

Simple Tuning of Arbitrary Controllers using Governors

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Abstract: The paper proposes a preliminary idea to an intuitive and straightforward mechanism for tuning arbitrary controllers and changing the closed-loop performance. While the structure and parameters of the original controller are kept unchanged, the inputs to the nominal controller are modified such that the closed-loop response becomes slower or faster. Such a governor setup implementation is advantageous, especially when re-tuning the original controller is impractical or impossible. The practicability and versatility of this approach is presented using several examples spanning from simple loops with PID controllers to complex nonlinear closed-loop systems with optimal and approximated explicit MPC.

Keywords: Tuning, constrained control, predictive control, controller structure, governor.

1. INTRODUCTION

Any controller or closed-loop system needs to be tuned for desired performance. Tuning of controllers depends on both the controller type and the performance criteria. Once the controller is in place, it needs maintenance and repeated tuning if the performance decreases. The most information on tuning exists for PID controllers that are applied in more than 90% of all installed control loops. Probably the first, and certainly the best known, are the Ziegler-Nichols rules (Ziegler and Nichols, 1942) but new tuning methods frequently appear; see O'Dwyer (2009); Somefun et al. (2021) for a comprehensive summary.

Some controllers are tuned during design. Other approaches make it possible to modify control performance on-line. For example, model predictive controllers (MPC) optimise performance functions on-line that can be changed by operators (García et al., 1989; Rawlings et al., 2017). But, if explicit variants of MPC control strategies are considered where the controller is calculated off-line and then embedded into the hardware, re-tuning is very difficult or impossible (Klaučo and Kvasnica, 2018; Oravec and Klaučo, 2022).

We propose an approach to tune controllers that uses a structure known as a governor. The governor usually manipulates the setpoint applied to the controller. Govern-

ors are extensively used for various purposes. Feedback controller tuning using a virtual reference was considered in Hjalmarsson (1999); Campi et al. (2002), where a setpoint trajectory is optimised to solve a model reference problem. Well-known is an approach to constraint handling where the reference is modified in such a way that input constraints are respected (Gilbert and Tan, 1991; Bemporad, 1998). Gilbert and Kolmanovsky have dealt with both linear and nonlinear systems as well as disturbances and uncertainties – see the recent survey paper (Garone et al., 2017) and references therein for a comprehensive treatment of this topic.

A popular application of reference governors is the so-called input shaping. Here, additional dynamics in the feedforward part of the controller modifies reference step changes to reduce unwanted oscillations for underdamped processes: cranes, movement of a liquid in tanks, robot control, etc. (Smith, 1957; Singh, 2010). Although this improves responses to reference changes, disturbance rejection is unaffected. There were some attempts to move the input shaper inside of the feedback loop. The simultaneous design of a PD controller with input shaping was considered in Huey and Singhose (2005, 2012). An optimal error governor for anti-windup was treated in Cavanini et al. (2021). In contrast to feedforward governors, such a scheme influences closed-loop stability.

Our proposed approach is aimed at tuning existing controllers that cannot be changed due to various reasons: black-box controllers, rule-based controllers, table-based controllers, etc. It is not restricted to any particular form of the controller. The main tuning objective is to change the desired speed of the closed-loop system using a single tuning parameter. This is implemented by a governor

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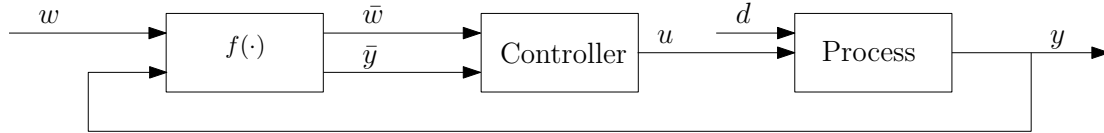


Fig. 1. Governor scheme for controller tuning.

structure that manipulates inputs to the actual controller. The resulting scheme retains properties of the existing controller, such as input constraint handling. The paper discusses the main principle of the method and provides several case studies that demonstrate its capabilities.

