

Subroutine EARTH - calculation of barycentric position and velocity

L Lindegren (1984 June 8)

Lund Observatory

Introduction

The Fortran subroutine EARTH is a self-contained implementation of the formulae given by Chapront, Francou and Morando in "The Calculation of the Positions and Velocities of the Earth During the HIPPARCOS Mission" (Bureau des Longitudes, May 1984). It calculates the Earth's barycentric position ( $\underline{r}$ ) and velocity ( $\underline{v}$ ) relative to the equatorial J2000.0 frame for any given time in a 2000-day interval from September 1987 to February 1993. The accuracy with respect to DE200 is 15 km and  $0.06 \text{ m s}^{-1}$  (max errors).

Argument list

SUBROUTINE earth( $t$ ,  $\underline{r}$ ,  $\underline{v}$ , ierr)

INPUT:  $t$  = time from J1988.0 = JD2447162.0 = 1988 Jan 1, 12 h TDT,  
[s] NOTE RESTRICTION IN RANGE BELOW!

OUTOUT:  $\underline{r}()$  = 3-vector containing the components of position in the  
equatorial J2000.0 frame, [m]

$\underline{v}()$  = 3-vector containing the components of velocity in the  
equatorial J2000.0 frame, [ $\text{m s}^{-1}$ ]

ierr = 0 on normal return  
-1 if  $t$  is not in the permitted range (see below)

NOTE 1:  $t$ ,  $\underline{r}$ , and  $\underline{v}$  are DOUBLE PRECISION (REAL\*8)

NOTE 2:  $\underline{r} = \underline{v} = 0$  are returned if ierr = -1

Restriction

The time argument ( $t$ ) must be in the range  $-10\ 540\ 800 \leq t < 162\ 259\ 200$ ,  
i.e.  $2447040 \leq \text{JD} < 2449040$  or  $87/09/01.5 \leq \text{date} < 93/02/21.5$ .

Execution time

On HP9000 at Lund Observatory, 36 ms per call.

Tests

The velocity  $\underline{v}$ , before conversion to equatorial coordinates, was compared with the portion of the BDL82 ephemerides listed by Chapront et al. The comparison was made for every 25th day (66 points per component) with the following results:

velocity component:	XPG	YPG	ZPG
rms difference:	0.0068	0.0071	$0.0036 \text{ m s}^{-1}$

The consistency between the calculated  $\underline{r}$  and  $\underline{v}$  was checked by computing, for every 5th day of the permitted interval (400 points per coordinate), the difference between  $\underline{v}$  and the numerical derivative of  $\underline{r}(t)$ . Results:

velocity component:	XPG	YPG	ZPG
rms difference:	0.0034	0.0042	$0.0012 \text{ m s}^{-1}$
maximum difference:	0.0302	0.0596	$0.0136 \text{ m s}^{-1}$

The subroutine makes use of five different Fourier approximations, depending on which 400-day interval contains  $t$ . Consequently, there are discontinuities at the separation times,  $\Delta\underline{r} = \underline{r}(t_{\text{sep}}^+) - \underline{r}(t_{\text{sep}}^-)$ , and similarly in  $\underline{v}$ :

$t_{\text{sep}}$ [s]	$\Delta XG$ [m]	$\Delta YG$ [m]	$\Delta ZG$ [m]	$\Delta XPG$ $[\text{m s}^{-1}]$	$\Delta YPG$ $[\text{m s}^{-1}]$	$\Delta ZPG$ $[\text{m s}^{-1}]$
24019200	4836	2049	1397	0.021	0.004	-0.012
58579200	-10092	-333	4549	-0.020	0.008	-0.002
93139200	12480	-18929	2684	-0.003	-0.005	-0.000
127699200	11149	11403	3506	-0.011	-0.008	0.002

The various tests are thus consistent with the accuracy claimed by Chapront et al. Still, a direct comparison with DE200 and with FAST's implementation of the same formulae should be made.

