(Short Communication)

Methodical notes.
All frames of reference with a homogeneous field of gravity

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Abstract

Uniformly accelerated reference systems are obtained. They are described by two types of metrics with nonzero curvature. These systems contain an arbitrary strictly positive differentiable function.

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1. Introduction

What do we know about systems with a uniform gravitational field? Very little.

The equations of motion in the general theory of relativity [1] allow one to calculate the covariant or contravariant acceleration vector of a fixed point

$$\frac{d^2x_i}{ds^2} - \Gamma_{\mathrm{i,kl}}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0\,;$$

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0.$$

In the stationary case, the acceleration vector α_i or $\alpha^i$is calculated by the equations

$$\alpha_i = \frac{d^2x_i}{dt^2} = c^2\Gamma_{\mathrm{i,00}}\,; \quad \alpha^i = \frac{d^2x^i}{dt^2} = -c^2\Gamma^i_{00} \qquad (1)$$

In [2] and [3], constant acceleration vectors for stationary bodies were found. Both metrics have non-zero curvature. Moreover, in the article [2] a metric with a constant covariant acceleration vector was found. Article [3] describes the simplest metric

$$ds^2 = \left(1 + \frac{\alpha x}{c^2}\right)dt^2 - (dx^2 + dy^2 + dz^2),$$

having constants $\alpha_i = -\alpha^i = \alpha$. Unfortunately, this metric has a homogeneous field only in a limited region

$$\frac{\alpha x}{c^2} > -1.$$

In this paper, we will mainly use the scalar acceleration value

$$a = \sqrt{-\alpha_i\alpha^i} \qquad (2)$$

2. Two classes of metrics with a homogeneous field

Consider a field in a metric of the form

$$ds^2 = s(x)dt^2 - p(x)(dx^2 + dy^2 + dz^2) \qquad (3)$$

and one-dimensional metric

$$ds^2 = s(x)dt^2 - p(x)dx^2 - dy^2 - dz^2. \qquad (4)$$

The results of calculating the parameters of metric (4) are given in Appendix 1. Metric (3) has a curvature tensor and its convolutions are much more complex, and we did not consider it necessary to present these data in full.

Calculations of vectors (1) give the same result for both metrics, and their scalar acceleration is related to the components of the fundamental tensor:

$$a^2 = \frac{s'(x)^2}{4p(x)} \qquad (4)$$

where

$$p(x) = \frac{s'(x)^2}{4a^2}.$$

This makes it possible to write finally homogeneous metrics

$$ds^2 = s(x)dt^2 - \frac{s'(x)^2}{4a^2}(dx^2 + dy^2 + dz^2) \qquad (5)$$

and

$$ds^2 = s(x)dt^2 - \frac{s'(x)^2}{4a^2}dx^2 - dy^2 - dz^2 \qquad (6)$$

where s(x) > 0 is an arbitrary function that has a continuous derivative. The main parameters of the metric (6) are given in Appendix 2.

We see that the resulting metrics have nonzero curvature. This suggests that uniformly accelerated metrics do not exist at all in flat space.

A good example of a uniformly accelerated frame of reference is the metric system (Appendix 3):

$$ds^2 = e^{2ax}dt^2 - e^{4ax}dx^2 - dy^2 - dz^2.$$

References

[1] Landau L D, Lifshitz E M. The Classical Theory of Fields Vol. 2 (4th ed.). Butterworth–Heinemann (1975).

[2] Morozov V B, Rüster S B. A critical analysis of Schwarzschild-like metrics. PJSE, v.7, n.5, (1-7), June 18, (2021)

[3] Morozov V B. On a uniformly accelerated frame of reference, Parana Journal of Science and Education, v.9, n.4, 2023, pp. 29-31.

Appendix

A1

$ds2 := -p(x1)\cdot\partial(x1)^2 - \left(\partial(x2)^2 + \partial(x3)^2\right) + s(x1)\cdot\partial(x4)^2$

$$ds2 := -p(x)\,\partial(x)^2 - \partial(y)^2 - \partial(z)^2 + s(x)\,\partial(t)^2 \qquad (4)$$

$Setup(metric = ds2)$

$$[metric = \{(1,1) = -p(x), (2,2) = -1, (3,3) = -1, (4,4) = s(x)\}] \qquad (5)$$

