Towards a Comprehensive Characterization of Grid Interpolation in Subgiant Stars

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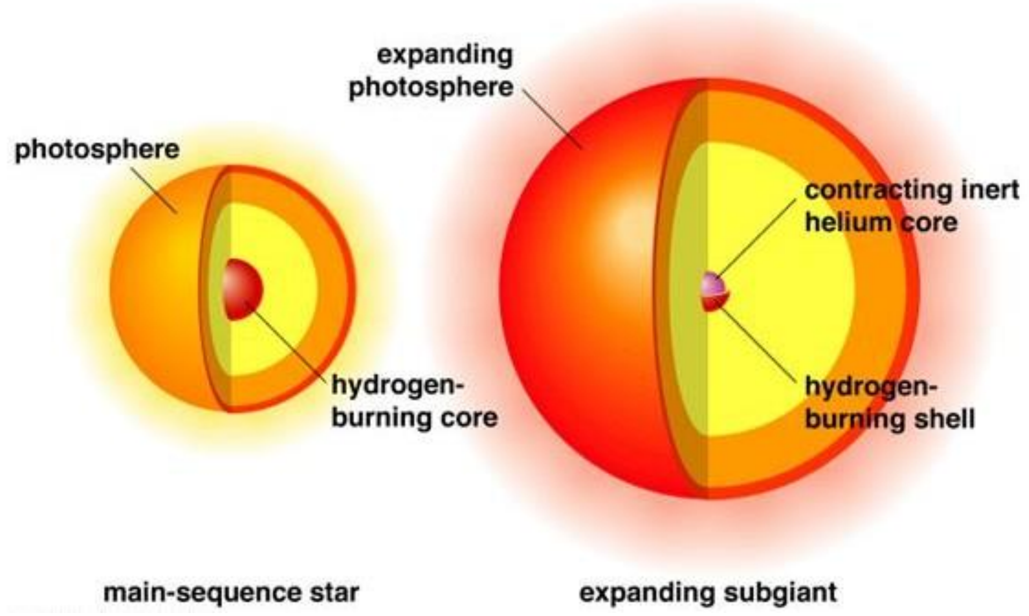
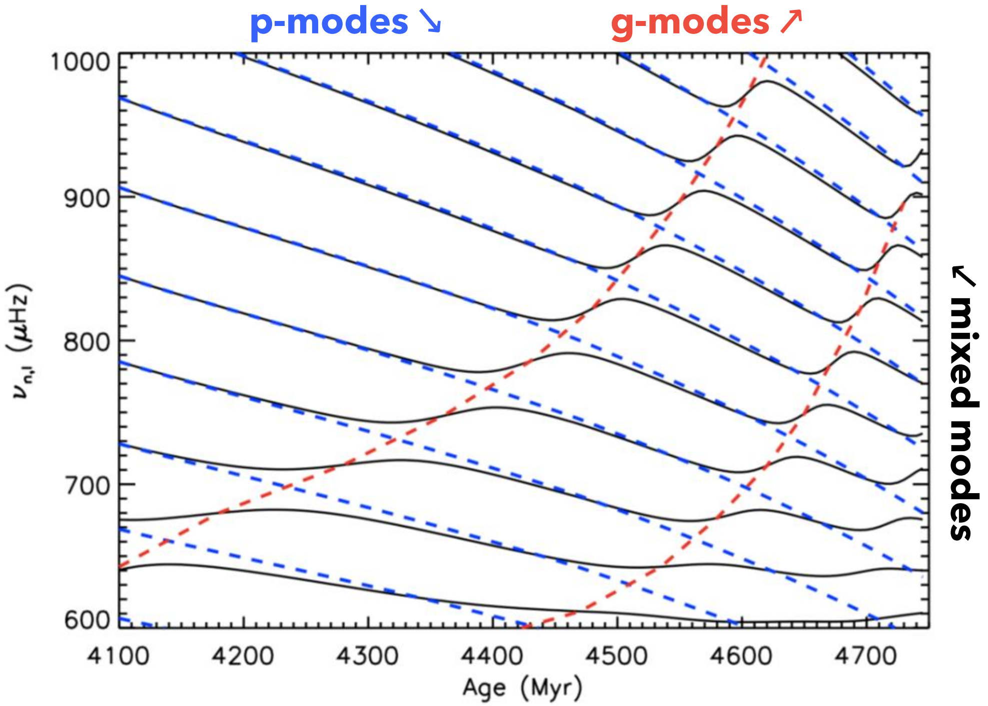
1. Introduction



↑ Internal structure of main-sequence and subgiant stars.



↑ Evolution of mixed modes in subgiant stars, as a result of the coupling between the different **p-** and **g-** mode components.

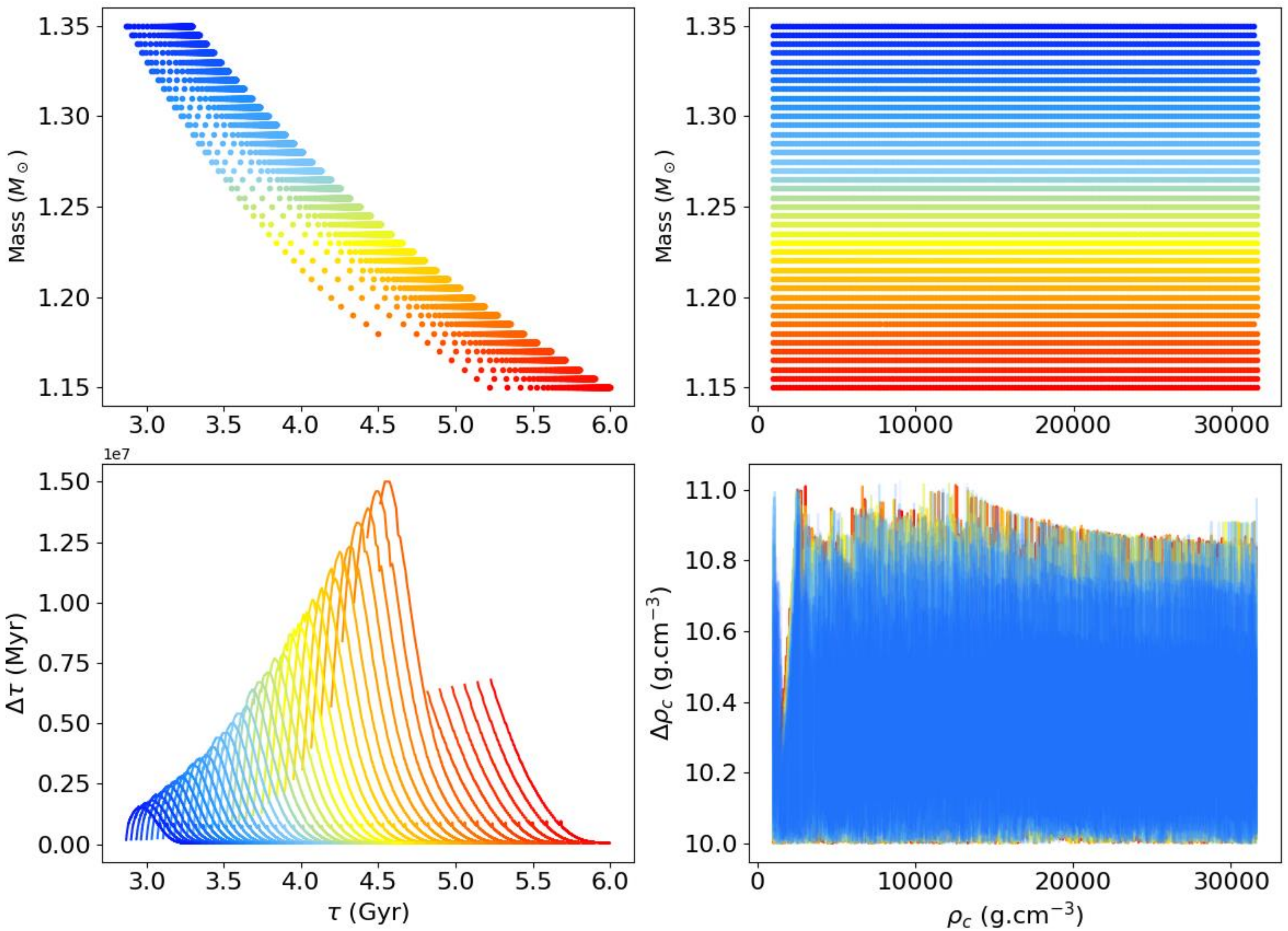
2. Characterization of the Interpolation Errors

Grid of stellar models:

- MESA's grid inputs:
  - $M = [(1.15{:}1.35), 0.005]\ \mathrm{M}_{\odot}$
  - $\rho_c = [(10^3{:}10^{4.5}), 10]\ \mathrm{g.cm}^{-3}$
- Distribution of models in the parameter space (every 20th) with τ (left) and ρ_c (right).
- $\ell = 0{,}1$ frequency modes within

$$\nu_{\ell,n} = \nu_{max} \pm 3\frac{0.66.\,\nu_{max}^{0.88}}{2\sqrt{2.\log(2)}}$$

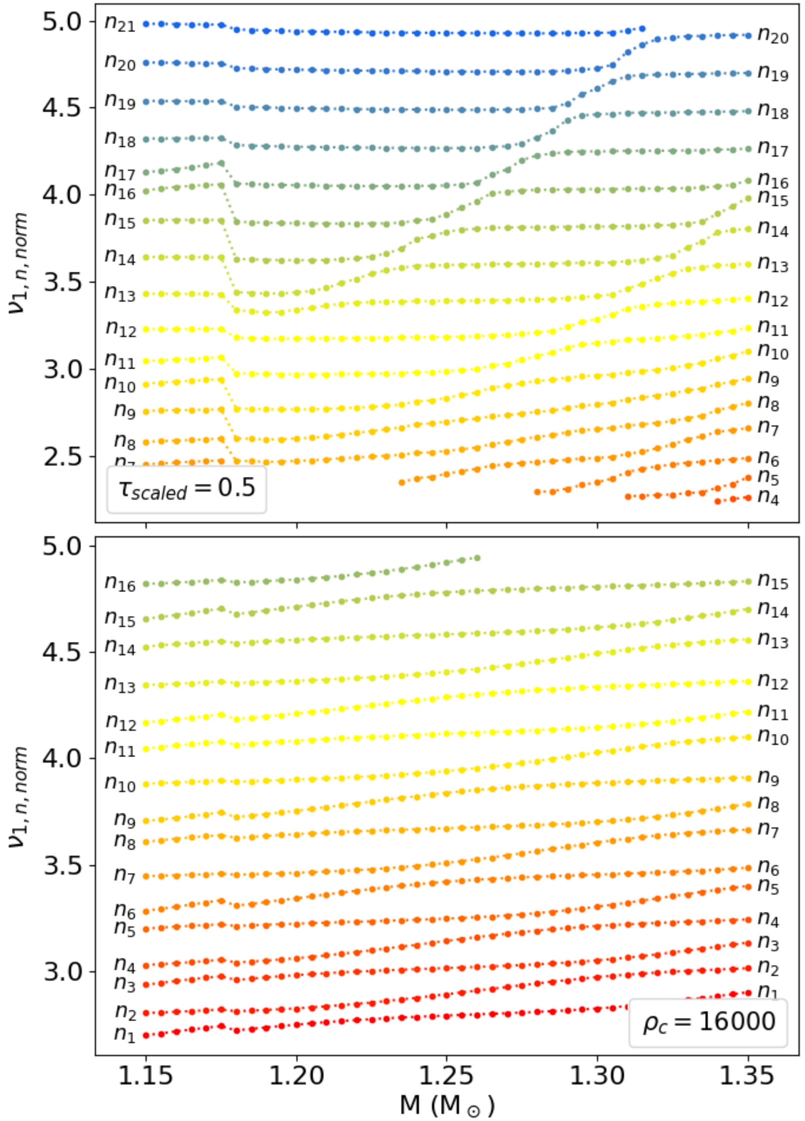
determined with GYRE.



2. Characterization of the Interpolation Errors

Evolution of non-radial modes with mass:

- τ_{scaled}=0.5 (top) and ρ_c=16000 g.cm^{-3} (bottom).
- $\nu_{1,n}(M)$ is not linear, but is smoothly resolved for the mass step of the grid.
- Exception for a gap between masses 1.175 and 1.180 $M_\odot$, caused by the transition between MS models with radiative and convective cores.



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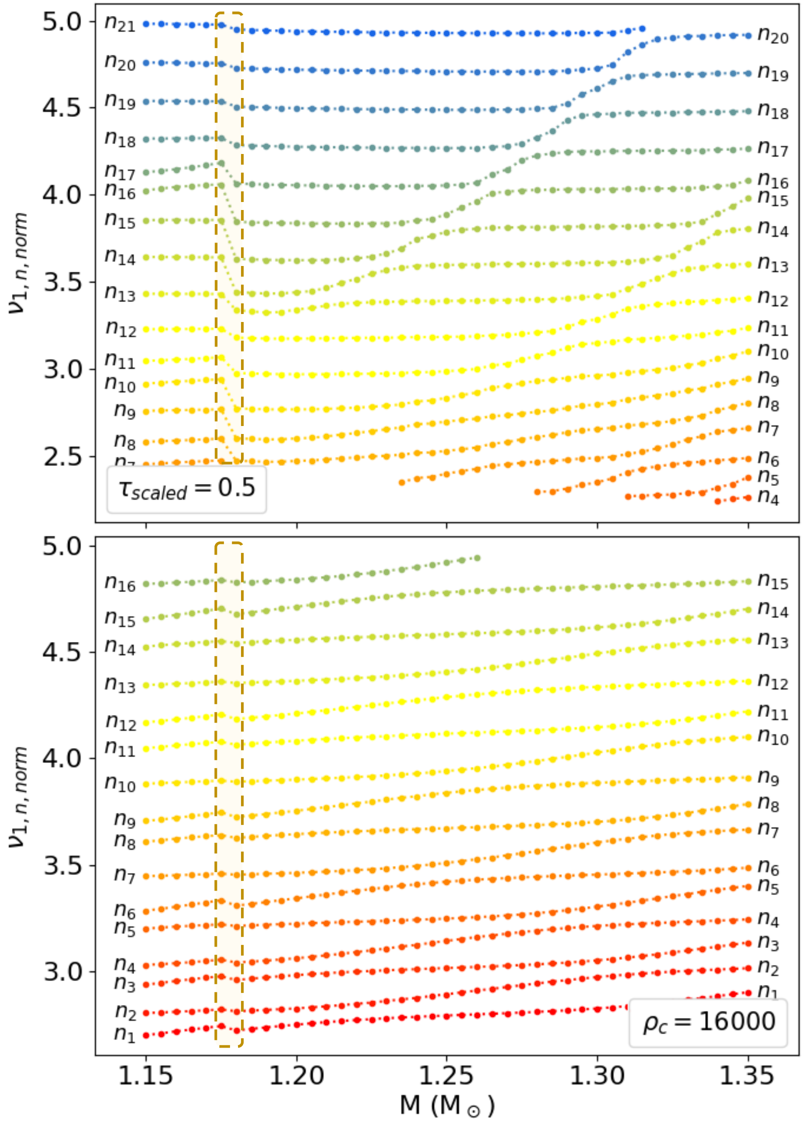


2. Characterization of the Interpolation Errors

2-step interpolation:

(i) age interpolation along evolutionary tracks;

(ii) interpolation across evolutionary tracks, at a fixed age proxy;

considering:

(i) linear regression or cubic splines;

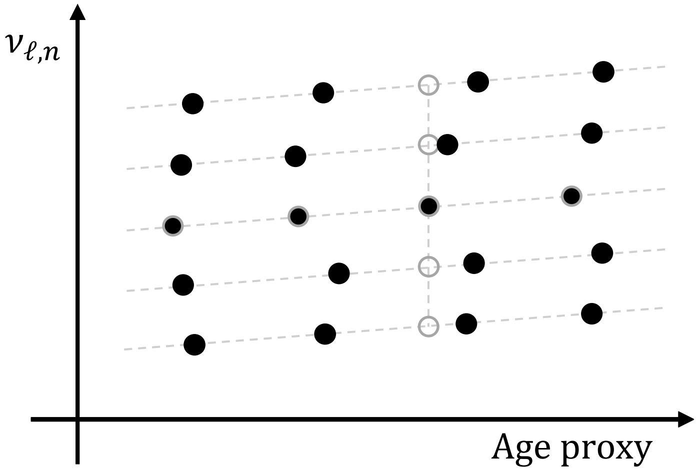
(ii) different age proxies: physical age (τ), scaled age (τ_{scaled}) or central density (ρ_c), where

$$\tau_{\text{scaled}} = \frac{t_{\text{model}} - t_{\text{TAMS}}}{t_{\text{TASG}} - t_{\text{TAMS}}}$$

Normalized frequencies: $\nu_{\ell,n,\text{norm}} = \nu_{\ell,n}/\sqrt{GM/R^3}$.