Jun 6 18:40 1984 earth.f Page 1

```
SUBROUTINE earth(t, r, v, ierr)
*****
c
c HIPPARCOS - NDAC - General routines - Earth ephemeris
c
c Calculates the Earth's barycentric position and velocity
c at a given time
c
c Input:
c   t      = time in seconds from J1988.0 (JD 2447162.0 12h TDT)
c             (-10540800 <= t < 162259200 or 2447040 <= JD < 2449040)
c
c Output:
c   r(i) = barycentric rectangular equatorial coordinates [km]
c   v(i) = barycentric equatorial velocity components [m/s]
c          (i = 1 to 3)
c   ierr = 0 on normal return, -1 if t is out of range
c
c Source: "The calculation of the positions and velocities of the
c           Earth during the HIPPARCOS mission", by J Chapront,
c           G Francou and B Morando (Bureau des Longitudes, 1984 May)
c
c Accuracy: < 10000 m and 0.05 m/s compared with BDL82
c
c Execution time: 36.2 ms per call (HP 9000, Lund Observatory)
c
c L Lindegren 1984 May 28
c
*****
IMPLICIT DOUBLE PRECISION (a-h,o-z)
DIMENSION r(3), v(3)
DIMENSION a(22), ap(22), an(22), phi(22)
DIMENSION b(8), bp(8), am(8), psi(8)
c
PARAMETER (pi = 3.141592653589793238d0, twopi = 2d0*pi)
c epsdeg = mean obliquity of equator at J2000.0 [deg]
PARAMETER (epsdeg = 23d0+26d0/60d0+21.448d0/3600d0,
            epsrad = epsdeg*pi/180d0)
seps = dcos(epsrad)
seps = dsin(epsrad)
c
Convert time argument to julian years from J2000.0:
c
tj = -12d0 + t/31557600d0
c-----
c Compute the Earth's position and velocity wrt Earth-Moon barycentre:
c   xt, yt, zt = rectangular ecliptical coordinates [km]
c   xpt, ypt, zpt = ecliptical velocity components [km/s]
c
DATA (    a(i),        ap(i),       an(i),       phi(i),   i=1,10)/
1      4653.7d0, .012387d0, 83.99684730d0, 0.66875d0,
2      383.2d0, .000009d0, 0.70993300d0, 1.45470d0,
3     128.5d0, .000681d0, 167.28376200d0, 3.02430d0,
4      75.8d0, .000028d0, 11.85622000d0, 2.05200d0,
5      39.1d0, -.000088d0, -71.43070000d0, 5.97900d0,
6      28.3d0, .000140d0, 156.13748000d0, 2.42700d0,
7      9.3d0, -.000025d0, -84.67248000d0, 3.69600d0,
8      9.2d0, .000070d0, 239.42439000d0, 4.78300d0,
9      7.5d0, .000018d0, 77.71377000d0, 0.73100d0,
a      7.3d0, .000021d0, 90.27992000d0, 3.76700d0/
DATA (    a(i),        ap(i),       an(i),       phi(i),   i=11,22)/
```

```
Jun 6 18:40.1 , earth.f age 2
```

```
1      5.3d0, .000042d0, 250.57068000d0, 5.38000d0,
2      3.2d0, .000002d0, 18.13930000d0, 2.00900d0,
3      3.6d0, .000011d0, 95.14313000d0, 4.40800d0,
4      2.8d0, -.000006d0, -65.14762000d0, 5.93600d0,
5      1.9d0, .000000d0, 6.99301000d0, 1.41200d0,
6      2.1d0, .000000d0, 6.28308000d0, 1.75400d0,
7      1.8d0, -.000005d0, -82.57698000d0, 2.24200d0,
8      1.4d0, .000007d0, 149.85440000d0, 2.47000d0,
9      1.1d0, .000006d0, 161.00869000d0, 3.06800d0,
a      0.9d0, .000000d0, -5.57314000d0, 4.80600d0,
1      0.8d0, .000004d0, 173.56684000d0, 6.12300d0,
2      0.9d0, .000009d0, 322.71131000d0, 0.85500d0/
c
DATA ( b(j), bp(j), am(j), psi(j), j=1,8) /
1      418.3d0, .001118d0, 84.33466200d0, 4.76950d0,
2      34.0d0, .000001d0, 1.04775000d0, 5.55600d0,
3      15.5d0, .000035d0, 71.09288000d0, 5.62700d0,
4      11.5d0, .000061d0, 167.62158000d0, 0.84200d0,
5      5.5d0, -.000002d0, -12.19403000d0, 3.27200d0,