$Ricci[scalar]$

$$\frac{2\,s''(x)\,p(x)\,s(x) - s'(x)^2\,p(x) - s'(x)\,p'(x)\,s(x)}{2\,s(x)^2\,p(x)^2} \qquad (6)$$

$Einstein[\sim mu, nu, nonzero]$

$$G^{\mu}{}_{\nu} = \left\{(2,2) = \frac{-2\,s''(x)\,p(x)\,s(x) + s'(x)^2\,p(x) + s'(x)\,p'(x)\,s(x)}{4\,s(x)^2\,p(x)^2}, (3,3) = \frac{-2\,s''(x)\,p(x)\,s(x) + s'(x)^2\,p(x) + s'(x)\,p'(x)\,s(x)}{4\,s(x)^2\,p(x)^2}\right\} \qquad (7)$$

$Einstein[nonzero]$

$$G_{\mu,\nu} = \left\{(2,2) = \frac{2\,s''(x)\,p(x)\,s(x) - s'(x)^2\,p(x) - s'(x)\,p'(x)\,s(x)}{4\,s(x)^2\,p(x)^2}, (3,3) = \frac{2\,s''(x)\,p(x)\,s(x) - s'(x)^2\,p(x) - s'(x)\,p'(x)\,s(x)}{4\,s(x)^2\,p(x)^2}\right\} \qquad (8)$$

$Christoffel[\sim alpha, mu, nu, nonzero]$

$$\Gamma^{\alpha}{}_{\mu,\nu} = \left\{(1,1,1) = \frac{p'(x)}{2\,p(x)}, (1,4,4) = \frac{s'(x)}{2\,p(x)}, (4,1,4) = \frac{s'(x)}{2\,s(x)}, (4,4,1) = \frac{s'(x)}{2\,s(x)}\right\} \qquad (9)$$

$Christoffel[nonzero]$

$$\Gamma_{\mu,\nu,\alpha} = \left\{(1,1,1) = -\frac{p'(x)}{2}, (1,4,4) = -\frac{s'(x)}{2}, (4,1,4) = \frac{s'(x)}{2}, (4,4,1) = \frac{s'(x)}{2}\right\} \qquad (10)$$

$Ricci[nonzero]$

$$R_{\mu,\nu} = \left\{(1,1) = \frac{-2\,s''(x)\,p(x)\,s(x) + s'(x)^2\,p(x) + s'(x)\,p'(x)\,s(x)}{4\,s(x)^2\,p(x)}, (4,4) = \frac{2\,s''(x)\,p(x)\,s(x) - s'(x)^2\,p(x) - s'(x)\,p'(x)\,s(x)}{4\,p(x)^2\,s(x)}\right\} \qquad (11)$$

$Riemann[nonzero]$

$$R_{\mu,\nu,\alpha,\beta} = \left\{(1,4,1,4) = \frac{-2\,s''(x)\,p(x)\,s(x) + s'(x)^2\,p(x) + s'(x)\,p'(x)\,s(x)}{4\,p(x)\,s(x)}, (1,4,4,1) = \frac{2\,s''(x)\,p(x)\,s(x) - s'(x)^2\,p(x) - s'(x)\,p'(x)\,s(x)}{4\,p(x)\,s(x)}, (4,1,1,4) = \frac{2\,s''(x)\,p(x)\,s(x) - s'(x)^2\,p(x) - s'(x)\,p'(x)\,s(x)}{4\,p(x)\,s(x)}, (4,1,4,1) = \frac{-2\,s''(x)\,p(x)\,s(x) + s'(x)^2\,p(x) + s'(x)\,p'(x)\,s(x)}{4\,p(x)\,s(x)}\right\} \qquad (12)$$

A2

$ds2 := -\frac{(s'(x))^2}{4 \cdot a^2} \cdot \partial(x1)^2 - \partial(x2)^2 - \partial(x3)^2 + s(x) \cdot \partial(x4)^2$

$$ds2 := -\frac{s'(x)^2\, \partial(x)^2}{4\, a^2} - \partial(y)^2 - \partial(z)^2 + s(x)\, \partial(t)^2 \quad (4)$$

$Setup(metric = ds2)$

$$\left[metric = \left\{(1,1) = -\frac{s'(x)^2}{4\, a^2},\ (2,2) = -1,\ (3,3) = -1,\ (4,4) = s(x)\right\}\right] \quad (5)$$

