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Numerical Analysis of Heat Transfer through a Pin Fin using RK5 and Euler Methods under the Tip Condition of Convective Heat Transfer

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ABSTRACT
Numerical determination of the temperature distribution along a circular pin fin and the fin heat transfer rate to the medium is presented. Both an Euler method and Butcher's fifth-order Runge- Kutta method (RK5) were used to solve the second-order governing differential equation for the temperature distribution. A shooting method was employed to convert the boundary value problem into an initial value problem. Computer code on SciLab package was written to perform the numerical iterations until the solution converges to the given convective heat transfer condition at the tip of the pin fin. Numerical results approximated the analytical solution well, while those from the RK5 method were almost identical to the exact solution. RK5 is thus by far more accurate than the Euler method. Parameters such as length, diameter and material of the pin fin were varied to improve the effectiveness of the pin fin for the heat transfer.

Keywords: *Euler method, RK5, Runge-Kutta*

INTRODUCTION

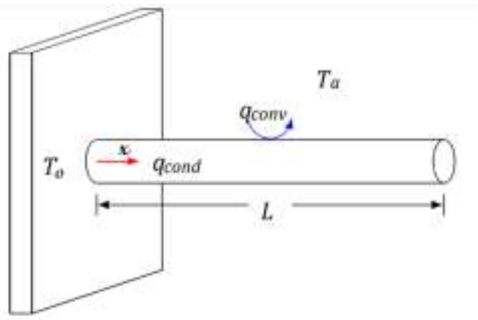
Many electrical components cannot withstand high temperatures of certain degrees.

In order to prevent component overheat and to dissipate the heat energy away from the electrical component through heat transfer by conduction and convection to the surrounding medium, a common method is to use fins attached to the electrical component to enhance the heat transfer by increasing the surface areas for heat convection to the surrounding liquid or gas medium, thereby lowering the temperature of the component.[1]

The fin geometry plays an important part in heat transfer process. Complex fin geometry has been put in place to serve this purpose.

However, complex geometry may not be solved analytically and numerical methods must be resorted to in the solution process.[1,2] This study uses a proposed pin fin design as an illustration with the objectives of:

1. Using two numerical methods, namely the Euler method and RK5 method, to solve problems of heat transfer process through a pin fin;
2. Examining how accurate the numerical methods are when compared with the analytical method; and
3. Suggesting ways of improvement to the heat transfer process by varying the parameters of the pin fin setting.



LITERATURE SURVEY

Literature survey was carried on the formulation of the heat transfer equations and the methods of solving these equations. The analytical method if available and two numerical methods (the Euler method and the Runge-Kutta method) along with the shooting method would be explored. [3,4]

Formulation of the Heat Transfer Equations

Newton's law of cooling states that the pace of cooling of a warm item at any second is corresponding to the temperature contrast (θ) between the article (T) and its encompassing medium (Ta).[3]

$$\frac{d\theta}{\theta} = constant\ at\ any\ instance \quad (1)$$

Or

$$\frac{\Delta\theta}{\theta} = constant\ per\ unit\ time \quad (2)$$

Newton’s statement cn be further expressed as:

$$Log\frac{\Delta t}{(\Delta t)o} = -\alpha\tau \quad (3)$$

where Δt is the excess temperature at time τ, $(\Delta t)_0$ is the initial excess temperature, and α is a constant. Differentiation of the equation gives:

$$\frac{d\ \Delta t}{d\tau} = \alpha\,\Delta t \quad (4)$$

This shows that greater the difference in temperatures between the object and surrounding, more is the rate of temperature drop of the object, cultivating the concept of heat transfer coefficient. In light of current knowledge, Newton's law applies to only small temperature excesses around twenty to thirty degrees Celsius.[3]

Fourier [4] extended the concept in Newton’s law of cooling. In his formulation, Fourier incorporated the concept of convective heat transfer coefficient at the base and introduced the conductive heat transfer coefficient to effectively decouple the conduction problem from the convection problem, both being as a mode of heat transfer. Various assumptions have been made to simplify the analysis of the heat transfer process, which include constant cross-sectional area of the pin fin, and constant fin thermal conductivity k and convection heat transfer coefficient h throughout the fin. Fourier established the resulting heat transfer equation for a heated projected object in the air medium at time τ as:

$$A_c\frac{\partial\theta}{\partial\tau} = \frac{kA_c}{\rho C_p}\frac{\partial^2\theta}{\partial x^2} - \frac{hP}{\rho C_p}\theta \quad (5)$$

where Cp is the specific heat capacity of a gas at constant pressure, ρ the density of the air, h the convective heat transfer coefficient and A_c the cross-sectional area of the fin, k the conductive heat transfer coefficient of the medium.

For the steady-state case when the heat flow and temperature profile along the pin fin is independent of time, $\frac{\partial\theta}{\partial\tau} = 0$ and a simplified equation is derived as follows:

$$\frac{d^2\theta}{dx^2} = \frac{hP}{kA_c}\theta \quad (6)$$

The amount of heat removed from the base plate due to pin fin is the heat conducted out and transferred to the fin, called the pin fin heat transfer rate q_f –

$$q_f = -kAc\frac{d}{dx}(T - T_a)|_{x=0} \quad (7)$$

Methods of Solving Heat Transfer Problems

Whenever available, analytical method to solve the ODE is used. Two numerical models, namely a fifth order Runge-Kutta method and an Euler method, are applied to solve the governing ODE for the heat transfer through a pin fin. Comparison of the numerical results with the analytical results is made to evaluate the accuracy of the two numerical methods.

