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Dynamic Behavior of Bernoulli-Euler Beam with Elastically Supported Boundary Conditions under Moving Distributed Masses and Resting on Constant Foundation

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Authors' contributions

This work was carried out in collaboration between both authors. Author AA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author AAS managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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Abstract

The dynamic behavior of uniform Bernoulli-Euler beam with elastically supported boundary conditions under moving distributed masses and resting on constant foundation is investigated in this research work. The governing equation is a fourth order partial differential equation with variable and singular co-efficients. In order to solve this equation, the method of Galerkin is used to reduce the governing differential equation to a sequence of coupled second order ordinary differential equation which is then simplified by applying the modified asymptotic method of Struble. The simplified equation is solved using the Laplace transform technique. The analysis of the closed form solution in this research work shows the conditions for resonance as well as the effects of beam parameters for moving force system only. The results in plotted graphs show that as the axial force, foundation modulus and shear modulus increase, the transverse deflection of the uniform Bernoulli-Euler beam with elastically supported boundary conditions decreases.

Keywords: Bi-parametric foundation; shear deformation; resonance; critical speed; natural frequency; beam; modified frequency.

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1 Introduction

Vibration that occurs in structural members, for instance bridges, when moving loads such as lorries, trains, cars etc move on them has motivated the interest of several researchers in applied mathematics and engineering. Most importantly, dynamic problems involving the behavior of beams on a foundation has been tackled earlier by various researchers. The dynamic responses of a beam acted upon by moving masses have been studied extensively in connection with the design of railway tracks and machining processes by Lee [1]. Oni and Awodola [2] investigated the dynamic behavior under moving concentrated masses of simply support rectangular plates resting variable Winkler elastic foundation. Kenny [3] took up the problem of investigating the dynamic response of infinite elastic beam on elastic foundation when the beam is under the influence of a dynamic load moving with constant speed. Awodola and Oni [4] investigated the dynamic response to moving masses of rectangular plates with general boundary conditions. Dynamic problem of a simply supported beam subjected to a constant force moving at a constant speed is analyzed by Olsson [5]. Cao and Zhong [6] solved the problem of a beam on a Pasternak foundation to distributed moving loads. Also Gbadeyan and Dada [7] investigated the dynamic response of plates on Pasternak foundation to distributed moving loads. Yin [8] worked on a reinforced Timoshenko beam on elastic foundation and derived its closed form solution. Yin [9] carried out work on a comparative modeling study of reinforced beam on elastic foundation. Awodola and Oni [10] considered a rectangular plates with general boundary conditions resting on a variable Winkler foundation. Adams [11] also considered the critical speeds and the response of a tensioned beam on an elastic foundation to repetitive moving loads. In the same vein, Ogunyebi [12] considered the problem of Bernoulli-Euler beam response to constant bi-parametric elastic foundation carrying moving distributed loads. Adeoye and Awodola [13] investigated the influence of rotatory inertial correction factor on the vibration of elastically supported non-uniform Rayleigh beam on variable foundation.

In all the aforementioned, no author has ever considered dynamic behavior of beam with elastically supported boundary conditions except recently Oni and Awodola [14] but his work was limited to Bernoulli-Euler beam on variable elastic foundation. None of the authors mentioned above has worked on the boundary conditions adopted in this research work.

2 Governing Equation

Considering the dynamic behavior of uniform Bernoulli-Euler beam with elastically supported boundary conditions under moving distributed masses and resting on constant foundation; the governing equation of motion is given by the fourth order partial differential equation as expressed below:

$$\frac{\partial^2}{\partial x^2}\left[EJ\frac{\partial^2}{\partial x^2}U(x,t)\right] - \mathrm{N}\frac{\partial^2}{\partial x^2}U(x,t) + \mu\frac{\partial^2}{\partial t^2}U(x,t) + K_0U(x,t) - G_0\frac{\partial^2}{\partial x^2}\mathrm{U(x,t)} = \mathrm{P}(x,t) \tag{2.1}$$

where x is the spatial co-ordinate, t is the time co-ordinate, U(x,t) is the transverse displacement, EI is the flexural rigidity of the structure, μ is the constant mass per unit length of the non-uniform beam, N is the constant axial force, $K_0$is the costant foundation modulus,$G_0$is the constant shear modulus, and P(x,t) is the moving distributed load.

