.
Inserting the term for particle energy of (38)f in (56) gives:

$$\frac{\boldsymbol{M}(e)}{g_a} = \frac{e\hbar c_0^2}{2W_e} = \frac{e\hbar c_0^2}{2}\frac{3\rho_e}{2b_0\Gamma_{1/3}} = ec_0\rho_e\left(\frac{\Gamma_{-1/3}}{3}\frac{3}{\Gamma_{-1/3}}\right)\frac{3[\hbar c_0/b_0]}{4\Gamma_{1/3}} = ec_0\rho_e\frac{\Gamma_{-1/3}}{3}\left[\frac{9[\alpha^{-1}]}{4\Gamma_{1/3}\Gamma_{-1/3}}\right] \approx \frac{ec_0\lambda_e}{4\pi} \tag{57}$$

The term on the right is obtained by expansion with $\Gamma_{-1/3}$/3 using (19) and (66) , to give an expression using λ_C . The last term could be expressed through the second term via W = hc_0/ λ_C.
A similar term will be obtained by a calculation starting directly from the fields as given by the following equation.
The relation of the values of E and B in an electromagnetic wave is given by B = E/c_0. This gives as first approximation for the value of $\boldsymbol{M}_n$ of a particle n:

$$\boldsymbol{M_n} \approx \frac{1}{\mu}\int_0^{\lambda_{C,n}} B(r)\Psi_n(r)^2 d^3r = \varepsilon c_0\int_0^{\lambda_{C,n}} E(r)\Psi_n(r)^2 d^3r = ec_0\rho_n\frac{\Gamma_{-1/3}}{3}\left[3^{0.5}\frac{3}{2}[\alpha]^{-1}\right] = ec_0\lambda_{C,n} \tag{58}$$

A simple ansatz with M = IA = ec_0πR^2/(2πR) with R = λ_c would give: M = ec_0 λ_c/2. The missing factor 4π, 2π, indicates a spherical symmetric particle state.

I 5 Coupling constants

I 5.1 Photon energy

In the following a term for length expressed via the Euler integral of (18) will be introduced for $\lambda_{C,n}$:

29 I.e. $W_{v,\,calc}$ ≈ 0.04eV, compare with m_v < 0.08 eV [17];

30 Note: to allow for comparison with tabulated values of M in units of [Am^2] the calculations in this chapter use e [C] not e_c [J], conversion factor: [m^{2}C/s] /[m^2 J/s] = e/e_c = 1/19.4 [C/J].

$$r_x = \int_0^{r_x} e^v\,dr \;=\; \rho_n/3 \int_{(\rho_n/r_x)^3}^{\infty} t^{-4/3} e^{-t}\,dt \;\approx \Gamma(-1/3, (\rho_n/r_x)^3)\;\; \rho_n/3 \tag{59}$$

In the limit $(\rho_x/r_x)^N$ -> 0

$$\Gamma(-1/N, (\rho_x/r_x)^N) = \int_{(\rho_x/r_x)^N}^{\infty} t^{-(1/N+1)} e^{-t}\,dt \;\approx N\,(\rho_x/r_x)^{-1} = N\,\sigma^{1/3}/2 \tag{60}$$

holds. Equation (60) inserted in the right side of (59) gives back r_x, however, (59)f may be seen as expressing r_x in terms useful for this model, i.e. ρ_n, σ_0 and Γ-functions.

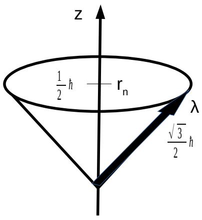

Fig. 2: r_n vs λ_C

The integration limits for calculating angular momentum in z-direction, r_n of J_z, (20)ff, and (Compton-)wavelength, λ_C, supposed to represent the rotating E-vector and in turn total angular momentum J should be related by the factor √3 of the ratio J/J_z:

$$\lambda_C / r_n = (1/2(1/2 + 1))^{0.5} / (1/2) \;= \sqrt{3} \tag{61}$$ [31]

Using equ. (60) for the incomplete Γ-function and multiplying r_x in the integration limit $(\rho_n/r_x)^3$ by √3 gives in good approximation (using (24)):

$$\lambda_{C,n} \approx 3^{1.5}\,\sigma_0^{1/3}/2\;\rho_n/3 \approx 3^{0.5}\,4\pi\,\Gamma_{-1/3}^{\;3}/3\,\rho_n \tag{62}$$

With (24) and (62) energy of a photon may be expressed as:

$$W_{Phot,n} = hc_0/\lambda_{C,n} \;=\; hc_0 \;/\; \int^{\lambda_{C,n}} e^v\,dr \;=\; \frac{2\,hc_0}{3^{0.5}\rho_n\sigma_0^{1/3}} \;\approx\; \frac{3\,hc_0}{3^{0.5}\,4\pi\,\Gamma_{-1/3}^3\,\rho_n} \tag{63}$$

I 5.2 Fine-structure constant, α

The energy of a particle is assumed to be the same in both photon and point charge description. Equating (39) with (63) gives:

$$W_{pc,n} = W_{Phot,n} = 2b_0\,\Gamma_{1/3}\,\rho_n^{-1}/3 \approx \frac{2\,hc_0}{3^{0.5}\rho_n\sigma_0^{1/3}} \;\approx\; \frac{3\,hc_0}{3^{0.5}\,4\pi\,\Gamma_{-1/3}^3\,\rho_n} \tag{64}$$

Using (19) for an argument of 1/3:

$$\Gamma_{+1/3}\,\Gamma_{-1/3} \;=\; 3^{0.5}\,2\pi \tag{65}$$

and the 1st term from right of equation (64) will give (note: h => ħ):

$$\alpha^{-1} \;=\; \frac{hc_0}{2\pi\,b_0} \;\approx\; \left(\frac{2\,\Gamma_{+1/3}}{3^{0.5}\,2\pi}\right)\left(\frac{4\pi}{3}\Gamma_{-1/3}^3\right) \approx\; \frac{2}{3}\frac{\Gamma_{-1/3}}{\Gamma_{+1/3}}\,4\pi\,\Gamma_{+1/3}\Gamma_{-1/3} \;\approx\; \boldsymbol{4\pi\,\Gamma_{+1/3}\Gamma_{-1/3}} \tag{66}$$

The last expression is emphasized since it has a simple interpretation in terms of the coefficients of the integrals over $\exp(-(\rho/r)^N)$. Equations (64)ff are based on the integral over a 3-dimensional point charge term modified by the exponential term according to (7) with N = 3, and a complementary integral - in 3D for length, λ_C - to yield a dimensionless constant.

This may be generalized to N dimensions (N ={3; 4}), to give a point charge term (S_N = geometric factor for N-dimensional surface, in case of 3D: 4π; 4D: $2\pi^2$):

$$\int_0^r e^{v(N)} r^{-2(N-1)}\;d^N r \;=\; S_N \int_0^r e^{v(N)}\,r^{-(N-1)}\,dr \tag{67}$$

that has to be multiplied by a complementary integral

31 Alternatively: $\lambda_{C,n} = 3\rho hc_0/(2b_0\Gamma_{1/3}) = 3\pi\,\alpha^{-1}\,\rho/\Gamma_{1/3}$; $r_n = 3/2\;\alpha^{-1}\;\rho\;\Gamma_{-1/3}/3$ => $\lambda_{C,n}/r_n = 6\pi/(\Gamma_{1/3}\Gamma_{-1/3}) = 6\pi/(2\pi\sqrt{3}) = 3^{0.5}$

$$\int_0^r e^{v(N)} r^{(N-3)} dr \qquad (68)$$

The exact result depends on the integration limit of the second integral, cf. II 5.1. However, in terms of the Γ-functions both electroweak coupling constants can be given in 1st approximation as

$$\alpha_N^{-1} = S_N \frac{\Gamma(+m/N)\Gamma(-m/N)}{m^2} = S_N \frac{\Gamma(+(N-2)/N)\Gamma(-(N-2)/N)}{(N-2)^2} \qquad \text{(m = N-2, cf. (18))} \qquad (69)$$

Dimension – space	coupling constant	Value of *inverse* of coupling constant, α_N^{-1}	
4D	$\alpha_4 = \alpha_{weak}$	$2\pi^2 \Gamma_{+1/2} \Gamma_{-1/2} /4 = \pi^3 =$	31.0
3D	$\alpha_3 = \alpha$	$4\pi \Gamma_{+1/3} \Gamma_{-1/3} = 4\pi \Gamma_{+1/3} \Gamma_{-1/3} =$	136.8

Table 3: Values of electroweak coupling constants

Since the values of the coupling constants depend on integrals over space they are in general dependent on the frame of reference, tab. 3 gives the values in a rest frame. Considering that the dominant contribution comes from the complementary integral, the values of the coupling "constants" should increase in non-rest frames, their inverse should decrease.
In chpt. II 5.1 a more detailed treatment of coupling constants in N dimensions will be given.

I 6 Wave function and differential equation

Quantum mechanics is a useful tool to describe phenomena of particles. Thus it is expectable that some congruence with the Kaluza-type model given here and quantum mechanical concepts has to exist.

I 6.1 Radial part

A solution for a wave such as (29)

$$e^{v(r)} = \varphi(r) = \exp\left(-\left(\frac{\rho^3}{2r^4} + \left[\left(\frac{\rho^3}{2r^4}\right)^2 - 4\frac{\rho^3}{\sigma 2r^5}\right]^{0.5}\right) r/2\right) \qquad (70)$$

might be deduced from 4 boundary conditions:
1.) To be able to apply φ(r) to a point charge φ(r = 0) = 0 is required.
2.) To ensure integrability an integration limit is needed. This may be achieved by φ(r) being in the form of a solution of a 2nd order differential equation of a general type of a damped oscillation.
3.) φ(r) should be applicable regardless of the expression chosen to describe the electromagnetic object. In particular requiring a point charge and a photon representation of a localized electromagnetic field (particle) to have the same energy, an exponent of r is required to be 3 in the approximation for small r.
4,) The Euler integrals for m/N ≤ 0 require an integration limit $(\rho/r)^3 = 8/\sigma$
In all integrals over $\varphi_n(r)$ equ. (71) may be used as approximation for (70)

$$\varphi_n(r < r_n) \approx \exp\left(\frac{-\rho_n^3}{r^3}\right) = e^{v(r)} \qquad (71)$$

φ(r) may be interpreted in a quantum mechanical way as acting as a probability amplitude on an electromagnetic field.
The function φ is a very good approximation of equation (70) and though it should be basically related to function Φ and the field equations of GR a differential equation resembling quantum mechanical expressions may be constructed from a differential equation of the damped oscillation type. The approximation ($r < r_n$), (σ -> 1) of equation (70) provides a solution with the following coefficients:

$$-\frac{r}{6}\frac{d^2\varphi(r)}{dr^2} + \frac{\rho^3}{2r^3}\frac{d\varphi(r)}{dr} - \frac{\rho^3}{r^4}\varphi(r) = 0 \quad ^{32} \qquad (72)$$

An equation such as (72) may be turned into a more conventional expression containing terms used in quantum mechanics by using a quantum mechanical operator for kinetic energy, T = $(\hbar c_0)^2 r /b_0$, i.e. $c_0^2 r /b_0$ representing 2/(2m) [33], and the 3rd term in the series expansion for energy, (84), for potential energy, V:

$$V(r) = b_0 \rho^3/(2 r^4) \qquad (73)$$

and a corresponding expansion by $(\hbar c_0)^2\alpha^2 /b_0^2$ for the 2nd order term of (72) (with one b_0-term used up in V(r)), an approximate differential equation for this model may be given that resembles quantum mechanical terms :

$$-\frac{(\hbar c_0)^2 r}{\alpha^{-2} b_0} \frac{d^2\varphi(r)}{dr^2} + r V(r) \frac{d\varphi(r)}{dr} - V(r)\varphi(r) = 0 \qquad (74)$$

A solution such as (70) might be related to a differential equation such as (72) if one applies the approach used in I 7.1.2, i.e. considering a subset of r terms (those given in (72)) to turn into constants. Though this should strictly be applied only for $r \geq \lambda_C$, one might treat these r-terms as varying sufficiently slowly for r approaching λ_C coming from $r < \lambda_C$, as well. This would suggest an interpretation related to a damped oscillation.
The range of validity of the approximation of a differential equation of type (72) has to be examined further.

