

## Determination of MSI corrections from flight data

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### Introduction

In order to check the implementation of MSI corrections, and in particular the orientation of the  $H$  axis in the matrix, a preliminary attempt was made to derive MSI corrections from flight data. The data consist of the complete transits of the two stars 109693 and 85222 in frames 4990968-77 and 4991010-19, respectively. The transverse scanfield index (ITRANS as defined in the RGO note 'Formats for IDT data reduction comparisons', 25-04-90) was 16 and 3 for the two transits. Since no definitive results from the corresponding great-circle reduction are available, only 'relative' MSI corrections (within each frame) can be determined. However, the formulation of the method used to derive the corrections includes GCR data, and is therefore applicable also to the determination of 'absolute' MSI corrections.

### Method

The principle for determining the MSI corrections is very simple: For transits of bright stars ( $H_p < 5$ ), compute residuals of the photon counts for individual samples with respect to a best fit light curve (without MSI), constraining the phase of the first harmonic (or the weighted phase, as the case may be) to the value determined by the GCR. Then accumulate for different scanfields two numbers, corresponding to the  $1 \times 1$  normal equation of each correction. After a sufficient number of bright transits, solve the MSI corrections by dividing the two numbers. It is *very* important that the light curve is computed with an accurate value for the modulation frequency, as obtained after the GCR improvement of the attitude and field-to-grid transformation. The practical procedure is as follows.

The GCR provides estimates of the mid-frame grid coordinates  $G_K$  (where  $K$  is the frame number and  $G$  is the grid coordinate increasing by 1 for each modulation period). These can be extracted from the GCR or they can be re-computed after the GCR by means of the star coordinates (abscissa, ordinate), attitude, and field-to-grid transformation, in the same way as is done in forming the rhs for the GCR. Inside each frame  $K$  the grid coordinates of individual samples ( $G_k$ ) can then be computed by linear or quadratic interpolation. ( $k$  is the sample index running from 1 to 2560.) Next, a light curve of the form

$$I_k = a_1 + a_2 \cos(2\pi G_k) + a_3 \cos(4\pi G_k) + a_4 \sin(4\pi G_k) \quad (1)$$

is fitted by ML adjustment of the parameters  $a_1$ - $a_4$  to the photon counts  $n_k$  of the relevant star in the frame.

Now suppose that  $\delta$  is the MSI correction that applies to sample  $k$ , expressed in nm. Then one should expect a better fit if  $G_k$  in (1) is replaced by  $G_k + \delta/8200$  (cf. [2.11] in Volume III; 8200 is the grid period in nm). This gives the following observation equation for the MSI correction:

$$\frac{2\pi}{8200} \left[ -a_2 \sin(2\pi G_k) - 2a_2 \sin(4\pi G_k) + 2a_4 \cos(4\pi G_k) \right] \delta \equiv C_k \delta \sim n_k - I_k \quad (2)$$

The formal standard error of the observation equation is  $\sqrt{I_k}$ . For each sample we thus compute  $I_k$  from the fitted light curve (1) and  $C_k$  from (2); then the following equation is accumulated to the normal equation for the relevant scanfield:

$$(C_k^2/I_k)\delta = C_k(n_k - I_k)/I_k \quad (3)$$

After processing a sufficient number of bright transits in this manner, the MSI correction for each scanfield can be obtained as

$$\delta = \frac{\sum C_k(n_k - I_k)/I_k}{\sum C_k^2/I_k} \quad (4)$$

with formal standard error

$$\sigma_\delta = \left( \sum C_k^2/I_k \right)^{1/2} \quad (5)$$

Note that (5) does not include the noise contributed by the attitude uncertainty of the GCR.

### Application without GCR data

For the present analysis the mid-frame grid coordinates  $G_K$  were not available from great-circle reductions but had to be derived from photon counts only. This is tantamount to estimating  $G_K$  simultaneously with  $a_1$ - $a_4$  in (1), i.e., to the normal 5-parametric IDT data analysis. Of course, this can only give the MSI corrections relative to the mean MSI correction of each frame. However, provided that the corrections within a frame are not too strongly correlated, this should already give at least some statistical indication about the MSI corrections.

### Results

For the transit of 109693 ( $H_p = 5.24$ ), MSI corrections were derived for 164 scanfields along ITRANS = 16. The formal standard errors range from 38 to 152 nm, with a median value of 45 nm.

For the transit of 85822 ( $H_p = 4.49$ ), 164 corrections were obtained along ITRANS = 3, with formal standard errors from 24 to 166 nm (median 38 nm).

Figures 1 and 2 show the derived MSI corrections plotted together with the tabulated corrections (from ESA-HIP-09894), assuming Case 4 (nominal orientation and sign) and Case 2 (reversed  $H$  axis).

In Figures 3 to 6 the derived correction is plotted against the tabulated MSI value for the four cases 2 (reversed  $H$ ), 4 (nominal), 6 (reversed  $G$  and  $H$ ) and 8 (reversed  $G$ ). (Case 1, 3, 5 and 7 are the same as above, except with the sign of the MSI reversed. If any of these cases were true, it would give a negative correlation with the derived values.) In these figures, only points with a formal standard error below 40 nm were plotted (about one third of the points). For comparison, Figure 7 gives a diagram similar to Figure 3, but with all points plotted, and Figure 8 shows the same data as in Figure 3 but with error bars.

Table 1 summarises the results of a linear regression of derived versus tabulated MSI for the different cases and with different upper limits on the formal standard error.

**Table 1** Sample correlation coefficient ( $r$ ), regression coefficient ( $c$ ) and unit weight residual ( $t$ ) for regression of derived MSI corrections against tabulated values for different orientation cases (1 to 4).  $\sigma_{\delta_{\max}}$  is the upper limit to the formal standard error of the derived correction;  $N$  is the number of scanfields used in the regression (having  $\sigma_{\delta} < \sigma_{\delta_{\max}}$ ).