Interpolation errors:

$$\delta\nu_{\ell,n} = \left(\nu_{\ell,n,\text{norm,teo}} - \delta\nu_{\ell,n,\text{norm,int}}\right) \times \sqrt{GM/R^3}\ (\mu\text{Hz}).$$

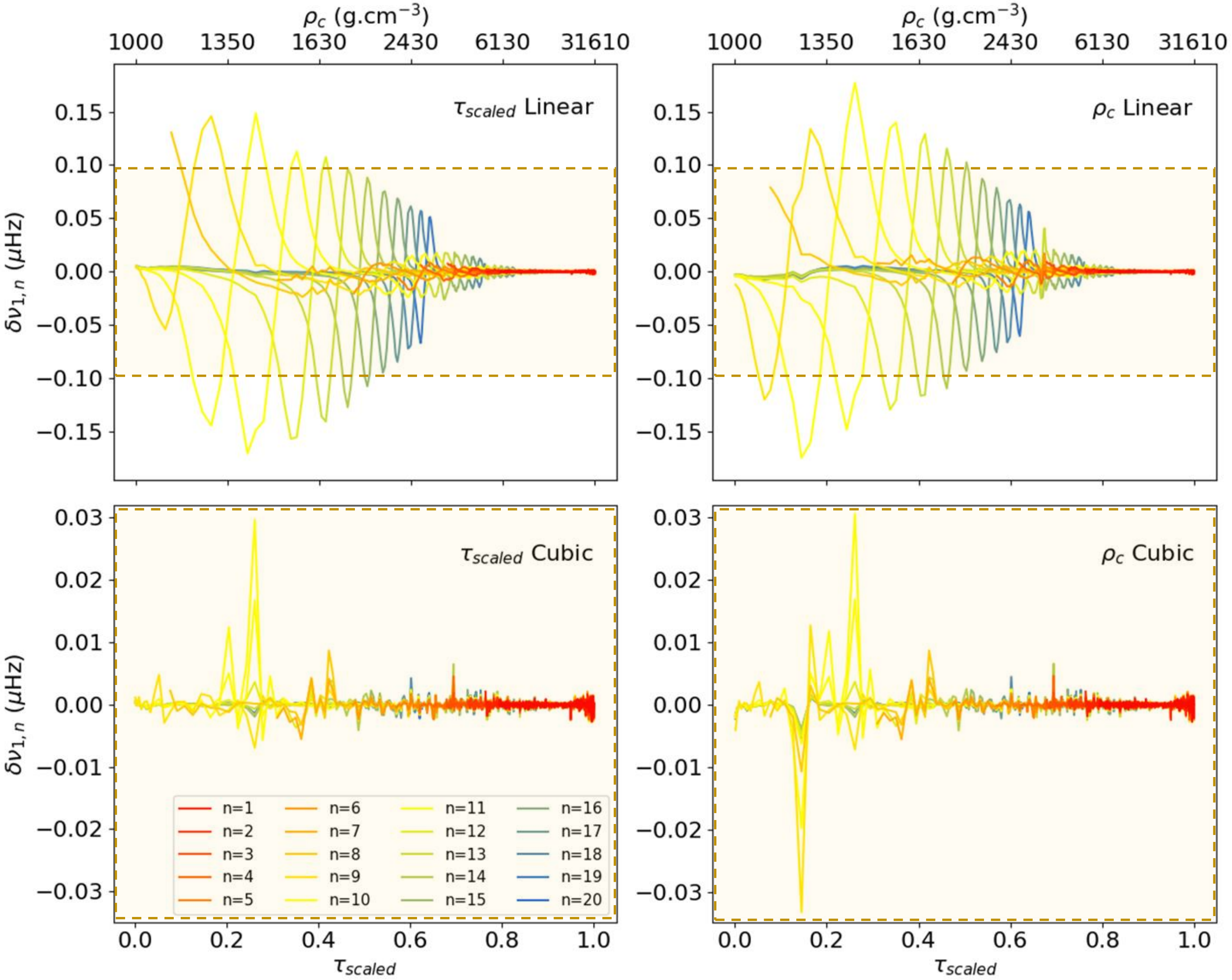


Scheme of the interpolation process.

2. Characterization of the Interpolation Errors

Interpolation **along** evolutionary tracks:

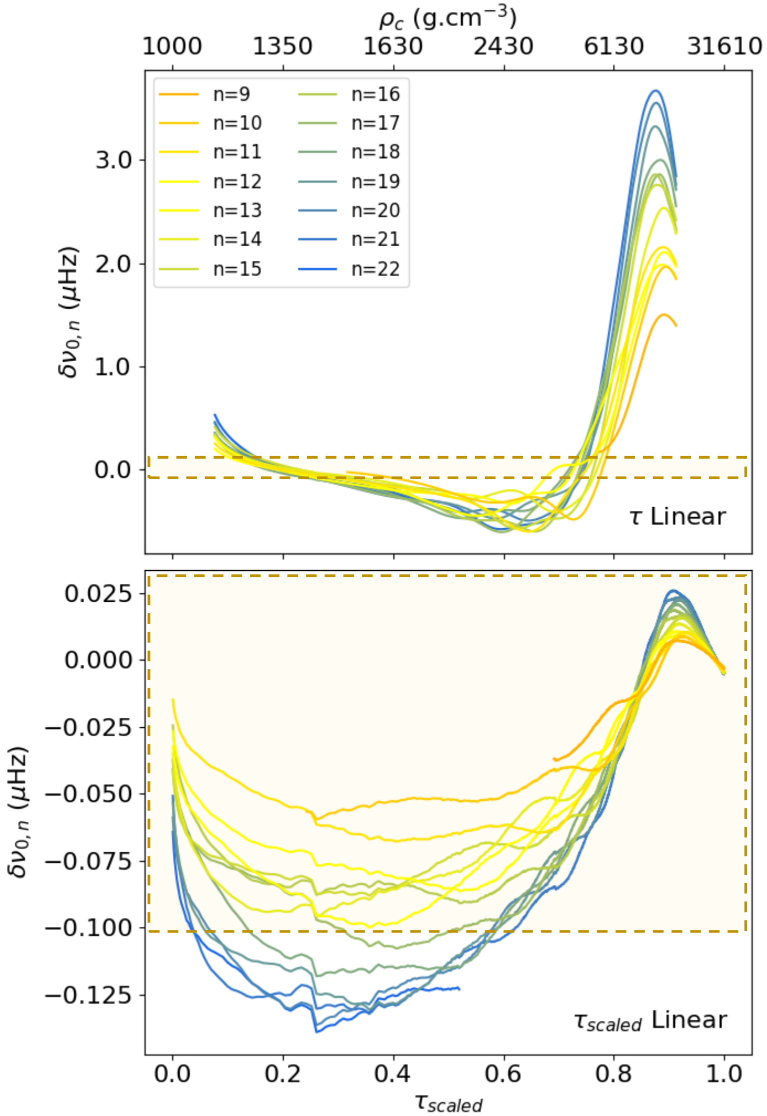
- $\ell = 1$ modes
- Age proxies: τ_{scaled} (left) & ρ_c (right)
- Linear regression (top) and cubic splines (bottom)
- Fixed mass 1.245 $\mathrm{M}_{\odot}$



2. Characterization of the Interpolation Errors

Interpolation **across** evolutionary tracks:

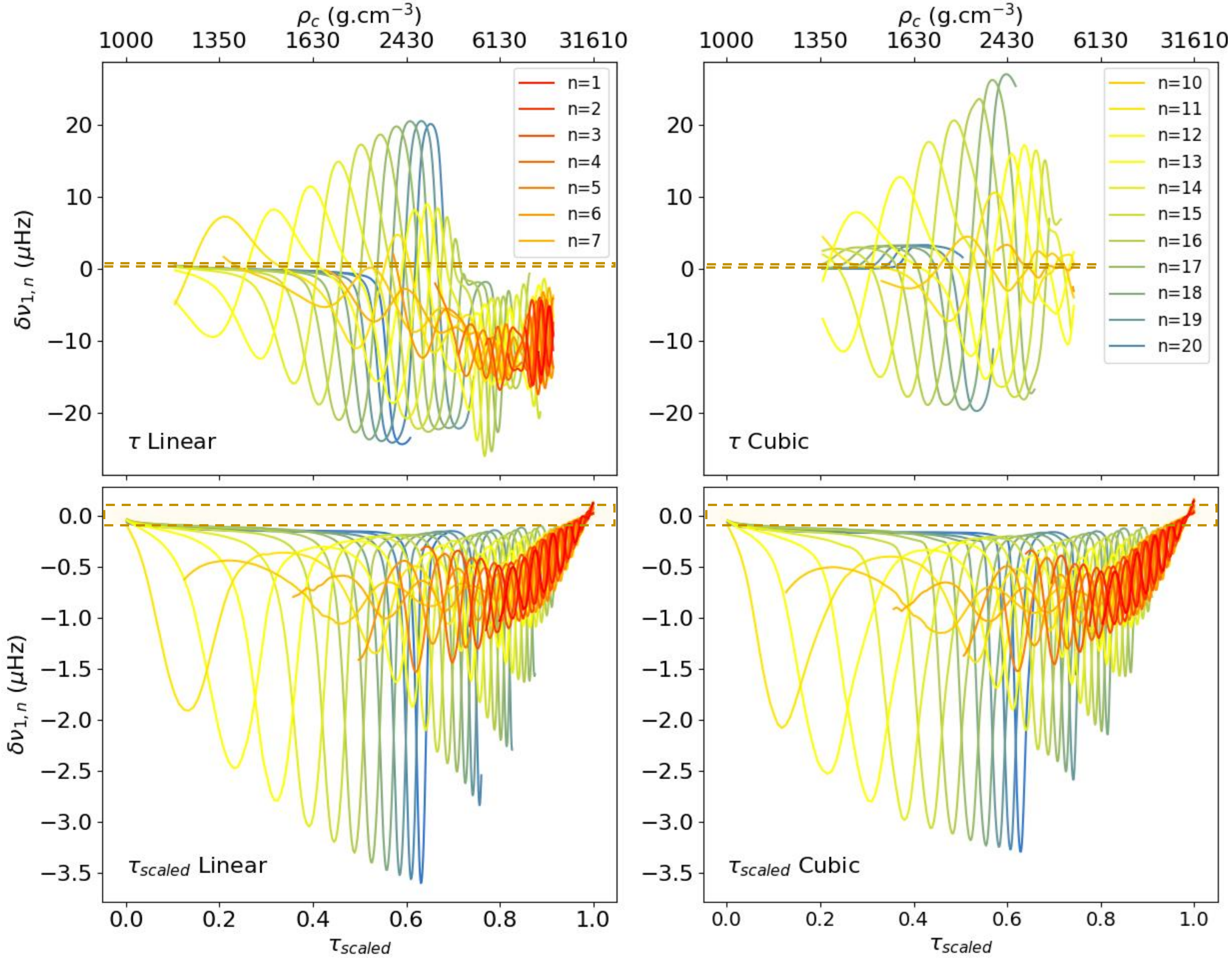
- $\ell = 0$ modes
- Age proxies: τ (top) and τ_{scaled} (bottom)
- Fixed mass 1.245 $\mathrm{M}_{\odot}$



2. Characterization of the Interpolation Errors

Interpolation **across** evolutionary tracks:

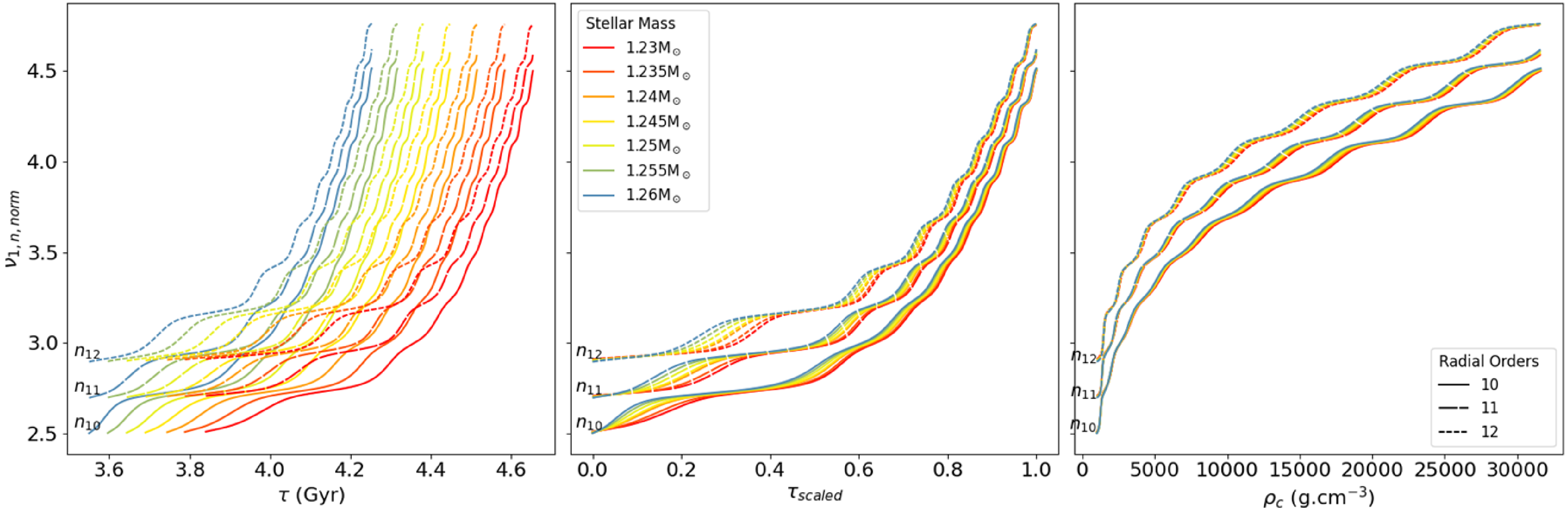
- $\ell = 1$ modes
- Age proxies: τ (top) and τ_{scaled} (bottom)
- Cubic splines along evolutionary tracks + Linear regression (left) and cubic splines (right) across evolutionary tracks
- Fixed mass 1.245 $M_{\odot}$



3. Search for Well-Motivated Parameters for a Grid

$$N^2 \sim \frac{g}{H_P} \sim \frac{\rho_c^3}{P_c} r^2 \sim \frac{\rho_c^2 \mu_c}{T_c} r^2$$

According to Deheuvels & Michels (2011), avoided crossings of different evolutionary tracks are more aligned in ρ_c than in the other age proxies, causing interpolation between different evolutionary tracks to occur between more similar phases of evolution of the modes.