Equation (2.1) can be re-written as

$$\frac{\partial^2}{\partial x^2}\left[EJ\frac{\partial^2}{\partial x^2}U(x,t)\right] - \mathrm{N}\frac{\partial^2}{\partial x^2}U(x,t) + \mu\frac{\partial^2}{\partial t^2}U(x,t) + K_0U(x,t) - G_0\frac{\partial^2}{\partial x^2}\mathrm{U(x,t)} =$$

$$\sum_{\mathrm{i}=1}^{\mathrm{N}}\mathrm{M_i gH(x - c_i t)}\left[1 - \frac{1}{\mathrm{g}}\frac{\mathrm{d}}{\mathrm{dt^2}}\mathrm{U(x,t)}\right] \tag{2.2}$$

Rewriting equation (2.2) further when considering a unit mass as

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2}{\partial x^2}U(x,t)\right] - \mathrm{N}\frac{\partial^2}{\partial x^2}U(x,t) + \mu\frac{\partial^2}{\partial t^2}U(x,t) + K_0U(x,t) - G_0\frac{\partial^2}{\partial x^2}\mathrm{U}(\mathrm{x},\mathrm{t})$$

$$+M\mathrm{H}(\mathrm{x}-ct)\left(\frac{\partial^2}{\partial t^2} + 2c\frac{\partial^2}{\partial x\partial t} + c^2\frac{\partial^2}{\partial x^2}\right)U(x,t) = Mg\mathrm{H}(\mathrm{x}-\mathrm{ct}) \quad (2.3)$$

The boundary conditions of the structure under consideration are first taken to be arbitrary.

The initial condition without any loss of generality is taken as;

$$\mathrm{U}(x,t) = \mathrm{U}(x,0) = 0 = \frac{\partial}{\partial t}U(x,t) = \frac{\partial}{\partial t}U(x,0) \quad (2.4)$$

3 Analytical Approximate Solution

In order to solve equation (2.3) subject to the condition (2.4), one applies a special technique called the generalized Galerkin method is employed. The generalized Galerkin method required that the solution of equation (2.1) be written in the form

$$U(x,t) = \sum_{m=1}^{n} Y_m(t)W_m(x) \quad (3.1)$$

where

$$W_m(x) = \sin\frac{\theta_m x}{L} + A_m\cos\frac{\theta_m x}{L} + B_m\sinh\frac{\theta_m x}{L} + C_m\cosh\frac{\theta_m x}{L} \quad (3.2)$$

is the beam function chosen so that the concerned boundary conditions are satisfied.

Substituting equation (3.2) into equation (2.3), one obtains

$$\begin{aligned}
EJ\frac{\partial^4}{\partial x^4}\sum_{m=1}^{n}Y_m(t)W_m(x) - N\frac{\partial^2}{\partial x^2} + \mu\frac{\partial^2}{\partial t^2}\sum_{m=1}^{n}Y_m(t)W_m(x)&\\
+ K_0\sum_{m=1}^{n}Y_m(t)W_m(x) - G_O\frac{\partial^2}{\partial x^2}\sum_{m=1}^{n}Y_m(t)W_m(x)&\\
+ \sum_{i=1}^{N}M\,\mathrm{H}(\mathrm{x}-\mathrm{ct})\Bigg[\frac{\partial^2}{\partial t^2}\sum_{m=1}^{n}Y_m(t)W_m(x) + 2c\frac{\partial^2}{\partial x\partial t}\sum_{m=1}^{n}Y_m(t)W_m(x)&\\
+ c^2\frac{\partial^2}{\partial x^2}\sum_{m=1}^{n}Y_m(t)W_m(x)\Bigg] = \sum_{m=1}^{n}MgH(x-ct)Y_m(t)W_m(x)& \quad (3.3)
\end{aligned}$$

rewriting equation (3.3), one obtains

$$\begin{aligned}
\sum_{m=1}^{n}\Bigg\{W_m(x)W_k(x)\ddot{Y}_m(t) + \frac{EJ}{\mu}W_m^{iv}(x)W_k(x)Y_m(t) - \frac{N}{\mu}W_m^{ii}(x)W_k(x)Y_m(t)&\\
+ \frac{K_0}{\mu}W_m(x)W_k(x)Y_m(t) - \frac{G_0}{\mu}W_m^{ii}(x)W_k(x)Y_m(t)&\\
+ \frac{M}{\mu}[W_m(x)W_k(x)\ddot{Y}_m(t) + 2c(\cos\omega tH(x-ct)W_m(x)W_k(x))\dot{Y}_m(t)&\\
+ c^2(\cos\omega tH(x-ct)W_m(x)W_k(x))Y_m(t)]\Bigg\}dx = \frac{M}{\mu}gW_k(x)dx& \quad (3.4)
\end{aligned}$$

In order to determine $Y_m(t)$, it is required that the expression on the left hand side of (3.4) be orthogonal to the function$W_k(x)$. Integrating (3.4), one obtains

$$\int_0^L \sum_{m=1}^{n} \Big\{ W_m(x)W_k(x)\ddot{Y}_m(t) + \frac{EJ}{\mu} W_m^{iv}(x)W_k(x)Y_m(t) - \frac{N}{\mu} W_m^{ii}(x)W_k(x)Y_m(t) + \frac{K_0}{\mu} W_m(x)W_k(x)Y_m(t) - \frac{G_0}{\mu} W_m^{ii}(x)W_k(x)Y_m(t) + \frac{M}{\mu}[W_m(x)W_k(x)\ddot{Y}_m(t) + 2c(\cos\omega t H(x-ct)W_m(x)W_k(x))\dot{Y}_m(t) + c^2(\cos\omega t H(x-ct)W_m(x)W_k(x))Y_m(t)] \Big\} dx = \frac{Mg}{\mu}\int_0^L W_k(x)dx \qquad (3.5)$$

equation (3.5) becomes

$$\sum_{m=1}^{n} \Big\{ A_1(m,k)\ddot{Y}_m(t) + [A_2(m,k) - A_3(m,k) + A_4(m,k) - A_5(m,k)]Y_m(t) + \frac{M_i g}{\mu L}[LA_6(m,k)\ddot{Y}_m(t) + 2cLA_7(m,k)\dot{Y}_m(t) + c^2 LA_8(m,k)Y_m(t)] \Big\} = \frac{Mg}{\mu} A_8(k) \qquad (3.6)$$