$\sigma_{\delta_{\max}}$ [nm]	$N$	Case	$r$	$c$	$t$
100	323	2	$0.16 \pm 0.06$	$0.79 \pm 0.24$	1.107
100	323	4	$0.10 \pm 0.06$	$0.56 \pm 0.24$	1.115
100	323	6	$-0.06 \pm 0.06$	$-0.16 \pm 0.26$	1.124
100	323	8	$0.06 \pm 0.06$	$0.18 \pm 0.24$	1.124
50	242	2	$0.19 \pm 0.06$	$0.82 \pm 0.25$	1.097
50	242	4	$0.16 \pm 0.06$	$0.63 \pm 0.25$	1.107
50	242	6	$-0.03 \pm 0.06$	$-0.57 \pm 0.27$	1.121
50	242	8	$0.04 \pm 0.06$	$0.15 \pm 0.26$	1.121
40	114	2	$0.25 \pm 0.09$	$0.92 \pm 0.33$	1.179
40	114	4	$0.26 \pm 0.09$	$0.79 \pm 0.35$	1.192
40	114	6	$0.00 \pm 0.09$	$0.06 \pm 0.38$	1.218
40	114	8	$-0.01 \pm 0.09$	$0.07 \pm 0.35$	1.218
35	67	2	$0.27 \pm 0.12$	$0.85 \pm 0.37$	1.057
35	67	4	$0.04 \pm 0.12$	$0.14 \pm 0.38$	1.098
35	67	6	$-0.00 \pm 0.12$	$0.03 \pm 0.39$	1.099
35	67	8	$0.10 \pm 0.12$	$0.35 \pm 0.39$	1.092

## Discussion

Looking at Figures 1 and 2 it is obvious that the noise in the derived data is much too high to permit any real conclusions. Also, there is clearly a strong correlation in the tabulated MSI values between lines with different transverse scanfield index (the four curves in Figures 1–2 correspond, from top to bottom, to transverse index 16, 31, 3 and 44 in the MSI table). This correlation, which is also shown in Figures 9–10, makes it difficult to distinguish between the two orientations of the  $H$  axis and explains why both cases 2 and 4 improve the internal fit (as reported in my e-mail of 1990 June 13).

From Figures 3–6 and Table 1 there seems to be a weak positive correlation in Case 2 and 4, which is absent in Case 6 and 8. This suggests that the nominal orientation of the  $G$  axis is correct (as indicated by previous analysis). On the whole, Case 2 gives a slightly stronger correlation and better fit than Case 4, indicating that the nominal orientation of the  $H$  axis may be inverted. This, too, is consistent with my previous findings (which is not surprising since it is the same data analysed in a slightly different way). The linear regression coefficient is of the order of unity, as it should be if there is some reality to the derived and tabulated MSI values. However, the differences between Case 2 and 4 do not seem to be statistically significant and the orientation of the  $H$  axis therefore cannot be determined from the present data.

A definitive determination of the matrix orientation should be feasible by similar analysis using a larger number of transits of even brighter stars.

## Prospects for a full in-flight calibration

For a meaningful in-flight calibration one should aim at an accuracy of 5 nm (or better) per scanfield. There are some 1700 stars in the Input Catalogue with  $H_p < 5$ , which gives us one bright field transit every 5 minutes on the average. The measurement error per scanfield from one such transit is probably about 50 nm (= 7.4 mas), including attitude errors after GCR. Considering that a single transit does not cover all 168 scanfields in a row, and the statistical distribution between the rows, one will need about 200 transits per row, or a total of 10,000 bright transits. With 65% collection efficiency this corresponds to a data interval of roughly two months. Extracting the raw photon counts for the bright transits (which is about 1% of the data) and performing the calibration is clearly a very big task, but not impossible!

## Final remarks

We do not really know how accurate the on-ground MSI calibration is. The only 'real' indication of the quality of the calibration is a comparison between two sets of measurements on the spare grid F9, made within a few months; the rms difference between these two sets is 11.6 nm. This is of the same order as the MSI values themselves (for F11). A graphical representation of the difference (Figure 3.3.a in ESA-HIP-09894) shows that a large fraction of it is in the form of irregularities on a scale intermediate to LSI and MSI, having a 'wavelength' of the order of 30-60 scanfields (in the  $G$  direction) and amplitudes up to 50 nm. Such irregularities are probably not easily modelled by a field-to-grid transformation of 3rd degree.

Obviously the errors in the MSI calibration cannot be much smaller than the MSI itself. Then it is highly questionable if we should at all apply the on-ground calibration. This is especially true if the calibration contains sizable intermediate-scale errors of the kind illustrated by Figure 3.3.a. Applying such corrections could possibly result in an improved fit on the sub-frame level, while simultaneously increasing the errors of the mid-frame coordinates! Additional tests should therefore consider the possibility that we may, for the time being, be better off not using the MSI table.