I 6.2 Angular part

For a differential equation of type (72) a separation of variables will in general not be possible, the spherical harmonics such as Y_{10} will not be a solution for (72). However, the agreement with particle energies, as seen in tab.1, seems to justify to use the first few spherical harmonics as an approximation.

I 7 Particle-particle interaction

The concept of forces of this model is quite different from that of the SM that is based on particle exchange. Within the framework of GR all forces should be traced back to space-time, curved by energy related quantities. Here electromagnetism is the starting point, additional forces should be part of a series expansion of the EM-terms. The 2nd order term will in general be attributed to gravitational effects for $r > \lambda_C$, for $r < \lambda_C$ it will be responsible for particle energy and thus might be vaguely related to effects traditionally attributed to the strong interaction. Weak interaction will be not part of the series expansion and has to be interpreted in different terms.
In short, the set of interactions gravitation, electromagnetic, weak, strong of the SM will turn into electromagnetic + next order term (= interaction based on r^{-4}-term in (84) for $r > \lambda_C$, gravitation for $r < \lambda_C$).

I 7.1 Gravitation

I 7.1.1 Planck scale

Expressing energy/mass in essentially electromagnetic terms suggests to test if mass interaction i.e. gravitational attraction can be derived from the corresponding terms. Assuming the expansion of the incomplete Γ-function for the integral over r^{-2}, $\Gamma(1/3,\rho_n/r^3)$ (83)f, might be an adequate starting point for gravitational attraction as well, implies that the Coulomb term b_0 will be part of the expression for F_G, i.e. the ratio between gravitational and Coulomb force, e.g. for the electron, $F_{G,e}/F_{C,e}$ = 2.41E-43, should be a term that can be given as completely separate, self-contained expression.
This is equivalent to assume that gravitational interaction is a higher order effect with respect to electromagnetic interaction and as such should be of less or equal strength compared to the latter. This suggests to use the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 /c_0^4 \qquad (75)$$

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32 [N15.1] $d\varphi(r)/dr = 3 \sigma\rho^3 r^{-4} \varphi(r)$
[N15.2] $d^2\varphi(r)/dr^2 = 9 (\sigma\rho^3)^2 r^{-8} \varphi(r) - 12 \sigma\rho^3 r^{-5} \varphi(r) + 6 \sigma\rho^3 r^{-5} \varphi(r)$ (polar coordinates)
[N15.1] -[N15.2] inserted in (72) gives:
[N15.3] $r/6 \{-9 (\sigma\rho^3)^2 r^{-8} + 6 \sigma\rho^3 r^{-5}\} + 3/2 (\sigma\rho^3)^2 r^{-7} - \sigma\rho^3 r^{-4} = 0$
[N15.4] $-3/2 (\sigma\rho^3)^2 r^{-7} + \sigma\rho^3 r^{-4} + 3/2 (\sigma\rho^3)^2 r^{-7} - \sigma\rho^3 r^{-4} = 0$
33 Assuming $W_{n,kin} = W_n/2$

as definition for Planck terms, giving for the Planck energy W_{Pl}:

$$W_{Pl} = c_0^2 (b_0/G)^{0.5} = c_0^2 (\alpha h c_0/G)^{0.5} \tag{76}$$

Using (76) gravitational attraction F_G in the classical limit can be expressed as:

$$F_G = \frac{b_0 W_n W_m}{W_{Pl}^2} \frac{1}{r^2} \tag{77}$$

The value of W_{Pl} according to definition (76) allows to give the ratio of W_e and W_{Pl} as (cf. (45), (47)f) [34]:

$$\frac{W_e}{W_{Pl}} = 4.9E\text{-}22 \approx \frac{\alpha_e}{2\alpha_{\lim}} \approx 1.5^2 \alpha^{10}/2 \stackrel{\text{def}}{=} \alpha_{Pl} \tag{78}$$

i.e. the relation between W_e and W_{Pl} is given by α_e, the electron coefficient in the exponent of e^v, (45), times the angular limit factor according to (47) divided by two [35].
The constant G may be given as:

$$G \approx \frac{\alpha_{Pl}^2 c_0^4 b_0}{W_e^2} \tag{79}$$

Since W_e may be expressed as function of π, $\Gamma_{1/3}$, $\Gamma_{-1/3}$ and e_c only, (53), (55), G may be expressed as a coefficient based on electromagnetic constants, $G \approx c_0^4 \alpha^{24}/(4\pi\varepsilon_c)$.

$$G_{calc} = \frac{c_0^4}{4\pi\varepsilon_c} \left(\frac{(4\pi)^3 \Gamma_{-1/3}^7 \;\; \alpha^{15}}{3\pi^{2/3} \Gamma_{1/3}} \right)^2 = 1.0013\, G_{\exp} \tag{80}$$

or

$$G_{calc} \approx \frac{c_0^4}{4\pi\varepsilon_c} \left(\frac{1}{3\pi^{2/3}} \left(\frac{\Gamma_{-1/3}}{\Gamma_{1/3}} \right)^4 \alpha^{12} \right)^2 \approx \frac{c_0^4}{4\pi\varepsilon_c} \frac{2}{3} \alpha^{24} = 1.0008\, G_{\exp} \tag{81}$$

I 7.1.2 Gravitation from series expansion of exponential function

Terms for gravitation may be recovered via a series expansion of either the Γ-function, see (83)f below, or the exponential e^v e.g. in (35)f. The latter gives for the first two terms of energy density:

$$w \approx \frac{\varepsilon_c \rho_0^2}{r^4} e^v \approx \varepsilon_c E^2 \left[1 + \sigma \alpha_{Pl} \left(\frac{e_c}{4\pi\varepsilon_c \boldsymbol{r}} \right)^3 \right] \tag{82}$$

which is a very good approximation for $r > \alpha\lambda_C$. The 1st term is the classical Coulomb term for energy density. The 2nd term contains by definition the ratio between Coulomb and gravitational terms for *one* particle, α_{Pl}. To turn this into the exact Coulomb / gravitation relationship requires

1) coefficient σ to approach unity, which may be approximately justified by considering the limit of chpt. I 3.3,
2) parameter r in $e_c/(4\pi\varepsilon_c r)$ to turn into a constant,
3) parameter r to approach the value $e_c/(4\pi\varepsilon_c)$.

For condition 2) one has to consider that r *in the exponential* may not be considered to be a free parameter for $r > \lambda_C$, the limit of a real solution for an equation such as (70). Using the limit of σ_{min} of (46) and inserting the Compton wavelength of the electron in (82) would give a value two orders of magnitude off to yield the expected value for the electrostatic / gravitation ratio. Since σ is essentially related to spin of a particle and it has to be assumed that spin does not play a role for $r >> \lambda_C$, one might omit this coefficient in (82) as well as in the term for λ_C and thus recover the exact gravitational term.

The same proceeding could be used for a N=2 solution of (7) giving a potential term in the metric (cf. I 1.3.1). The general expression for the series expansion would be:

Coulomb-term $(1 + \alpha_{Pl})$.

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34 Within the precision of the model parameters, such as (24)f and in particular of the extra factor of ≈ 2/3 of W_e.
35 A factor 2 might correspond to relate only the electrostatic contributions of (39) for the electron with the electrostatically defined value of a Planck state, see I 3.3.1 as well.

Particle interaction would be given by the square of the α_{Pl} term multiplied by appropriate coefficients from the α-series according to (44) for particles of spherical symmetry in a rest system. Since the 2nd term of such a series expansion should not exceed the 1st, electromagnetic one, the maximum relativistic mass for such particles would be defined by α_e^{-1}, while the inverse of the maximum angular term, i.e. α_{lim}^{-1} as given in (47) secures that particles that are not spherical symmetric in a rest system can not exceed the Planck limit either, this gives relation (78).

The approach using assumptions 1) - 3), is supported by the considerations of chpt. I 8.1, yielding a term for the cosmological constant in the correct order of magnitude.

I 7.1.3 Series expansion of $\Gamma(1/3,(\rho_n/r_n)^3)$

The series expansion of $\Gamma(1/3,(\rho_n/r_n)^3)$ in the equation for calculating particle energy (38)f gives [8]:

$$\Gamma(1/3,\ (\rho_n/r)^3) \approx \Gamma_{1/3} - 3\left(\frac{\rho_n}{r}\right) + \frac{3}{4}\left(\frac{\rho_n}{r}\right)^4 - \frac{3}{7}\left(\frac{\rho_n}{r}\right)^7 + \ldots \tag{83}$$

and for $W_n(r)$:

$$W_n(r) \approx W_n - 2b_0\frac{3\rho_n}{3\rho_n r} + 2b_0\frac{3}{4}\frac{\rho_n^4}{3\rho_n r^4} = W_n - \frac{2b_0}{r} + b_0\frac{\rho_n^3}{2r^4} \tag{84}$$

The 2nd term in (84) drops the particle specific factor ρ_n and gives twice [36] the electrostatic energy of two elementary charges at distance r. The 3rd term is an appropriate choice for the 0th order term of the differential equation (cf. I 6) as potential energy term. It is supposed to be responsible for the localized character of a particle state and may be identified with the “strong force” of the standard model as observable e.g. in particle scattering.

I 7.1.4 Virtual superposition states

Within this model particles might interact via direct contact in place of interaction via boson-exchange. The particles are not expected to exhibit a rigid radius. Within the limits of charge and energy conservation a superposition of many states might be conceivable, extending the particle in space with radius ~ $r_{m,n}$, r_n appropriate for energy of each virtual particle state (VS) [37], providing a source of energy at a distance r_{VS} from the primary particle and in turn contributing to the stress-energy tensor responsible for curvature of space-time that manifests itself in gravitational attraction.

In general VS are not supposed to consist of analogues of e.g. spherical symmetric states covering the complete angular range of 4π but to be an instantaneous, short term extension of the (rotating) E-vector thus requiring the angular limit factor of (46). A long range effect of the 3rd, the strong interaction term, of (84) may be exerted via virtual particle states. To estimate such an effect in first approximation the following will be used;

- the 3rd term of equ. (84) with ρ according to (49)f,
- the angular limit state of σ_{min} according to (46) approximated as ≈ 1,
- $(e_c/(4\pi\ \varepsilon_c))^3 \approx (\alpha^{-1} r_e)^3$, which might be considered to represent the cube of a natural unit of length. ρ

For any VS at $r = \alpha^{-1}\ r_{VS} = \Pi_{W,VS}^{1/3}\ e_c/(4\pi\varepsilon_c)$ [38], i.e. the radius of the VS in natural units, equ. (85) will hold:

$$W_{VS}(r) \approx b_0\frac{\rho_{VS}^3}{(\alpha^{-1}\ r_{VS})^4} \approx \frac{b_0\alpha_0\Pi_{W,VS}(\alpha^{-1}r_e)^3}{(\alpha^{-1}\ r_{VS})^3(\alpha^{-1}\ r_{VS})} \approx \frac{b_0\alpha_0\Pi_{W,VS}(\alpha^{-1}r_e)^3}{(\Pi_{W,VS}^{1/3}\alpha^{-1}\ r_e)^3(\alpha^{-1}\ r_{VS})} \approx \frac{b_0\alpha_0}{(\alpha^{-1}\ r_{VS})} \approx \frac{b_0}{(\alpha^{-1}r_{VS})}\left(\frac{F_{G,e}}{F_{C,e}}\right)^{0.5} \tag{85}$$

Considering that the composition of the stress-energy tensor from virtual states is expected to be based on a much more complex mechanism requiring consideration of all possible virtual states at a particular point and appropriate averaging, (85) seems to be a quite acceptable first approximation.