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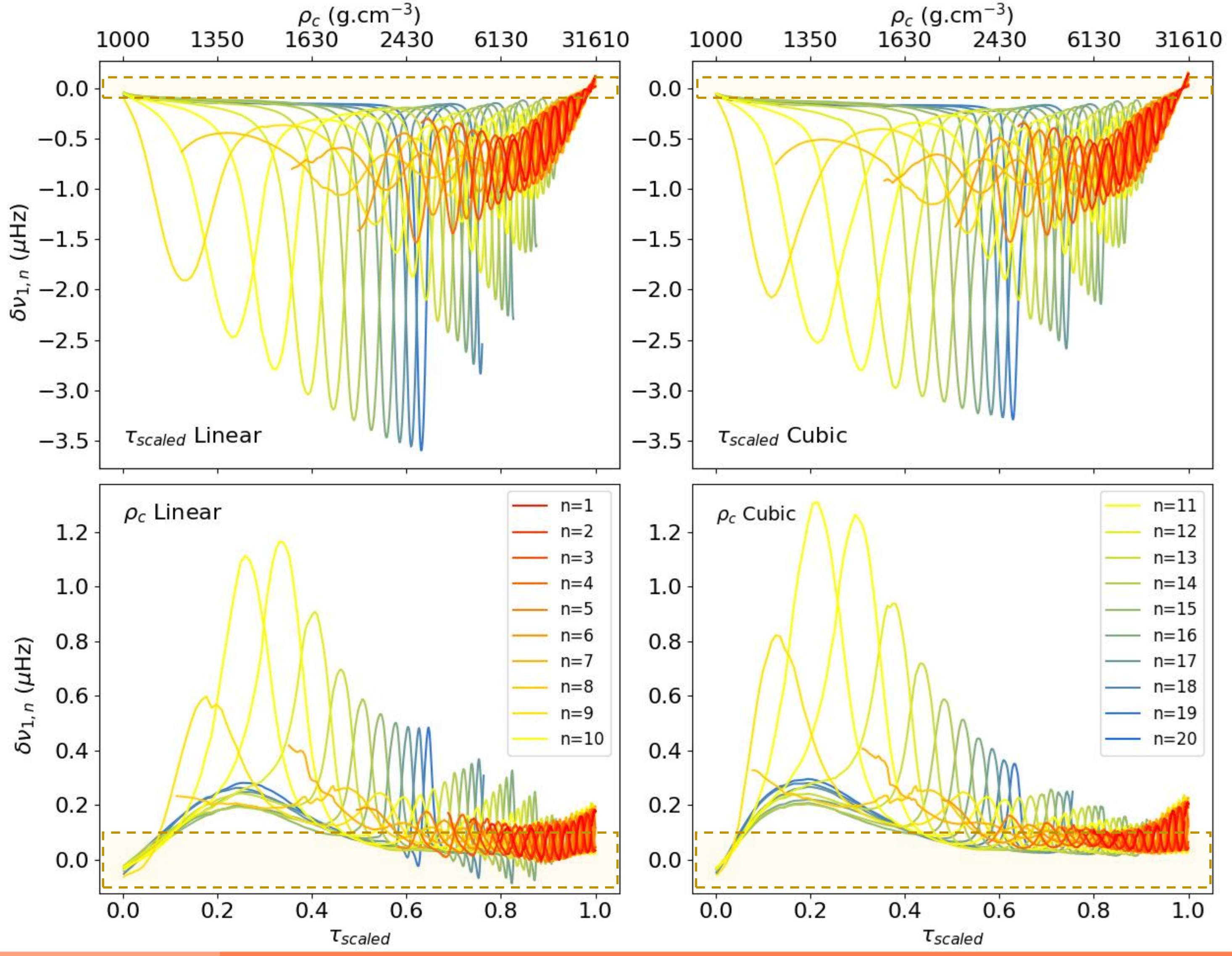
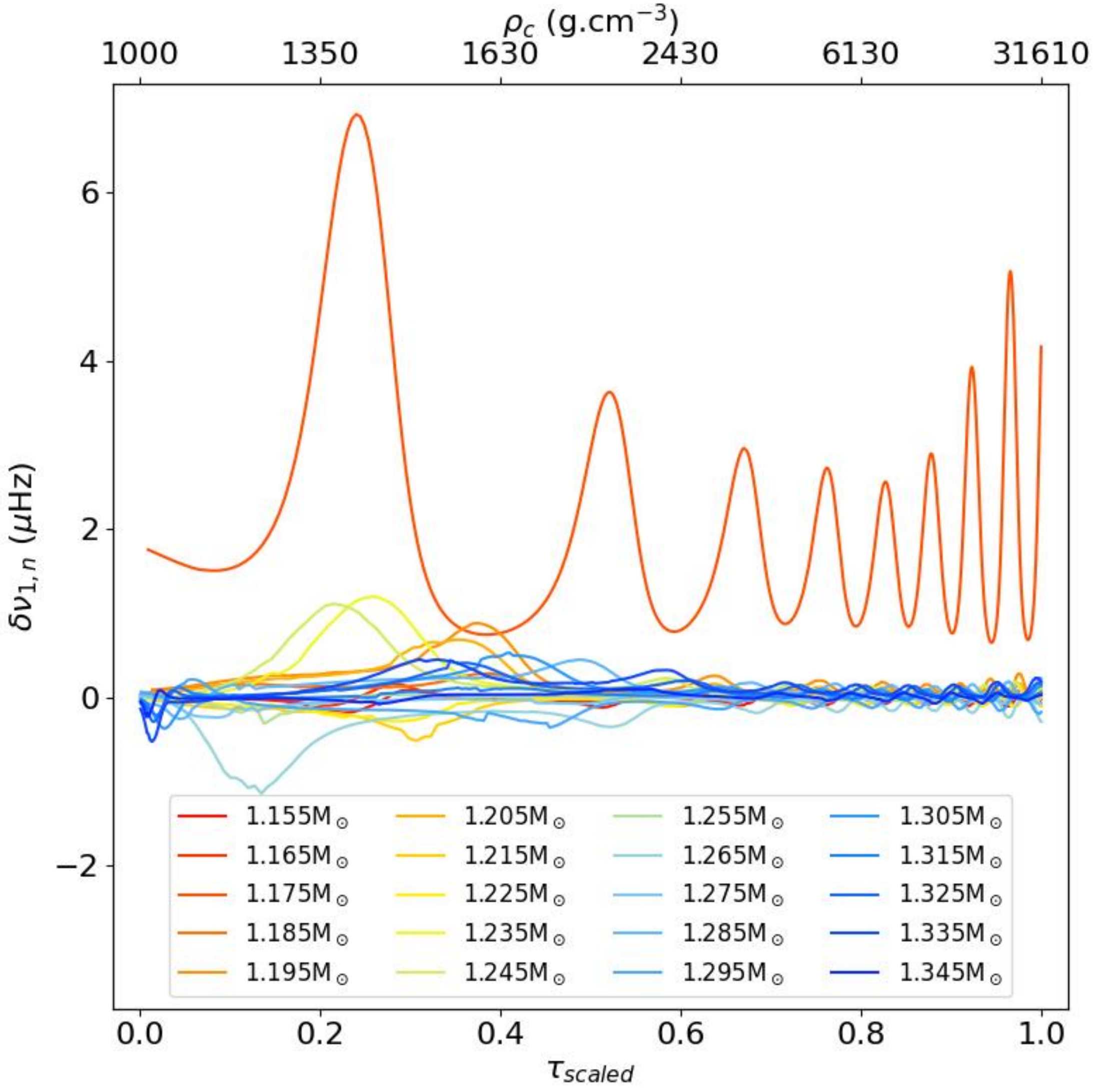


4. The Case of 1.175 $M_\odot$

Full interpolation of the stellar grid:

- All masses with the frequency mode $\nu_{1,11}$
- Cubic splines along evolutionary tracks + Linear regression across evolutionary tracks
- Age proxies: ρ_c



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Evolution of non-radial modes with mass:

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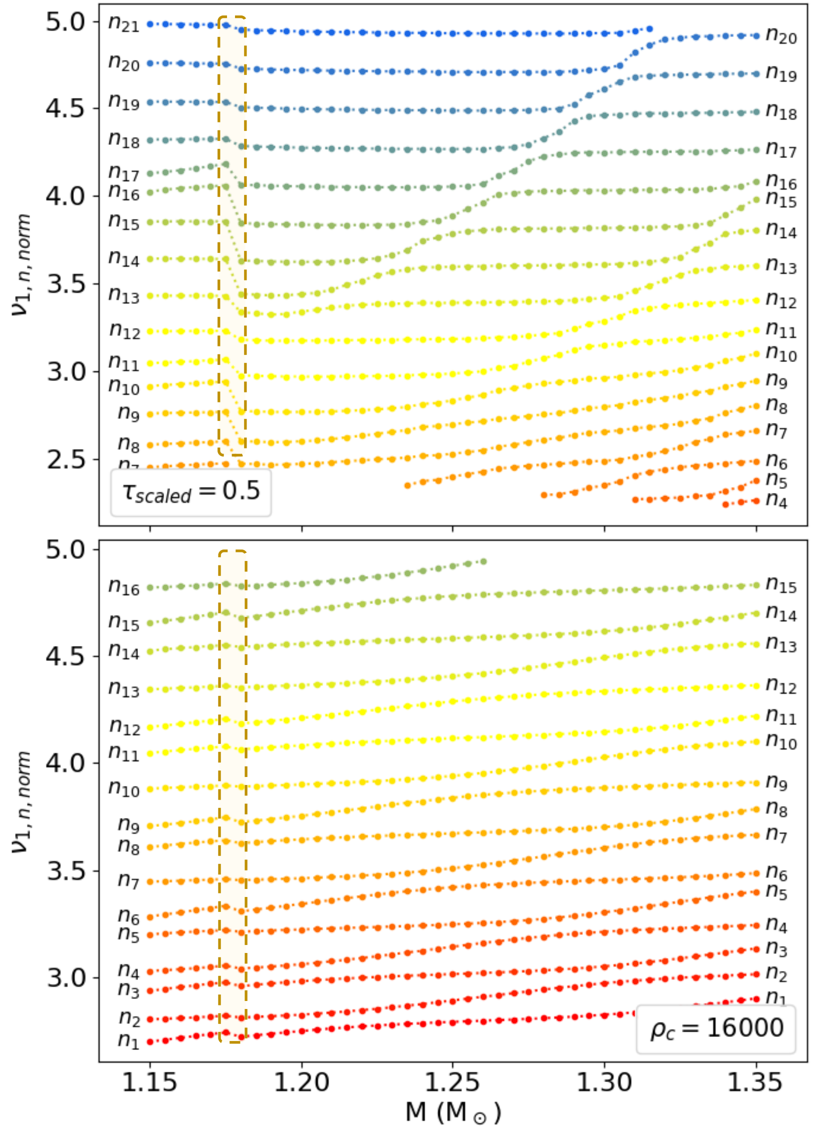