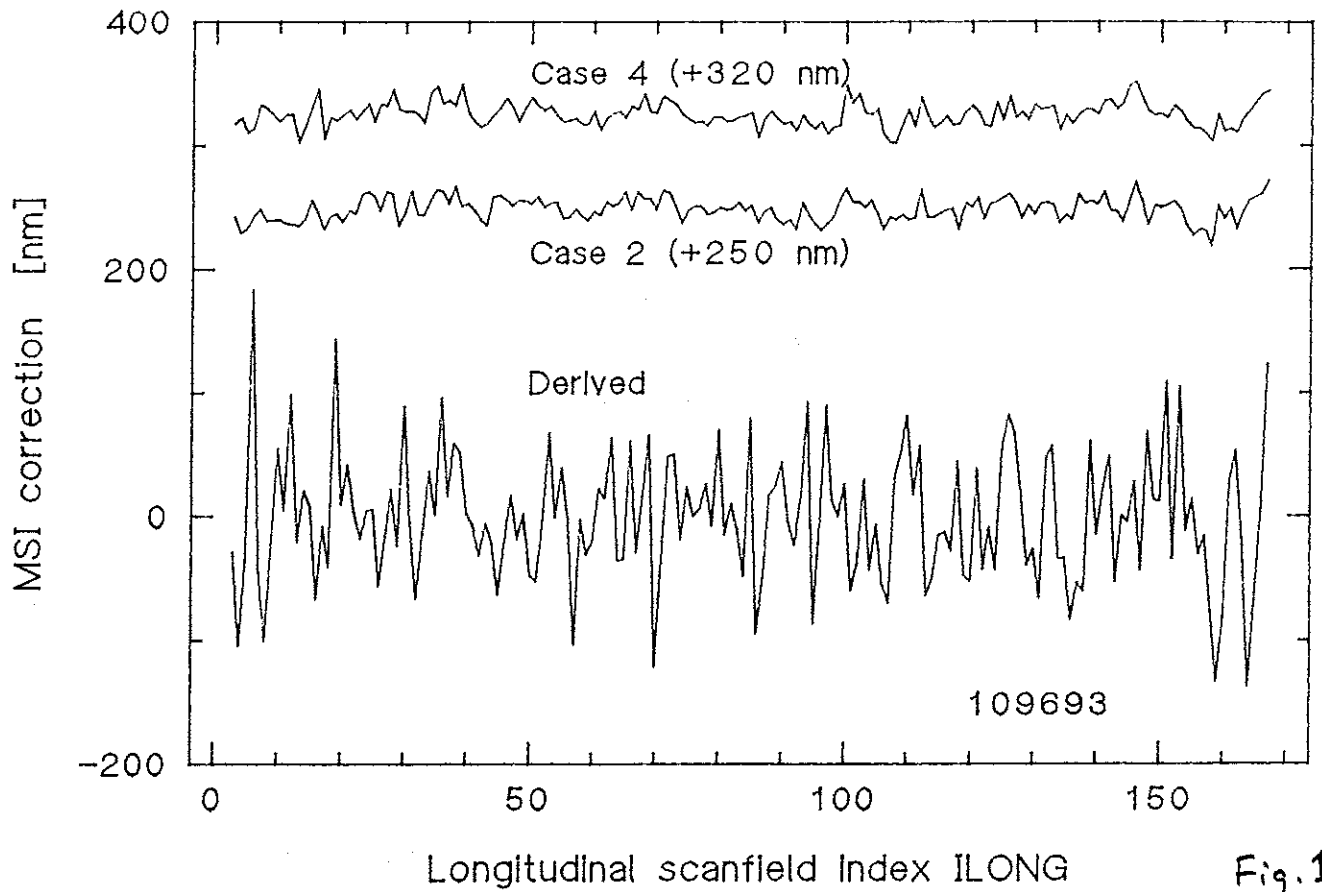


Fig. 1.