The pin fin problem in the study provides one initial value condition at the component base end. Another boundary condition can only be ascertained for the other end. However, for both numerical methods, two initial value conditions are required to enable the stepwise numerical calculations to proceed; a shooting method is adopted to iterate the tip end condition to the correct initial value.[5]

Analytical method

The given boundary condition is that of convective heat loss at the tip of the fin, which, at x=L, is given as,

$$h\theta_L = -k\frac{d\theta}{dx}|_{x=L} \quad (8)$$

The analytical solution for temperature distribution of θ/θ_0 is

$$\frac{\cosh m(L-x) + \left(\frac{h}{mk}\right)\sinh m(L-x)}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL} \quad (9)$$

where $m^2 = hP/k\,Ac$.

The pin fin heat transfer rate q_f is

$$M \frac{\sinh m(L-x)+\left(\frac{h}{mk}\right)\cosh m(L-x)}{\sinh mL+\left(\frac{h}{mk}\right)\cosh mL} \quad (10)$$

where $M = \sqrt{hPkA_c}\theta_0$

Numerical methods

In simple fin geometry, such as uniform cross-section, and constant boundary conditions, analytical solution is possible for evaluating temperature distribution and heat transfer rates.[2] However, when the fin geometry is complicated and boundary conditions, medium condition or fin properties are varying, numerical methods have the advantages of handling the widely varying problem parameters and complicated problems by using an incremental numerical approach to solve an otherwise analytically unsolvable governing ODE.

Euler method

Hoffma [5] detailed the Euler method for solving second order ODE numerically with known initial values. In general, a second order differential equation with initial values can be expressed as:

$$\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right) \text{ with initial values } y(x_0) = y_0 \text{ and } \frac{dy}{dx}\Big|_{x=x_0} = y_0' \quad (11)$$

It can be decoupled into two first order ODE:

$$\frac{dz}{dx} = f(x, y, z) \quad \text{and} \quad (12)$$

$$z = \frac{dy}{dx} \quad (13)$$

With a step size of h, starting from the initial condition x_0 and z_0, the value of x is increased by the step size of h. Linear approximation using the instantaneous gradient can find the next value of y. With only a single constant of the instantaneous gradient for linear approximation, Euler method is a first-order approximation. This process is iterated to find the value of y for a particular x.

$$x_1 = x_0 + h \quad (14)$$
$$y_1 = y_0 + h\, z_0 \quad (15)$$

$$z_1 = z_0 + hf(x_0, y_0, z_0) \quad (16)$$

In general terms, the iterative Euler process can be expressed as:

$$x_{i+1} = x_i + h \quad (17)$$

$$y_{i+1} = y_i + h\, z_i \quad (18)$$

$$z_{i+1} = z_i + hf(x_i, y_i, z_i) \quad (19)$$

Fifth order Runge-Kutta method

Runge-Kutta methods can solve initial value problems numerically for the system of two differential equations. The approximation is no more first order in the linear form, but to the fifth order for greater accuracy. There are several versions of fifth-order Runge-Kutta methods. Butcher [6] presented his fifth order Runge-Kutta method applied to the following system of differential equations:

$$\frac{dy}{dx} = P(x, y, z) \tag{20}$$

$$\frac{dz}{dx} = Q(x, y, z) \tag{21}$$

with the initial value conditions:

$$y(x_0) = y_0, z(x_0) = z_0$$

The Butcher's fifth order Runge-Kutta method is an iterative process with a step of h in x. Upon each increment of h in x, the subsequent y and z can be obtained as expressed below:

$$y_{i+1} = y_i + \frac{h}{90}(7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6) \tag{22}$$

$$z_{i+1} = z_i + \frac{h}{90}(7l_1 + 32l_3 + 12l_4 + 32l_5 + 7l_6) \tag{23}$$

where

$$x_{i+1} = x_i + h \tag{24}$$

$$k_1 = P(x_i, y_i, z_i) \tag{25}$$

$$l_1 = Q(x_i, y_i, z_i) \tag{26}$$

$$k_2 = P\left(x_i + \frac{1}{4}h,\ y_i + \frac{1}{8}hk_1,\ z_i + \frac{1}{8}hl_1\right) \tag{27}$$

$$l_2 = Q\left(x_i + \frac{1}{4}h,\ y_i + \frac{1}{8}hk_1,\ z_i + \frac{1}{8}hl_1\right) \tag{28}$$

$$k_3 = P\left(x_i + \frac{1}{4}h,\ y_i + \frac{1}{8}hk_1 + \frac{1}{8}hk_2,\ z_i + \frac{1}{8}hl_1 + \frac{1}{8}hl_2\right) \tag{29}$$

$$l_3 = Q\left(x_i + \frac{1}{4}h,\ y_i + \frac{1}{8}hk_1 + \frac{1}{8}hk_2,\ z_i + \frac{1}{8}hl_1 + \frac{1}{8}hl_2\right) \tag{30}$$