$Ricci[scalar]$

$$-\frac{2\, a^2}{s(x)^2} \quad (6)$$

$Einstein[\sim mu, nu, nonzero]$

$$G^{\mu}{}_{\nu} = \left\{(2,2) = \frac{a^2}{s(x)^2},\ (3,3) = \frac{a^2}{s(x)^2}\right\} \quad (7)$$

$Einstein[nonzero]$

$$G_{\mu,\nu} = \left\{(2,2) = -\frac{a^2}{s(x)^2},\ (3,3) = -\frac{a^2}{s(x)^2}\right\} \quad (8)$$

$Christoffel[\sim alpha, mu, nu, nonzero]$

$$\Gamma^{\alpha}{}_{\mu,\nu} = \left\{(1,1,1) = \frac{s''(x)}{s'(x)},\ (1,4,4) = \frac{2\, a^2}{s'(x)},\ (4,1,4) = \frac{s'(x)}{2\, s(x)},\ (4,4,1) = \frac{s'(x)}{2\, s(x)}\right\} \quad (9)$$

$Christoffel[nonzero]$

$$\Gamma_{\mu,\nu,\alpha} = \left\{(1,1,1) = -\frac{s'(x)\, s''(x)}{4\, a^2},\ (1,4,4) = -\frac{s'(x)}{2},\ (4,1,4) = \frac{s'(x)}{2},\ (4,4,1) = \frac{s'(x)}{2}\right\} \quad (10)$$

$Ricci[nonzero]$

$$R_{\mu,\nu} = \left\{(1,1) = \frac{s'(x)^2}{4\, s(x)^2},\ (4,4) = -\frac{g}{2\, s(x)}\right\} \quad (11)$$

$Riemann[nonzero]$

$$R_{\mu,\nu,\alpha,\beta} = \left\{(1,4,1,4) = \frac{s'(x)^2\, g}{2\, s(x)},\ (1,4,4,1) = -\frac{s'(x)^2\, g}{2\, s(x)},\ (4,1,1,4) = -\frac{s'(x)^2\, g}{2\, s(x)},\ (4,1,4,1) = \frac{s'(x)^2\, g}{2\, s(x)}\right\} \quad (12)$$

A3

$ds2 := -e^{-4ax} \cdot \partial(x1)^2 - \partial(x2)^2 - \partial(x3)^2 + e^{-2ax} \cdot \partial(x4)^2$

$$ds2 := -e^{-4ax}\,\partial(x)^2 - \partial(y)^2 - \partial(z)^2 + e^{-2ax}\,\partial(t)^2 \quad (4)$$

$Setup(metric = ds2)$

$$[metric = \{(1,1) = -e^{-4ax}, (2,2) = -1, (3,3) = -1, (4,4) = e^{-2ax}\}] \quad (5)$$

$Ricci[scalar]$

$$-2\,a^2\,e^{4ax} \quad (6)$$

$Einstein[\sim\text{mu, nu, nonzero}]$

$$G^{\mu}{}_{\nu} = \{(2,2) = a^2 e^{4ax}, (3,3) = a^2 e^{4ax}\} \quad (7)$$

$Einstein[\text{nonzero}]$

$$G_{\mu,\nu} = \{(2,2) = -a^2 e^{4ax}, (3,3) = -a^2 e^{4ax}\} \quad (8)$$

$Christoffel[\sim\text{alpha, mu, nu, } nonzero]$

$$\Gamma^{\alpha}{}_{\mu,\nu} = \{(1,1,1) = -2\,a, (1,4,4) = -e^{2ax}\,a\ , (4,1,4) = -a, (4,4,1) = -a\} \quad (9)$$

$Christoffel[nonzero]$

$$\Gamma_{\mu,\nu,\alpha} = \{(1,1,1) = 2\,e^{-4ax}\,a, (1,4,4) = e^{-2ax}\,a, (4,1,4) = -e^{-2ax}\,a, (4,4,1) = -e^{-2ax}\,a\} \quad (10)$$

$Ricci[nonzero]$

$$R_{\mu,\nu} = \{(1,1) = a^2, (4,4) = -e^{2ax}\,a^2\} \quad (11)$$

$Riemann[\text{nonzero}]$

$$R_{\mu,\nu,\alpha,\beta} = \{(1,4,1,4) = e^{-2ax}\,a^2, (1,4,4,1) = -e^{-2ax}\,a^2, (4,1,1,4) = -e^{-2ax}\,a^2, (4,1,4,1) = e^{-2ax}\,a^2\} \quad (12)$$