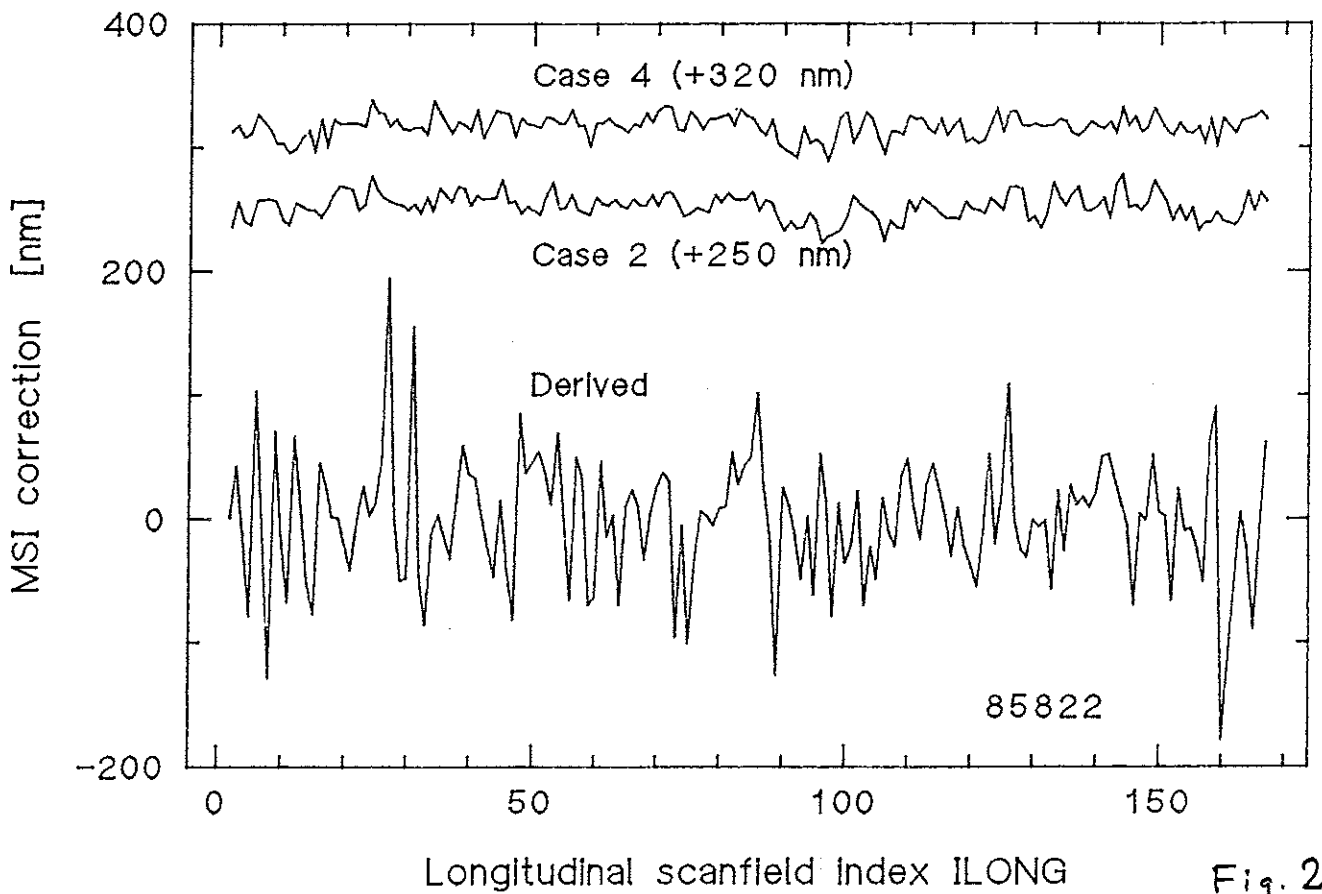
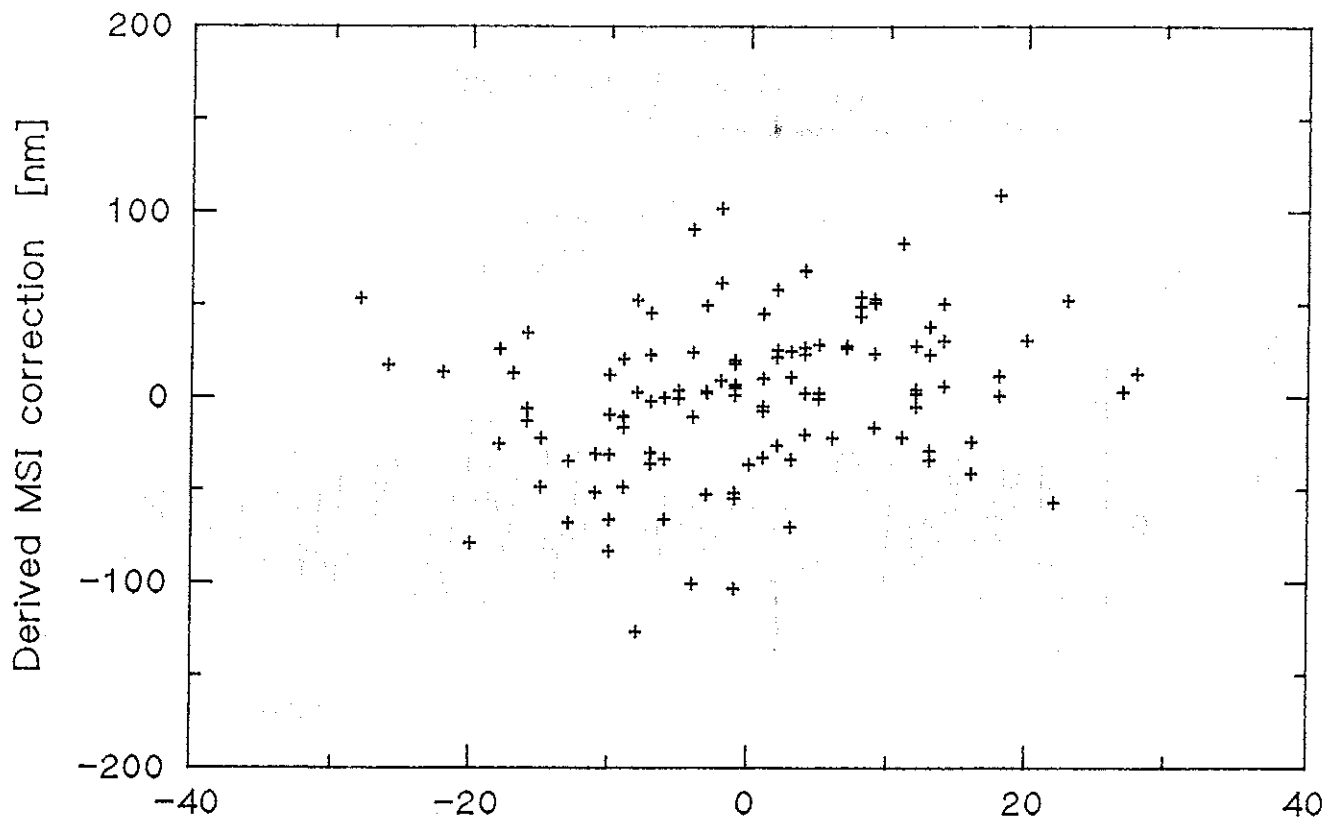
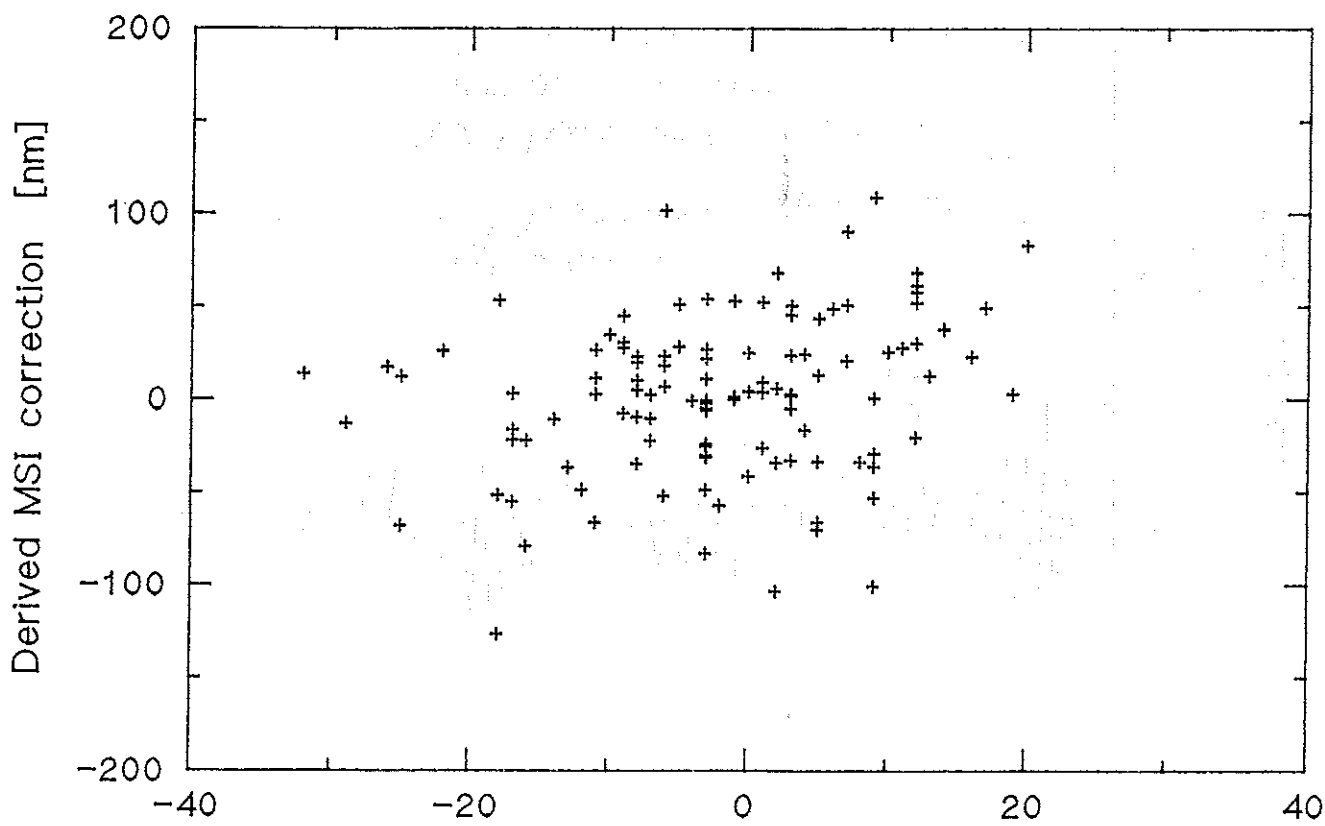