$$k_4 = P\left(x_i + \frac{1}{2}h,\ y_i - \frac{1}{2}hk_2 + hk_3,\ z_i - \frac{1}{2}hl_2 + hl_3\right) \tag{31}$$

$$l_4 = Q\left(x_i + \frac{1}{2}h,\ y_i - \frac{1}{2}hk_2 + hk_3,\ z_i - \frac{1}{2}hl_2 + hl_3\right) \tag{32}$$

$$k_5 = P\left(x_i + \frac{3}{4}h,\ y_i + \frac{3}{16}hk_1 + \frac{9}{16}hk_4,\ z_i + \frac{3}{16}hl_1 + \frac{9}{16}hl_4\right) \tag{33}$$

$$l_5 = Q\left(x_i + \frac{3}{4}h,\ y_i + \frac{3}{16}hk_1 + \frac{9}{16}hk_4,\ z_i + \frac{3}{16}hl_1 + \frac{9}{16}hl_4\right) \tag{34}$$

$$k_6 = P\left(x_i + h,\ y_i - \frac{3}{7}hk_1 + \frac{2}{7}hk_2 + \frac{12}{7}hk_3 - \frac{12}{7}hk_4 + \frac{8}{7}hk_5,\ z_i - \frac{3}{7}hl_1 + \frac{2}{7}hl_2 + \frac{12}{7}hl_3 - \frac{12}{7}hl_4 + \frac{8}{7}hl_5\right) \tag{35}$$

$$l_6 = Q\left(x_i + h,\ y_i - \frac{3}{7}hk_1 + \frac{2}{7}hk_2 + \frac{12}{7}hk_3 - \frac{12}{7}hk_4 + \frac{8}{7}hk_5,\ z_i - \frac{3}{7}hl_1 + \frac{2}{7}hl_2 + \frac{12}{7}hl_3 - \frac{12}{7}hl_4 + \frac{8}{7}hl_5\right) \tag{36}$$

For i=1, 2, 3, ……..

Shooting method – the secant formula

The secant method is a more sophisticated shooting method to iterate and refine the initial condition, in which the root of a function f(w) is approximated by a secant line through two points on the graph of f(w), as opposed to a tangent line through one point on the graph as in the case of the Euler method.[5] In the study the boundary condition at the pin fin tip is f(w)=0 where w is the initial temperature gradient at x=0.

As a secant line requires two points on the graph of f(x), it is necessary to choose two initial iterates w_0 and w_1. The next iterate w_2 is computed as the w-value at which the secant line connecting the points (w_0, $f(w_0)$) and (w_1, $f(w_1)$) has a y-coordinate of zero, resulting in the following equation:

$$\frac{f(w_1) - f(w_0)}{w_1 - w_0}(w_2 - w_1) + f(w_1) = 0 \tag{37}$$

w_2 can be expressed as:

$$w_2 = w_1 - \frac{f(w_1)(w_1 - w_0)}{f(w_1) - f(w_0)} \tag{38}$$

The same iteration can carry on to find successive x as follows:

$$w_{i+1} = w_i - \frac{f(w_i)(w_i - w_{i-1})}{f(w_i) - f(w_{i-1})} \tag{39}$$

The target is to get f(w) <0.001, close enough to zero, in which the boundary condition at the tip is fulfilled.

IMPLEMENTATION OF THE MODELS AND NUMERICAL ANALYSIS

The problem statement gives the following parameters:

$$T_0 = 60°C,\ T_a = 24°C,\ k = 19\ W/mK,\ h = 20W/m^2K,\ d = 0.001m,\ l = 0.03m$$

With the given data, the governing ODE becomes:

$$\frac{d^2T}{dx^2} - 4201.526316\ (T - 24) = 0 \tag{40}$$

According to the brief, the tip of the pin was considered as convection hear transfer. Therefore, the rate of change of temperature at the tip of the pin fin is equal to:

$$\frac{dT}{dx} = -1.05263158\, T_L \tag{41}$$

which is the boundary condition to be fulfilled.

Both Euler and RK5 methods were employed to estimate the temperature profile. For both numerical methods and the exact solution, computer code was written in SciLab to perform the calculations and iterations. The complete code with explanations is included in Appendix 1. A step size of 0.0003m long with 100 steps was used to generate the temperature profile for each numerical method.

Since the heat transfer equation for pin fin is a second order ODE, two initial value conditions are required at the base end of the fin pin. In this problem the temperature at the base (x=0) was given as T_0= 60, however the rate of change of temperature at the base was yet to be determined. The secant formula was employed to refine the rate of change of temperature at the base by using the boundary condition at the tip through iterations. The secant formula requires two initial iterates; two rates of change of temperature at the tip are computed. The value of the computed *dT/dx* was refined after every iteration by the secant formula, to arrive at the calculated *dT/dx* at *x=0* while fulfilling the second boundary condition. The initial condition was considered acceptable when the boundary condition was satisfied with an error of less than 0.001. This was done with a conditional statement IF to detect the error in an FOR loop statement. The two initial iterates used are -4210 and -2100. [6]

RESULTS AND DISCUSSION

Results from the Euler's method and RK 5 method along with the analytical solution are presented in Figure 1. It can be seen that both numerical methods closely approximate the analytical method. Figure 2 is an enlarged view of Figure 1 at the fin tip, which shows that RK5 gives results almost identical to the analytical method.

