I read your publication, especially its descriptive part, because, frankly speaking, I am not strong like you and other authors in similar formulas related to computational and mathematical geometry, trigonometry, fractal analysis and related areas. I also had to look at a multidimensional array of data on this topic on the Internet.

When the number of dimensions exceeds two, the number of possible shapes increases dramatically. Considering spaces with a large number of dimensions, we must allow movements in directions that we are not able to visualize.

I note that we are not talking about those directions that lie, say, between the direction to the north and the direction to the west (for example, to the northwest) and not even about the directions of the “north through northwest” type. We are talking about such directions that can be indicated only by going beyond the coordinate system familiar to us, keeping the path along the axis that has yet to be drawn.

Descartes showed that thinking in the language of coordinates is much more productive than geometric constructions. The coordinate system he created, which is now called Cartesian, combined algebra and geometry. In a narrow sense, Descartes showed that by constructing three axes (x, y, and z) perpendicular to each other and intersecting at one point, one can precisely indicate the position of any point in three-dimensional space using three numbers: x, y, and z, called coordinates . But in fact, Descartes' contribution is much wider - in one brilliant gesture, he greatly expanded the field of study of geometry.

Using this approach, you can work with a space of any dimension - not necessarily (x, y, z), the dimension of each particular space is determined by the number of coordinates needed to indicate the position of a point in this space. Using this approach, one can consider spaces of any dimension and perform various calculations in them, without worrying about how these spaces are depicted.

Riemann, working on the geometry of curved (non-Euclidean) spaces, found that such spaces are not limited in terms of the number of dimensions. He also showed how distance, curvature, and other characteristics could be accurately calculated in these spaces, thereby working on a mathematical theory capable of tying together electricity, magnetism, light, and gravity - thus anticipating a task that haunts scientists to this day.

The elements of Euclid's geometry are the point (zero-dimensional space) - an object without length, width and height; line - an object that is characterized by length, but does not have width and height; plane - an object that is described by length and width, but does not have a height property; body is an object with length, width and height. These concepts were developed by mankind on the basis of practical experience with material objects. From a physical point of view, a line appears when a point moves. If a point moves along a plane or in space without changing the direction of movement (translation), then it moves in a straight line.

Although Riemann succeeded in freeing space from the constraints of the Euclidean plane and three dimensions, physicists ignored his ideas for decades. The lack of any interest on their part can be explained by the absence of any experimental evidence to conclude that space is curved or that there are additional dimensions beyond three.

Thus, Riemann's innovative mathematical constructions were so far ahead of the physics of that time that it took almost another fifty years for physicists (or at least one of the physicists) to be able to use his ideas. This physicist was Albert Einstein.

Einstein was the first to reduce physics to mathematical geometry. In his view, gravity is just a curvature of space. Heavy bodies bend the space around them, and, roughly speaking, along these depressions of the stage, they roll towards each other, as if attracted, or not, as it were, because they are really attracted, this is how we perceive it. It turned out that not only gravity, but also all other physical phenomena are only a consequence of the geometry of our multidimensional world.

When developing the special theory of relativity, which was first presented in 1905 and developed into the general theory of relativity in subsequent years, Einstein drew attention to the idea of the German mathematician Hermann Minkowski, which consists in the fact that time is inextricably linked with three spatial dimensions, forming with them a new geometric construct known as spacetime. So suddenly, time acquired the status of the fourth dimension, which decades earlier had been included by Riemann in his elegant equations.

Minkowski, in a speech delivered in 1908, gave a mathematical justification for his pretentious statement: "From now on, time itself and space itself become an empty fiction, and only their unity retains a chance for reality." A reasonable justification for the fact that these two concepts (space and time) were connected by something like a marriage union - if, of course, the conclusion of a marriage union needs justification at all - is that any object moves not only through space, but also through time. Therefore, in order to describe any event in a four-dimensional space-time continuum, not three, but four coordinates are needed - three spatial and one temporal: (x, y, z, t).

It resembles a “symmetric”, ternary number system, the values of its digits (-1, 0 and 1) on the number axis are located symmetrically with respect to zero, with the added value of time (t), that is, three elements and time, there are also three sides in Pythagorean triangles, through which global calculations can be made, just as the electron, proton and neutron are the three elements to which the fourth value of time (t) can be applied.

Its advantages as a symmetric system are that, firstly, it is not necessary to somehow specially mark the sign of the number - the number is negative if its leading digit is negative, and vice versa, and the inversion (sign change) of the number is done by inverting all its digits ; secondly, rounding here does not require any special rules and is performed by simply zeroing the least significant digits.