Fig. 2.



Tabulated MSI correction (Case 2) [nm]

Fig. 3.



Tabulated MSI correction (Case 4) [nm]

Fig. 4.

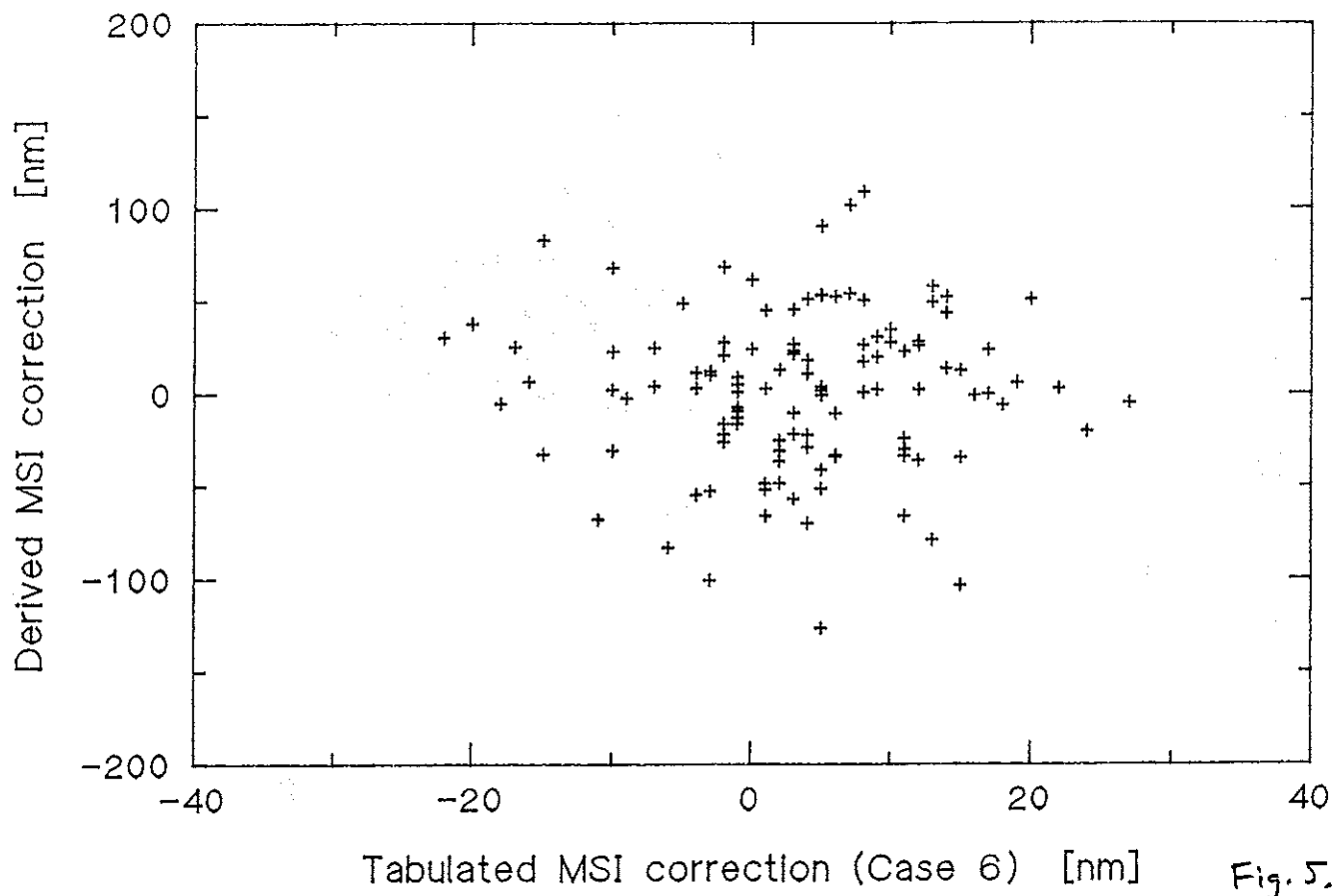


Fig. 5.