Where,

$$A_1(m,k) = \int_0^L \mathrm{W_m(x)W_k(x)}, \quad A_2(m,k) = \frac{EI}{\mu}\int_0^L \mathrm{W_m^{iv}(x)W_k(x)dx} \qquad (3.7)$$

$$A_3(m,k) = \frac{N}{\mu}\int_0^L \mathrm{W_m^{ii}(x)W_k(x)dx} \quad , \quad A_4(m,k) = \frac{K_0}{\mu}\int_0^L \mathrm{W_m(x)W_k(x)\,dx} \qquad (3.8)$$

$$A_5(m,k) = \frac{G_0}{\mu}\int_0^L \mathrm{W_m^{\mathit{ii}}(x)W_k(x)dx} \quad , A_6(m,k) = \int_0^L \mathrm{W_m(x)W_k(x)\,dx} \qquad (3.9)$$

$$A_7(m,k) = \int_0^L W_m^i\mathrm{(x)W_k(x)\,dx}\,, A_8(m,k) = \int_0^L W_m^{ii}\mathrm{(x)W_k(x)\,dx} \qquad (3.10)$$

$$A_9(k) = \int_0^L \mathrm{W_k(x)\,dx} \qquad (3.11)$$

In order to evaluate the integrals in (3.7) to (3.11), one makes use of the Fourier series representation for the Heaviside function in the form

$$H(x-ct) = \frac{1}{4} + \frac{1}{\pi}\sum_{n=0}^{\infty} \frac{\sin\big((2n+1)\pi(x-ct)\big)}{2n+1}, \qquad 0 < x < 1 \qquad (3.12)$$

Thus, substituting equation (3.12) into the equation (3.6),after some simplifications and rearrangement one obtains

$$\sum_{m=1}^{n} A_1(m,k)\ddot{Y}_m(t) + Q_0(m,k)Y_m\,(t) + \lambda_0[(\frac{1}{4}\Delta_1(m,k) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_2(m,k) - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_3(m,k))\ddot{Y}_m(t)$$
$$+2c\left(\frac{1}{4}\Delta_4(m,k) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_5(m,k) - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_6(m,k)\right)\dot{Y}_m(t)$$
$$+c^2\left(\frac{1}{4}\Delta_7(m,k) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_8(m,k) - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_9(m,k)\right)Y_m(t)]$$
$$= \frac{\mathrm{Mg}L}{\mu\lambda_k}\Big[-\cos\theta_k + A_k\sin\theta_k + B_k\cosh\theta_k + C_k\sinh\theta_k + \cos\frac{\theta_k ct}{L} - A_k\sin\frac{\theta_k ct}{L} - B_k\cosh\frac{\theta_k ct}{L} - C_k\sinh\frac{\theta_k ct}{L}\Big] \qquad (3.13)$$

where,

$$\lambda_0 = \frac{\mathrm{M}}{\mu\mathrm{L}} \qquad (3.14)$$

$$\Delta_1(m,k) = L\int_0^L W_m(x)W_k(x)\,dx \qquad (3.15)$$

$$\Delta_2(m,k) = L\int_0^L \sin(2n+1)\pi x W_m\,W_k dx \qquad (3.16)$$

$$\Delta_3(m,k) = L\int_0^L \cos(2n+1)\,\pi x W_m W_k dx \qquad (3.17)$$

$$\Delta_4(m,k) = L\int_0^L W_m^I\,(x)W_k(x)dx \qquad (3.18)$$

$$\Delta_5(m,k) = L\int_0^L \sin(2n+1)\pi x W_m^i\,(x)W_k(x)dx \qquad (3.19)$$

$$\Delta_6(m,k) = L\int_0^L \cos(2n+1)\pi x W_m^i\,(x)W_k(x)dx \qquad (3.20)$$

$$\Delta_7(m,k) = L\int_0^L W_m^{ii}\,(x)W_k(x)dx \qquad (3.21)$$

$$\Delta_8(m,k) = L\int_0^L \sin(2n+1)\pi x W_m^{ii}\,(x)W_k(x)dx \qquad (3.22)$$

$$\Delta_9(m,k) = L\int_0^L \cos(2n+1)\pi x W_m^{ii}\,(x)W_k(x)dx \qquad (3.23)$$