6      2.5d0, .000013d0, 156.47529000d0, 0.24500d0,
7      0.8d0, -.000002d0, -64.80981000d0, 3.75400d0,
8      0.8d0, .000006d0, 239.76220000d0, 2.60000d0/
c
xt = 0d0
yt = 0d0
zt = 0d0
xpt = 0d0
ypt = 0d0
zpt = 0d0
c
DO 10 i = 1, 22
    arg = an(i)*tj + phi(i)
    sarg = dsin(arg)
    carg = dcos(arg)
    xt = xt + a(i)*carg
    yt = yt + a(i)*sarg
    xpt = xpt - ap(i)*sarg
    ypt = ypt + ap(i)*carg
10 CONTINUE
c
DO 20 j = 1, 8
    arg = am(j)*tj + psi(j)
    zt = zt + b(j)*dsin(arg)
    zpt = zpt + bp(j)*dcos(arg)
20 CONTINUE
c-----
c Compute the position and velocity of the Earth-Moon barycentre
c wrt the Solar System barycentre:
c   x, y, z = rectangular ecliptical coordinates [km]
c   xp, yp, zp = ecliptical velocity components [km/s]
c Different Fourier approximations apply in each 400-day interval
c-----
q1 = twopi*dmod(tj,1.d0)
q2 = 2d0*q1
q3 = 3d0*q1
q4 = 4d0*q1
c
k = int(1.305d0 + t/34560000d0)
GO TO (100, 200, 300, 400, 500) k
GO TO 9000
c
```

```

c 1st interval: JD = 2447040 to 2447440 :-
c
c 100 CONTINUE
c
      x = 1767306d0 + 149873097d0*dsin(q1+3.32150163d0)
      , + 1214054d0*dsin(q2+3.23701927d0)
      , + 15497d0*dsin(q3+3.234605d0)
      , + 247d0*dsin(q4+3.2907d0)
      , + ( 124679.53d0 + 57333.93d0*dsin(q1+2.198456d0)
      , + 5708.16d0*dsin(q2+1.087967d0) )*tj
c
      y = -7054773d0 + 149880435d0*dsin(q1+1.75395672d0)
      , + 1275547d0*dsin(q2+1.69998796d0)
      , + 15615d0*dsin(q3+1.677850d0)
      , + 213d0*dsin(q4+1.6298d0)
      , + (-318062.22d0 + 27255.88d0*dsin(q1+1.374492d0)
      , + 2489.36d0*dsin(q2+1.118104d0) )*tj
c
      z = 6904d0 + 20490d0*dsin(q1+5.88658069d0)
      , + 4698d0*dsin(q2+4.71744233d0)
      , + 20d0*dsin(q3+4.700716d0)
      , + 4d0*dsin(q4+0.9551d0)
      , + ( 438.07d0 + 1960.55d0*dsin(q1+5.746100d0)
      , + 401.74d0*dsin(q2+4.688597d0) )*tj
c
      xp = 0.004117d0 + 29.824788d0*dsin(q1+4.89225134d0)
      , + 0.480865d0*dsin(q2+4.82075070d0)
      , + 0.009243d0*dsin(q3+4.807645d0)
      , + 0.000194d0*dsin(q4+4.8437d0)
      , + ( .000000228d0 + .01101472d0*dsin(q1+3.672392d0)
      , + .00199431d0*dsin(q2+2.430282d0) )*tj
c
      yp = 0.0192229d0 + 29.803685d0*dsin(q1+3.32424850d0)
      , + 0.499325d0*dsin(q2+3.28330890d0)
      , + 0.009297d0*dsin(q3+3.249232d0)
      , + 0.000171d0*dsin(q4+3.1732d0)
      , + ( .00247608d0 + .00368281d0*dsin(q1+2.282847d0)
      , + .00011645d0*dsin(q2+3.346957d0) )*tj
c
      zp = 0.0111116d0 + 0.017059d0*dsin(q1+0.34820501d0)
      , + 0.004363d0*dsin(q2+5.42413917d0)
      , + 0.000017d0*dsin(q3+5.791498d0)
      , + 0.000003d0*dsin(q4+2.2206d0)
      , + ( .00094315d0 + .00151018d0*dsin(q1+0.336428d0)
      , + .00037042d0*dsin(q2+5.399171d0) )*tj
      GO TO 600
c
c 2nd interval: JD = 2447440 to 2447840 :-
c
c 200 CONTINUE
c
      x = 3468606d0 + 149558278d0*dsin(q1+3.32272388d0)
      , + 1217620d0*dsin(q2+3.28720143d0)
      , + 15732d0*dsin(q3+3.258030d0)
      , + 227d0*dsin(q4+3.2954d0)
      , + ( 274530.85d0 + 37482.42d0*dsin(q1+1.703852d0)
      , + 2908.22d0*dsin(q2+0.009521d0) )*tj
c
      y = -4959263d0 + 149934542d0*dsin(q1+1.75149582d0)
      , + 1190532d0*dsin(q2+1.68412246d0)
      , + 15663d0*dsin(q3+1.683766d0)