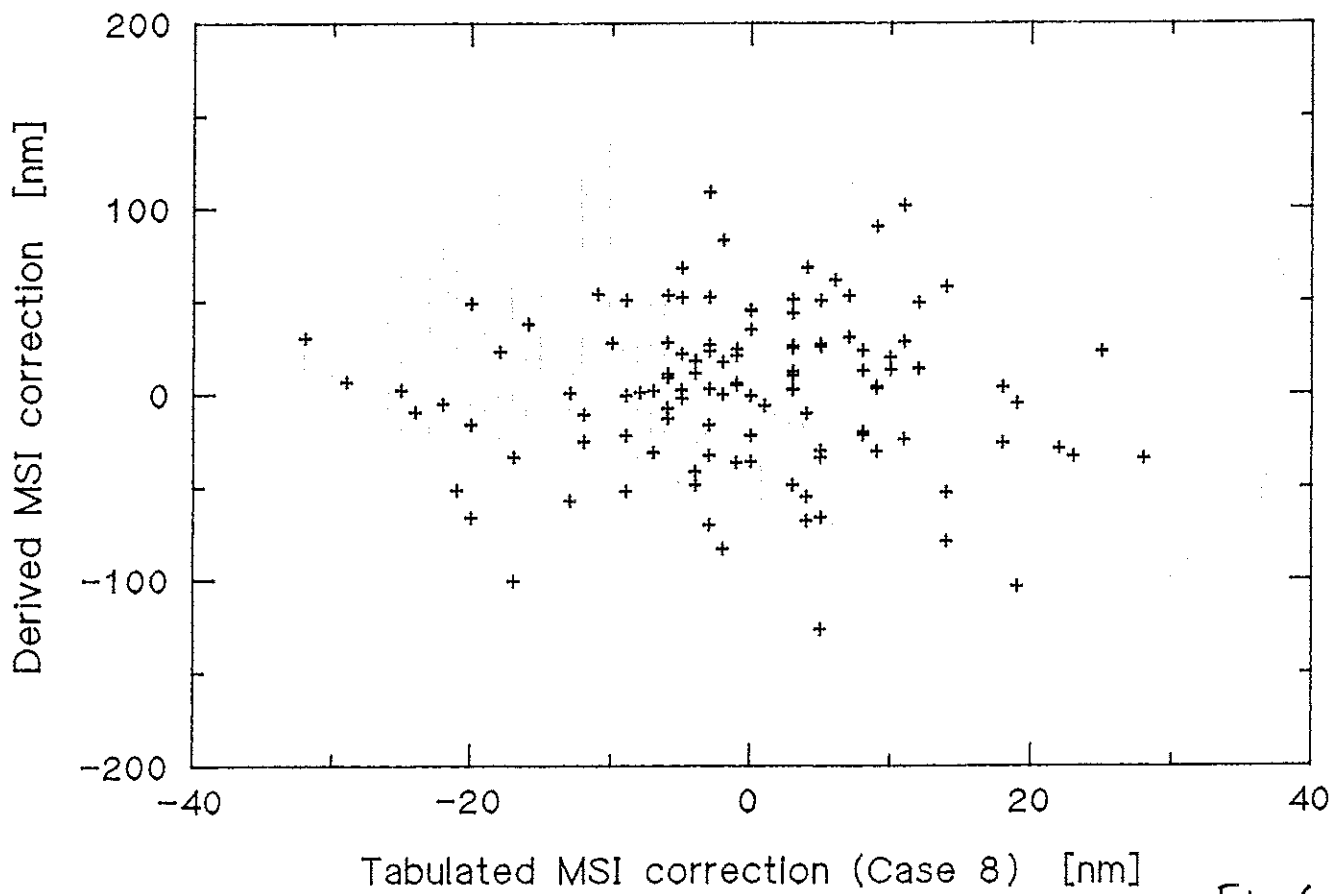
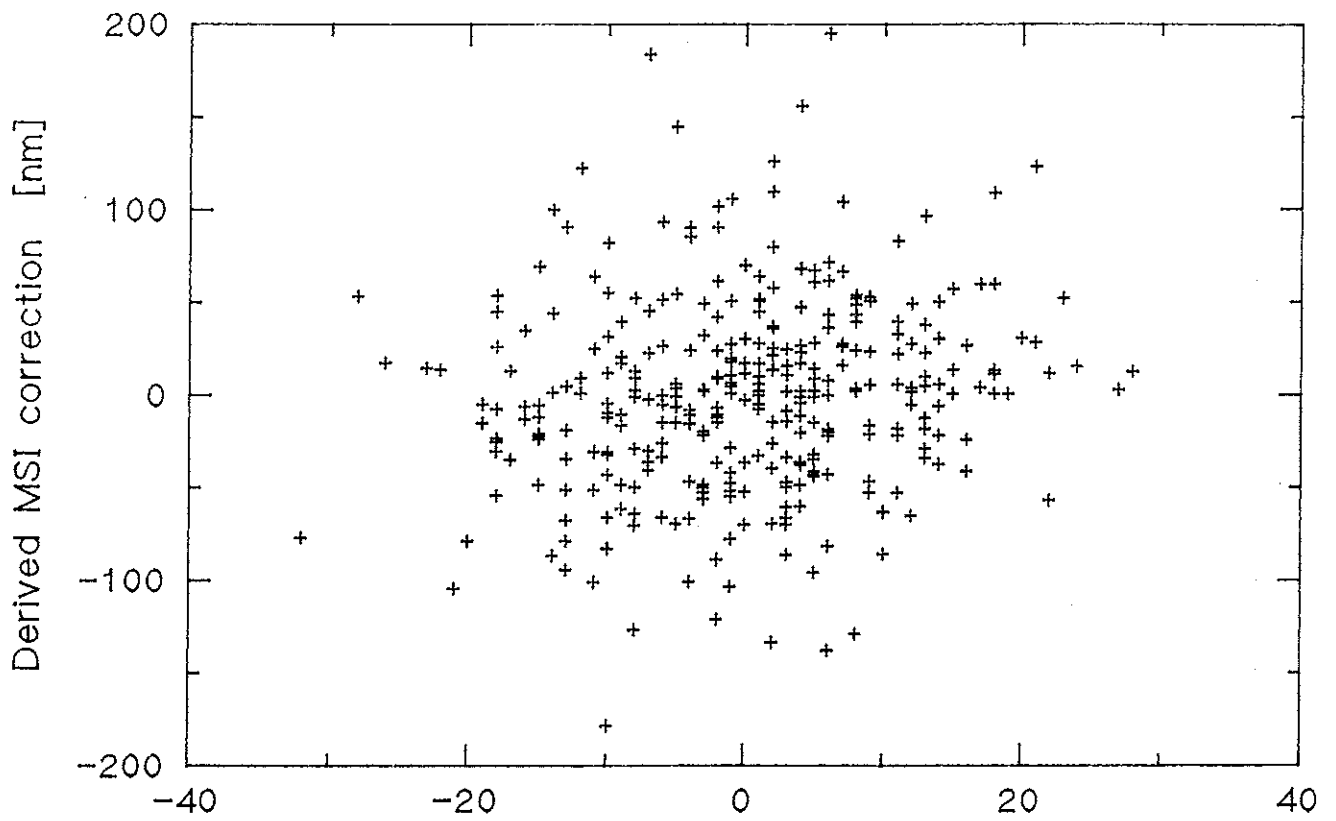