$$Q_0(m,k) = A_2(m,k) - A_3(m,k) + A_4(m,k) - A_5(m,k) \qquad (3.24)$$

Setting n = m, in equation (3.13), one obtains

$$A_1(m,k)\ddot{Y}_m(t) + Q_0(m,k)Y_m(t) + \lambda_0[(\frac{1}{4}\Delta_1(m,k) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_2(m,k) - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_3(m,k))\ddot{Y}_m(t)$$
$$+2c\left(\frac{1}{4}\Delta_4(m,k) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_5(m,k) - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_6(m,k)\right)\dot{Y}_m(t)$$
$$+c^2\left(\frac{1}{4}\Delta_7(m,k) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_8(m,k) - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_9(m,k)\right)Y_m(t)]$$
$$\frac{\text{Mg}L}{\mu\lambda_k}\Big[-\cos\theta_k + A_k\sin\theta_k + B_k\cosh\theta_k + C_k\sinh\theta_k + \cos\frac{\theta_k ct}{L} - A_k\sin\frac{\theta_k ct}{L} - B_k\cosh\frac{\theta_k ct}{L} - C_k\sinh\frac{\theta_k ct}{L}\Big] \tag{3.25}$$

Equation (3.25) is the transformed equation governing the problem of the dynamic response to -moving distributed masses of elastically supported Bernoulli-Euler beam resting on bi-parametric elastic foundation. This coupled non-homogeneous second order differential equation holds for all variant of the classical and non-classical boundary conditions. In what follows, two special cases of the equation (3.13) are considered.

3.1 Moving force problem

In moving force problem, only the load is being transferred to the structure. That is the inertia effect is negligible. So setting $\lambda_0 = 0$ in the transformed equation (3.25), one obtains

$$A_1(m,k)\ddot{Y}_m(t) + Q_0(m,k)Y_m(t) = \frac{\text{Mg}L}{\mu\lambda_k}[-\cos\theta_k + A_k\sin\theta_k + B_k\cosh\theta_k + C_k\sinh\theta_k + \cos\frac{\theta_k ct}{L} - A_k\sin\frac{\theta_k ct}{L} - B_k\cosh\frac{\theta_k ct}{L} - C_k\sinh\frac{\theta_k ct}{L}] \tag{3.26}$$

Simplifying further, one obtains

$$\ddot{Y}_m(t) + \beta_r^2 Y_m(t) = F_m[\theta_{mp} + \cos\alpha_z - A_k\sin\alpha_z t - B_k\cosh\alpha_z t - C_k\sinh\alpha_z t] \tag{3.27}$$

Where

$$\theta_{mp} = -\cos\theta_k + A_k\sin\theta_k + B_k\cosh\theta_k + C_k\sinh\theta_k \tag{3.28}$$

$$F_m = \frac{MgL}{\mu\lambda_k A_1(m,k)} \tag{3.29}$$

$$\beta_r^2 = \frac{Q_0(m,k)}{A_1(m,k)}, \qquad \alpha_z = \frac{\lambda_k c}{L} \tag{3.30}$$

equation (3.27) is an approximate model, which assumes the inertia effect of the moving mass as negligible.

Solving equation (3.27) using Laplace transform and convolution theory and taking into account equation (3.1), one obtains

$$U(x,t) = \sum_{m=1}^{n} \frac{\mathrm{MgL}}{\mu\lambda_k A_1(\mathrm{m,k})} \times \frac{1}{\beta_r^2(\beta_r^4 - \alpha_z^4)} \{\theta_{mp}(1-\cos\beta_r t)(\beta_r^4 - \alpha_z^4) + (\beta_r^2 - \alpha_z^2)$$
$$[\beta_r^2(\cos\beta_r t - \cos\alpha_z t) - A_k\beta_r(\alpha_z \sin\beta_r t - \beta_r \sin\alpha_z t)] - (\beta_r^2 - \alpha_z^2)$$
$$[\mathrm{B_k}\beta_r^2(\cosh\alpha_z t - \cos\beta_r t) + \mathrm{C_k}\beta_r(\beta_r \sinh\alpha_z t - \alpha_z \sin\beta_z t)]\}$$
$$\times [\sin\frac{\lambda_k x}{L} + \mathrm{A_k}\cos\frac{\lambda_k x}{L} + B_k \sinh\frac{\lambda_k x}{L} + \mathrm{C_k}\cosh\frac{\lambda_k x}{L}] \tag{3.31}$$

equation (3.31) represents the transverse-displacement of Bernoulli-Euler beam with elastically supported end conditions on a bi-parametric elastic foundation.