2. MAIN IDEA

The main idea of the proposed approach is to modify the input signals to the nominal controller. We insert an additional block preceding the controller to the loop, the governor. If the original signals processed by the controller are the setpoint $w(t)$ and process output $y(t)$, the governor replaces them with the corresponding modified versions $\bar{w}(t), \bar{y}(t)$ using the functional form

$$(\bar{w}(t), \bar{y}(t)) = f(w(t), y(t)), \quad (1)$$

as shown in Fig. 1.

There are several possible candidates for the transformation function $f(\cdot)$ in (1). In order to keep the governor simple and intuitive, a static gain is proposed if the controller directly processes the tracking error $e(t) = w(t) - y(t)$

$$\bar{e}(t) = K_e e(t), \quad (2)$$

where $\bar{e}(t)$ is fed to the controller input instead of $e(t)$. This acts as an error governor. The dimensionless tuning constant K_e is smaller than 1 if the closed-loop response should be slowed down. The opposite case of $K_e > 1$ makes the controller more aggressive and the closed-loop faster.

If the controller handles setpoint and output signals separately, several approaches can be proposed. We start with (2) and process the tracking error by modifying both setpoint and output signals

$$\bar{e}(t) = \bar{w}(t) - \bar{y}(t) = K_e(w(t) - y(t)), \quad (3)$$

$$\bar{w}(t) = K_e w(t), \quad \bar{y}(t) = K_e y(t). \quad (4)$$

It is also possible to modify only one of the signals. For example, if one only manipulates the reference value and leaves the process output unchanged, the reference governor results in

$$\bar{e}(t) = \bar{w}(t) - \bar{y}(t) = K_w(w(t) - y(t)), \quad (5)$$

$$\bar{w}(t) = K_w w(t) + (1 - K_w)y(t), \quad \bar{y}(t) = y(t). \quad (6)$$

Yet another possibility is to manipulate the process output, which yields the output governor form

$$\bar{e}(t) = \bar{w}(t) - \bar{y}(t) = K_y(w(t) - y(t)), \quad (7)$$

$$\bar{w}(t) = w(t), \quad \bar{y}(t) = K_y y(t) + (1 - K_y)w(t). \quad (8)$$

In general, the error governor modifies both input signals, whereas reference and output governors modify only one of them. If both signals are left unchanged, the original closed-loop response remains intact.

Other controller types can be handled similarly. For example, a state feedback controller (with integral action) of the form

$$u(t) = -\mathbf{K}\mathbf{x}(t) - K_i \int_0^t e(\tau) d\tau \quad (9)$$

can be thought of as a system with inputs $\mathbf{x}(t), e(t)$. The error governor will provide modified signals $\bar{\mathbf{x}}(t), \bar{e}(t)$ that will be fed into the original controller, where

$$\bar{\mathbf{x}}(t) = K_e \mathbf{x}(t), \quad \bar{e}(t) = K_e e(t), \quad (10)$$

and K_e is the tuning parameter. If the reference governor is to be implemented, the states will remain intact, and only the reference that enters the integral term will be modified as in (6).

3. CASE STUDIES

We will demonstrate the applicability of the proposed approach to selected case studies. The first one deals with constrained PI control containing an anti-windup part. The second case study solves a multivariable inverted decoupling problem.

The last two cases deal with model predictive controllers. Constrained MPC of a double integrator assumes that the MPC controller is in its explicit form (Bemporad et al., 2002) and is calculated off-line. Finally, we apply the governor to an approximated MPC controller by a neural network.

3.1 Nonlinear Process with PI Control

We will control a nonlinear process consisting of two tanks in series with interaction (Mikleš and Fikar, 2007). Each tank is of a vertical construction with vertical walls and a cross-sectional area F that holds a liquid of the height $h(t)$ with the volume $Fh(t)$. The process parameters include cross-sectional areas F_1, F_2 , valve constants k_{11}, k_{22} . There are two freely adjustable liquid streams entering the tanks on their top: volumetric flow-rates $q_{0,1}(t)$ to the first tank (manipulated variable u) and $q_{0,2}(t)$ to the second tank (disturbance variable d). We will assume that the manipulated input flow-rate $q_{0,1}$ is constrained: $u_{\min} = 0.05 \text{ m}^3/\text{s}$, $u_{\max} = 2 \text{ m}^3/\text{s}$. We can only measure the level in the second tank with some measurement noise and that the process is initially at the steady state h_1^s, h_2^s . To simulate disturbances, the uncontrolled flow-rate $q_{0,2}$ changes at time $t = 35 \text{ s}$ from $q_{0,2} = 0 \text{ m}^3/\text{s}$ to $q_{0,2} = 0.1 \text{ m}^3/\text{s}$.