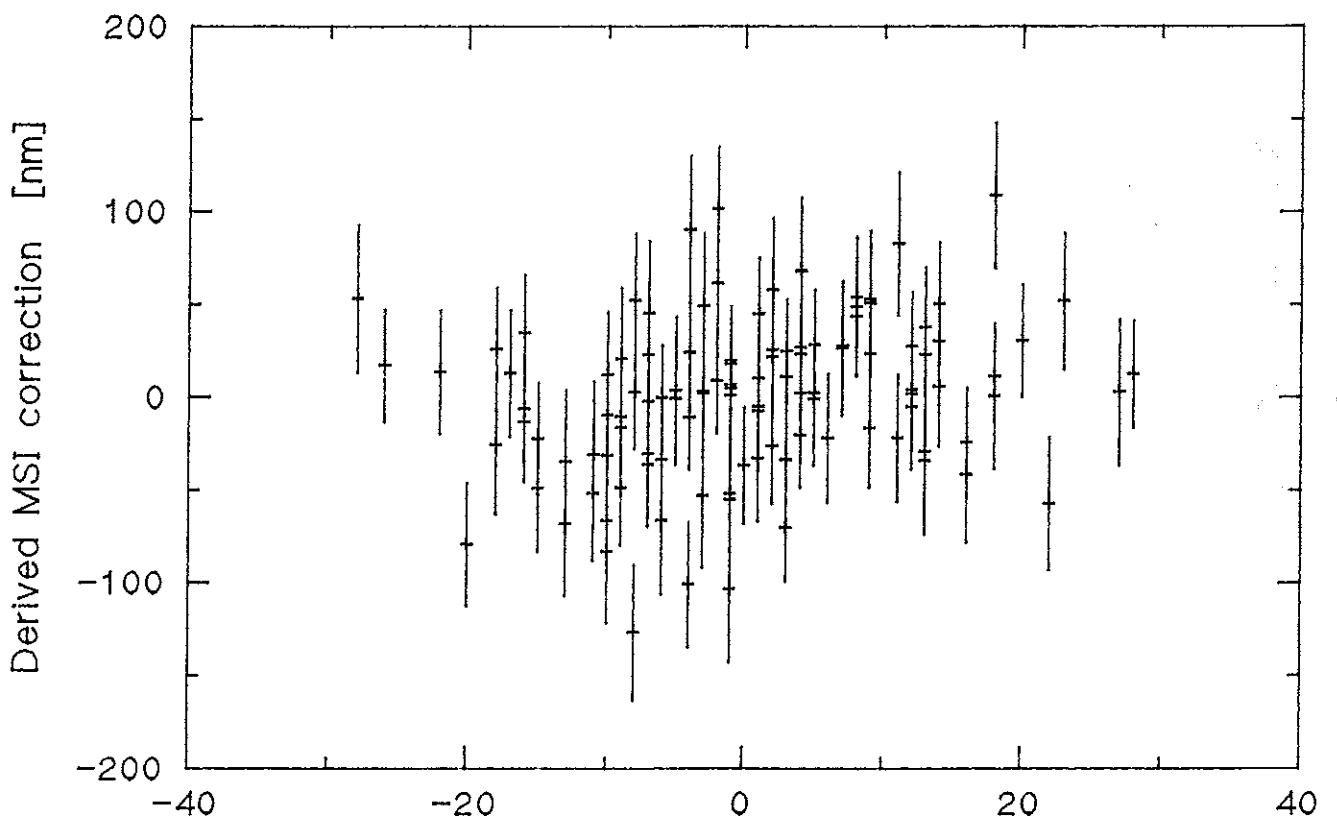