The crucial factor that turns the r^{-4} dependence of the strong interaction term into r^{-1} of gravitational interaction is the proportionality of ρ_n to any characteristic particle length, r_n, $r_{m,n}$, $\lambda_{C,n}$ etc. which is valid for

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36 Due to adding up the electromagnetic contributions in (39): $W_n = 2W_{n,el} = 2W_{n,mag} = W_{n,el} + W_{n,mag}$

37 The superposition states considered here would be not virtual in a Heisenberg sense, the energy is provided by the source particle.

38 $\Pi_{W,VS} = \Pi_{k=1}^{VS}\alpha$^(-3/$3^k$) ; i.e. the product of all spherical symmetric particle coefficients except for electron in the energy representation of equ. (44);

each particle state subject to the relations of this model.

Equ. (85) is a representation of the gravitational energy of the electron, terms for other particles may be obtained by inserting values according to (52) in (85) which might be interpreted as the intensity/frequency of emergence of virtual states being proportional to the energy of the primary particle.

Gravitational attraction, $F_{m,n;\,r}$ between two particles m and n at a distance $R_{m,n}$ would be given as:

$$F_{m,n;r} \approx \frac{1}{b_0} \; W_{VS(m,r)} W_{VS(n,r)} \approx b_0 \; \frac{W_m W_n}{W_e^2 \; R_{m,n}^2} \left(\frac{F_{G,e}}{F_{C,e}} \right) \approx b_0 \; \frac{\Pi_{W,m} \Pi_{W,n}}{R_{m,n}^2} \; \alpha_0^2 \qquad (86)$$

It has to be noted that the result of (85) corresponds to the reasoning of I 7.1.2, i.e. in the limit of Planck energy defined relative to the electron by α_0, the gravitational term will equal the electrostatic term, i.e.

$$b_0 \alpha_0^2 \Pi_{VS(n)}^2 \leqslant b_0 \qquad (87)$$

has to hold, which requires the maximum possible energy state to exhibit a coefficient in (44) of:

$$\Pi_{W,max} = \alpha_0^{-1} \qquad (88)$$

Through equ. (85) a relation between the ground state of the series given in chpt. I 2.5 and the upper limit term, the 3rd term of the series expansion, equ. (84) is established. All the relevant relationships given above can thus be derived from either the assumption that the electron constitutes a ground state or that the 3rd term of equ. (84) represents gravitational interaction.

The qualitative features of such a model may be summarized as:

- particle energy and radius are not static, generation of virtual states with an intensity proportional to energy of the primary particle, W_0, provides energy at a distance, resulting in a contribution to the stress-energy tensor proportional to W_0/r_{VS} and the associated gravitational effect;
- the 3rd term in the energy equation (84) that is responsible for effects associated with the strong force at short distances is identical with the term responsible for gravitational effects at long distance;
- energy and distance (and implicitly t) are intrinsically connected (=> energy-space-time).

I 7.2 Strong interaction

In the SM “strong interaction” is responsible for bonding of quarks in nuclei and nuclear bonding.

There is no need for the first in case of this model.

The r^{-4}-term term in the expansion of energy, (84), chpt. I 7.1.3, as used in the differential equation, chpt. I 6, as well, may be interpreted in terms of a potential that, since it is the decisive term for forming a particle state and is significant only for $r < \lambda_C$, might be considered to play the equivalent role of “strong interaction” for this case.

As for bonding in nuclei a mechanism with exchange of particles should be compatible with this model. More direct EM-interaction might be considered as well (cf. II 3). [39]

I 7.3 Weak interaction

Weak interaction does not fit in the expansion scheme given above and thus can not have any interpretation similar to the one given for strong interaction. Particle exchange as underlying mechanism might be compatible with this model in this case as well. In general there seems to be a close relationship between this model and electroweak theory, however, based on different principles, to be discussed in II.

39 According to the former interpretation of this model it would be suggestive to interpret strong interaction as evidenced in scattering events to be in 1st approximation due to direct interaction of the respective electric and magnetic fields. Such an interaction should depend on: 1) comparable overall spatial extension and strength of the fields (i.e. essentially energy density), 2) comparable spatial extension and field strength of volume elements attributable to partial charge. Condition 1) should prevent neutrinos or the electron to exhibit effective interaction with hadrons, condition 2) prevents interaction of the τ which is at the end of the partial product series for y_1^0 and should exhibit a high, potentially infinite number of radial nodes, separating densely spaced volume elements of alternating charge. A special relationship of “leptons” to neutrinos might be due to their weak electromagnetic interaction compared to that of the hadrons.

I 8 Cosmology

I 8.1 Cosmological constant Λ

The 2nd term on the right side of the full 5D equation (5), $\sim 1/\Phi\left(\nabla_\alpha(\partial_\alpha \Phi) - g_{\alpha\beta}\Box\Phi\right)$, might be considered to be a natural candidate for the cosmological constant term, $g_{\alpha\beta}\Lambda$. Its exact expression will depend on the complete 4D or 5D metric used. Nevertheless it will have to contain terms of type $g_{\alpha\beta}\Phi''/\Phi$ such as $\rho_n^3/\mathbf{r}^5$ or $\rho_n^6/\mathbf{r}^8$ with all r originating from derivatives of the exponential only. Using $\mathbf{r} = e_c/(4\pi\varepsilon_c)$ as upper bound of r, as suggested in I 7.1.2 will yield approximate values in the order of magnitude of critical, vacuum density, ρ_c, ρ_{vac} and of Λ:

$$\frac{\Phi''}{\Phi} \approx \frac{\rho^3}{\mathbf{r}^5} \approx \frac{\alpha_{Pl}}{(e_c/(4\pi\varepsilon_c))^5}\left(\frac{e_c}{4\pi\varepsilon_c}\right)^3 = \alpha_{Pl}\left(\frac{4\pi\varepsilon_c}{e_c}\right)^2 = 0.089\ [\mathrm{m}^{-2}] \qquad (89)$$

Multiplied by ε_c this gives an energy density of 2.97E-10 [J/m^3].

Multiplied by the conversion factor for the electromagnetic and gravitational equations, equ. (2), $8\pi\varepsilon_c G/c_0^4$ equ. (89) gives as estimate for Λ:

$$\alpha_{Pl}\frac{(4\pi)^2\varepsilon_c^3}{e_c^2}\frac{8\pi G}{c_0^4} \approx 6.17\mathrm{E}{-53}\ [\mathrm{m}^{-2}] \qquad (90)$$ [40]

The first term of (15) in chpt. I 1.3.1 should represent a vacuum term and thus be the most likely source for a term such as $\rho_n^3/\mathbf{r}^5$.
[41]

Dark energy is related to Λ and considered to be responsible for the accelerated expansion of the universe. An “electromagnetic version of Λ, such as given by (89), should have drastically stronger effects, maybe having some relationship to cosmic inflation.

I 8.2 Dark matter

Dark matter refers to the concept to attribute gravitational effects that cannot be explained by conventional theories of gravity to a yet undetected type of matter that contributes a multiple of ordinary, baryonic mass.

An alternate approach to explain the underlying phenomena is to modify the standard laws for gravitation. In particular MOND is able to explain many phenomena in the range of low strength of gravitation and correlate them with the presence of baryonic matter [19], e.g. the baryonic-Tully–Fisher relation, $v_\phi^4 \sim GM_{gal}$.

Term such as $(1 + a_0/a)$ might indicate that a series expansion such as used in this model could be of some use to gain a MOND-like expression.

I 8.3 Relationships to black holes

The size of the ring singularity of an electron in the Kerr–Newman metric will be:

$$R_{a,e} = \hbar c_0/(2W_e) = \lambda_{C,e}/(4\pi) \qquad (91)$$

40 $\Lambda \approx 1.11\mathrm{E}{-52}$ [m^{-2}] with Hubble constant $H_0 = 67.66$ [km/s/Mpc] [18]

41 An interpretation such as given in I 7.1.4 implies that at least part of the energy in the “vacuum” is supplied by virtual particle states. Such a contribution to vacuum energy is not constant but scales with the magnitude of gravitational effects (i.e. W_0/r_{VS}).In this model energy of a VS is intrinsically connected to distance itself. If spacetime is directly constituted by the presence of energy, the expansion of the gravitational field via VS contributes to the expansion of the universe. From this perspective a possible expansion of the universe is restricted by the available total energy and infinite expansion would require a complete conversion of matter into radiation.

II Terms based on Euclidean Geometry

II 1 Quaternion-based quark-like model

The model as described above emphasizes a Kaluza-like ansatz with spin as boundary condition. Reversing the main focus, emphasizing angular momentum and implicitly assuming curvature of space as necessary boundary condition for localization is a straight forward alternate way to get additional information about the states of this model [20].

II 1.1 Rotating orthogonal vector triple of E, B and C – EBC-triple

Defining particles as being based on an orthogonal vector triple of E, B and C of the propagation velocity [42] rotating in 3D with the E-vector constantly oriented to a fixed point in a local coordinate system (EBC-triple), the vectors E, B and C being supposed to be locally orthogonal and subject to standard Maxwell equations has the following consequences [43]:

1) Such a rotation is related to the group *SO(3)* (and ***SU(2)*** as important special case). In the following a quaternion ansatz will be used for modeling the respective rotations.

2) E-vector constantly oriented to a fixed point implies ***charge***. As implicitly assumed above, neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of reversed E-vector orientation and opposite polarity.

3) A local coordinate system = rest system implies ***mass***.

4) In case of any lateral extension of the E-field, for r -> 0 the overlap of a rotating E-vector implies rising energy density, resulting in rising curvature of space-time according to GR or its modification as of equ. (3).

5) As essentially electromagnetic waves such states are consistent with a “point-like” structure function on the other hand imply a spatial distribution of energy density and angular momentum / spin.

6) Simple rotations of such an EBC-triple with half the angular frequency of E, B, or C may serve as model for S =1/2. The trajectory of the E-vector encloses a spherical cone or -mirrored at the center – a spherical double cone with a surface area relative to a sphere of 2/3, 1/3, 1/3, respectively, or equivalently the complementary toroidal wedges of surface fraction 1/3, 2/3, 2/3. Each of the 3 pairs of cone and complementary toroid are identical to a sphere, and may correspond to a charge 1 or 0. i.e. the basic **geometry of the solutions corresponds to 6 leptons and 6 quarks** of the SM

7) **Chirality**, Helicity, Spin

Geometry allows for 2 different chiral orientations of the EBC-triple (right- left-handed), each with 2 different spin orientations. This has the following consequences:

- Chirality is an absolute attribute independent of reference frame;
- a pair of chiral states may exist in **3 triplet-like states**, “LL”, “RR” and 1/√2 (LR+RL) taking the **equivalent role of “color”** in decays;
- If each particle (-component) has a well defined chirality, there have to exist forbidden transitions for different chiral particles, being a possible explanation for unobserved transitions such as μ -> e + γ or the stability of the proton;
- Phenomena such as “handedness” in electromagnetism, the “chirality” of the weak interaction and matter-antimatter asymmetry might be based on a universal preference for one chiral set of states.

II 1.2 Quaternion U, D, S-components

It is suggestive to implement the model described in II 1.1 in the form of quaternions. In the following a standard algorithm for rotation with quaternions will be used.

A “dreibein” of three orthonormal vectors E, B, C (EBC-triple), described as imaginary part of a quaternion with real parts set to 0, will be subject to alternate, incremental rotations around the axes E, B and C.

For each E, B and C the following variables will be defined:

- de, db, dc: incremental step for rotation angle,
- de_sum, db_sum, dc_sum: total rotation angle,

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42 Orthogonal spatial „Dreibein“

43 This is a simplified picture neglecting the role of 4th and 5th dimension that might have an influence in attributing states of this model relative to states of the SM.