The nonlinear process model is determined from the mass balance of the system, where constant density is assumed

$$F_1 \frac{dh_1}{dt} = q_{0,1} - k_{11} \sqrt{h_1 - h_2}, \quad (11a)$$

$$F_2 \frac{dh_2}{dt} = q_{0,2} + k_{11} \sqrt{h_1 - h_2} - k_{22} \sqrt{h_2}, \quad (11b)$$

and the steady-state levels are given as

$$h_1^s = \left(\frac{q_{0,1}^s}{k_{11}} \right)^2 + h_2^s, \quad h_2^s = \left(\frac{q_{0,1}^s + q_{0,2}^s}{k_{22}} \right)^2. \quad (12a)$$

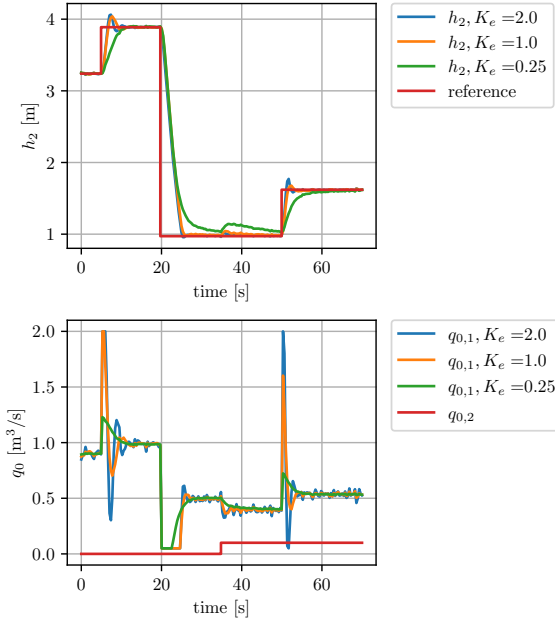


Fig. 2. PI control with error governor control of the nonlinear process.

The concrete values of parameters and signals of the process are as follows: $q_{0,1}^s = 0.9 \text{ m}^3/\text{s}$, $q_{0,2}^s = 0 \text{ m}^3/\text{s}$, $F_1 = 0.5 \text{ m}^2$, $F_2 = 0.6 \text{ m}^2$, $k_{11} = 0.8 \text{ m}^{2.5}/\text{s}$, and $k_{22} = 0.5 \text{ m}^{2.5}/\text{s}$.

We consider that the process is controlled using a PI controller with anti-windup compensation of the form

$$\bar{u}(t) = K_p e(t) + u_b(t), u_b(t) = \frac{1}{T_i s + 1} u_{\text{SAT}}, \quad (13a)$$

$$u_{\text{SAT}} = \max(\min(\bar{u}(t), u_{\text{max}}), u_{\text{min}}), u(t) = u_{\text{SAT}}, \quad (13b)$$

with $K_p = 2$, $T_i = 8.6 \text{ min}$.

Simulation results are shown in Fig. 2. The error governor (2) with $K_e = \{0.25, 1.0, 2.0\}$ was used. The original controller properties are satisfied in all cases: the controller tracks setpoint changes, rejects disturbances (a step change in $q_{0,2}$ at $t = 35 \text{ s}$), and respects the input constraints without windup effects. The tuning parameter changes the controller response as requested without any change to the original closed-loop. Aggressive tuning with $K_e = 2$ results in an oscillatory behaviour of control actions while reducing the settling time. On the other hand, the closed-loop response is slowed down considerably with $K_e = 0.25$. The controller is insensitive to measurement noise and heavily dampens its actions. The settling time is larger, and the closed-loop behaviour is overdamped.