4 Discussion of the Analytical Solutions

For this undamped system, it is desirable to examine the phenomenon of resonance. From equation

$$\beta_r = \alpha_z \tag{4.1}$$

Where

$$\alpha_z = \frac{\lambda_k c}{L} \tag{4.2}$$

that is

$$\beta_r = \frac{\lambda_k c}{L} \tag{4.3}$$

5 Illustrative Examples

5.1 Clamped-elastic boundary conditions

At a clamped end, both deflection and slope vanish. Thus, when the Bernoulli-Euler beam is clamped at $x = 0$ and elastically supported at $x = L$, the conditions are expressed as

$$U(0,t) = 0 = U^i(0,t) \tag{5.1}$$

at the end $x = 0$

and

$$U^{ii} - k_1 U^i(L,t) = 0 = U^{iii}(L,t) + k_2 U(L,t) \tag{5.2}$$

at the end $x = L$

and for the normal modes

$$W_m(0) = 0 = W_m^i(0) \tag{5.3}$$

at the end $x = 0$

and

$$W_m^{ii}(L) - k_1 W_m^i(L) = 0 = W_m^{iii}(L) + k_2 W_m(L) \tag{5.4}$$

at end $x = L$

which implies that

$$W_k(0) = 0 = W_k^i(0) \tag{5.5}$$

and

$$W_k^{ii}(L) - k_1 W_k^i(L) = 0 = W_k^{iii}(L) + k_2 W_k(L) \tag{5.6}$$

Using (5.3) and (5.4), it can be shown that at $x = 0$,

$$A_m = -C_m \text{and} \quad B_m = -1 \tag{5.7}$$

and at $x = L$, one obtains

$$A_m = \frac{\frac{\lambda_m}{L}[\sin\lambda_m + \sinh\lambda_m] + k_1[\cos\lambda_m - \cosh\lambda_m]}{\frac{\lambda_m}{L}[\cos\lambda_m + \cosh\lambda_m] - k_1[\sin\lambda_m + \sinh\lambda_m]} = \frac{\frac{\lambda_m^3}{L^3}[\cos\lambda_m + \cosh\lambda_m] + k_2[\sinh\lambda_m - \sin\lambda_m]}{\frac{-\lambda_m^3}{L^3}[\sin\lambda_m - \sinh\lambda_m] + k_2[\cos\lambda_m - \cosh\lambda_m]}$$
$$= -C_m \tag{5.8}$$

From (5.8)one obtains

$$\tan\lambda_m = \tanh\lambda_m \tag{5.9}$$

Hence, we have

$$\lambda_1 = 3.927,\ \lambda_2 = 7.069,\ \lambda_3 = 10.21, \ldots \tag{5.10}$$

Using (5.7), (5.8) and (5.10) in equation (3.40), one obtains the displacement response respectively to a moving force and a moving mass of clamped-elastic ends Bernoulli-Euler beam on a constant foundation.

5.2 Free elastic boundary conditions

For free end at $x = 0$ and elastically supported at $x = L$, the conditions are expressed as

$$U^{ii}(0,t) = 0 = U^{iii}(0,t) \tag{5.11}$$

at the end $x = 0$

and

$$U^{ii}(L,t) - k_1 U^i(L,t) = 0 = U^{iii}(L,t) + k_2 U(L,t) \tag{5.12}$$

at the end $x = L$

For normal modes

$$W_m^{ii}(0) = 0 = W_m^{iii}(0) \tag{5.13}$$

at$x = 0$

and

$$W_m^{ii}(L) - k_1 W_m^i(L) = 0 = W_m^{iii}(L) + k_2 W_m(L) \tag{5.14}$$

at end $x = L$

which implies that

$$W_k^{ii}(0) = 0 = W_k^{iii}(0) \tag{5.15}$$

and

$$W_k^{ii}(L) - k_1 W_k^1(L) = 0 = W_k^{iii}(L) + k_2 W_k(L) \tag{5.16}$$

Thus, it can be shown that

$B_m = 1$ and

$$A_m = \frac{\frac{\lambda_m}{L}[\sinh\lambda_m - \sin\lambda_m] + k_1[\cosh\lambda_m + \cos\lambda_m]}{\frac{\lambda_m}{L}[\cosh\lambda_m - \cos\lambda_m] - k_1[\sinh\lambda_m - \sin\lambda_m]}$$
$$= \frac{\frac{\lambda_m^3}{L^3}[\cos\lambda_m - \cosh\lambda_m] - k_2[\sin\lambda_m + \sinh\lambda_m]}{\frac{\lambda_m^3}{L^3}[\sin\lambda_m + \sinh\lambda_m] + k_2[\cos\lambda_m + \cosh\lambda_m]} = C_m \tag{5.17}$$

From (5.17)one obtains

$$\tan\lambda_m = \tanh\lambda_m \tag{5.18}$$

Hence, we have

$$\lambda_1 = 3.927,\ \lambda_2 = 7.069,\ \lambda_3 = 10.21, \ldots \tag{5.19}$$

Using (5.17) and (5.19) in equation (3.40), one obtains the displacement response respectively to a moving force of free-elastic ends Bernoulli-Euler beam on a constant foundation.

6 Numerical Results and Discussion

To illustrate the analysis presented in this, Bernoulli-Euler beam is taken to be of length L = 12.192m, the load velocity c = 8.128m/s and modulus of elasticity $E = 2.109 \times 10^9 kg/m$, the moment of inertia $J = 2.37698 \times 10^{-3} m^4$.