- ex, ey, ez, bx, by, bz, cx, cy, cz: cartesian components of the respective vectors,
- eex, eey, eez, bbx, bby, bbz, ccx, ccy, ccz: cartesian components of the respective vectors to be buffered until rotation around the axes E, B and C is complete,
- uu, sih, qw, qx, qy, qz: internal variables for quaternion-rotation calculation.

The following part of the algorithm gives the rotation of B around the E axis for an incremental step de:
de_sum = de_sum + de; uu = Sqr(ex ^ 2 + ey ^ 2 + ez ^ 2); sih = Sin(de / 2); qw = Cos(de / 2); qx = (ex / uu) * sih qy = (ey / uu) * sih; qz = (ez / uu) * sih; bx = bbx; by = bby; bz = bbz;
bxx = bx * (qx * qx + qw * qw - qy * qy - qz * qz) + by * (2 * qx * qy - 2 * qw * qz) + bz * (2 * qx * qz + 2 * qw * qy);
byy = bx * (2 * qw * qz + 2 * qx * qy) + by * (qw * qw - qx * qx + qy * qy - qz * qz) + bz * (-2 * qw * qx + 2 * qy * qz);
bzz = bx * (-2 * qw * qy + 2 * qx * qz) + by * (2 * qw * qx + 2 * qy * qz) + bz * (qw * qw - qx * qx - qy * qy + qz * qz);
bx = bxx; by = byy; bz = bzz;

This has to be followed by rotation of C around the E axis; and equivalent routines for the rotation of E, B around the C axis and the rotation of E, C around the B axis. After each incremental step for de, db, dc the Cartesian components of the E, B, C vectors may be stored in a list, tab. 5 gives an example. A rotation is considered complete if all vectors regain there starting values.

The vectors are thought to indicate spatial orientation only, *polarity orientation of E and B has to be considered in the analysis of the results*. Orientation of angular momentum remains a free parameter.
In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin J = 1/2.
There are 3 possible solutions for de, db and dv respectively, to be called U, D, S:

de = 0.5 db = 0.5 dc				0.5 de = db = 0.5 dc				0.5 de = 0.5 db = dc			
E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg
2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3
U				D				S			

Tab. 4; UDS-States, including average of the x, y, z-components and total average of the E-and B- field for complete rotation;

The average of the x, y, z-components of the field are multiples of 1/9th, the average total E- and B-field is 1/3rd or 2/3rd, respectively, the surface ratio of the enclosed spherical cap is 1- E-avg, 1-B-avg.

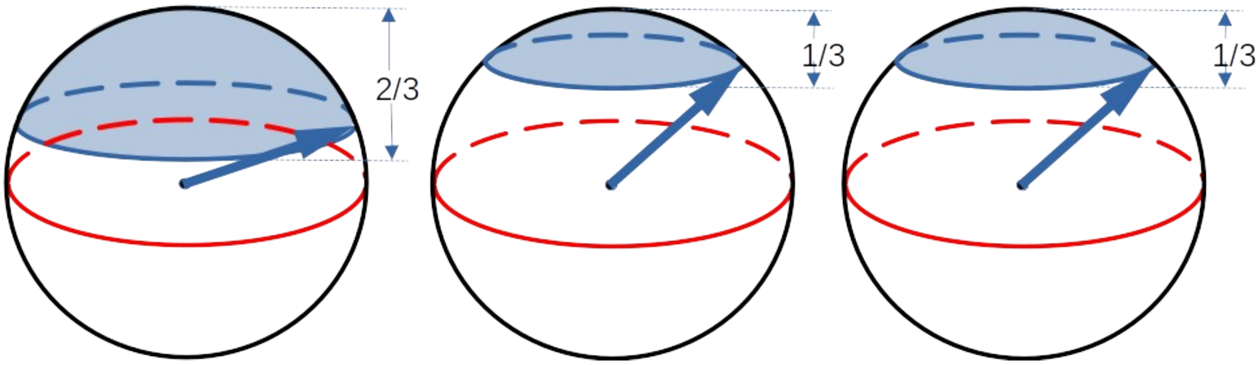

Fig. 3; Rotation geometry for e.g. E-field; E-avg = 1/3, 2/3, 2/3; Relative area of the enclosed spherical cap = 2/3, 1/3, 1/3; Schematic, not to scale;

It has to be noted that the U and D components are symmetric in their E and B-components, while in S E- and B-components are symmetric to each other.
A typical table for results of a U-rotation with starting values E(1,0,0), B(0,1,0), C(0,0,1) will look like this:

			E			B			C		
de_sum	**db_sum**	**dc_sum**	**x**	**y**	**z**	**x**	**y**	**z**	**x**	**y**	**z**
0	**0**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**1**
0.5	1.0	1.0	1.000	0.018	-0.017	-0.017	1.000	0.009	0.018	-0.009	1.000
1.0	2.0	2.0	0.999	0.035	-0.035	-0.035	0.999	0.018	0.035	-0.017	0.999
1.5	3.0	3.0	0.997	0.053	-0.052	-0.052	0.998	0.028	0.053	-0.025	0.998
2.0	4.0	4.0	0.995	0.071	-0.068	-0.068	0.997	0.037	0.071	-0.032	0.997
,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,
,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,	,,,
117.5	235.0	235.0	0.992	-0.085	0.089	0.089	0.995	-0.040	-0.085	0.047	0.995
118.0	236.0	236.0	0.995	-0.068	0.071	0.071	0.997	-0.032	-0.068	0.037	0.997
118.5	237.0	237.0	0.997	-0.052	0.053	0.053	0.998	-0.025	-0.052	0.027	0.998
119.0	238.0	238.0	0.999	-0.035	0.035	0.035	0.999	-0.017	-0.035	0.018	0.999
119.5	239.0	239.0	1.000	-0.017	0.017	0.017	1.000	-0.009	-0.017	0.009	1.000
120.0	240.0	240.0	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000
Sum field component			**0.111**	**0.222**	**0.222**	**0.222**	**0.444**	**0.444**	**0.444**	**0.444**	**0.444**

Table 5: U-rotation with starting values E(1,0,0), B(0,1,0), C(0,0,1)

The diagram for the E,B, C-components as function of dc will look like this:

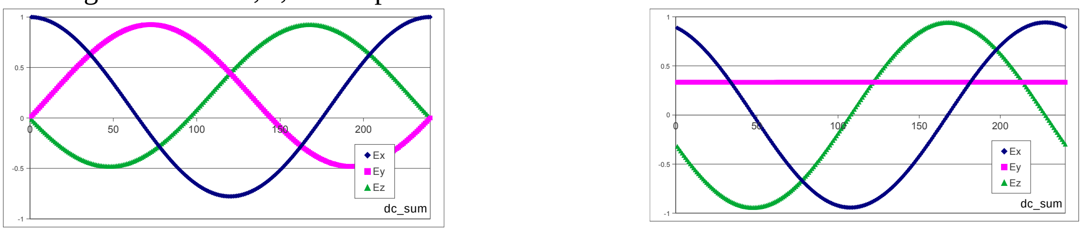

Fig. 4.: a) E-components for Cartesian starting values b) E-components after coordinate transformation

From a coordinate transformation to a representation with one Cartesian coordinate as axis of rotation (in fig 4 rotation +26,6°, z-axis, -41,8° x-axis, to give y-axis as axis of rotation) one can infer that the E-vector circumvents a spherical cap of area 2πr 2/3r. Mirroring at the center of rotation gives a value of 2/3 of the surface of a sphere, which according to Gauss' law represents 2/3 of a full point charge. The analogue procedure yields a value of 1/3 of a point charge for D and S-rotations.

U, D and S-components thus have the same spin and partial charge as u, d and s-quarks, however, are considered to identify parts of a coherent electromagnetic wave. This is on the one hand consistent with a "point-like" structure function on the other hand implies a spatial distribution of energy density and angular momentum / spin imparting a certain volume to the "particle", a feature necessary to interpret scattering experiments such as [21, 22], which the quark model realizes via the concept of sea quarks + gluons.

There is no need for phenomenona such as "confinement", "color charge" or "gluons". However, the possibility to change orientation in the E, B, C-dreibein from left to right handed has to be considered, see II 5.3.

II 2 Magnetic moments of baryons from U, D, S-components

There is a crucial test for the applicability of such a quaternion-based model: magnetic moments of uds-baryons.

To calculate these three components of U, D, S will be combined that represent orthonormal starting conditions for E, B, C. Spin / angular moment of the 3 components has to add up to J = 3/2 for the omega-baryon, to J = 1/2 for all other baryons discussed below. Within this model this is not an assumption but may be calculated in principle in detail. In the following it will be sufficient to have two components sharing the same orientation of the axis of rotation, i.e. both can be transformed according to fig 4. with the same set of rotation angels or -in a trivial case- include 2 identical components. Together with the freedom in choosing direction of rotation, allowing for cancelling or adding up spin as needed, this will be sufficient to obtain J =

1/2, J = 3/2 baryons.

Table 6 gives an example for UUD and DDU.

	UUD	Proton		DDU	Neutron				
	U_1			D_1			D_1_inv		
Start value	-Ez	-Bx	Cy	-Ex	-Bz	Cy	Ex	Bz	Cy
Bx, By, Bz	**-0.444444**	**0.444444**	**-0.222222**	**-0.222222**	**0.222222**	**-0.111111**			
	E	B		E	B		E	B	
Rot_Z_axis	-45	135		-45	135		45	45	
Rot_X_axis	19.47	19.47		19.47	19.47		-19.47	109.5	
	U_2			D_2			D_2_inv		
Start value	-Ex	By	-Cz	Ey	-Bx	-Cz	-Ey	Bx	-Cz
Bx, By, Bz	**-0.222222**	**0.444444**	**-0.444444**	**-0.111111**	**0.222222**	**-0.222222**			
	E	B		E	B		E	B	
Rot_Z_axis	-26.57	116.56		-26.57	116.56		26.57	26.57	
Rot_X_axis	41.82	41.81		41.82	41.81		-41.8	131.8	
E, B inverted	D_inv			U_inv			U		
Start value	-Ey	-Bz	Cx	-Ez	-By	Cx	Ez	By	Cx
	E	B		E	B		E	B	
Rot_Z_axis	-45	135		-26.57	116.56		45	45	
Rot_X_axis	19.47	19.47		41.82	41.82		19.47	109.5	
	D			U			U		
Start value	Ey	Bz	Cx	Ez	By	Cx	Ez	By	Cx
Bx, By, Bz	**0.222222**	**0.222222**	**0.111111**	**0.444444**	**0.444444**	**0.222222**	**0.444444**	**0.444444**	**0.222222**
Bx, By, Bz Avg(UUD)	-0.148148	0.37037	-0.185185	0.037037	0.296296	-0.037037			
B_Avg			**0.439790**			**0.300890**			

Tab. 6: Example for appropriate combinations of U- and D-components for proton and neutron

In D_inv and U_inv the sign of E- and B-components is inverted. The D and U for calculation of the effective B-field include the appropriate sign from their charge while U_inv, D_inv components represent the actual geometric orientation of the E, B-vector only, which is needed for calculation of the angular momentum J from the square of the electromagnetic fields. In table 6 ”Rot_X_axis” and ”Rot_Z_axis” give the angle of rotation needed to transform to a representation with y-coordinate as axis of rotation for the B-field. From this one can see that for the proton U_1 and D_inv as well as for the neutron D_2 and U_inv are equivalent, i.e. they posses identical orientation of spin. Since orientation of rotation is a free parameter opposite spin will cancel both contributions, leaving the 3rd component’s spin of J = 1/2 as total spin of the nucleon.

The U and D components are complementary with respect to the sign and relative value of the components of the E- and B- fields (given in tab. 6 only for the Bx, By, Bz-components relevant for calculating a geometry-based average value of B, B_Avg) attributable to proton and neutron. The starting values of E, B, C are given for reference only.