3.2 Decoupling Multivariable Control

We consider a model of a quadruple tank process, which represents a multivariable system with 2 inputs (pump powers) and 2 outputs (liquid levels in two tanks) described by the matrix of transfer functions

$$G(s) = \begin{pmatrix} \frac{2.6}{62s+1} & \frac{1.5}{(23s+1)(62s+1)} \\ \frac{1.4}{(30s+1)(90s+1)} & \frac{2.8}{90s+1} \end{pmatrix}. \quad (14)$$

We adopted the model and control strategy from Hägglund et al. (2022), where an inverted decoupling control scheme

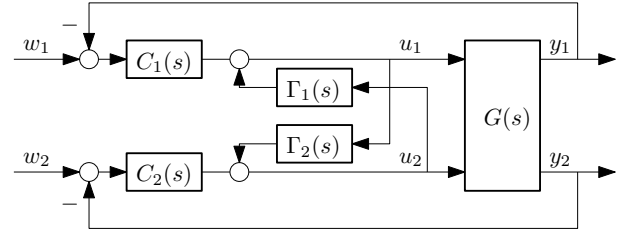


Fig. 3. Inverted decoupling multivariable control scheme.

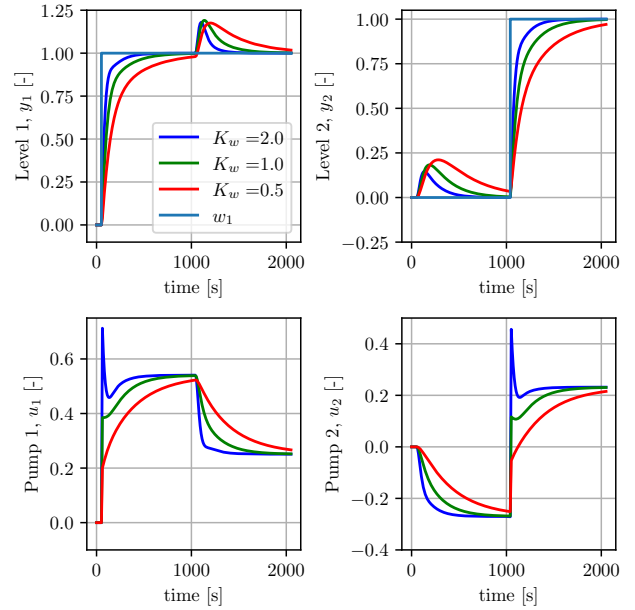


Fig. 4. Reference governor PI (no decoupling) control of the quadruple tank system.

featuring two interaction PI controllers was proposed. The particular installation of the decoupling control is visualised in Fig. 3, and individual PI controllers are given by the following expressions

$$C_1(s) = 0.385\beta \left(1 + \frac{1}{62s}\right), \quad \Gamma_1(s) = \frac{\gamma_1}{(\tau s + 1)^2}, \quad (15a)$$

$$C_2(s) = 0.357\beta \left(1 + \frac{1}{90s}\right), \quad \Gamma_2(s) = \frac{\gamma_2}{(\tau s + 1)^2}, \quad (15b)$$

where $\tau = 62/200$, $\gamma_1 = -0.577$, $\gamma_2 = -0.500$, $\beta = 1 - \gamma_1\gamma_2$. Both PI controllers $C_1(s), C_2(s)$ are used and the terms $\Gamma_1(s), \Gamma_2(s)$ are compensators in the inverted decoupling scheme. The decoupling feature is inhibited with $\gamma_1 = \gamma_2 = 0$.

A reference governor was applied to both versions of the decoupling scheme, i.e., to the control scheme without the decoupler and to the scheme with the decoupler. The same weights $K_w = \{0.5, 1.0, 2.0\}$ were chosen for evaluating performance of both loops. Fig. 4 shows the reference tracking for step changes in references for the controller without decoupling. It can indeed be observed that there are significant interactions between both process outputs. The green line represents the nominal controller, the blue line the faster response, and the red line the slower response. The proposed governor behaves intuitively and can vary the response speed.