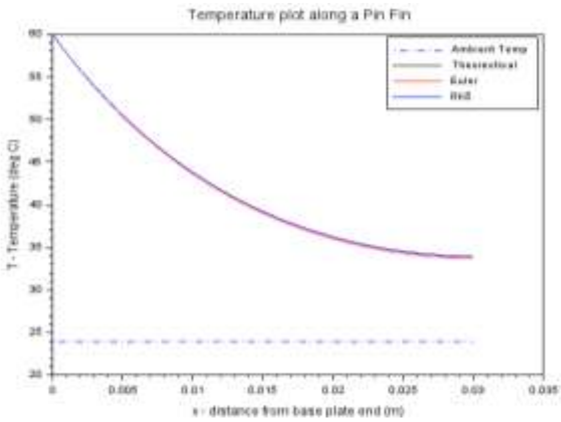


Fig. 1: *Temperature plot across the entire pin fin for different solution methods.*

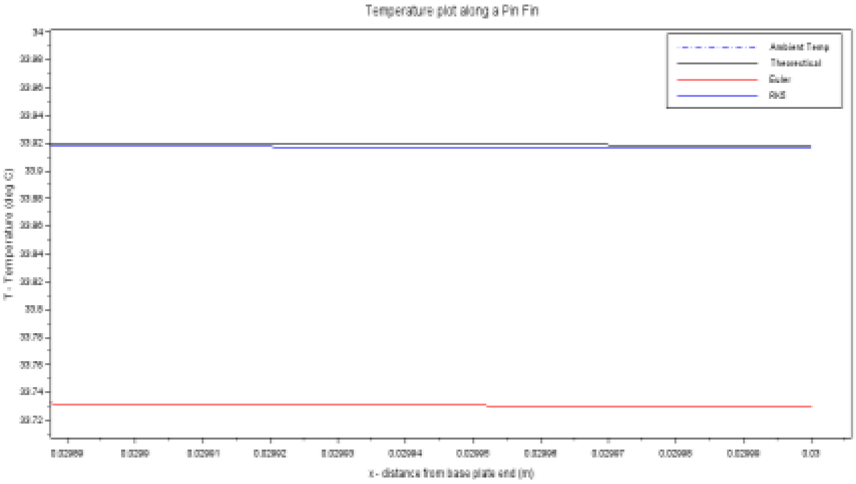


Fig. 2: *Enlarged temperature plot around the tip of the pin fin.*

Accuracy of the models

Results at the fin tip from the two models and the analytical solutions are tabulated in Table 1 below for comparison.

Table 1: *Comparison of model results using different iteration steps.*

Method	**Temp at tip**	**Error in temperature at tip**	**Temp above ambient at tip**	**Error in temperature above ambient at tip**
Analytical	33.918305	NA	24.918305	NA
Euler	33.729829 (n=100) 33.880567 (n=500) 33.899381 (n=1000)	-0.5557% -0.1113% -0.05579%	24.729829 (n=100) 24.880567 (n=500) 24.899381 (n=1000)	-0.7564% -0.1514% -0.07593%
RK5	33.918190 (n=10) 33.918189 (n=50) 33.918185 (n=100)	-0.0003391% -0.0003420% -0.0003538%	24.918190 (n=10) 24.918189 (n=50) 24.918185 (n=100)	-0.0004615% -0.0004655% -0.0004816%

As the calculations methods are based on the temperature above the ambient one, it would only meaningful to compare the error of temperatures above the ambient temperature between the exact solution and the numerical methods. The error percentages for temperatures above the ambient under different number of iterations are plotted in Figure 3 below.

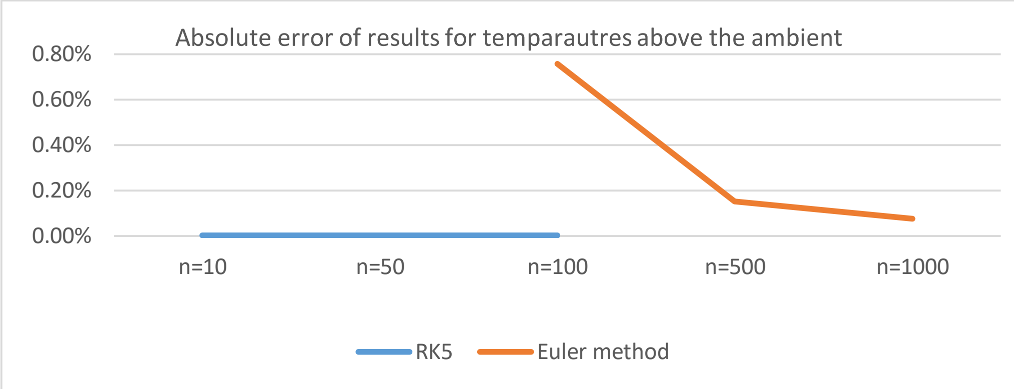


***Fig. 3:** Error of results for numerical methods under different number of iterations.*

From Table 1, it can be seen that the accuracy of RK5 results with 10 iterations is far greater than the Euler results even with 1000 iterations. It shows that both the accuracy and the efficiency of computation with RK5 are much superior to the Euler method, signifying that higher order numerical method such as RK5 can model the solution much more accurately.