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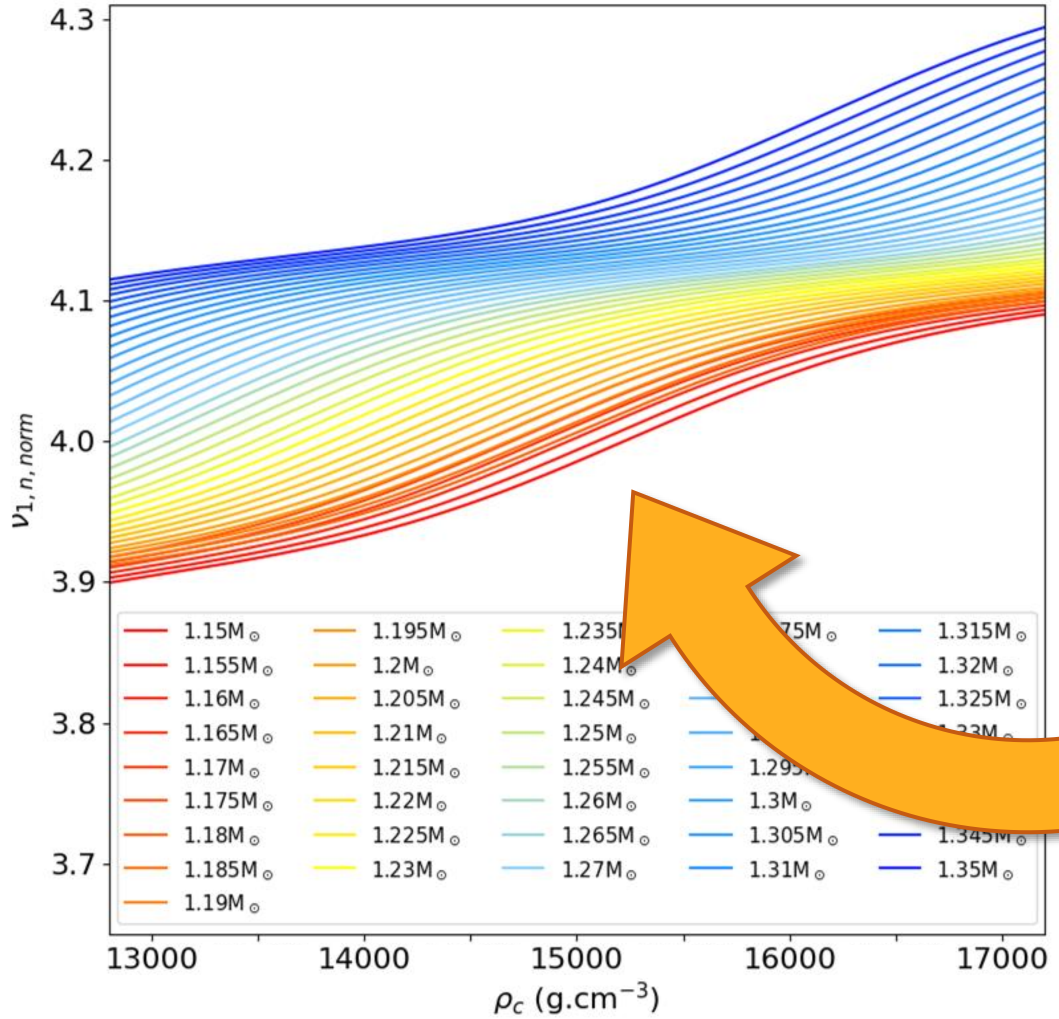

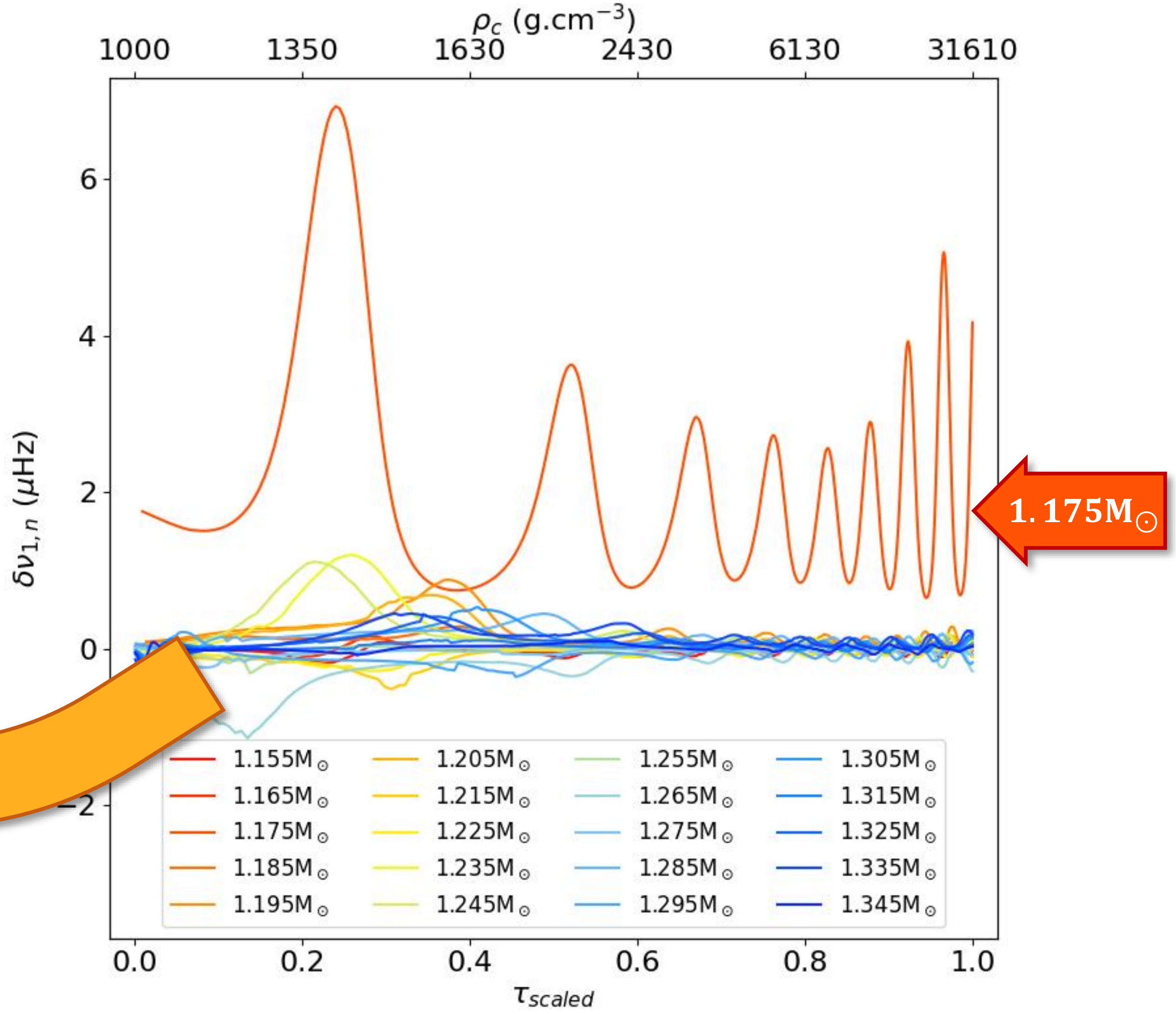

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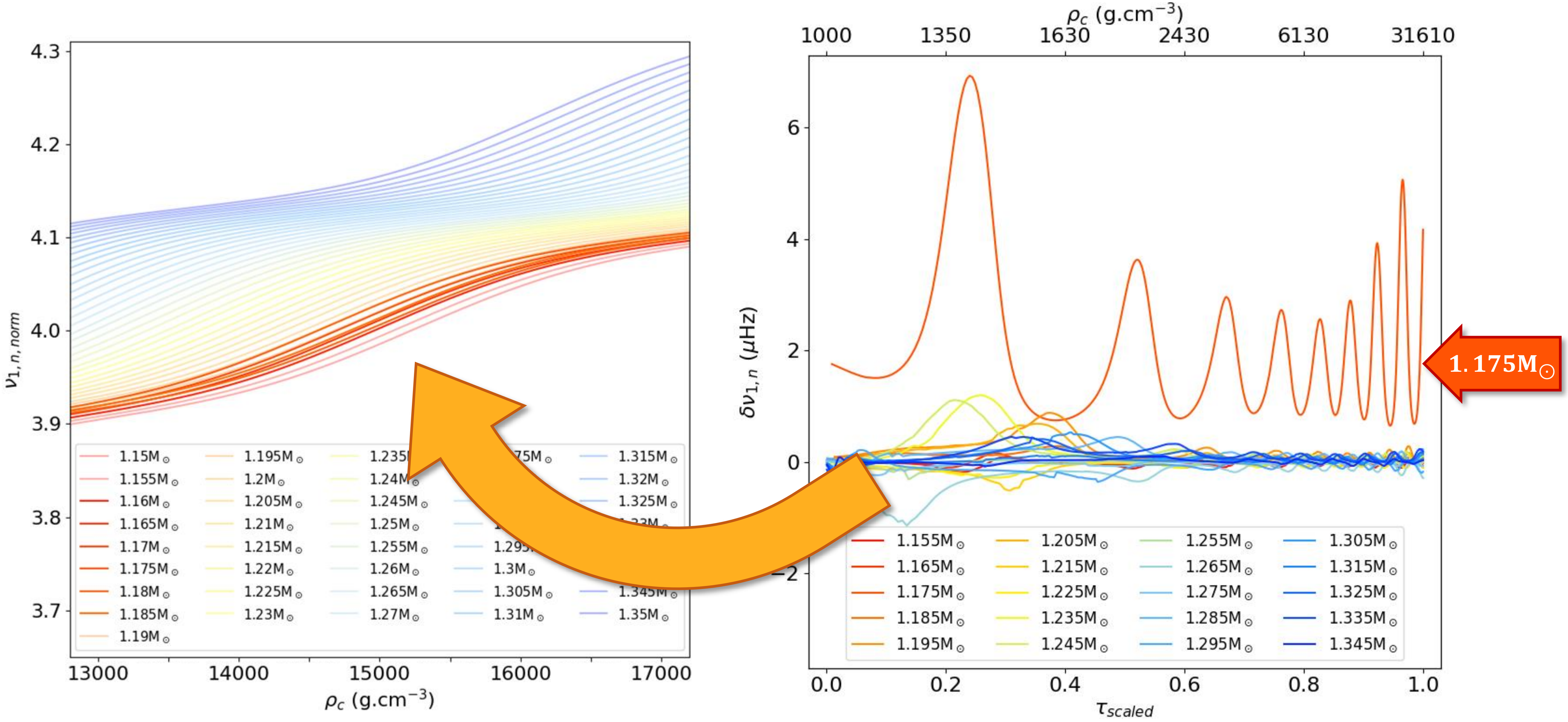