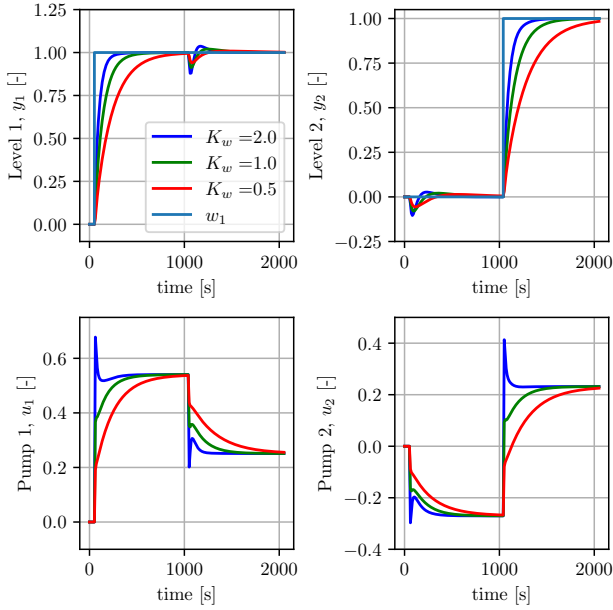


Fig. 5. Reference governor PI (with inverted decoupling) control of the quadruple tank system.

Simulations with the decoupler are shown in Fig. 5. Here we note that the interactions were greatly reduced. This holds for all three considered cases of the faster, slower, and nominal controllers.

3.3 Regulation Problem with Model Predictive Control

We consider the discrete-time double integrator with the state-space model

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}u_k, \quad (16a)$$

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}. \quad (16b)$$

The process is controlled from nonzero initial conditions to the origin using an explicit model predictive controller using the following parameters: constraints $-5 \leq \mathbf{x}_k \leq 5$, $-0.5 \leq u_k \leq 0.5$, quadratic cost function using prediction horizon $N = 5$ with state and control weighting matrices $\mathbf{Q} = \mathbf{I}$, $R = 1$, respectively. The resulting controller is defined on 33 state-space regions. To tune the closed-loop behaviour, process states are modified as in (10)

$$\bar{\mathbf{x}}(t) = K_x \mathbf{x}(t), \quad (17)$$

and the modified states are fed as inputs to the explicit MPC controller.

Fig. 6 shows state and control trajectories for initial conditions $\mathbf{x}_0 = (1, -1)$ and $K_x = \{0.6, 0.8, 1.0, 1.2, 1.6\}$. The green line denotes the nominal controller. We can see in the state portrait that it is possible to make the closed-loop system both faster and slower. This is also indicated in the control trajectories, K_x has an impact on the magnitude of the control moves. Any value of the tuning parameter guarantees the nominal controller objective. Regulation of the process towards the origin is still achieved.

As requested, the controller respects the upper control constraint $u_k = 0.5$ for values of $K_x > 0.8$. Note also the difference in state trajectories with $K_x = \{1.2, 1.6\}$.

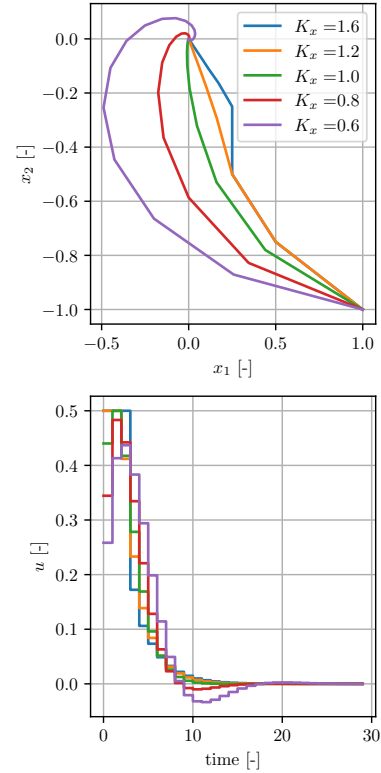


Fig. 6. State governor explicit MPC control of the double integrator system.

While the control is constrained in the first three steps, both state trajectories are the same. Once the control is unconstrained, the tuning parameter influences the speed of the regulation.

Additional care has to be taken for a more aggressive controller with K_x and the feasible domain of states for which was the nominal controller calculated. A substantial K_x can result in modified states being outside of the feasible region. Therefore, we suggest either finding a minimum admissible K_x using projection to the feasible state region or application of the nominal controller with $K_x = 1$.