Improvement to the Effectiveness of the Pin Fin

There are various parameters in the governing ODE which would affect how effective the pin fin can transfer heat away from the base plate. These parameters include the diameter and length of the pin fin, the thermal conductivity of the pin fin material, the convective heat transfer coefficient of the medium, and the tip condition. As the tip condition and the medium material are outside the scope of the study, only the diameter and the length of the pin fin, and the thermal conductivity of the pin fin material would be explored to improve the effectiveness of the pin fin in transferring the heat away.

Effect of the diameter of the pin fin

Diameters of 1, 2, 3 and 4 mm were tested for the effect on the pin fin heat transfer rate. The SciLab results are tabulated in Table 2.

***Table 2:** Effect of pin fin diameters on tip temperature and pin fin heat transfer rate.*

Diameter (mm)	**Temp at tip (degC)**	**Temp at tip above ambient (degC)**	**Pin fin heat transfer rate q_f (W)**	**Pin fin heat transfer rate as compared to 1mm diameter**
1	33.918	9.918	0.03351	1
2	40.75	16.75	0.08728	2.6
3	44.70	20.70	0.1482	4.4
4	47.24	23.24	0.2131	6.4

The SciLab output for the temperature profile along the pin fin for different diameters is shown in Figure 4.

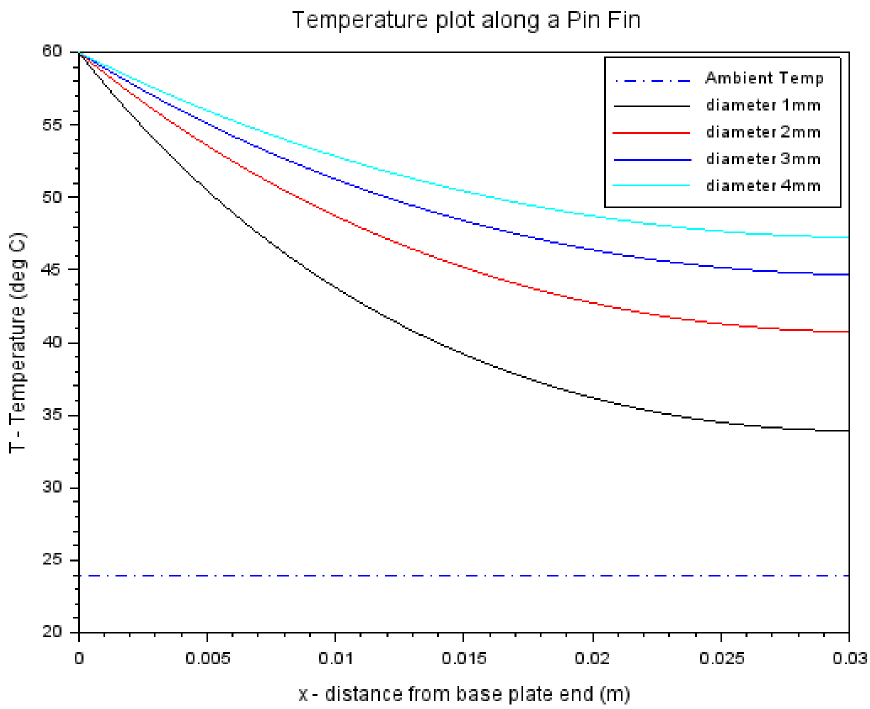


***Fig. 4:** Temperature plot along pin fin for different diameters.*

The relative effectiveness of heat transfer rate is plotted for different diameters of pin pins in Figure 5.

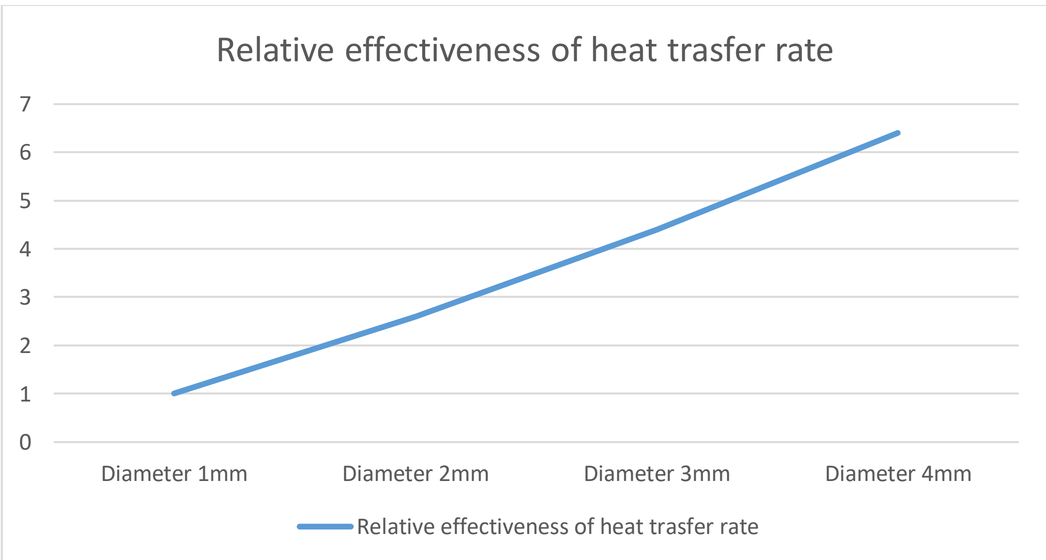


***Fig. 5:** Relative effectiveness of heat transfer rate for different diameters of pin fins.*

It can be seen from Figure 5 that with larger diameters, more heat will go to the tip resulting in higher tip temperatures and heat being more effectively transferred to the surrounding by convection giving rise to higher pin fin heat transfer rates.