The results for U and D are exceptional in regard to a large number of such solutions and the exchangeability of U and D [44]. In rare cases where a U solution does not match a D-solution and vice versa a closer examination of a control sample gives components where the condition J = 1/2 is not met. Exchangeability of components for particle pairs is in general not found for particles including an S-component, maybe due to its different internal symmetry compared to U and D. In the case of the solutions examined, compliance with condition J = 1/2 for the lambda-particle (UDS) can be maintained by using a spin-cancelling UD-solution in combination with an S-component, for UUS, DDS, USS-combinations trivial solutions with two identical U, D, S-components exist, in the case of DSS, xi$^-$, one can resort to the method used for the nucleons to find a J = 1/2 solution.

The results given are from a subgroup of results, a comprehensive study of all solutions is pending. Control

44 Each triple of UDS solutions yields roughly 50+ solutions for B-avg in a range ~ 0- 0.555, so on average any magnetic moment is expectable to be hit by chance with an accuracy of ~1%. The solutions for p and n are within 0.2% of the experimental value, which is hardly convincing. On the other hand, the probability that an exchange of the UDS-parameters gives the corresponding partner is ~ 1/50+. Other solutions have to be checked for their resulting spin and some of the other solutions might actually be values attributable to other particles.

samples have been made to check that a) in rare cases where a U and D solutions do not match a UUD/DDU pair the condition for S = 1/2 is not met; b) all U and D- components shown in the tables in combination with an S are components appearing in UUD/DDU-pairs as well.

Results for appropriate USD-combinations are shown in tab. 7.

	USD	**Lambda**		**UUS**	**Sigma +**		**DDS**	**Sigma -**		**USS**	**Xi 0**		**DSS**	**Xi -**		**SSS**	**Omega -**	
	U			**U**			**D**			**S**			**S**			**S**		
Bx,By,Bz	-0.444	0.444	-0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	0.444	-0.222	0.444	-0.222	0.444	0.444
	S			**U**			**D**			**S**			**S**			**S**		
Bx,By,Bz	0.444	-0.444	0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	-0.444	0.444	-0.222	-0.222	0.444	0.444
	D			**S**			**S**			**U**			**D**			**S**		
Bx,By,Bz	0.222	0.222	0.111	0.4444	0.4444	0.222	0.444	0.444	0.222	0.444	0.444	0.222	0.222	-0.222	0.111	0.444	0.444	0.222
Bx,By,Bz Avg(UUD)	0.074	0.074	0.037	0.000	0.444	-0.222	0.074	0.000	0.222	0.000	-0.148	-0.222	0.074	0.000	0.111	0.000	0.444	0.370
B_Avg			**0.111**			**0.497**			**0.234**			**0.267**			**0.134**			**0.579**

Table 7: Combinations of USD-components for calculating magnetic moments of baryons.

To calculate magnetic moments, above factors of B_avg, derived from the quaternion model, have to be multiplied by a factor considering the absolute strength of fields. Using the simple model of a current loop, M = I*A , gives for magnetic moments of baryons with S = 1/2 [45]:

$$\boldsymbol{M_n} \approx e\,c_0 \lambda_C / 2 \;*\; \text{B\_avg} \quad (= 2\pi \mu_{Bohr} * \text{B\_avg}) \qquad (92)$$

Factor 2π of the Bohr magneton, applicable for the electron and muon, is considered to represent a degree of rotational freedom of simple particles that more complex structures composed of several U, D, S-components do not exhibit.

		λ_C	$e\,c_0 * \lambda_C /2$	B_Avg	|M|Calc = $ec_0\lambda_C\,B_{avg}/2$	|M|Exp[Am^2]	|M|Calc/ |M|Exp	|M|Calc/|M|Exp Const. quark
p^{+-}	UUD	1.32E-15	3.17E-26	0.440	1.39E-26	1.41E-26	0.988	-
n	DDU	1,32E-15	3.17E-26	0.301	9.55E-27	9.66E-27	0.988	0.973*
Λ^0	UDS	1.10E-15	2.64E-26	0.111	2.94E-27	3.10E-27	0.949	-
Σ^+	UUS	1.04E-15	2.50E-26	0.497	1.24E-26	1.24E-26	1.002	1.090
Σ^-	DDS	1.04E-15	2.50E-26	0.234	5.83E-27	5.86E-27	0.994	0.897
Ξ^0	USS	9.43E-16	2.26E-26	0.267	6.05E-27	6.31E-27	0.958	1.152
Ξ^-	DSS	9.38E-16	2.25E-26	0.134	3.01E-27	3.06E-27	0.983	0.784
Ω^-	SSS	7.41E-16	1.78E-26	0.579	1.03E-26	1.02E-26	1.010	0.909

Tab. 8: Magnetic moments for UDS-Baryons; col.3: Compton wavelength; col.4: magnetic moment for current loop; col.5: average B-component from quaternion calc.; col.6: values from experiment [9]; col.7: ratio calculated / experiment value; col8: ratio (calculated constituent quark model) / experiment value [9]

Minor systematic errors have to be expected in this model. The ratio of particle magnetic moments for pairs of particles from the same family gives:

	|M|Calc – Col.6	B_avg – Col.5	Const. Quarks
M(p/n)_Calc/M(p/n)_Exp	0.999809	1.001187	0.973*
M(Σ^+/Σ^-)_Calc/M(Σ^+/Σ^-)_Exp	1.007813	1.001111	1.115
M(Ξ^0/Ξ^-)_Calc/M(Ξ^0/Ξ^-)_Exp	0.974652	0.969601	1.470

Table 9: Ratio of particle magnetic moments of baryon pairs compared for calculated (based on B_avg, col.5 in Tab. 3) and experimental values [9]; col. 3 after [9], (* p/n calc. via Clebsch-Gordan coefficients, Σ, Ξ via fit based on p, n, Λ^0);

The role of S-components needs some more research. The deviation of the nucleons and sigmas is in the order of QED corrections. This is not the case for the xis which are not part of one of the α-series for energy.

If one looks at interchangeability of B-components only, (4/9, 4/9, 2/9-solutions characterized as 4 in the following, 2/9, 2/9, 1/9 as 2), it is possible to obtain symmetric solutions (i.e. exchange of the components) between 4 and 2 for all particles.

45 No experimental data for Σ^0. Using the value for Λ^0 would give 2.8E-27[Am^2]

B-comp		B-Avg	Charge	Particle	B-comp		B-Avg	Charge	Particle	Calc/Lit
442	UUD	0.440	+1	p	22**4**	DDU	0.301	0	n	1.000
444	UUS	0.474	+1	Σ^+	222	DDD	0.237	-1	Σ^-	0.943
444	USS	0.267	0	Ξ^0	222	DDD	0.134	-1	Ξ^-	0.975
442	USD	0.444	0	Λ^0, Σ^0						

Tab. 10: B-comp: short for 4/9, 4/9, 2/9 and 2/9, 2/9, 1/9 solutions, bold indicates U with charge 2/3, all other charges 1/3; Calc/Lit indicates the ratio of magnetic moments of particle pairs compared for calculated and literature values.

However, this would involve DDD solutions for the negative baryons.

II 3 Nucleons – stability, bonding in nuclei, scattering

Apart from the quantitative results for partial charges and magnetic moments some qualitative trends for nucleon properties may be inferred from the quaternion-based model.

The spin-cancelling of a UD-unit involves components with opposite charges occupying the same spatial area, which is energetically favorable. This suggests among other things:

1) Comparatively lower energy for particles with UD-component;
2) High stability / life time of the nucleons;
3) A possible contribution to bonding in nuclei via UD-U—D-UD, a direct U-D-bond even without meson intermediate;
4) If such an inter-nucleon UD-bond plays a role in bonding in nuclei this would suggest a significant change in UD-structure between isolated and bound nucleons, which might play a role in the “EMC-effect” [21];
5) In DIS-experiments the ratio of the structure functions of neutron and proton, $F_2^n(x)/F_2^p(x)$ approaches 1 for x -> 0 (x = Bjorken-scale) which would be in agreement with a supposed identical field distribution of E and B-fields in the nucleons. For x -> 1 this model predicts the ratio $F_2^n(x)/F_2^p(x)$ to approach
$(z(UD)^2 + Z(D)^2)/(z(UD)^2 + Z(U)^2) = ((+1/3)^2 + (-1/3)^2)/((+1/3)^2 + (+2/3)^2) = 2/5$
which is in good agreement with high precision scattering experiments which yield values in the range 0.4 – 0.5 [22].

II 4 Six geometrical objects for leptons and hadrons

Older versions of this work operated under the tacit understanding that equivalent to concepts of quantum mechanics a rising “quantum number”, n, in equations such as (52) would concur with a rising number of spatial “nodes” of the associated wave, finally reaching infinity in particles such as the delta or tau [46]. While such an assumption is supported somewhat particularly in the progression in the first couple of particles, i.e. neutrinos, e and μ to mesons and to baryons as well as the increasing number of particles with high values of spin, it does deviate considerably from the SM concepts in case of n => ∞. Experimental properties of the delta particles may be explained sufficiently with uds-quarks and are in contradiction to the assumptions for the tauon showing lepton properties given in the note below. As demonstrated in II 1, II 2 the UDS-scheme works well for baryons, not requiring an increasing number of “nodes”. An approach such as used in I 3.8.2 for going from the pion to the kaons and finally an eta may be the way to match both approaches: a basic half-integer spin object may be described with the “cone” geometry of II 1 which might in general be mirrored at the origin, giving a p-orbital-like symmetry and occupying approximately 1/3 of the total sphere,

46 Suggesting the following explanation for the outstanding character of the τ: According to this model it is suggestive to interpret strong interaction as evidenced in scattering events to be in 1st approximation due to direct interaction of the respective electric and magnetic fields. Such an interaction should depend on: 1) comparable overall spatial extension and strength of the fields (i.e. essentially energy density), 2) comparable spatial extension and field strength of volume elements attributable to partial charge. Condition 1) should prevent neutrinos or the electron to exhibit effective interaction with hadrons, condition 2) prevents interaction of the τ which is at the end of the partial product series for y_1^0 and should exhibit a high, potentially infinite number of radial nodes, separating densely spaced volume elements of alternating charge. A special relationship of “leptons” to neutrinos might be due to their weak electromagnetic interaction compared to that of the hadrons.

resulting in factor $3^{1/3}$ of I 3.4. Only if such an object is able to occupy the total space of a sphere at least approximately this may result in a fit of the spherical alpha series for energy.

I.e. the approach of chpt. I, an approximation neglecting angular terms / magnetic vector potential / time as variable, is too simplistic to be used to obtain more information about the internal structure of particles.

It may be left undecided, if an equivalent of a ”linear combination” of UDS-components refers to an actual “steady state“ spatial configuration (favoured for hadrons) or a temporal average of an object with a consistent phase relation (sufficient for leptons).

This concept will be discussed in the following. The basic principles will be constructive interference and matching chirality.

I 4.1 General classification system for particles

U, D, S-components for mesons and leptons

The reasoning given above with ortohogonal E,B,C-vectors rotating around a central point yields 3 solutions that may be attributed to J = 1/2, each representing a rotation with half speed around one of the axes E, B, C.
The trajectory is considered to be the envelope of a cone, composing 3 such cones with orthogonality of all E, B-fields involved as boundary condition, yields the magnetic moments as demonstrated above.
Regarding only the trajectory, this model lacks “volume”, which one might get by assuming that the inner part of the cone is filled with the respective fields as well, either by some allowed extension, fluctuation range, by the uncertainty principle or whatever. The surface of a cone turns into a spherical cone as elementary geometric object as used above. Referring the 2/3, 1/3 charges to the full sphere would require a double spherical cone.

