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Derivation of the Starmapper geometry in field coordinates

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1. INTRODUCTION

For the main processing of HIPPARCOS data as well as for TYCHO astrometry, it is assumed that the position of the Starmapper (SM) slits is known with respect to the telescope system Z. More precisely, we need the functions $w^*_{fg}(z)$ of the group centre lines (or *fiducial lines*) in the field coordinates (w, z). [$f = \pm 1$ for preceding/following field, $g = 1, 2$ for the vertical, chevron slit group.] Such information can be derived from four different sources, viz.:

(a) laboratory measurements on the grid

(b) great-circle (set) solutions during commissioning or in the main processing

(c) theoretical payload models

(d) residuals of SM transits in the attitude reconstitution or TYCHO astrometry processing.

Within NDAC, only method (d) has been addressed thus far (NDAC/LO/040, 043) and certain steps have been taken (?) towards including SM distortion parameters in the attitude reconstitution.

In this report I shall outline the kind of information provided by (a) – (c) and how these sources might be combined in order to derive $w^*_{fg}(z)$.

2. LABORATORY MEASUREMENTS

Let (G^*, H^*) be physical coordinates (e.g. in mm) in the tangent plane of the grid. The projection is normal to this plane, and the origin is assumed to coincide with the tangential point, but not necessarily with the optical centre on the grid. I assume furthermore that the measuring axes are orthogonal and identical in scale. [These latter conditions can be ensured by averaging two sets of measurements, with the grid rotated

90° in between.] However, the axes need not be exactly aligned with the slits.

Among these measurements, the SM fiducial lines can be identified as the functions

$$G^* = G^*_g(H^*), \quad g = 1, 2 \tag{1}$$

derived in tabular form by averaging the G^*-coordinates of the four slits at a certain H^*.

Introducing the *optical* grid coordinate G (being an integer at the centre of each slit in the primary field, similar to the G used in the NDAC processing), the geometry of the primary grid is similarly expressed in physical coordinates by means of the transformation

$$G = \bar{G}(G^*, H^*) + d(G^*, H^*) \tag{2}$$

Here, $\bar{G}$ is a smoothed grid coordinate represented by a low-order polynomial, while d is a small-scale correction in tabular form (e.g. according to scanfield). $\bar{G}$ and d must be orthogonal. For simlicity, let us assume a second-order expression for $\bar{G}$,

$$\bar{G}(G^*, H^*) = G_{00} + G_{10}G^* + G_{01}H^* + G_{20}G^{*2} + G_{11}G^*H^* + G_{02}H^{*2} \tag{3}$$

We assume that $G^*_g(H^*)$ and $\bar{G}(G^*, H^*)$ are thus known, and that d is accounted for in the IDT preprocessing and therefore does not appear any further.

3. GREAT-CIRCLE SOLUTIONS

The set solutions will determine the large-scale distortion of the primary field in the form of coefficients g_{mn}, h_{mn}, g'_{mn}, h'_{mn} (NDAC/LO/051). For the sake of clarity, assume the following simplified expression for the smoothed grid coordinate:

$$\bar{G} = \bar{G}(f, w, z) = fh_{00} + g_{10}w + g_{01}z + g_{20}w^2 + g_{11}wz + g_{02}z^2 \tag{4}$$

in which chromatic terms and f-dependent linear and quadratic terms have been omitted. (Note that $g_{00} = 0$ by definition.)