Effect of pin fin length

Fin pin lengths of 3, 6, 9 and 12 cm were tested for the effect on the pin fin heat transfer rate. The following results of tip temperatures and pin fin heat transfer rates were obtained in Table 3.

Table 3: *Effectiveness of hat transfer rate for different lengths of pin fins.*

Pin fin length (cm)	**Temp at tip (^{0}C)**	**Temp at tip above ambient (^{0}C)**	**Pin fin heat transfer rate q_f(W)**	**Pin fin heat transfer rate as compared to length of 3cm**
3	33.918	9.918	0.03351	**1**
6	25.44	1.44	0.03483	1.039
9	24.21	0.21	0.03486	1.040
12	24.03	0.03	0.03487	1.041

The SciLab output for the temperature profile along the pin fin for different pin fin lengths is shown in Figure 6.

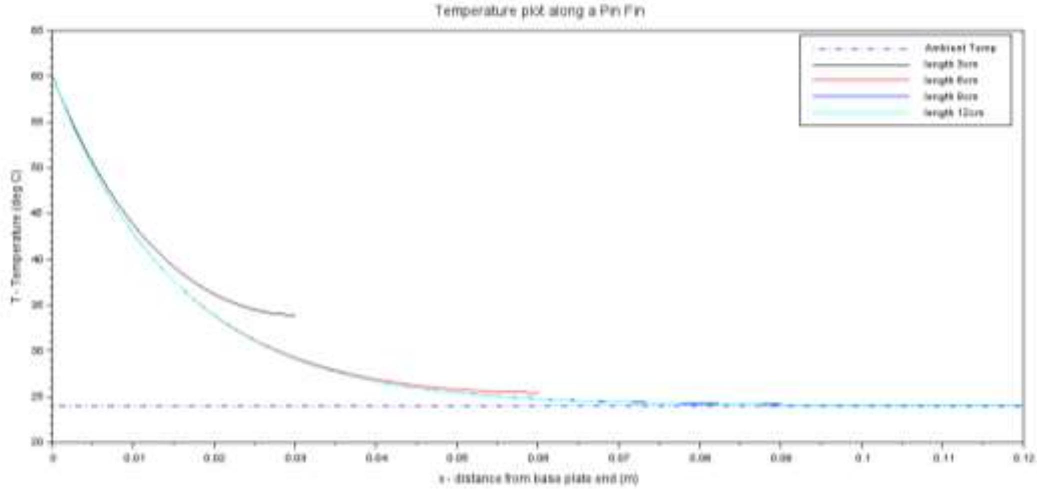


Fig. 6: *Temperature along the pin fin for different pin fin lengths.*

It can be seen from the Figure 6 that with greater lengths, only very small additional heat will be transferred. There is no point to increase the length of the pin fin beyond 9cm, as no noticeable gain in the heat transfer rate can be obtained, as the tip has reached the ambient temperature.

Effect of pin fin material

Materials of different thermal conductivities may affect the effectiveness of the heat fin. Five more other materials with thermal conductivities ranging between 14.9 to 386 W/mK were tested for the effect on the heat transfer rate. The results are included in Table 4.

Table 4: *Effectiveness of hat transfer rate for different materials of pin fins.*

Material	**Thermal conductivity (W/mK)**	**Temp at tip (degC)**	**Temp at tip above ambient (degC)**	**Pin fin heat transfer rate q_f(W)**	**Pin fin heat transfer rate as compared to Titanium**
Stainless steel	14.9	31.57	7.57	0.03145	0.90
Titanium	19	33.73	9.73	0.03350	**1**
Chrome steel (1%C)	59	45.21	21.21	0.04948	1.48
Tin	67	46.41	22.41	0.05106	1.52
Aluminium	204	46.41	22.41	0.1541	4.6
Copper	386	46.41	22.41	0.2910	8.69

The SciLab output for the temperature along the pin fin for different pin fin materials is presented in Figure 7.