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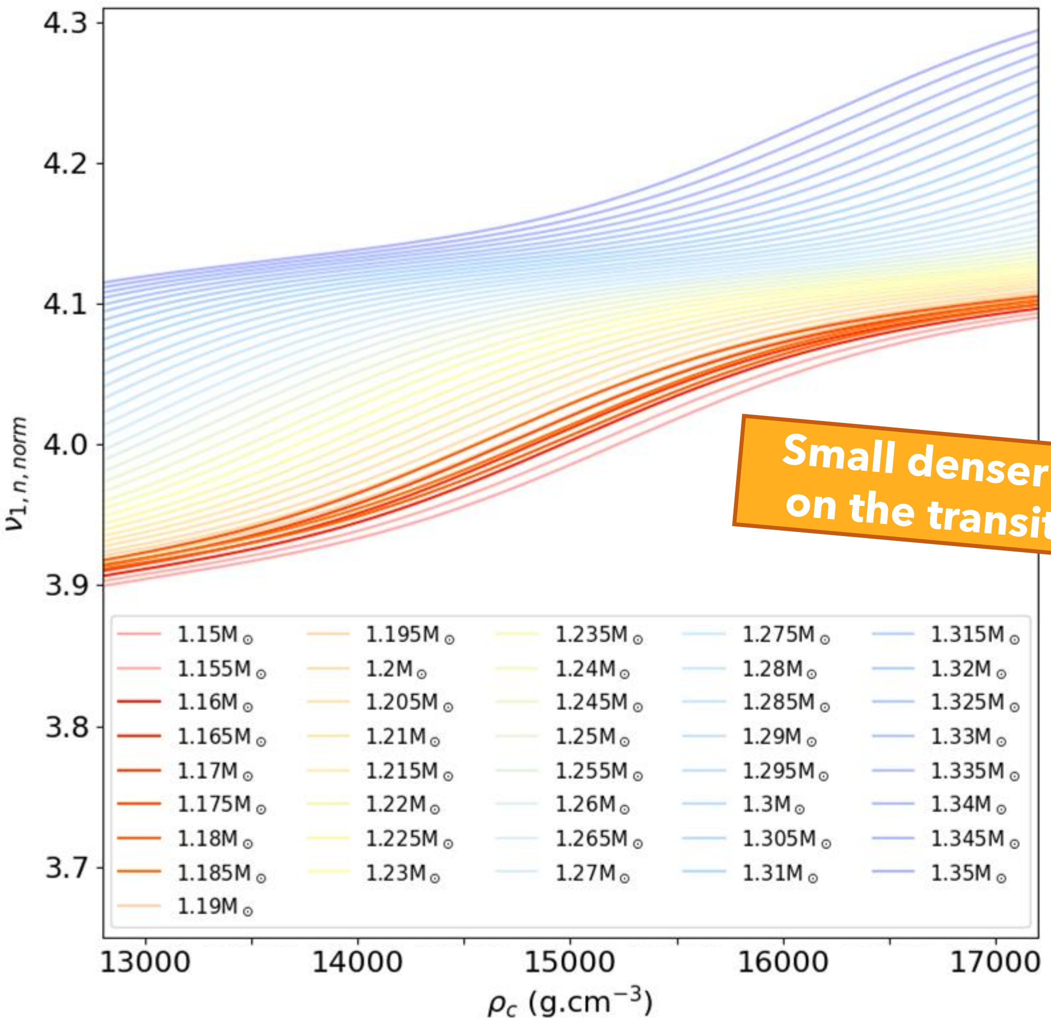

Small denser grid focusing on the transitional masses

Subgrid of stellar models:

- MESA's grid inputs:
  - M = [(1.170:1.180), 0.001] $M_\odot$
  - $\rho_c = [(10^3:10^{4.5}),10]$ g.cm^{-3}

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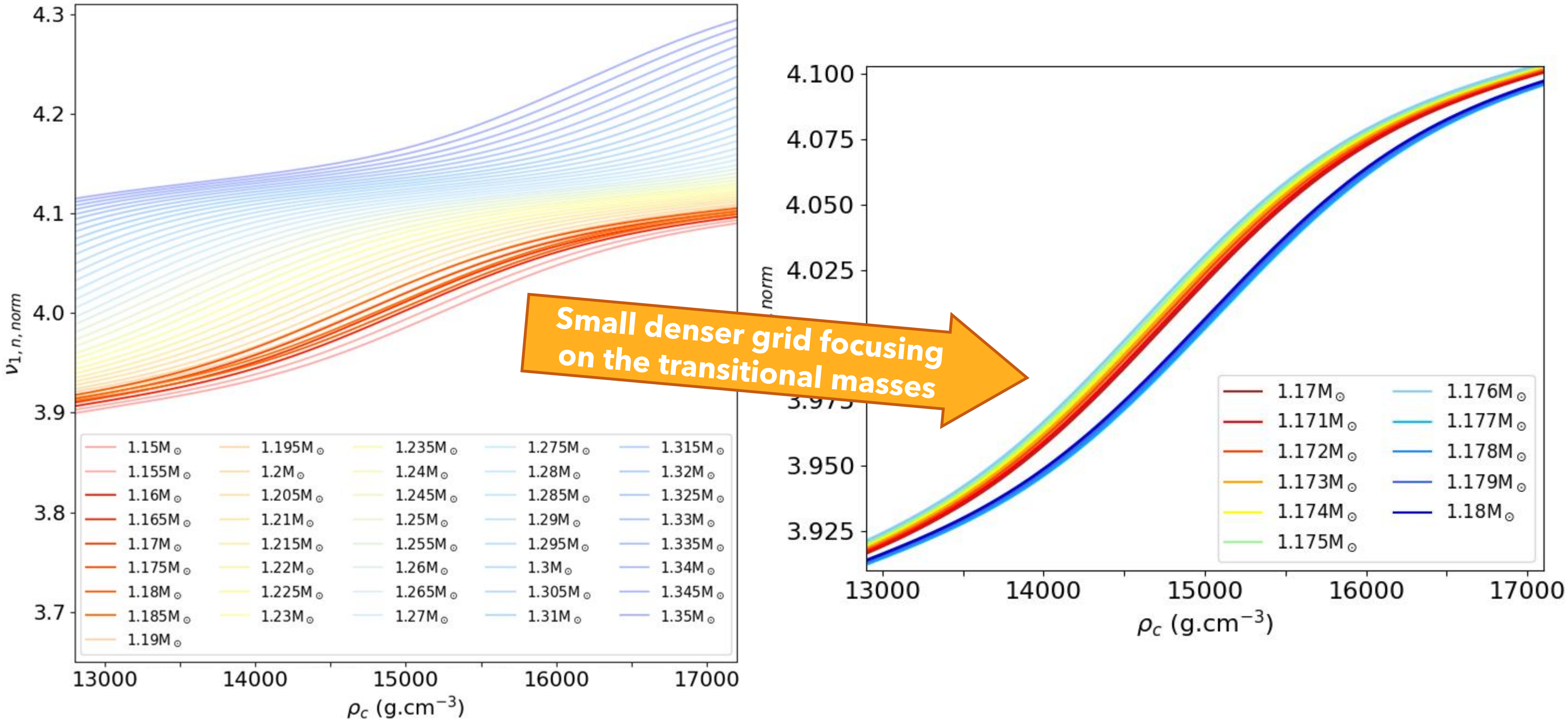