6.1 Graphs for clamped-elastic boundary conditions

Fig. 1 display the effect of foundation modulus on the displacement profile of clamped elastic Bernoulli-Euler beam under the action of load moving at constant velocity for moving distributed forces. The graph shows that the response amplitudes decrease as the value of **Ko** increases.

Fig. 2 display the effect of shear modulus on the deflection profile of clamped elastic Bernoulli-Euler beam under the action of load moving at constant velocity for moving distributed forces. The graph shows that the response amplitudes decrease as the value of **Go** increases.

Fig. 3 display the effect of axial force on the displacement profile of clamped elastic Bernoulli-Euler beam under the action of load moving at constant velocity for moving distributed forces. The graph shows that the response amplitudes decrease as the value of **No** increases.

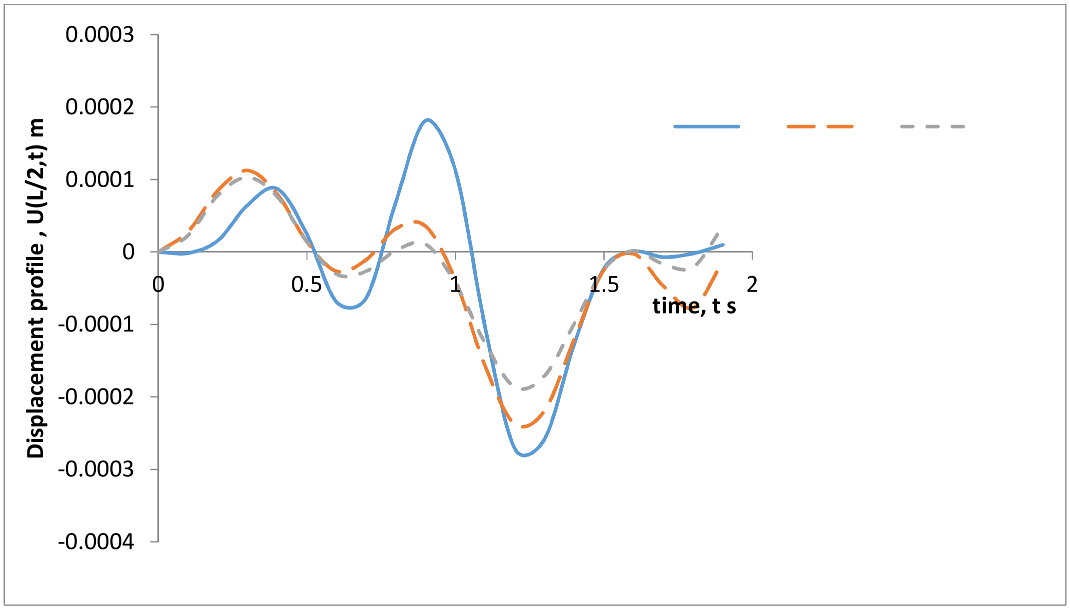

Fig. 1. Displacement profile of a clamped elastic uniform Bernoulli- Euler beam on variable foundation and traverse by moving distributed for for fixed values of No and Go and various values of Ko

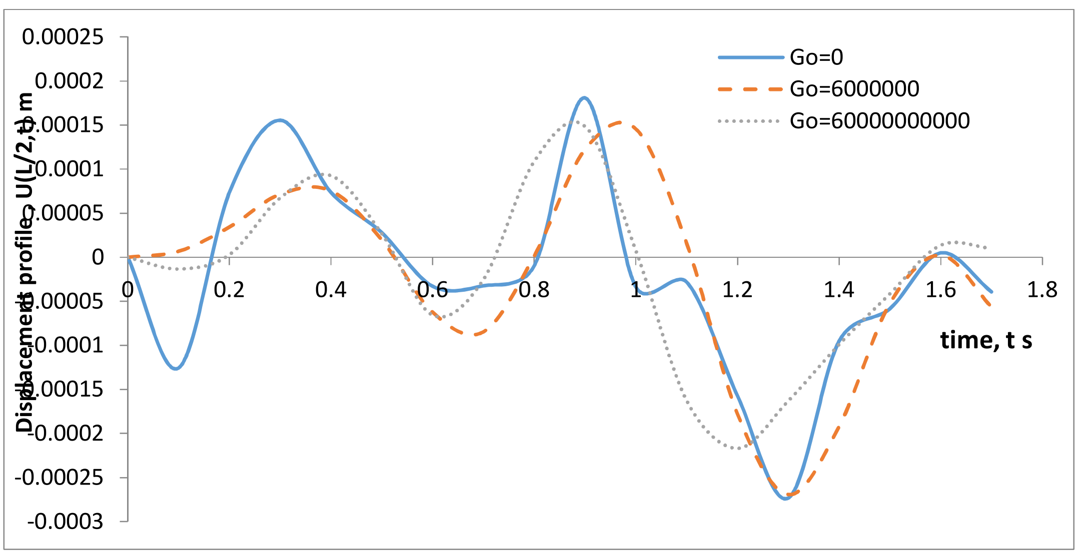

Fig. 2. Displacement profile of a clamped elastic uniform Bernoulli- Euler beam on constant foundation and traverse by moving distributed force for fixed values of No and Ko and various values of Go

6.2 Graphs for free-elastic boundary conditions

Fig. 4 display the effect of foundation modulus on the displacement profile of free elastic Bernoulli-Euler beam under the action of load moving at constant velocity for moving distributed forces. The graph shows that the response amplitudes decrease as the value of **Ko** increases.