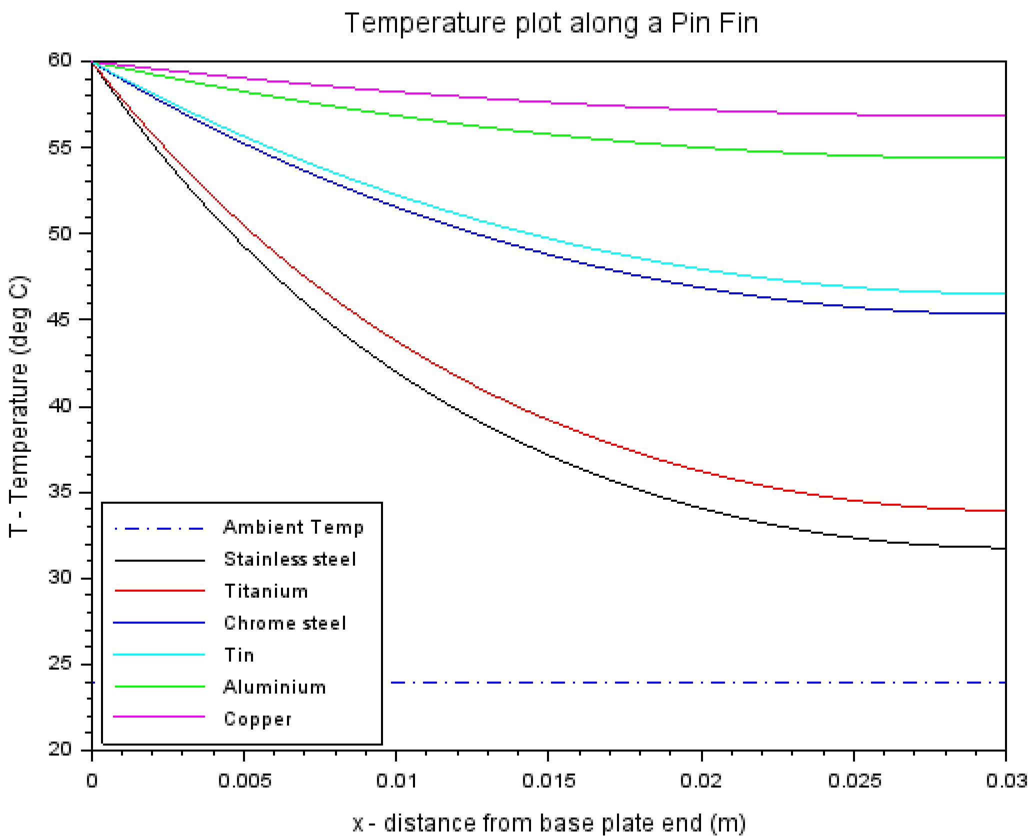


Fig. 7: *Temperature along the pin fin for different pin fin materials.*

It can be seen from Figure 7 that with materials of larger thermal conductivities, the pin fin will be more effective in the heat transfer rate. When the thermal conductivity of the pin fin is high, more heat will be conducted to the tip giving rise to higher temperatures along the pin fin, thus facilitating heat to be carried away by convection to the medium. Among the six common metals as heat conductor, copper provides the best heat transfer rate, followed by aluminium.

CONCLUSION

In this study, the Euler method and Butcher's fifth order Runge-Kutta method (RK5) along with the secant method were applied to solve heat transfer problems which are expressed as second order differential equations with given boundary conditions. While both methods approximate well the exact solution, RK5 generates results almost identical to the exact solution even using only a few numbers of iterations. RK5 is thus much superior to the Euler method in terms of accuracy and efficiency of computation. To get more accurate results from the Euler method, the step size must be reduced to small values which will result in large numbers of iterations. Higher order approximation method such as RK5 is able to produce results much more accurate than lower first order method, such as Euler.

Effects of the parameters in the pin fin heat transfer governing ODE were studied. It can be concluded that larger the pin fin diameter, longer the pin fin length and higher thermal conductivity of the pin fin material would contribute to more heat transfer to the medium. However, it should be noted that there is a limit on the length of the pin fin, beyond which no more noticeable additional heat will be transferred when the pin fin tip has reached the ambient temperature.