In addition, of all positional number systems, ternary is the most economical - it can write more numbers than in any other system, with an equal number of characters used: for example, in decimal, to represent numbers from 0 to 999, it will take 30 characters (three digits, ten possible values for each), in the binary system the same thirty characters can encode numbers in the range from 0 to 32767, and in the ternary system - from 0 to 59048. The most economical would be a number system with a base equal to the Euler number (e = 2.718…), and 3 is the closest integer to it.

Unlike the generally accepted, binary number system used in programming, it is rarely used anywhere.

What about the fact that the zeta function looks like dipole lines of force? Isn't there some not mathematical, but physical, connection with fundamental physics?

Take harmonic vibrations. In mathematics, they are described by the rotation of a segment of unit length and, depending on the phase (reference point), we get two functions of the sine or cosine of the angle, or more simply, the value of one of the sides of a right-angled triangle, the hypotenuse of which is the segment of unit length mentioned above.

Everything is clear with harmonic oscillations, they have a constant frequency and period, i.e. a segment that rotates at a constant speed occupies a certain position at regular intervals, so they say that the sine and cosine are periodic functions.

The task can be complicated by adding several harmonic oscillations with different, but constant frequencies and periods. We can take different initial phases and lengths of the segments and get a complex oscillation. If we add the segments according to the rules of vector addition, then the end of the resulting vector at regular intervals will also occupy a certain position, despite the fact that its length will constantly change, i.e. we again get a periodic oscillation.

Now let's complicate the task even more, take not a constant, but a variable frequency of rotation of the segments, we will slightly increase it. In this case, we will get a complex oscillation, but it will no longer be either harmonic or periodic, i.e. the end of the resulting vector at regular intervals will not occupy a certain position.

Approximately such an oscillation is described by the Riemann function, i.e. it is not possible to mathematically determine how this function behaves at infinity, and therefore it is impossible to say where its zeros are.

Riemann suggested that all zeros of this function are possible only in the symmetrical case, when two sets of segments oscillate, the lengths of which are equal to the reciprocals of the roots of natural numbers. This is how the number 1/2 arises, and everything else is the rotation of these segments.

At present, the most difficult task is to show that the zeros of this function of rotating segments remain real, i.e. the function changes sign even if these zeros are critically close.

A few words about the Mandelbrot set from mathematical and philosophical points of view.

Mandelbrot found "fractal order" where only disorder had been seen before. The main method for constructing mathematical fractals is iteration, that is, the repeated repetition of a certain geometric operation, and since geometric figures are space-dependent, in a philosophical context they are placed between the sensory world and the ideal world.

This, generally speaking, is not surprising, since in abstract mathematics many of the intelligible mathematical entities turn out to be spatially representable at the same time, that is, they have analogues in the sensually perceived world.

The interpretation of fractal structures in specific cognitive settings made philosophers of mathematics re-evaluate well-known mathematical concepts and essences, in particular, various types of dimensions, measurement paradoxes, as well as such a simple fractal as the famous Cantor set that arose when trying to solve the problem of the continuum hypothesis.

From a philosophical point of view, the concept of the Mandelbrot fractal distances itself from such traditional concepts of setting and describing a geometric shape as border, length, width. These semantic concepts are simply not there, since they cease to work within the concept of a fractal, and therefore it is completely unclear how to apply them.

These observations received an unexpected continuation in a new area of mathematical knowledge - the theory of fractals, which shifts cognitive attitudes from strict rationality to intuitive-figurative thinking. From a mathematical point of view, it is interesting that in order to introduce the concept of "fractal" it was not necessary to invent any absolutely new formalizations or mathematical concepts.

The modern geometrical aspect of the theory of scale invariance was developed by Benoit Mandelbrot before the mathematical theory of new geometric forms having a fractional dimension, which, with good reason, was called by him "fractal geometry". This scientific direction, which initially relied on mathematical paradoxes, quite unexpectedly gave unique intellectual and aesthetic impressions of the beauty of many fractals. They are connected with the property of fractal sets to “look” approximately the same at any scale, which is now called “scale invariance”.

On this occasion, Mandelbrot wrote: “My passion for scale invariance was constantly fueled by new enthusiasm and, enriched by new tools and ideas, thanks to changes in the field of research, gradually led me to the creation of a full-fledged general theory”.

The Mandelbrot set, whose boundary is a fractal set, describes specific processes and phenomena of reality associated with the properties of stability and chaos in dynamic systems, thereby stimulating the study of fractal sets on the complex plane.