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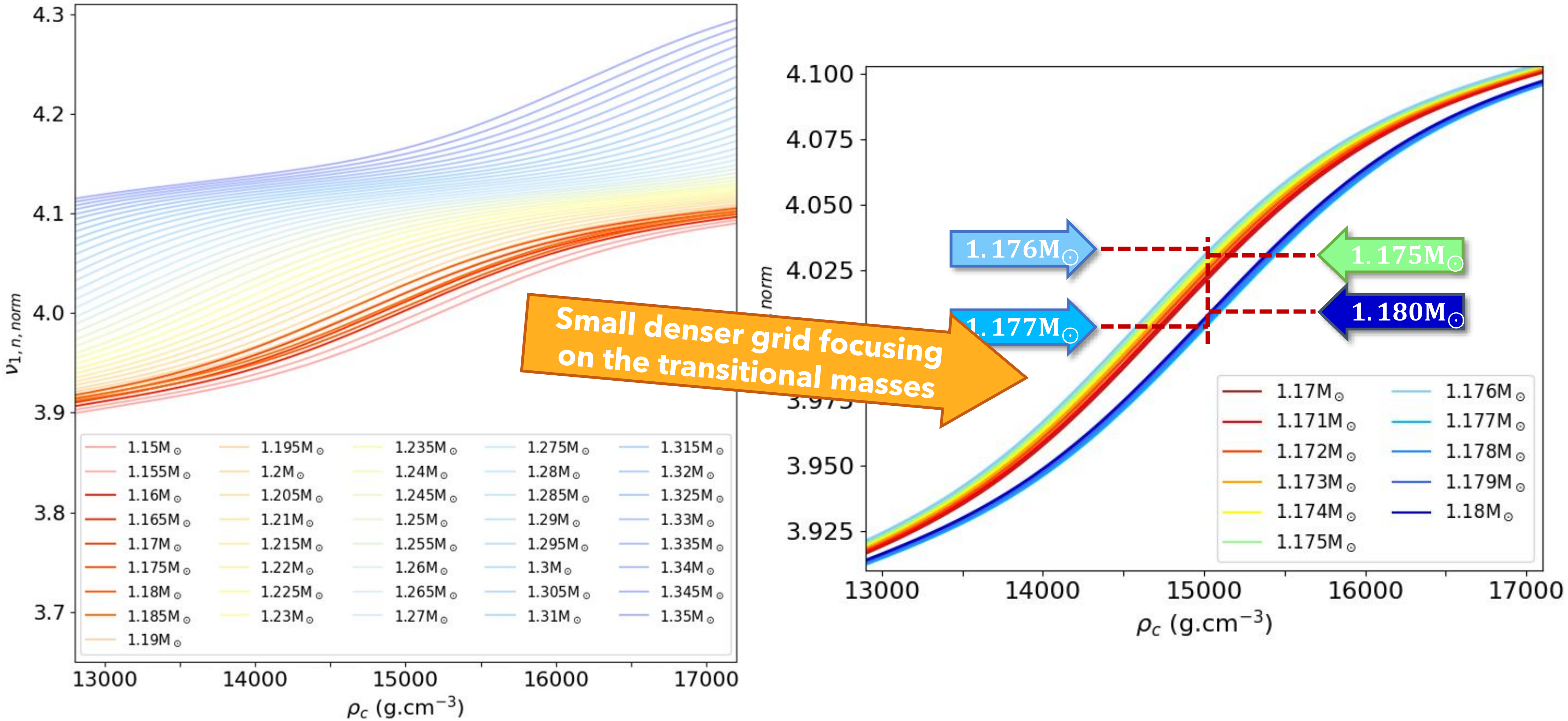

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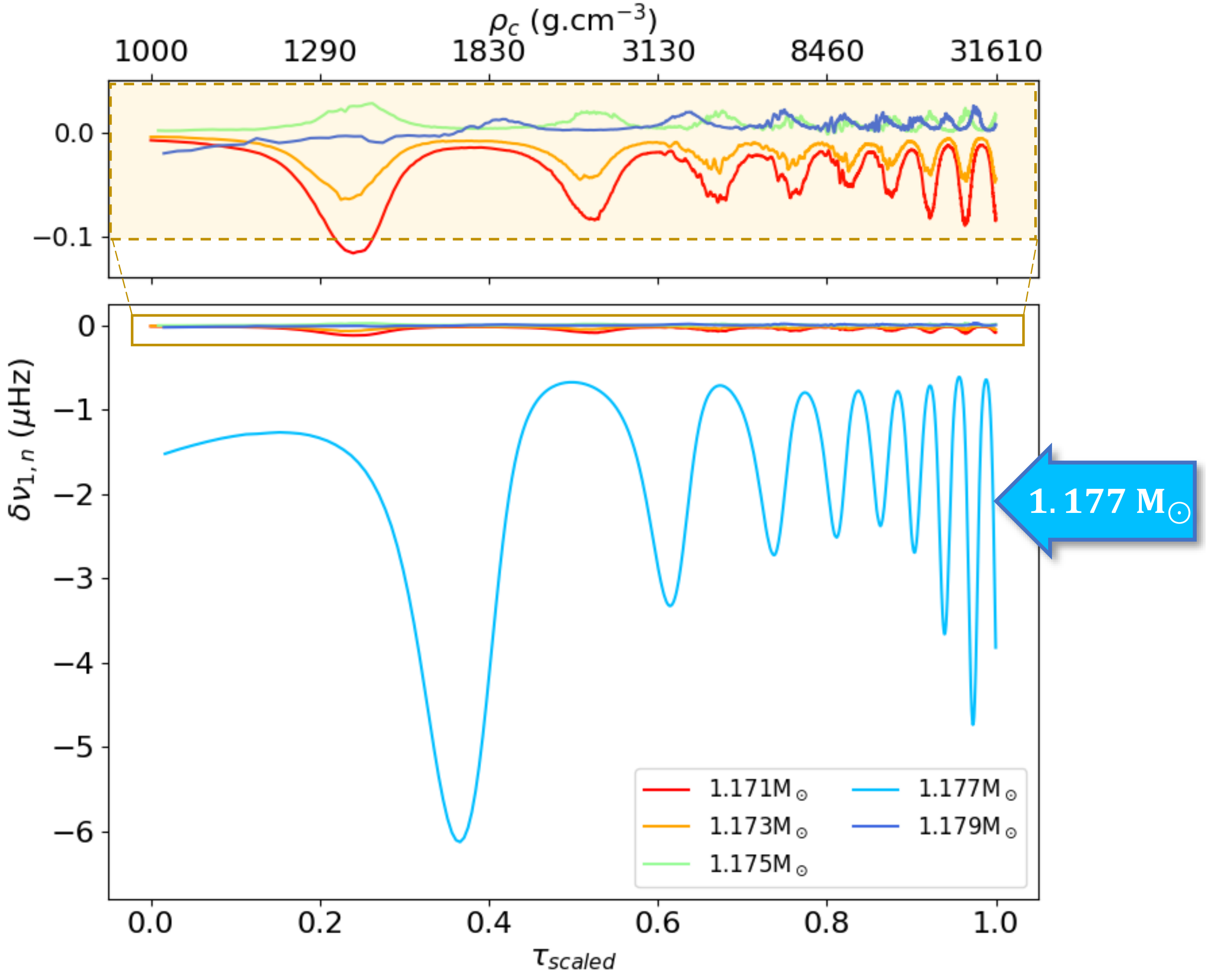
  - $\rho_c = [(10^3{:}10^{4.5}), 10]\ g.cm^{-3}$

Full interpolation of the stellar mini-grid:

- All masses, frequency mode $\nu_{1,11}$
- Age proxies: ρ_c
- Cubic splines along evolutionary tracks + Linear regression across evolutionary tracks



5. Conclusions

- For the typical mass step of a stellar grid, and the current interpolation algorithms considered, it is essentially unfeasible to obtain results within the accuracy limits associated with the typical observations (0.1μHz).

- The best interpolation algorithm considers:
  - Cubic splines along evolutionary tracks
  - Linear regression across evolutionary tracks
  - ρ_c as the age proxy

- There is a discontinuity, caused by the different core properties during the MS, that influences the evolution of the modes during the subgiant phase and the interpolation that cannot be resolved by increasing the grid resolution with the adopted physics.