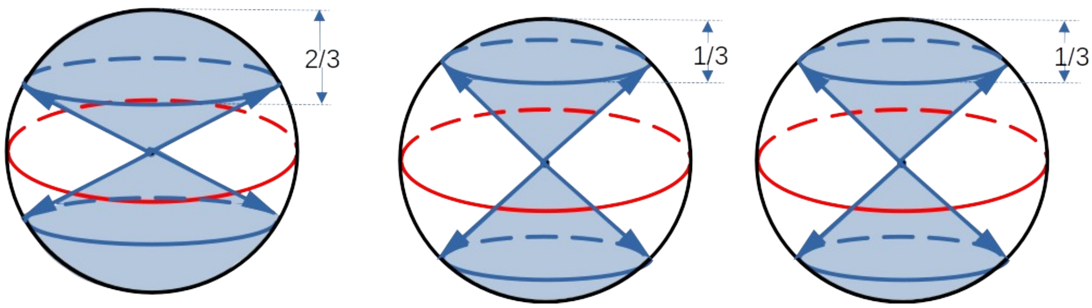

Fig. 5: Double cone (blue) and toroidal wedge (white) for each of the J = 1/2 trajectories of the E- or B-fields

Looking at this one can see that the surface of revolution as given by the trajectory of the E-vector defines actually 2 different objects, the double spherical cone (blue) and the complement of the full ball (white), to be called a toroidal wedge in the following. It would be arbitrary to assume that the inner part of the cone may be “filled” with E-field, while the inner part of the toroidal wedge may be not. Thus the trajectories of the E- (and of course the B-) field for spin 1/2 define in fact 6 different 3D-geometric objects:
3 x (double) spherical cone = U,D,S; charge 2/3, 1/3, 1/3

3 x toroidal wedge = U-Complement, D-Complement, S-Complement; charge 1/3, 2/3, 2/3.

This suggests to use the U, D, S-Complements as candidates for an equivalent of the c,b,t-quarks.

There are two points that support such an interpretation:
1) Due to its extension/geometry the fields of different toroidal wedges should interact more than those of spherical cones, a trend to higher energies should be expected, in particular for the Complement-S = T.
2) The simplest version for a particle of same phase, same angular momentum, same chirality would just be given by a double spherical cone and its complement, which would be nothing else than a ball - in 1st approximation, there still would be a difference in symmetry due to locally different distribution of field density.

The balls of 2) should be the objects of lowest energy, for any such an object with charge 1 there could exist an uncharged one of the same symmetry. They might thus be identified with the leptons.
An electron might be considered e.g. as an D + D-Complement = anti-C particle, however, unlike a D-meson with spin 1/2. While this is not possible with quarks, i.e. objects with particle character, it is possible with an

electromagnetic wave [47].

The non-spherical symmetry of the tauon might be considered due to a distortion associated with its S-complement.

In such a picture the neutrinos might represent a state where the center of rotation is not at the “tip” of an E-vector, but at its “center” resulting in neutral particles

Only if combinations of these elements with different spin, chirality are formed, and nodes aka field-free areas separate them, a differentiation will be necessary, resulting in mesons and baryons.

Such a concept of 3 generations differs from that of the SM that groups related states differing in charge by 1 unit which is a necessity for a process involving a charged W-boson.

II 5 Relationship to electroweak phenomena

The series expansion for energy according to (84) does not provide a term identifiable with electroweak interaction. However, there are some indications that this model incorporates elements of electroweak and Higgs mechanism, in particular if considering a 4^{th} spatial dimension, i.e. a 5D space-time ansatz.

1.) This model originates from establishing a relation between two rotating objects [48], one attributable to SO(2), U(1) symmetry - a photon [49] - and one attributed to SO(3), SU(2) symmetry - particles, i.e. the model is based on the symmetries of electroweak interaction.

2.) SU(2) symmetry is directly related to the property “mass”.

3.) The values for the fine-structure and the weak coupling constant may be combined in a single expression with dimension (4, 3) as parameter, see I 5 and II 5.1.

4.) An interpretation in different dimensions suggests to classify the objects of this model into groups that may be identified with the particles of the Higgs mechanism, see II 5.2.

II 5.1 Coupling constants in N dimensions - geometry

The exact result of the integrals given in I 5 depends on the integration limit of the second integral.

The integration limits for calculating angular momentum in z-direction, r_n of J_z, (22)ff, and (Compton-)wavelength, λ_C, supposed to represent the rotating E-vector and in turn total angular momentum J should be related by the factor √3 of the ratio J/J_z:, see chpt. I 5.1.

3D case:

The 3D case of the coupling constant is easy to interpret, for the 4D-case some assumptions have to be made concerning the integration limit. The following gives an alternative, more detailed interpretation than given in I 5 ($\varphi_N = \exp(-(\rho/r)^N)$).

The exact value of the product of the integrals (67)f, depends on the integration limit relevant for the second integral, i.e. the lower integration limit of the Euler integrals, which can be expressed as 3D volume with $\Gamma_{-1/3}$ as radius (24):

$$\rho_n^3/\lambda_{C,n}^3 = 8/(3^{1.5}\sigma_0) = \left(3^{0.5}\frac{4\pi}{3}\ \Gamma_{-1/3}{}^3\right)^{-3} \qquad (93)$$

The additional factor $3^{0.5}$ may be interpreted as the ratio between r_n of equ. (22)ff and $\lambda_{C,n}$ as required in the expression for photon *energy*. This gives $\Gamma(-1/3, 1/\sigma_0) \approx 36\pi^2\Gamma_{-1/3}$ and

$$2\int_0^r \varphi_3 r^{-2} dr \int_0^r \varphi_3 dr \approx 2\left[\frac{\Gamma_{1/3}}{3}\right]\left[2\pi\, 2\pi\, 9\frac{\Gamma_{-1/3}}{3}\right] = 4\pi\,\Gamma_{1/3}\Gamma_{-1/3}\ \mathbf{2\pi} = \mathbf{2\pi}\ \alpha^{-1} \quad [50] \qquad (94)$$

The result of (94) yields a dimensionless constant $\alpha' = h\, c_0\, 4\pi\, \varepsilon/e^2$ and it is a matter of choice to include 2π in

47 In the simplest case it might be viewed only as a time average of its (hypothetical) constituents.

48 If not indicated otherwise, rotation is relating to the E-field.

49 U(1) is the symmetry group for electromagnetism for general reasons. The isomorphic rotation group SO(2) does characterize a photon of spin 1 if one considers the projection of the rotation of E (and B) vector with respect to the axis of propagation on the plane orthogonal to it.

50 Factor 2 from adding electric and magnetic contributions to energy;

the dimensionless coupling constant. Factor 9 cancels the corresponding factors from the Euler integrals. The remaining factor of 4π is needed to yield the correct value of α. A general N-dimensional version of (93) may be given as:

$$8/\sigma_N = \left(3^{0.5\delta} V_N \ \left(\Gamma(-1/\mathrm{N})\right)^N\right)^{-N/(N-2)} \tag{95}$$

V_N is the coefficient for volume in N-D, coefficient $3^{0.5}$ will be omitted in 4D where coordinate r is considered to be directly related to energy via $r_n \sim 1/W_n$ and r_n might be directly identified with $\lambda_{C,n}$; subscript in σ_N corresponds to dimension in the following.

4D case:

Using φ_4 according to the definition (7) and (95) for 4D

$$\rho_n^4/r_n^4 = 8/\sigma_4 = \left(\frac{\pi^2}{2}\ \left(\Gamma_{-1/4}\right)^4\right)^{-2} = 1.232\text{E-}7 \tag{96}$$

as integration limit, with (18) the non-point-charge integral in 4D will be given by:

$$\int_0^r \varphi_4 r\, dr \sim \Gamma(-1/2, 8/\sigma_4) = \int_{8/\sigma_4}^{\infty} t^{-1.5} e^{-t} dt = 5687 \approx 16\pi^4 \Gamma_{-1/2} \tag{97}$$

The 4D equivalent of (94) will be:

$$2\int_0^r \varphi_4 r^{-3} dr \int_0^r \varphi_4 r\, dr \approx 2\left[\frac{\Gamma_{1/2}}{4}\right]\left[16\pi^4 \frac{\Gamma_{-1/2}}{4}\right] = \frac{\pi^2}{2}\Gamma_{1/2}\Gamma_{-1/2}\ \mathbf{4\pi^2} = \pi^3\,\mathbf{4\pi^2} = \alpha_{weak}^{-1}\mathbf{4\pi^2} \tag{98}$$

The interpretation is the same as in the 3D-case:
A $4\pi^2$ term originating from the second integral of equation (98) is required for turning h^2 into $\hbar^2$ since the integral refers to ρ_n^2 and thus to the square of energy and h, ħ. Factor 16 cancels the corresponding factors from the Euler integrals. The remaining factor of $\pi^2/2$ is needed to yield the correct value of α_{weak}.

2D case:

the 2D case is not as straightforward as the 4D case. The integral over the 1D point charge

$$\int_0^r \varphi_2 r^{-1} dr = \Gamma(0, \rho_n^2/r_2^2)\ /2 \tag{99}$$

features Γ(0, x), with Γ(0, x) -> ∞ for x -> 0 and m = N-2 = 0 in the equations above. Setting nevertheless m=1 in the 2D equivalent of the integration limit

$$\rho_n^2/\lambda_{C,n}^2 = 8/(\sigma_2) = \left(3^{0.5}\pi\ \ \Gamma_{-1/2}{}^2\right)^{-2} \approx 1/4676 \tag{100}$$

and calculating $\Gamma(0, \rho_2^2/r_2^2)$ numerically gives $\int\varphi_2 r^{-1}\, dr \approx \Gamma(0, \rho_2^2/r_2^2)/2 = 7.872/2$. In the 2D case the complementary integral would be identical to the point charge integral, giving $2(\int\varphi_2 r^{-1}\, dr)^2 \approx 4\pi^3/4 = \pi^3$, i.e. the same value as 4D, maybe giving an alternate candidate for α_{weak}.

II 5.2 Electroweak bosons / Interpretation in 4D space

The mapping of the electroweak constants to 3 and 4 dimensions invites speculation if electroweak interaction and the Higgs mechanism may be interpreted in a 4D spatial scheme. Identifying the 4th spatial dimension with energy or equivalently curvature of space-time implies a relation of the Higgs field with x_4.
The Higgs mechanism is based on the symmetry breaking of the Higgs field. The most obvious symmetry breaking associated with the creation of a "localized photon" in this model is the generation of +/- charge due to the persistent orientation of the E-vector towards the origin, corresponding to SO(3) or related symmetry and implying curvature of space and non-orthogonality to the x_4 / energy coordinate.

In general rest-mass is supposed to correspond to [51]:

- curvature of 4D space time
- non-orthogonality to x_4
- a coherent object with 3D spatial distribution of (field-) energy
- SO(N) symmetry

Above considerations suggest to classify all particles in such an EBC-quaternion scheme. The basic

51 The first 3 points more or less paraphrase the same content.

requirements will be a slight variation of chapter X:

- E, B, C are orthogonal
- overlapping fields correspond to spatial variation of energy density / varying 5th dimension contribution / rest system / mass
- rotation with respect to a fixed point,
- coherence.

The condition "constant orientation of the E-vector to a fixed point" will be dropped. This allows an analogue rotation where the center of rotation might be interpreted to mark the "center" of a E and B vector, resulting in neutral particles.

A solution for J = 1 may be achieved either by an equal rotation of all 3 of the EBC-components (assumed to correspond to W-bosons), or by an equal rotation of the EB-components, with the C-component being independent from the EB-rotation. An obvious solution for the latter is represented by E ~ B, C(x,y,z) = constant (= straight, light ray), the photon. A second solution might be possible, where the rotation of the C component is decoupled from that of E,B, i.e. EB rotate around the C-axis while the rotation direction of C is constant, describing a plain circle locally orthogonal to the EB-plane. This might be a reason for the Z^0 to be characterizable by the Γ-term $(\Gamma_{-1/3}/3)^2$.

The electroweak bosons show some numerical relationship with the dimensionless coefficients related to geometry, the Γ-functions, see tab. 11, that might represent a minimum value of energy for the respective symmetry with respect to the vacuum value (point).