```

Jun 6 18:40 34 earth. Page 4

, + (-131259.53d0 + 230d0\*dsin(q4+1.6841d0)  
, + 54628.96d0\*dsin(q1+0.819366d0)  
, + 6407.51d0\*dsin(q2+5.360163d0) )\*tj  
c z = -46452d0 + 5215d0\*dsin(q1+3.40926807d0).  
, + 911d0\*dsin(q2+1.66974121d0)  
, + 3d0\*dsin(q3+1.336907d0)  
, + (-4245.41d0 + 575.77d0\*dsin(q1+4.017750d0)  
, + 82.36d0\*dsin(q2+1.624835d0) )\*tj  
c xp = 0.003195d0 + 29.776074d0\*dsin(q1+4.89324372d0)  
, + 0.482264d0\*dsin(q2+4.85804766d0)  
, + 0.009387d0\*dsin(q3+4.828933d0)  
, + 0.000180d0\*dsin(q4+4.8568d0)  
, + (-.00051043d0 + .00823289d0\*dsin(q1+3.277024d0)  
, + .00139938d0\*dsin(q2+1.603583d0) )\*tj  
c yp = 0.066823d0 + 29.772314d0\*dsin(q1+3.32390337d0)  
, + 0.490560d0\*dsin(q2+3.28392412d0)  
, + 0.009409d0\*dsin(q3+3.256187d0)  
, + 0.000184d0\*dsin(q4+3.2917d0)  
, + (.006644128d0 + .00413311d0\*dsin(q1+1.553978d0)  
, + .00062284d0\*dsin(q2+0.471226d0) )\*tj  
c zp = 0.001095d0 + 0.002349d0\*dsin(q1+5.57468719d0)  
, + 0.000674d0\*dsin(q2+3.65889425d0)  
, + 0.000002d0\*dsin(q3+3.214643d0)  
, + (.00011493d0 + .00026059d0\*dsin(q1+5.790680d0)  
, + .00006189d0\*dsin(q2+3.631155d0) )\*tj  
GO TO 600

c 3rd interval: JD = 2447840 to 2448240 :-

c 300 CONTINUE

x = 3434782d0 + 149791038d0\*dsin(q1+3.32378708d0)  
, + 1185517d0\*dsin(q2+3.25370020d0)  
, + 15642d0\*dsin(q3+3.252063d0)  
, + 223d0\*dsin(q4+3.3131d0)  
, + ( 269070.21d0 + 32946.48d0\*dsin(q1+2.508579d0)  
, + 7455.48d0\*dsin(q2+0.612894d0) )\*tj  
y = -2850095d0 + 149731333d0\*dsin(q1+1.75166435d0)  
, + 1201906d0\*dsin(q2+1.71143688d0)  
, + 15772d0\*dsin(q3+1.675158d0)  
, + 240d0\*dsin(q4+1.6575d0)  
, + ( 76770.38d0 + 46662.85d0\*dsin(q1+0.504115d0)  
, + 4749.42d0\*dsin(q2+4.827788d0) )\*tj  
z = -80605d0 + 37774d0\*dsin(q1+1.99266914d0)  
, + 9950d0\*dsin(q2+5.74625001d0)  
, + 38d0\*dsin(q3+4.345647d0)  
, + 5d0\*dsin(q4+4.3059d0)  
, + (-7765.04d0 + 3596.47d0\*dsin(q1+1.990821d0)  
, + 1041.75d0\*dsin(q2+5.713825d0) )\*tj  
xp = 0.129646d0 + 29.963203d0\*dsin(q1+4.89701209d0)  
, + 0.460855d0\*dsin(q2+4.74921897d0)  
, + 0.009241d0\*dsin(q3+4.819603d0)  
, + 0.000170d0\*dsin(q4+4.8203d0)  
, + (.01261208d0 + .01938423d0\*dsin(q1+5.032456d0)