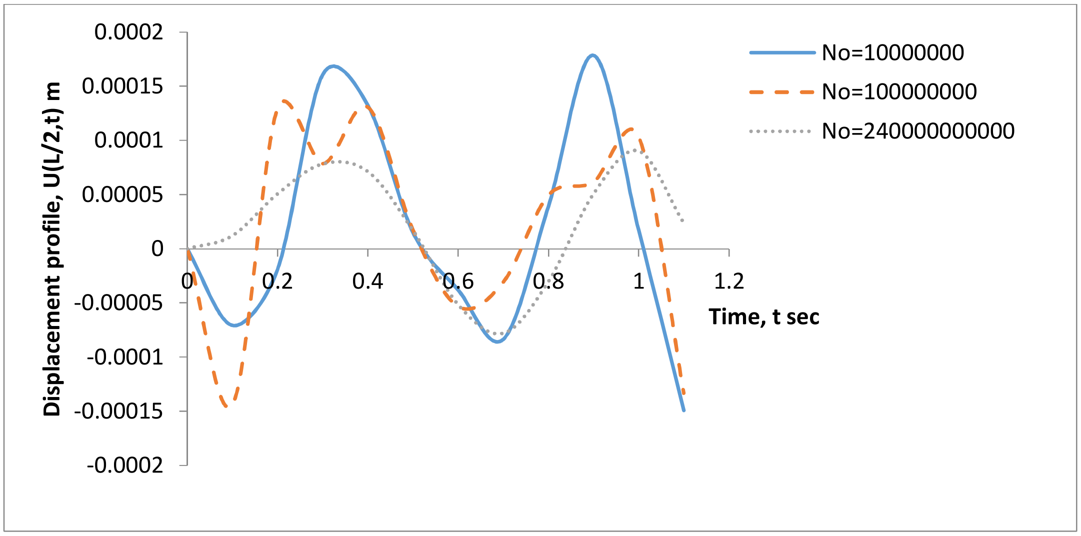

Fig. 3. Displacement profile of a clamped elastic uniform Bernoulli- Euler beam on constant foundation and traverse by moving distributed force for fixed values of Go and Ko and various values of No

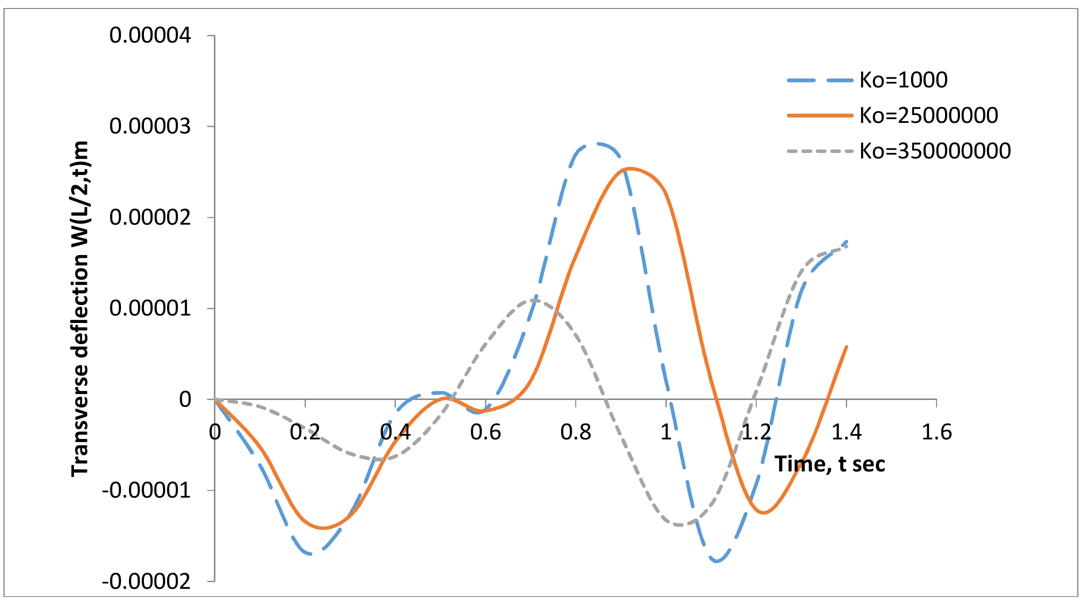

Fig. 4. Displacement profile of a free elastic uniform Bernoulli- Euler beam on constant foundation and traverse by moving distributed for for fixed values of No and Go and various values of Ko

Fig. 5 displays the effect of shear modulus on the displacement profile of free elastic Bernoulli-Euler beam under the action of load moving at constant velocity for moving distributed forces. The graph shows that the response amplitudes decrease as the value of **Go** increases.