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Appendix

Computer code on SciLab

```
//Heat transfer through a pin fin - on SciLab
//Theoretical method, Euler method and RK
//Input parameters
H=20  //convective heat transfer coefficient (20W/m2K)
K=19  //conductive heat transfer coefficient (W/mK)
d=0.001 //diameter of pin fin (m)
L=0.03 //length of pin fin (m)
T0=60 //temperature at base plate (degC)
Ta=24 //ambient temperature (deg C)

A=%pi*(d/2)^2 //calculate cross sectional area of pin fin
P=%pi*d    //calculate perimeter of pin fin
M=(H*P)/(K*A)
m=M^0.5

//Set boundary values and parameters for running SciLab programme
x0=0    //set x0 as starting x
y0=60    //set y0 as base temperature
znn = zeros(21,1) //initialise temperature gradient vector
fznn= zeros(21,1) //initialise the function vector in Secant method
znn(1)=-4210  //set first temperature gradient iterate
znn(2)=-2100  //set second temperature gradient iterate
h=0.0003    //set step size
N=100      //set no. of steps (iterations)

//Theoretical (exact) solution

i=[0:0.0003:L]
Temp=(cosh(m*(L-i))+(H/(m*K))*sinh(m*(L-i)))/((cosh(m*L))+(H/(m*K))*sinh(m*L))
Temperature=Temp*(T0-Ta)+Ta
```

```
plot (i,Ta, '-.')
plot (i, Temperature, 'k')
xtitle("Temperatur plot across a Pin Fin","x - distance from base plate end (m)", "T - Temperature (deg C)" )

//Euler Mathod

function ydot=p(x, y, z)
 ydot=z
endfunction

function zdot=q(x, y, z)
 zdot=4210.526316*(y-24)
endfunction

function [x, y, z]=euler(p, q, x0, y0, z0, h, N)
 x = zeros(N+1, 1)
 y = zeros(N+1, 1)
 z = zeros(N+1, 1)
 x(1) = x0
 y(1) = y0
 z(1) = z0
 for j = 1:N
  x(j+1) = x(j) + h
  y(j + 1) = y(j) + h*p(x(j), y(j), z(j))
  z(j + 1) = z(j) + h*q(x(j), y(j), z(j))
 end
endfunction

//Secant method
//Set the first initial iterate znn(1) and find the first fznn(1)
z0=znn(1)
[x, ye, ze]=euler(p, q, x0, y0, z0, h, N)
fznn(1)=ze(N+1)+1.05263158*(ye(N+1)-Ta)

//Set the second initial iterate znn(2) and find the second fznn(2)
z0=znn(2)
[x, ye, ze]=euler(p, q, x0, y0, z0, h, N)
fznn(2)=ze(N+1)+1.05263158*(ye(N+1)-Ta)

//Use Secant formula for iterating to fulfil the boundary condition fznn(i)=0,
//thus finding the temperature along the pin fin
for i=3:10
 if abs(fznn(i-1))>0.001 then
 znn(i)=znn(i-1)-fznn(i-1)*(znn(i-1)-znn(i-2))/(fznn(i-1)-fznn(i-2))
 z0=znn(i)
 [x, ye, ze]=euler(p, q, x0, y0, z0, h, N), end
end
```

```
//Plot temperature profile using Euler method
plot(x,ye,'r')

//RK5 method

function [x, y, z]=rk5(p, q, x0, y0, z0, h, N)

 x = zeros(N+1, 1)
 y = zeros(N+1, 1)
 z = zeros(N+1, 1)
 x(1) = x0
 y(1) = y0
 z(1) = z0

 for i = 1:N

  k1=p(x(i),y(i),z(i))
  m1=q(x(i),y(i),z(i))
  k2=p(x(i)+h/4, y(i)+h*k1/4, z(i)+h*m1/4)
  m2=q(x(i)+h/4, y(i)+h*k1/4, z(i)+h*m1/4)
  k3=p(x(i)+h/4, y(i)+h*k1/8+h*k2/8, z(i)+h*m1/8+h*m2/8)
  m3=q(x(i)+h/4, y(i)+h*k1/8+h*k2/8, z(i)+h*m1/8+h*m2/8)
  k4=p(x(i)+h/2, y(i)-h*k2/2+h*k3, z(i)-h*m2/2+h*m3)
  m4=q(x(i)+h/2, y(i)-h*k2/2+h*k3, z(i)-h*m2/2+h*m3)
  k5=p(x(i)+3*h/4, y(i)+3*h*k1/16+9*h*k4/16, z(i)+3*h*m1/16+9*h*m4/16)
  m5=q(x(i)+3*h/4, y(i)+3*h*k1/16+9*h*k4/16, z(i)+3*h*m1/16+9*h*m4/16)
  k6=p(x(i)+h, y(i)-3*h*k1/7+2*h*k2/7+12*h*k3/7-12*h*k4/7+8*h*k5/7, z(i)-3*h*m1/7+2*h*m2/7+12*h*m3/7-12*h*m4/7+8*h*m5/7)
  m6=q(x(i)+h, y(i)-3*h*k1/7+2*h*k2/7+12*h*k3/7-12*h*k4/7+8*h*k5/7, z(i)-3*h*m1/7+2*h*m2/7+12*h*m3/7-12*h*m4/7+8*h*m5/7)

  x(i+1) = x(i) + h
  y(i+1) = y(i) + h*(7*k1 + 32*k3+ 12*k4 + 32*k5 + 7*k6)/90
  z(i+1) = z(i) + h*(7*m1 + 32*m3+ 12*m4 + 32*m5 + 7*m6)/90

 end
endfunction

//Secant method
//Set the first initial iterate znn(1) and find the first fznn(1)
z0=znn(1)
[x, ye, ze]=rk5(p, q, x0, y0, z0, h, N)
fznn(1)=ze(N+1)+1.05263158*(ye(N+1)-Ta)

//Set the second initial iterate znn(2) and find the second fznn(2)
z0=znn(2)
[x, ye, ze]=rk5(p, q, x0, y0, z0, h, N)
fznn(2)=ze(N+1)+1.05263158*(ye(N+1)-Ta)
```

```
//Use Secant formula for iterating to fulfil the boundary condition fznn(i)=0,
//thus finding the temperature along the pin fin
for i=3:10
 if abs(fznn(i-1))>0.001 then
 znn(i)=znn(i-1)-fznn(i-1)*(znn(i-1)-znn(i-2))/(fznn(i-1)-fznn(i-2))
 z0=znn(i)
 fznn(i-1)=ze(N+1)+1.05263158*(ye(N+1)-Ta)
 [x, ye, ze]=rk5(p, q, x0, y0, z0, h, N)
 disp(-K*A*z0),end  //Heat transfer rate
end

//Plot temperature profile using RK5 method
plot(x,ye,'b')
legend ("Ambient Temp","Theorectical", "Euler","RK5")
```