Fig. 6.



Tabulated MSI correction (Case 2) [nm]

Fig. 7



Tabulated MSI correction (Case 2) [nm]

Fig. 8.



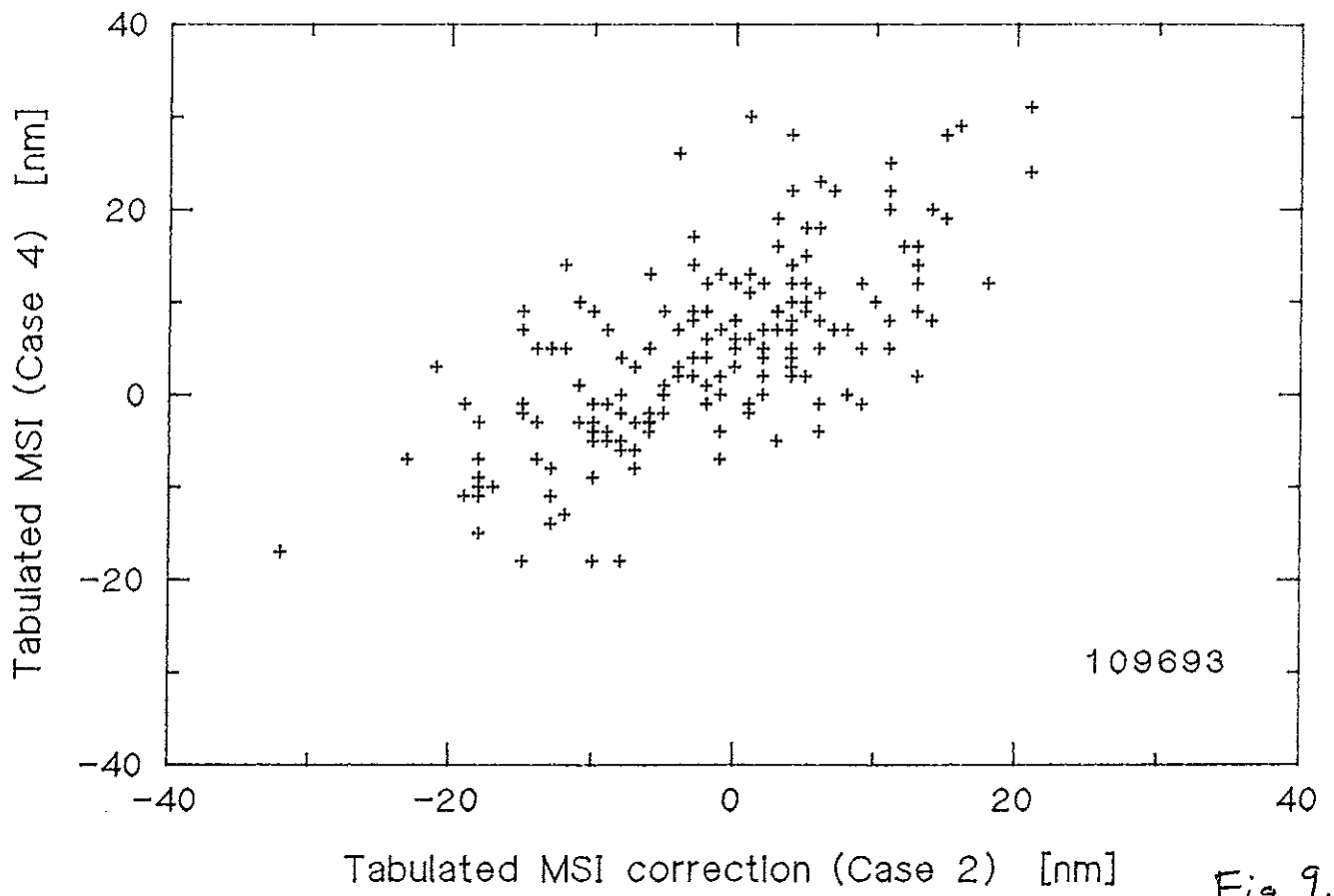


Fig. 9.

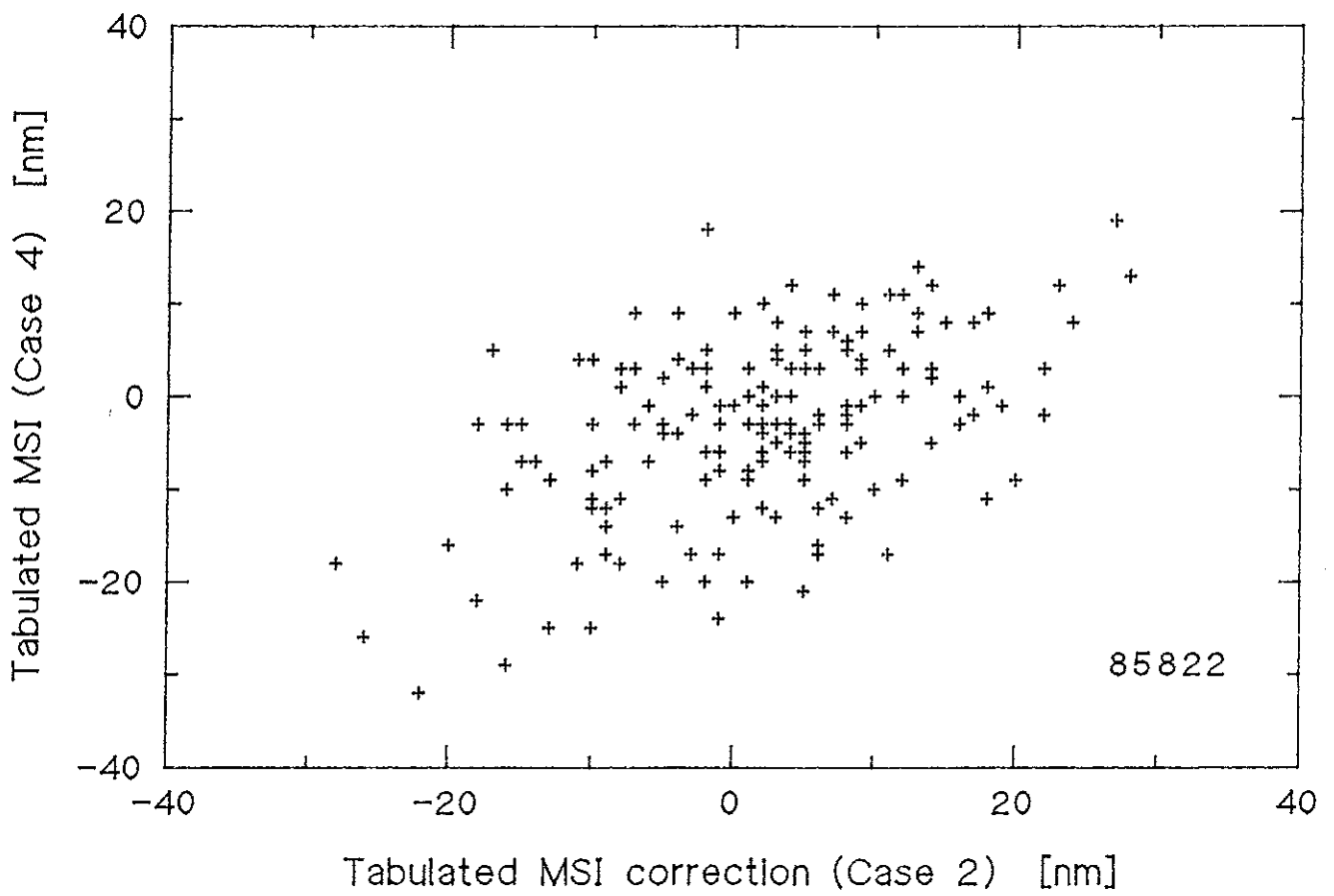


Fig. 10.