Jun 6 18:40 1984 earth.f Page 5

```
,           + .00642273d0*dsin(q2+2.576186d0) )*tj
c
c      yp =  0.008197d0 + 29.812790d0*dsin(q1+3.32276099d0)
,           + 0.478856d0*dsin(q2+3.27535145d0)
,           + 0.009408d0*dsin(q3+3.245216d0)
,           + 0.000190d0*dsin(q4+3.2153d0)
,           + (.00060873d0 + .00845346d0*dsin(q1+2.140822d0)
,           + .00190251d0*dsin(q2+0.306032d0) )*tj
c
c      zp = -0.016073d0 + 0.022537d0*dsin(q1+2.35251161d0)
,           + 0.006040d0*dsin(q2+0.0248595d0)
,           + 0.000028d0*dsin(q3+5.316494d0)
,           + 0.000005d0*dsin(q4+5.4737d0)
,           + (-.00165417d0 + .00231196d0*dsin(q1+2.313273d0)
,           + .00062765d0*dsin(q2+6.278171d0) )*tj
60 TO 600
c
c 4th interval: JD = 2448240 to 2448640 :-
```

400 CONTINUE

```
c      x = 2453135d0 + 149831003d0*dsin(q1+3.32403036d0)
,           + 1221225d0*dsin(q2+3.29783518d0)
,           + 15727d0*dsin(q3+3.239650d0)
,           + 225d0*dsin(q4+3.2510d0)
,           + (-159138.26d0 + 36890.79d0*dsin(q1+2.713965d0)
,           + 3910.10d0*dsin(q2+5.793020d0) )*tj
c      y = -1483650d0 + 149565140d0*dsin(q1+1.75231844d0)
,           + 1264764d0*dsin(q2+1.73564478d0)
,           + 15687d0*dsin(q3+1.663479d0)
,           + 226d0*dsin(q4+1.4912d0)
,           + (227342.80d0 + 35945.90d0*dsin(q1+0.107701d0)
,           + 3953.37d0*dsin(q2+2.758656d0) )*tj
c      z = -55340d0 + 26988d0*dsin(q1+1.34308134d0)
,           + 6815d0*dsin(q2+4.73987633d0)
,           + 32d0*dsin(q3+2.798720d0)
,           + 8d0*dsin(q4+3.7217d0)
,           + (-5254.83d0 + 2851.15d0*dsin(q1+1.268804d0)
,           + 800.76d0*dsin(q2+4.696222d0) )*tj
c      xp = 0.039223d0 + 29.875045d0*dsin(q1+4.89455784d0)
,           + 0.476036d0*dsin(q2+4.87106881d0)
,           + 0.009397d0*dsin(q3+4.813998d0)
,           + 0.000180d0*dsin(q4+4.8448d0)
,           + (.00402069d0 + .01230399d0*dsin(q1+4.465194d0)
,           + .00269216d0*dsin(q2+1.301180d0) )*tj
c      yp = 0.124112d0 + 29.806498d0*dsin(q1+3.32814873d0)
,           + 0.498888d0*dsin(q2+3.23590760d0)
,           + 0.009256d0*dsin(q3+3.236412d0)
,           + 0.000166d0*dsin(q4+3.0664d0)
,           + (.01375967d0 + .01090787d0*dsin(q1+4.604063d0)
,           + .00275150d0*dsin(q2+1.712047d0) )*tj
c      zp = 0.002982d0 + 0.003646d0*dsin(q1+3.99570075d0)
,           + 0.001640d0*dsin(q2+0.60794671d0)
,           + 0.000011d0*dsin(q3+4.406153d0)
,           + 0.000005d0*dsin(q4+5.4806d0)
,           + (.00036588d0 + .00037673d0*dsin(q1+4.092182d0)
```