	Point charge			Bosons						
Dimension - space		Value of charge ~ $\alpha^{1/2}$	Value relative to g	Electroweak bosons + VEV	W [GeV]	Γ-coefficient relative to VEV	VEV/√2 divided by Γ-coeff.	W(calc)/ W(Lit)	W relative to VEV/√2	Kaluza coeff. N for Φ
4D	g	$\boldsymbol{\pi^{-3/2}}$	1	VEV/√2	174.1	$2W^{+/-}/g$	160.8	0.924		4
1D				Higgs	125.4	**Γ_/3**	128.6	1.026	0.720	1
2D	g'	$\pi^{-4/2}$	0.541	Z^0	91.2	*(Γ_/3)²*	95.0	1.041	0.542	2
3D	e	$\boldsymbol{(4\pi\Gamma_+\Gamma_-)^{-0.5}}$	0.476	$W^{+/-}$	80.4	$\Gamma_-^{\ 2}\ /(3\Gamma_+)$	*84.8*	1.055	0.462	3

Table 11: Comparison of point charge coupling constant values with electroweak energy scale
Relationships from EW-theory grey background (only approximate equivalent for VEV=2W/g); W/Z = e/g'; √2 originating from relationship with G_F

The terms indicated in bold are direct consequences of this model, italic indicates a conjecture fitting this model. The energy values of the Higgs and Z^0 bosons can be derived from the expectation value of the Higgs field, VEV/√2 = 246GeV/√2, according to the 1 and 2D dimensionless coefficients for length (col. 7). A corresponding factor for the W-bosons is not obvious [52]. In case of the Higgs boson the corresponding integral for N = 1 in (18) gives a length corresponding to 1D and no point charge.

This suggests to interpret the Higgs-boson as a 1-dimensional object, related to the limiting case of rotating states where the angular extension approaches zero. This might reduce such an object to a "pure" E-vector in a rest system with no propagation and thus no B-field.
The Z^0 would represent an object where the VEV is distributed over a plane area.

The Higgs-boson is subject to the same 1D interpretation from the progression of the particle series:
The "rotating E-vector" of chpt. 3 may be interpreted to cover the whole angular range in the case of y_0^0 while a y_1^0 object might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the limit l -> ∞, a state of minimal angular extension representing the original (E-)v ector, extending in both directions from the origin. Considering only „half" such a state, extending in one direction only, would feature the energy of the Higgs boson, $W \approx W_{Higgs}$. "Minimal angular extension" might imply that the contribution of the magnetic field to total particle energy according to (39) does vanish, giving an alternate interpretation for the factor of 2 appearing in connection with extremal particle state such as in (78).
Zero charge and parity +1 might be explainable by radial nodal planes.

52 For $W^{+/-}$ $3/\Gamma_{1/3}$ is used as a first guess for the boson energy relation, since $\Gamma_{1/3}$ is the characteristic coefficient for energy and W ~1/r holds.

Higgs and the other electroweak bosons do not represent extended quantum fields but the EM-fields with the smallest possible extension. The vacuum expectation value might be considered to be something point-like, a transition state for the E-vector to switch its orientation. Such a process might in itself be seen as some kind of 5D-oscillation.

II 5.3 Aspects related to chirality

II 5.3.1 Left- and right handedness, Chirality

The orthogonal vectors of the **EBC**-triple allow for two mirror symmetric versions, right-/left-handed, i.e. "chiral" versions. To avoid confusion with the more specific definition of "chirality" in particle physics in the following the term "handedness" will be used. The handedness is *not* a consequence of the cross product of **E** X **B** but defined by the 3rd component, velocity **C** or energy flux respectively, being an independent physical parameter, i.e. **S** = **E** X **B** will be replaced by **S** = ± **E** X **B** in a more general case.
In case of a circular polarized photon 4 different states are possible, left- and right handed, each with two different circular polarisation states (= angular momentum states = spin states), and might be expressed as a rank 4 vector equivalent to a spinor. Particles are modeled by rotation of the **EBC** - tripel and posses the same 4 possible states.
Experimental evidence shows that only half of the above discussed, geometrically feasible states are realized. In electrodynamics, which in all practical cases is based on the properties of electrons and protons, this is associated with concepts such as the "Right-Hand-Rule" and the plus sign in **S** = + **E** X **B**, in particle physics with the concept of "chirality" and chiral theories. Both phenomena would be based on the realization of only one of two possible geometric solutions in nature.
As far as particles are concerned one of the 2 possible solutions, "antimatter", seems to be strongly suppressed on the level of the universe.
This hints at the phenomena "Right-Hand-Rule" in EM, Chirality in particle physics and matter/antimatter asymmetry in the universe being all based on the same asymmetry.

Identifying particles and antiparticles with handedness allows to define "chirality" independent of mass and reference frame.

Handedness and polarisation state orientation have to play a role in the realisation of possible decay channels.

II 5.3.2 Relationship with Dirac-Spinor

A Spinor is characterized by requiring a rotation of 720° to go back to its original state. Neither reduction of a particle to a point in space nor plane wave solutions are required for the Dirac equation or its solution, the formalism may be referred to an actual 3D-rotation, cf. e.g. Ohanian [23], Battey-Pratt and Racey [24]. The formalism above conforms to such approaches. The interpretation in form of 2 spin states is straightforward. Concerning the additional 2 states in a Dirac-spinor, that conventionally are associated with antiparticles, this may be covered by this model as well. Antiparticles are supposed to exhibit opposite charge and handedness („chirality") compared to the corresponding particle. Applying the quaternion-based rotation model given above, a change of handedness / chiral orientation may be achieved not by changing the rotation speed of one *pair* of EB, etc. vectors with respect to the 3rd component, but changing the rotation speed of one *vector only*, e.g. E or B, etc. with respect to the 3rd component. This works i.a. with a reduction by 0.5 of rotation speed, i.e. a rotation of 720° to go back to the original state of a spinor.
I.e. *all* 4 components of a spinor may be associated with actual 720° rotations.

II 5.3.3 Color

The two possible chiral configurations, right-handed "R" and left-handed "L" suggest to be a possible source for a factor 3 frequently appearing in the quantitative interpretation of processes involving a quark-antiquark-pair, such as in the decay, e.g. of the W- or Z-boson, or in the coefficient R of electron-positron-annihilation. While this is attributed to the 3 "colors" of quarks in the SM, the same factor would result for any UDS-pair having the possibility to exist in triplet-like states, "LL", "RR" and 1/√2 (LR+RL) [53] (referring to an axial vector representing the EBC-configuration).

53 With a singlet state corresponding to destructive interference;

II 6 Miscellaneous

II 6.1 Γ-functions

Γ-functions, which are central in the equations of this model, represent a generalized factorial function. There are hints that relationships between Γ-functions with arguments 1/2, 1/3 and 1/4 exist that might be useful in interpreting aspects of this model in 3- and 4-spatial dimensions.

For e.g. $\Gamma_{+1/3}$, $\Gamma_{-1/3}$ and $\Gamma_{-1/4}$ of the integral over $\varphi_4(r)\,dr$

$$\Gamma_{-1/4} = 1.002\ \Gamma_{+1/3}\,\Gamma_{-1/3}^{2}\,/9 \tag{101}$$

holds, i.e. since the ρ terms cancel for spherical symmetric states, the integral $\int\varphi_4(r)dr$ yields essentially the same value as the product of $\int\varphi_3(r)dr\ (\int\varphi_3(r)dr)^2$ which might represent 3D curvature of space (cf. W-boson in tab. 11). For a static case one might interpret this as the product of 3D curvature of space multiplied by the inverse of the integral over the 4th spatial coordinate, which itself might represent energy, to be a constant ≈1.

$$\int_0^r \varphi_3 r^{-2} dr \left(\int_0^r \varphi_3 r^{-2} dr\right)^2 \left(\int_0^r \varphi_4 dr\right)^{-1} = \frac{4}{27}\,\frac{\Gamma_{+1/3}\Gamma_{-1/3}^{\ \ 2}}{\Gamma_{-1/4}} \approx \frac{4}{3} \tag{102}$$

(Pending appropriate integration limits)

Calculating R_4 from the 4D-surface, $2\pi^2 R_4^3 = 1.5\ \alpha^{-2.5}$ gives a value of

$$R_4 = 2\ \pi\Gamma_{-1/3} \tag{103}$$

the Higgs vacuum state might be interpreted as a corresponding pole of the hypersphere.

II 6.2 5D space-time

This is in part some extension of II 5.2 to get additional food for thought.

The line element in 5D will be $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 +/- d\lambda^2$.

The 4D-photon may be characterized by symmetry U(1) representing a rotation of E and B fields in a xy-plane around the axis of propagation (z), which is a straight line in 3D-space; E⊥B⊥C. The following refers to an appropriate rotating coordinate system. The E- and B-fields do not overlap in 3D-space. They will have some “natural” progression [54] along the path of their coordinate and may be considered to be a vector of constant length for sake of simplicity as will the 5th coordinate to be characterized by the wavelength, λ, of the photon. The coordinates z and t are not constant but may be characterized equivalently via their relation to λ. For certain symmetric cases, like the 4D-photon itself or spherical symmetric particles, one might think of all ct, x, y, z, λ-vectors to have the same length.

This suggests to search for a geometric explanation of the relationship between particle energies using Lorentz transformation.

One way to explain the concepts of this model is the “rotating E-vector”. According to the considerations given above the state of maximum energy (particle rest frame) that coincides with that of the Higgs boson / VEV, represents an extremal state for such a “rotating localized photon”. In the simple picture given above, the “rotating E-vector” does not simply spread out the associated energy over a larger volume, there has to be some more complex mechanism based on the modified field equations of GR. For example, the relationship between the delta particle and the Higgs-boson, the endpoint of the spherical symmetric and purely linear series, is given by a difference in wave length / energy equivalent to the symmetry factor $4\pi\Gamma_{-1/3}^{2}$ (except for factor 2 considered for the moment to be due to missing B-field of the boson). I.e. the difference in energy is proportional to volume while within the energy series and in the first different y_l^m-states it is proportional to $V^{1/3}$.

One might either start out from the “maximum curvature of spacetime”of a Higgs-state, that may be spread over a larger volume - as seen from flat space - resulting in particles of less average curvature and thus less energy. In other words, if a vector of given energy-/density/curvature would rotate/spread over the complete sphere, the lower energy density in turn would lead to a less curved space. In complete spherical symmetry “curvature” is spread out most evenly corresponding to the lowest energy. The coefficient $\Gamma_{-1/3}$, attributed to integrals over φ(r)dr to yield lengths should appear as term $4\pi\Gamma_{-1/3}^{3}/3$ in the expressions for a spherical symmetric object.

---

54 E.g. something of the kind given in III 3;

Alternatively one may start from assuming e_c to be a distinguished natural unit for energy, being a characteristic constant for “minimal curvature of spacetime” (involving “charge”). Angular momentum of particles requires the relationship given in chpt. I 2.5, considering the volume $4\pi\Gamma_{-1/3}^3/3$ would give the energy of the electron. W_{Higgs} would represent the corresponding maximum of energy / curvature. An “α-free” solution for particles with no charge, $e_c\,/(4\pi\Gamma_{-1/3}^3/3)$ might tentatively be considered as lower limit representing a particle with no (partial) charge, maybe a neutrino.

Considering the electron to be equivalent to a spherical symmetric object “containing” *one* Higgs boson (or one “unit” of VEV) spreading out, one might ponder what the energy of a spherical object of “maximum curvature of spacetime” is, i.e. a spherical symmetric electron-type object containing not one Higgs boson but being filled up with Higgs boson-like curvature. Such a hypothetical object might be constructed by raising the energy ratio Higgs / electron to a power of 3, i.e. filling the whole 3D volume with Higgs particles [55]. The resulting energy would be close to the Planck energy (≈ 1.8 W_{Pl}; Table 2 gives a corresponding estimate in terms of powers of α), implying both a Planck particle and a Higgs boson to represent some kind of “maximum curvature of spacetime” though in different symmetry.