4. THEORETICAL INSTRUMENT MODEL

Given a large set of parameters representing the position, orientation and deformation of each optical component, one can in principle establish a one-to-one relationship between the physical grid coordinates (G^*, H^*) and the field coordinates (f, w, z). Comparison between the physical grid geometry, represented here by eqn (3), and its appearance in the field [eqn (4)], might then yield a certain information about the instrument parameters which in turn could be used in mapping the physical SM grid onto the field coordinates. At first sight, this procedure would seem totally unrealistic due to the very large number of instrument parameters involved and their probable indeterminacy. Fortunately, however, it appears that optical distortions arise almost entirely from mechanical displacement and tilt of the grid as a whole with respect to the reference position and direction defined by the beam combiner and spherical mirror. Polishing errors and mirror deformations produce other kinds of aberration, such as chromaticity, which are largely constant within each field, but contribute very little to the large scale distortion. (Cf. MAT.HIP.7697, issue 2, where the derived LSC polynomials in load cases 2.15-2.17 directly allow to recompute the defocus and rotation angles; load case 2.14 on the other hand is considerably discrepant.)

In a previous note (1983 May 6) I derived the theoretical relations between field coordinates and physical grid coordinates for an idealized Schmidt telescope with displaced and tilted grid. With some adjustment of notations the resulting polynomials are

$$G^*_o = R(1+\tfrac{1}{2}\theta^2)w + R(\psi + \phi\theta)z \;(-)\;\tfrac{1}{2}R\,\theta\,w^2 - R\,\phi wz \;(+)\;\tfrac{1}{2}R\,\theta\,z^2 \tag{5a}$$

$$H^*_o = -R\psi w + R(1+\tfrac{1}{2}\phi^2)z + \tfrac{1}{2}R\,\phi w^2 + R\,\theta wz - \tfrac{1}{2}R\,\phi\,z^2 \tag{5b}$$

in which R is the effective focal length (depending on defocus) and ϕ, θ, ψ the rotational displacements of the grid around X, Y, Z axes (as defined by MATRA). When deriving (5), the origin of the physical grid coordinates was placed at the optical centre $(w, z) = (0, 0)$; hence G^*_o and H^*_o must be further transformed to the arbitrary origin of the laboratory measurements. I have found that this latter transformation is

$$G^* = -R\sigma + (1 - \tfrac{1}{2}\sigma^2)G^*_o - \tfrac{1}{2}\sigma\tau H^*_o + \frac{\sigma}{2R}(G^{*2}_o + H^{*2}_o) \tag{6a}$$

$$H^* = -R\tau - \tfrac{1}{2}\sigma\tau G^*_o + (1 - \tfrac{1}{2}\tau^2)H^*_o + \frac{\tau}{2R}(G^{*2}_o + H^{*2}_o) \tag{6b}$$

if the origin of the laboratory measurements is at $(G_O^*, H_O^*) = (R\sigma, R\tau)$. Combining (5) and (6) we get the following general model for the field--to-grid coordinate transformation:

$$G^* = -R\sigma + R[1 - \tfrac{1}{2}(\sigma^2 - \theta^2)]\omega + R(\psi + \phi\theta - \tfrac{1}{2}\sigma\tau)z +$$

$$+ \tfrac{1}{2}R(\sigma - \theta)\omega^2 - R\phi\omega z + \tfrac{1}{2}R(\sigma + \theta)z^2 - \tfrac{1}{2}fR\varepsilon \qquad (7a)$$

$$H^* = -R\tau - R(\psi + \tfrac{1}{2}\sigma\tau)\omega + R[1 - \tfrac{1}{2}(\tau^2 - \phi^2)]z +$$

$$+ \tfrac{1}{2}R(\tau + \phi)\omega^2 + R\theta\omega z + \tfrac{1}{2}R(\tau - \phi)z^2 \qquad (7b)$$

In (7a) the last term has been added to include the correction ε to the value of the basic angle used in relating the field coordinates to the telescope system.