Jun 6 18:40 1994 earth.f page 6

```
,          + .00019304d0*dsin(q2+0.569187d0) )*tj
      GO TO 600
c 5th interval: JD = 2448640 to 2449040 :-
c
500 CONTINUE
c
x = 1032343d0 + 149754551d0*dsin(q1+3.32272226d0)
,          + 1243372d0*dsin(q2+3.31720790d0)
,          + 15628d0*dsin(q3+3.232144d0)
,          + 219d0*dsin(q4+3.1443d0)
, + (-20067.11d0 + 53514.43d0*dsin(q1+2.211798d0)
,          + 5608.91d0*dsin(q2+4.976948d0) )*tj
c
y = -927242d0 + 149600320d0*dsin(q1+1.75285079d0)
,          + 1245714d0*dsin(q2+1.72502437d0)
,          + 15576d0*dsin(q3+1.663514d0)
,          + 217d0*dsin(q4+1.6018d0)
, + ( 296301.36d0 + 27928.78d0*dsin(q1+0.258491d0)
,          + 2030.17d0*dsin(q2+3.537113d0) )*tj
c
z = -6394d0 + 5951d0*dsin(q1+6.05884037d0)
,          + 853d0*dsin(q2+2.29746327d0)
,          + 3d0*dsin(q3+0.563079d0)
, + ( 727.23d0 + 993.98d0*dsin(q1+5.696901d0)
,          + 111.56d0*dsin(q2+2.260274d0) )*tj
c
xp = -.031907d0 + 29.787908d0*dsin(q1+4.89452802d0)
,          + 0.495963d0*dsin(q2+4.86851833d0)
,          + 0.009305d0*dsin(q3+4.805229d0)
,          + 0.000173d0*dsin(q4+4.7430d0)
, + (-.00423128d0 + .00543338d0*dsin(q1+3.535918d0)
,          + .00093172d0*dsin(q2+0.371781d0) )*tj
c
yp = -0.006312d0 + 29.771006d0*dsin(q1+3.32313779d0)
,          + 0.500667d0*dsin(q2+3.29871124d0)
,          + 0.009299d0*dsin(q3+3.232478d0)
,          + 0.000171d0*dsin(q4+3.1637d0)
, + (-.00211952d0 + .00769075d0*dsin(q1+1.558711d0)
,          + .00104167d0*dsin(q2+4.441790d0) )*tj
c
zp = 0.000986d0 + 0.000177d0*dsin(q1+3.52171756d0)
,          + 0.000049d0*dsin(q2+2.73655236d0)
, + ( .00012945d0 + .00004583d0*dsin(q1+0.092686d0)
,          + .00000544d0*dsin(q2+2.560430d0) )*tj
c
c 600 CONTINUE
c
c Add the two components of motion to get the Earth's barycentric
c ecliptical motion:
c
xg = x + xt
yg = y + yt
zg = z + zt
xpg = xp + xpt
ypg = yp + ypt
zpg = zp + zpt
c
c Convert to [m], [m/s] and equatorial coordinates:
c
```

Jun 6 18:40 1984 earth.f Page 7

```
r(1) = 1000d0*xg
r(2) = 1000d0*(ceps*yg - seps*zg)
r(3) = 1000d0*(seps*yg + ceps*zg)
v(1) = 1000d0*xpg
v(2) = 1000d0*(ceps*ypg - seps*zpg)
v(3) = 1000d0*(seps*ypg + ceps*zpg)
c
ierr = 0
RETURN
c-----
c Error condition: t not in allowable interval
c-----
9000 CONTINUE
DO 9010 i = 1, 3
    r(i) = 0d0
    v(i) = 0d0
9010 CONTINUE
c
ierr = -1
RETURN
c
c
END
```