Fig. 6 display the effect of axial force on the displacement profile of free elastic Bernoulli-Euler beam under the action of load moving at constant velocity for moving distributed forces. The graph shows that the response amplitudes decrease as the value of **No** increases.

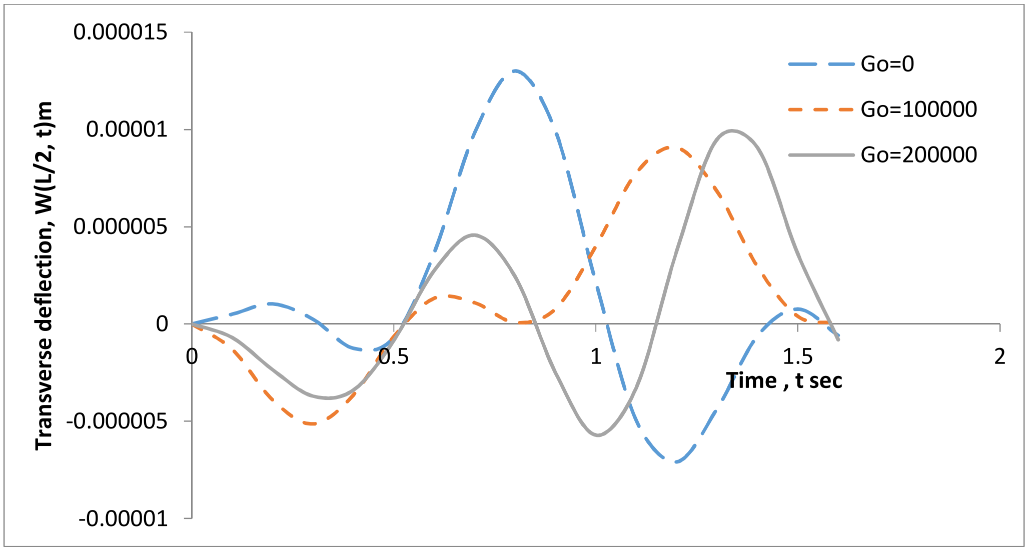

Fig. 5. Displacement profile of a free elastic uniform Bernoulli- Euler beam on constant foundation and traverse by moving distributed for for fixed values of No and Ko and various values of Go

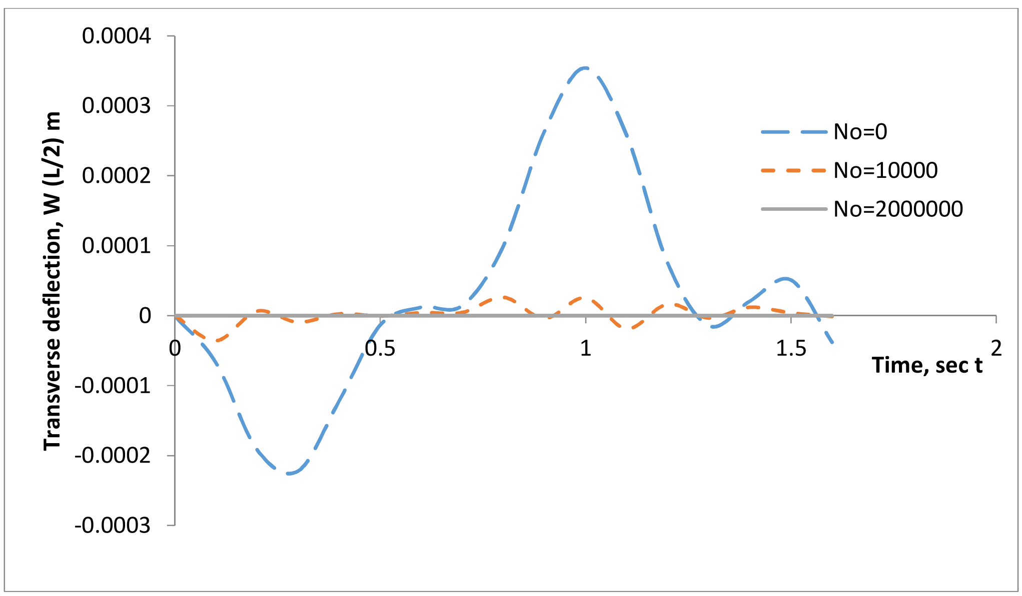

Fig. 6. Displacement profile of a free elastic uniform Bernoulli- Euler beam on constant foundation and traverse by moving distributed for for fixed values of Ko and Go and various values of No

7 Conclusion

In this research work, the problem of assessing the dynamic behavior of uniform Bernoulli-Euler beam with elastically supported boundary conditions under moving distributed masses and resting on constant foundation is considered. The close form solution of the governing fourth order partial differential equation with variable and singular coefficients of uniform Bernoulli-Euler beam for moving force is presented. The solutions are analyzed and resonance conditions are obtained for the problem. The results in plotted curves show the effects of axial force, shear modulus and foundation modulus on the beam for moving force problem only.

Competing Interests

Authors have declared that no competing interests exist.

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