5. SYNTHESIS

Inserting (7) into (3) and identifying the terms with (4) we get the following expressions for the observable distortion coefficients:

$$0 \quad = G_{00} - R(G_{10}\sigma + G_{01}\tau) + R^2(G_{20}\sigma^2 + G_{11}\sigma\tau + G_{02}\tau^2) \qquad (8a)$$

$$h_{00} = -\tfrac{1}{2}G_{10}R\varepsilon \qquad (8b)$$

$$g_{10} = -G_{01}R\psi + G_{10}R[1 - \tfrac{1}{2}(\sigma^2 - \theta^2)] - 2G_{20}R^2\sigma - G_{11}R^2\tau \qquad (8c)$$

$$g_{01} = G_{01}R + G_{10}R(\psi + \phi\theta - \tfrac{1}{2}\sigma\tau) - G_{11}R^2\sigma - 2G_{02}R^2\tau \qquad (8d)$$

$$g_{20} = \tfrac{1}{2}G_{10}R(\sigma - \theta) + G_{20}R^2 - G_{11}R^2\psi \qquad (8e)$$

$$g_{11} = -G_{10}R\phi + 2G_{20}R^2\psi + G_{11}R^2 - 2G_{02}R^2\psi \qquad (8f)$$

$$g_{02} = \tfrac{1}{2}G_{10}R(\sigma + \theta) + G_{11}R^2\psi + G_{02}R^2 \qquad (8g)$$

Two more equations are obtained from the laboratory coordinates of the SM apex (G_A^*, H_A^*), at which point we have by definition $z = 0$, while $\omega = \omega_A + \tfrac{1}{2}f\varepsilon$ (say). Thus,

$$G^*_A = -R\sigma + R[1 - \tfrac{1}{2}(\sigma^2-\theta^2)]w_A + \tfrac{1}{2}R(\sigma - \theta)w^2_A \tag{8h}$$

$$H^*_A = -R\tau - R(\psi + \tfrac{1}{2}\sigma\tau)w_A + \tfrac{1}{2}R(\tau + \theta)w^2_A \tag{8i}$$

We have then nine equations for the eight unknown instrument parameters R, ε, σ, τ, ϕ, θ, ψ, w_A. Remembering that G_{10} is by far the dominating coefficient in (3), we readily see that eqns (8a) - (8i) principally depend respectively on σ, ε, R, ψ, θ, ϕ, θ, w_A, and τ, and that the parameters are thus determinable with a slight redundancy. Inserting the parameter values into (7) and inverting the transformation [expressing (w, z) in terms of (G^*, H^*)] it is then a simple matter to map the fiducial lines onto field coordinates and thus determine $w^*_{fg}(z)$. With

$$\hat{w} = G^*/R + \sigma + \tfrac{1}{2}f\varepsilon \tag{9a}$$

$$\hat{z} = H^*/R + \tau \tag{9b}$$

denoting a first-order approximation of the field coordinates, the inverse transformation becomes

$$w = [1 + \tfrac{1}{2}(\sigma^2-\theta^2)]\hat{w} - (\psi + \phi\theta - \tfrac{1}{2}\sigma\tau)\hat{z} - \tfrac{1}{2}(\sigma - \theta)\hat{w}^2 + \phi\hat{w}\hat{z} - \tfrac{1}{2}(\sigma + \theta)\hat{z}^2 \tag{9c}$$

$$z = [1 + \tfrac{1}{2}(\tau^2-\phi^2)]\hat{z} + (\psi + \tfrac{1}{2}\sigma\tau)\hat{w} - \tfrac{1}{2}(\tau + \phi)\hat{w}^2 - \theta\hat{w}\hat{z} - \tfrac{1}{2}(\tau - \phi)\hat{z}^2 \tag{9d}$$

It is questionable whether the laboratory measurements will be able to determine the second-order terms [G_{20}, G_{11}, G_{02} in (3)] to any useful accuracy. Satisfyingly, the procedure described above is fully applicable also to a first-order model. Inspection of (8) shows that the only consequence is the inability to determine the tilt parameters ϕ and θ. But since these angles only appear as ϕ^2, $\phi\theta$, and θ^2 in the linear terms of (9), it should be clear that the solution does not depend critically on the determination of quadratic distortion terms.