What is the conserved quantity in all processes?

III Miscellaneous

III 1 Particle decay / mean lifetime

To check if the model yields any information about mean lifetimes (MLT) the particles attributed to y_0^0 and y_1^0 are arranged according to their α-exponent index n and indicated for different types of particle families in fig. 6. There seems to be a tendency for charged particles to be significantly more stable than neutral ones and for y_1^0- lifetimes to be lower than y_0^0- lifetimes. [56]

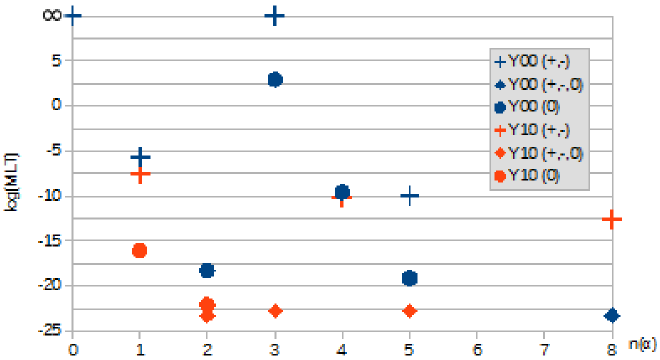

Figure 6: Mean lifetime for y_0^0 (blue) and y_1^0 (red) particles; Box: charged only (+,-), neutral only (0), charged and neutral particle families with near identical MLT (+,-,0).

	MLT [s]	log(MLT)	n(alpha)
e^{+-}	∞		0
μ^{+-}	2.20E-06	-5,7	1
η	5.00E-19	-18,3	2
p	∞		3
n	8.80E+02	2,9	3
Λ^0	2.60E-10	-9,6	4
Σ^0	7.40E-20	-19,1	5
Σ^{+-}	8.00E-11	-10,1	5
Δ	5.60E-24	-23,3	∞
π^{+-}	2.60E-08	-7,6	1
π^0	8.50E-17	-16,1	1
ρ^{0+-}	4.50E-24	-23,3	2
ω^0	7.80E-23	-22,1	2
Σ^{*0+-}	1.80E-23	-22,7	3
Ω^-	8.20E-11	-10,1	4
N(1720)	1.70E-23	-22,8	5
tau^{+-}	2.90E-13	-12,5	∞

Table 12: Values for mean lifetime [9] used in fig. 6

55 Implying a 4th power relationship between W_{Pl} and W_e; The interpretation of a Planck particle as 3D object is supported by the possible description of such a state as a black hole.

56 In [7] a dependence of MLT on α is given, however, there seems not to be a direct relation to the α-coefficients of this work.

In this model a stable particle above the ground state requires a symmetry-forbidden transition to prevent it to decay to a lower state of energy. A decay of the electron would obviously violate charge conservation. A decay of the proton might be strongly inhibited, if conservation laws for its partial components have to be observed.
Electronic transition times in atoms and molecules may differ by up to 17 orders of magnitude depending on selection rules and might be exceeded significantly in the case of particles. Thus 17 orders of magnitude relative to the free neutron, i.e. ≈1E+20 [s] would not be a particular unreasonable estimation and place the MLT of a free proton already well above the currently estimated age of the universe of ≈1E+17 [s]. Experiments measuring the lifetime of a proton *in an H-atom* of water (e.g. Super-Kamiokande [25]) give results of ≈ 1E+41 [s] [57].

The stability of the nucleons (and Λ^0) may be attributable to the UD-combination discussed in II 3.

With regard to parity violation in electroweak decays, due to the character of particles as extended electromagnetic objects, asymmetry in decays involving inhomogeneous EM-field settings, e.g. polarized nuclei, would have to be expected.

III 2 Scattering

U, D and S-components have the same spin and partial charge as u, d and s-quarks, however, are considered to identify parts of a coherent electromagnetic wave. This is on the one hand consistent with a “point-like” structure function and the original SLAC experiments [27] on the other hand implies a spatial distribution of energy density and angular momentum / spin imparting a certain volume to the “particle”, a feature necessary to interpret scattering experiments such as EMC [28] or MARATHON [22], which the quark model realizes via the concept of sea quarks + gluons.

In DIS-experiments the ratio of the structure functions of neutron and proton, $F_2^n(x)/F_2^p(x)$ approaches 1 for x -> 0 (x = Bjorken-scale) which would be in agreement with a supposed identical field distribution of E and B-fields in the nucleons, while for x -> 1 $F_2^n(x)/F_2^p(x)$ is expected to approach $(z(UD)^2 + Z(D)^2)/(z(UD)^2 + Z(U)^2) = ((+1/3)^2 + (-1/3)^2)/((+1/3)^2 + (+2/3)^2) = 2/5$ which is in good agreement with high precision scattering experiments which yield values in the range 0.4 – 0.5 (EMC [28], MARATHON [22]).

A possible contribution to bonding in nuclei via a UD-U—D-UD inter-nucleon bond might suggest a significant change in UD-structure between isolated and bound nucleons, which might play a role in the “EMC-effect” [21].

III 3 Free particle

Omitting the 0^{th} order term in the differential equations might produce the equation of a free particle. Using the following version of equ. (72) for the electron gives:

$$\frac{r}{6}\frac{d^2\varphi(r)}{dr^2} - \frac{\rho^3}{2r^3}\frac{d\varphi(r)}{dr} = 0 \tag{104}$$

$$\frac{d^2\varphi(r)}{dr^2} \approx \frac{3\rho^3}{r^4}\frac{d\varphi(r)}{dr} +\ldots \quad ?? \tag{105}$$

indicating there could exist a function in the general form of (106) for a photon, maybe describing the decrease of the electromagnetic fields perpendicular to wave propagation.

$$\varphi(r) \approx \exp\left(\frac{-\rho^3}{r^3}\right) + \ldots. \tag{106}$$

III 4 Elementary charge

III 4.1 Electrical charge

As φ(r) approaches 1 for r −> r_n the Gauss integral $\varepsilon_0 \int E(r)\varphi(r)\, dA$ approaches the limit of the elementary charge e. Since for r −> 0 the term E(r)φ(r) goes to zero, there is no 'point charge' at the origin.

57 The electron in the H-atom of water has a nonzero probability at the proton position. It is well known that the environment influences decay (neutron), electromagnetic forces influence decay (neutral pion) and last not least the proton can react directly with an electron as evidenced by electron capture. All these effects alter the lifetime of particles and should alter MLT relative to a free proton to some extent.

At a distance of r_m marking the approximate maximum of W(r), φ(r) attains a value of 0.667 yielding a calculated charge of 2/3 e and a value of W_n of $W_n = W_n/4$ [58].

The parameters of the electron in this model approximately fit the Schwinger limit $m_e^2 c_o^3/(e\ \hbar)$ =1.3E+18 [V/m] in form of $W_e/(er_e)$ ≈ 3.6E+17 [V/m] [29].

III 4.2 Magnetic charge

It might be an artefact of an overly simplified model, but actually the quarternion-ansatz of chpt. II does not distinguish electric and magnetic fields, both field vectors have the same center of rotation, outward orientation and same phase.

III 5 Pauli exclusion principle, Spin–statistics theorem

IV Discussion

IV 1 General relationship with QM

Quantum mechanical theories seem to be effective theories that may start on scales of approximately the particle. The wave function will have to be related to the wave properties of electromagnetic fields, its square giving a probability density that will be equivalent to the square of E, B, i.e. energy density. The parameter mass will give a sufficient average for the integral over electromagnetic energy density.

General features of quantum mechanics that emerge from this Kaluza ansatz include quantization of energy or the pivotal constant of quantum mechanics, Planck’s constant, h, that may be derived from the electromagnetic constants and geometry as expressed in the derivation of α.

Some applications of QM such as QED [59] or the Schrödinger equation, the foundation of theoretical chemistry, are extremely efficient and it seems unlikely they could be replaced by anything better. However, QCD seems to lack this kind of performance.

It might be no surprise that a theory which can not give any fundamental quantitative information about elementary mass struggles to give a model for mass-mass interaction.

IV 2 Relationship with the standard model of particle physics

The standard model of particle physics (SM) originated from the observation that the symmetries of certain particles, the hadrons, can be ascribed to the composition from 6 distinct objects possessing appropriate properties such as partial charge and spin 1/2 After some hesitation these objects were accepted as elementary *particles*. The combination of 2 gives mesons, 3 give baryons. Particles that do not fit into the scheme are considered separate entities, leptons, elementary as well though with quite different properties than quarks. Both leptons and quarks are featured in two distinct theories, electroweak theory (EW) and quantum chromodynamics (QCD).

It requires extraordinary evidence to justify a distinction of particles such as electron and proton that for the most part are characterized by a similar set of properties. Such extraordinary evidence is considered to be given by the possibility to explain a wide range of particle phenomena based on the properties of these objects.

Quantitative calculations in EW and QCD may be quite precise, however, apart from a rather complex formalism, require the input of appropriate free parameters, such as the Weinberg angle, Higgs VEV, electroweak coupling constants, mass of elementary particles, etc.

This article is divided into 2 parts. The second part, Euclidean Geometry, reproduces the lepton/quark scheme of the SM with the corresponding set of properties, thereby allowing to hijack associated concepts, e.g. for modeling scattering or particle decay if necessary. However, there exist some fundamental differences compared to the SM concept.

1) The basic entities of the model are electromagnetic waves not particles, eliminating the need for features

58 For the pair e, μ the value of r_m is also distinguished by the relation $r_\mu = r_{m,e}$, see I 2.6.
59 Considerations such as given in I 7.1.4 for virtual particles and the high precision of QED results for effects involving point charges could be an indication that effects of virtual states might involve the presence of point charges.

such as confinement, color charges [60], etc.
2) The basic ordering scheme is much simpler. It relies on the 3 possibilities for a EBC-triple to yield a spin 1/2 object, in turn leading to 6 geometric objects that can be used to define leptons, mesons and hadrons. I.e. the leptons are not leftovers but an integral part of the classification scheme.
3) The most basic properties such as the existence of 3 neutrinos, 3 charged leptons and 6 quark-like objects, as well as the charges 2/3 and 1/3 of quark-like objects are not a postulate inferred from experimental particle properties but are inherent in the model itself.
4) Even on its simplest level the model can be used to precisely calculate complex properties such as the relative values of magnetic moments of baryons ab initio.

Still such a quaternion-based model might have the smell of a toy model if not the Kaluza ansatz of part I would provide the most important ingredient for such objects to exist: the appropriate Non-Euclidean geometry. The model can be based on the solid foundation of the formalism of GR and it is possible to calculate the values for the free parameters needed in the SM.

The major deviation from conventional GR is dropping the constant of gravitation in the field equations, a minor thing from a mathematical point of view. Kaluza's original ansatz was able to produce correct expressions for both GR and EM but failed to reconcile the huge difference in order of magnitude of both effects. The remedy proposed here to reunite both is an almost trivial one, series expansion. That the coefficients used in this approach have some significance is attested by both the results for particles as well as the possibility to produce a reasonable term for the cosmological constant.
Thus this ansatz seems to be an opportunity to gain a new perspective to address problems at the scale of cosmology as well. Another example is the reduction in the set of values of natural constants:

e, c_0, ε, μ, h, G, α, α_{weak}, energies of elementary particles => e and c_0

which downsizes the space for speculations about concepts such as the anthropic principle or fine-tuned physical constants significantly.

Formally not only GR and EM are based on differential equations of at most 2nd order but QM-terms are as well and the formalism of non-Euclidean geometry of GR might be the most flexible one, able to incorporate the other two. The ability of a Kaluza model to yield a simple, coherent, comprehensive and first of all quantitative description of phenomena related to particles, absorbing phenomena attributed to "strong" and "weak" forces underscores its claim to be a step towards a TOE.

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