3.4 Multivariable Neural Network Controller

We consider a tank where cold and hot inlet streams are mixed (Klaučo and Kvasnica, 2019). The dynamics of the process are given by

$$Fh(t) \frac{dT(t)}{dt} = \alpha_c(t)q_{c,\max}(T_{c,0} - T(t)) + \alpha_h(t)q_{h,\max}(T_{h,0} - T(t)), \quad (18a)$$

$$F \frac{dh(t)}{dt} = \alpha_c(t)q_{c,\max} + \alpha_h(t)q_{h,\max} - \mu\sqrt{h(t)}. \quad (18b)$$

The controlled variables are $\mathbf{y}^\top = (T(t), h(t))$ and represent the temperature of the liquid inside the tank and the level of the liquid in the tank, respectively. Manipulated variables are valve openings for the hot and cold inlet streams, denoted by $\mathbf{u}^\top = (\alpha_c(t), \alpha_h(t)) \in [0, 1]$. The physical properties of the tank are given by the base $F = 1 \text{ m}^2$, the maximum height $h_{\max} = 1 \text{ m}$, the output valve

aggregated coefficient $\mu = 0.015$. We consider that the maximum inlet streams are limited to $q_{h,\max} = 0.02 \text{ m}^3 \text{ s}^{-1}$ and $q_{c,\max} = 0.03 \text{ m}^3 \text{ s}^{-1}$.

This process was controlled using a deep neural network controller that is based on supervised learning of a control policy by imitation of an MPC controller. The original MPC controller was constructed using a linearised model of the process using a convex quadratic objective function, box constraints on controlled variables and manipulated variables, and with a slew rate penalisation to ensure offset free reference tracking.

The neural network MPC-based controller (NNMPC) learned from the training data set with alternating initial conditions $\mathbf{z} = [\mathbf{y}(k)^\top, \mathbf{u}(k-1)^\top, \mathbf{w}(k)^\top]^\top$ with optimal control action \mathbf{u}^* that create data set with 50 000 samples. The data set was later split into training and testing sets with a ratio of 4 : 1. The neural network used here consists of an input layer with six inputs, an output layer with two outputs, and two hidden layers with 24 and 48 neurons, respectively. We used the ReLU activation function in each block. The training started with randomly initialised weights using Adam optimiser with an exponential decay learning rate with five steps with 500 epochs starting at value 0.1 and finishing at value 0.0001. The training was stopped early and fine-tuned for another 10 epochs leaving no constraints violated according to the test data set.

If this controller had to be re-tuned, the entire training procedure would have to be repeated. To prevent it, the approximated MPC with the neural network was implemented in closed-loop via the suggested governor.

The closed-loop performance of the neural network controller is depicted in Fig. 7 and Fig. 8, where the nominal case and the cases with reference governors with $K_w = 0.6$ and $K_w = 2.0$ are presented. In all cases, the control and state constraints are respected and the neural network controller tuning works as expected.

4. CONCLUSIONS

This paper introduced a procedure that can be used to tune an arbitrary controller. The main concept lies in inserting a governor term as a static gain that can be intuitively changed to tune the speed of the closed-loop response. This results in a governor structure that manipulates the setpoints or process outputs that are seen by the nominal controller. Hence, the governor can be thought of as supervisory control above the primary closed-loop system.

A similar principle is used in cascade control configuration. However, cascade control assumes that there are two process outputs. The primary controller handles the main controlled variable, and the secondary controller is applied to compensate for some faster dynamics measured using the secondary variable.

The proposed outer governor loop is also reminiscent of a two-degree-of-freedom controller design. There, however, the feedforward part dynamically manipulates strictly reference variables which is indeed the pure reference governor. As such, it cannot handle process disturbances that are treated using the feedback part only.