The analytical characteristics of mathematical objects are expressed through the algebraic, topological and ordinal structures of mathematics, although the "fractional" world of fractals is a non-standard generalization of the classical concept of dimension, completely different from the linearity of mathematical objects with a single dimension.

In fractal objects, modern problems of mathematical knowledge in general are intertwined - logic and calculation, independence from a person and dependence on him, complexity and simplicity. The visualization of such a complex object as the Mandelbrot set has become possible only thanks to a modern computer, but this does not mean that it can be fully depicted. Since the theoretical structures of modern mathematics are too complex for "fine" models, the transition to computer simulation stimulates the adoption of relatively "rough" models, which have begun to replace reality.

The philosophy of fractal geometry rationalizes and concretizes the Eastern principle "one in all and all in one", pointing to the area of its applicability and at the same time methodologically revealing the simplicity of the complex. Therefore, thanks to this new branch of mathematics, knowing the course of processes on a small scale, it is possible to discover the possibility of determining them on a large scale and, accordingly, vice versa. Note that the number of different length scales of natural objects for practical purposes is infinite, since reality has not only greater complexity, but, generally speaking, the complexity of another level.

In a certain sense, the Mandelbrot set and similar fractal sets cannot be considered the creation of a computer, since the computational process must continue indefinitely, and therefore these sets cannot be finally and accurately calculated. The existence attributed to the Mandelbrot set can be interpreted as a property of its absolute nature. When Mandelbrot saw the very first computer images, he considered the resulting fuzzy structures to be the result of a failure and only then became convinced that they really are part of the set being built, although the complex structure of the Mandelbrot set in all its details cannot be covered by anyone, since it is impossible to completely display on the computer.

The result of the study of any mathematician working on a computer will be an approximation to a single fundamental mathematical structure that already exists somewhere outside of us. In this case, even the most sophisticated mathematical intuition, perhaps, is just some knowledge of the behavior of mathematical objects in "virtual reality", giving the impression of a person about being in an artificially created world. At the worldview level, the fractal is already becoming a philosophical category, and "according to the degree of complexity, the following types of fractal formations are traditionally distinguished: geometric, algebraic, stochastic".

The philosophical concept of the Mandelbrot fractal has also changed the computerized socio-cultural reality, in which virtual reality reflects the integral unity of the subjective and objective in a person.

Modern computational mathematics, which combines in its methodological substantiation the formalism of theoretical mathematics and the constructivism of applied mathematics, taking into account more and more improved computers, is, generally speaking, not limited in its movement towards reliability and consistency with the available proofs of mathematical existence.

The supporting structure of modern mathematics can be represented as a gradual accumulation of "uncorrected structures", which undoubtedly include number theory, analytic and algebraic geometry, differential and integral equations, real, complex and functional analysis, and many other theories.

A distinctive feature of the complete structural stabilization of developed mathematical theories is their axiomatization. But if modern mathematics is understood only as the science of abstract axiomatic structures, then most of the computational mathematics clearly does not fit into it. Therefore, in the context of a systematic approach to the substantiation of mathematical theories, one should strive for their unification, synthesis and closing them, in the end, into a systemic integrity.

From the point of view of philosophical and mathematical approaches to the substantiation of mathematical theories, the discovered effects are not only non-trivial, but also fundamental to a certain extent in the modern theory of knowledge. Although fractal objects have been known for a long time, cognitive interest in them appeared after the active popularization of Benoit

Mandelbrot, thanks to which mathematicians and computer scientists were able to discover a wonderful world, taking a fresh look at well-known mathematical objects and phenomena.

With the help of fractal objects, nature in the language of mathematics demonstrates not just a much higher degree of complexity corresponding to the modern level of development of science, but a fundamentally different level. When additional motivation appears in the context of new philosophical approaches to the representation of the fractal geometry of nature, created primarily for the needs of natural science and playing a leading role in the revival of the theory of iterations, sometimes it is possible to move away from the template that is usually followed in the study of various topics of classical mathematics, and return to the analysis of new mathematical objects with less prejudice.

Of particular interest in the philosophy of mathematics is one of the basic principles of the methodology of modern science - the "principle of non-linearity", according to which any complex developing system cannot be explicated in an additive way, since its integrity cannot be described through the summativity of its constituent parts.

Therefore, the philosophical component in the practical implementation of mathematics acts as a constructive methodological guideline, which is very important for the formation of the cognitive and transformative worldview of students, since in each specific period of the historical evolution of mathematical knowledge, it is possible to identify the philosophical and methodological coexistence of mathematical methods that are at different stages of evolution.

Thank you.