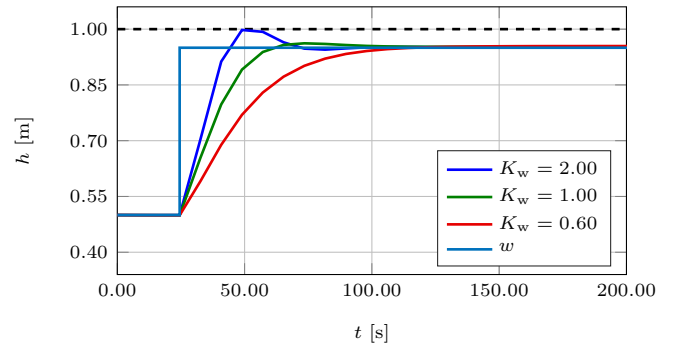
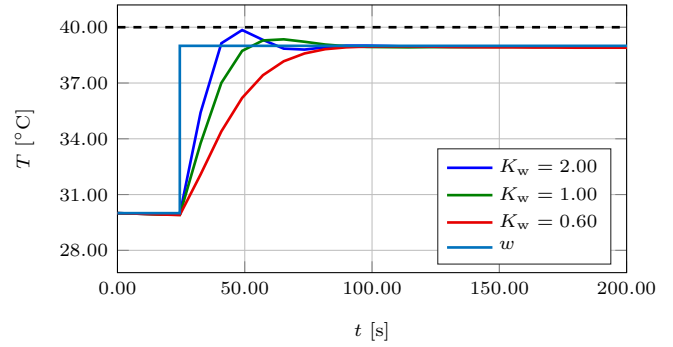


Fig. 7. Evolution of controlled variables under the authority of nominal NNMPC (green), aggressive governor and NNMPC (blue), and detuned governor with NNMPC (red).

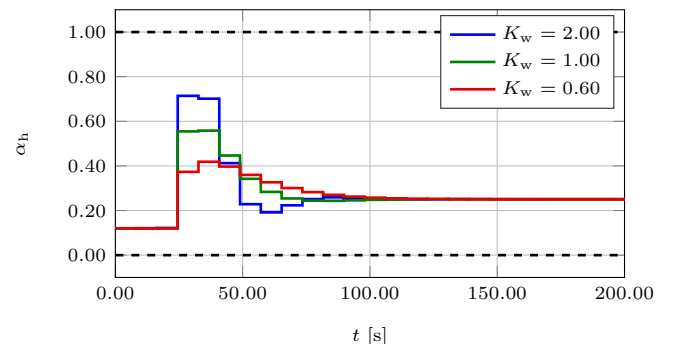
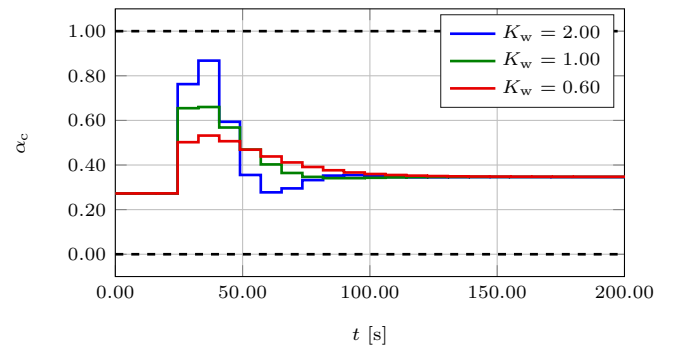


Fig. 8. Evolution of manipulated variables under the authority of nominal NNMPC (green), aggressive governor and NNMPC (blue), and detuned governor with NNMPC (red).

The paper presented the basic principle of the governor for closed-loop tuning, demonstrated in both linear and nonlinear examples, involving both SISO and MIMO con-

trol loops. An experimental evaluation for a simple height control reference governor was presented in (Dyrška et al., 2023). In all presented cases, the governor-enabled closed-loop system showed consistent results. Although tuning using a single parameter has some drawbacks and limits the range of achievable performance, it satisfies the primary design aim – to make the closed-loop system slightly slower or faster compared to the nominal conditions and irrespective of the existing controller. One of the most significant advantages of such a governor implementation is that it removes the need for costly and highly impractical re-tuning of optimal and approximated explicit MPCs.

There are several drawbacks of the scheme in its present state. It is not obvious how to transform the goal of the desired closed-loop performance to the choice of the tuning gain, as this strongly depends on the nominal controller. Therefore, only the trial-and-error method can be applied. Also, theoretical issues arise in model predictive control approaches, if it is possible to guarantee recursive feasibility and closed-loop stability. Similar issues can be observed with neural network controllers, whether the governor pushes the controller outside the trained area.

A more thorough treatment will follow. It will focus on the stability of the closed-loop system and the comparison to other tuning procedures.

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